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July 2013

Mathematics 10

NOVA SCOTIA

CURRICULUM GUIDE

Mathematics 10

Implementation Draft
July 2013

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Contents

Introduction

Background and Rationale

Mathematics curriculum is shaped by a vision that fosters the development of mathematically literate students who can extend and apply their learning and who are effective participants in society. It is essential that the mathematics curriculum reflect current research in mathematics instruction. To achieve this goal, Western and Northern Canadian Protocol's (WNCP) *The Common Curriculum Framework for Grades 10–12 Mathematics* (2008) has been adopted as the basis for the new mathematics curriculum in Nova Scotia.

The Common Curriculum Framework was developed by the seven ministries of education (Alberta, British Columbia, Manitoba, Northwest Territories, Nunavut, Saskatchewan, and Yukon Territory) in collaboration with teachers, administrators, parents, business representatives, post-secondary educators, and others. The framework identifies beliefs about mathematics, general and specific student outcomes, and performance indicators agreed upon by the seven jurisdictions. The outcomes and performance indicators have been adapted for Nova Scotia. This document is based on both national and international research by the WNCP and the National Council of Teachers of Mathematics (NCTM).

There is an emphasis in the Nova Scotia curriculum on particular key concepts at each grade that will result in greater depth of understanding and, ultimately, stronger student achievement. There is also a greater emphasis on number sense and operations concepts in the early grades to ensure students develop a solid foundation in numeracy.

Purpose

This document provides sets of outcomes and performance indicators to be used as a mandated common base for defining mathematics curriculum expectations. This common base should result in consistent student outcomes in mathematics within the province of Nova Scotia. It should also enable easier transfer for students moving within the province or from any jurisdiction that has adopted the WNCP framework. This document is intended to clearly communicate to all education partners across the province the high expectations for students' mathematical learning.

Program Design and Components

Pathways

The *Common Curriculum Framework for Grades 10–12 Mathematics*, on which the Nova Scotia Mathematics 10–12 curriculum is based, includes pathways and topics rather than strands as in *The Common Curriculum Framework for K–9 Mathematics*. In Nova Scotia, four pathways are available: Mathematics Essentials, Mathematics at Work, Mathematics, and Pre-Calculus.

Each topic area requires that students develop a conceptual knowledge base and skill set that will be useful to whatever pathway they have chosen. The topics covered within a pathway are meant to build upon previous knowledge and to progress from simple to more complex conceptual understandings.

Goals of Pathways

The goals of all four pathways are to provide prerequisite attitudes, knowledge, skills, and understandings for specific post-secondary programs or direct entry into the work force. All four pathways provide students with mathematical understandings and critical-thinking skills. It is the choice of topics through which those understandings and skills are developed that varies among pathways. When choosing a pathway, students should consider their interests, both current and future. Students, parents, and educators are encouraged to research the admission requirements for post-secondary programs of study as they vary by institution and by year.

Design of Pathways

Each pathway is designed to provide students with the mathematical understandings, rigour, and critical-thinking skills that have been identified for specific post-secondary programs of study and for direct entry into the work force.

The content of the Mathematics Essentials courses was designed in Nova Scotia to fill a specific need for Nova Scotia students. The content of each of the Mathematics at Work, Mathematics, and Pre-Calculus pathways has been based on the *Western and Northern Canadian Protocol (WNCP) Consultation with Post-Secondary Institutions, Business and Industry Regarding Their Requirements for High School Mathematics: Final Report on Findings* (Alberta Education 2006) and on consultations with mathematics teachers.

MATHEMATICS ESSENTIALS (GRADUATION)

This pathway is designed to provide students with the development of the skills and understandings required in the workplace, as well as those required for everyday life at home and in the community. Students will become better equipped to deal with mathematics in the real world and will become more confident in their mathematical abilities.

MATHEMATICS AT WORK (GRADUATION)

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for entry into some college programs and for direct entry into the work force. Topics include financial mathematics, algebra, geometry, measurement, number, and statistics and probability.

MATHEMATICS (ACADEMIC)

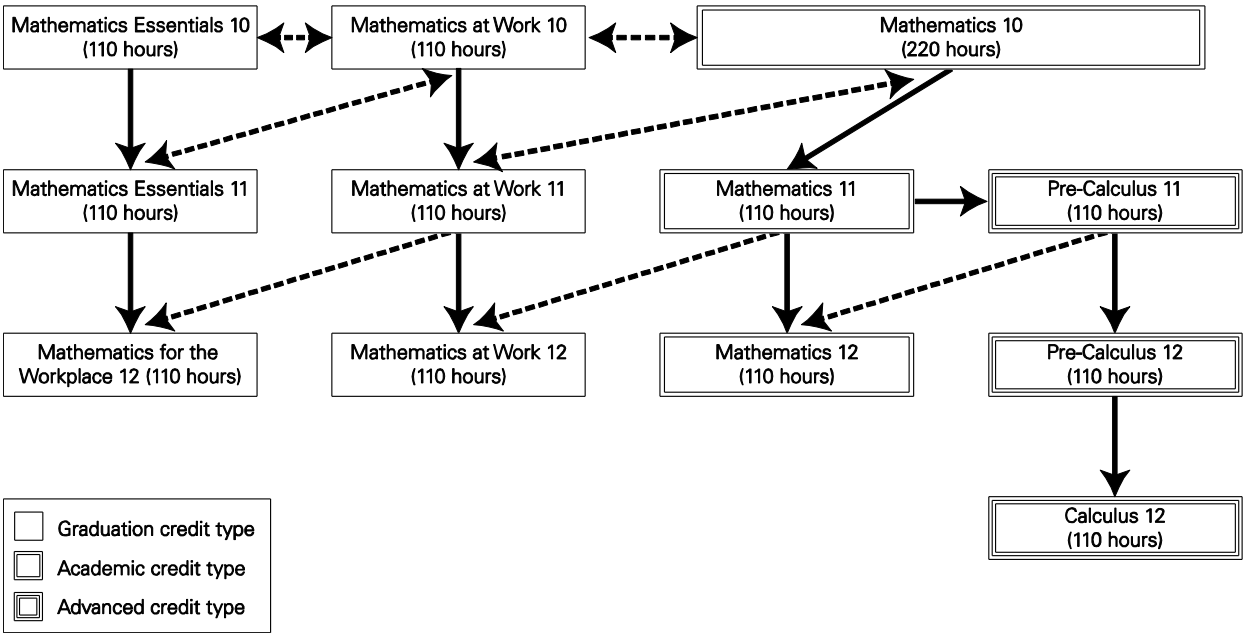
This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for post-secondary studies in programs that require an academic or pre-calculus mathematics credit. Topics include financial mathematics, geometry, measurement, number, logical reasoning, relations and functions, and statistics and probability. **Note:** After completion of Mathematics 11, students have the choice of an academic or pre-calculus pathway.

PRE-CALCULUS (ADVANCED)

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for entry into post-secondary programs that require the study of theoretical calculus. Topics include algebra and number, measurement, relations and functions, trigonometry, and permutations, combinations, and binomial theorem.

Pathways and Courses

The graphic below summarizes the pathways and courses offered.



Instructional Focus

Each pathway in senior high mathematics pathways is arranged by topics. Students should be engaged in making connections among concepts both within and across topics to make mathematical learning experiences meaningful.

Teachers should consider the following points when planning for instruction and assessment.

- The mathematical processes that are identified with the outcome are intended to help teachers select effective pedagogical approaches for the teaching and learning of the outcome.
- All seven mathematical processes must be integrated throughout teaching and learning approaches and should support the intent of the outcomes.
- Wherever possible, meaningful contexts should be used in examples, problems, and projects.
- Instruction should flow from simple to complex and from concrete to abstract.
- The assessment plan for the course should be a balance of assessment for learning and assessment of learning.

The focus of student learning should be on developing a conceptual and procedural understanding of mathematics. Students' conceptual understanding and procedural understanding must be directly related.

Assessment

Ongoing assessment for learning is essential to effective teaching and learning. Research has shown that assessment for learning (formative assessment) practices produce significant and often substantial learning gains, close achievement gaps, and build students' ability to learn new skills (Black & Wiliam 1998; OECD 2006). Student involvement in assessment promotes learning. Timely and effective teacher feedback and student self-assessment allow students to reflect on and articulate their understanding of mathematical concepts and ideas.

Assessment in the classroom includes

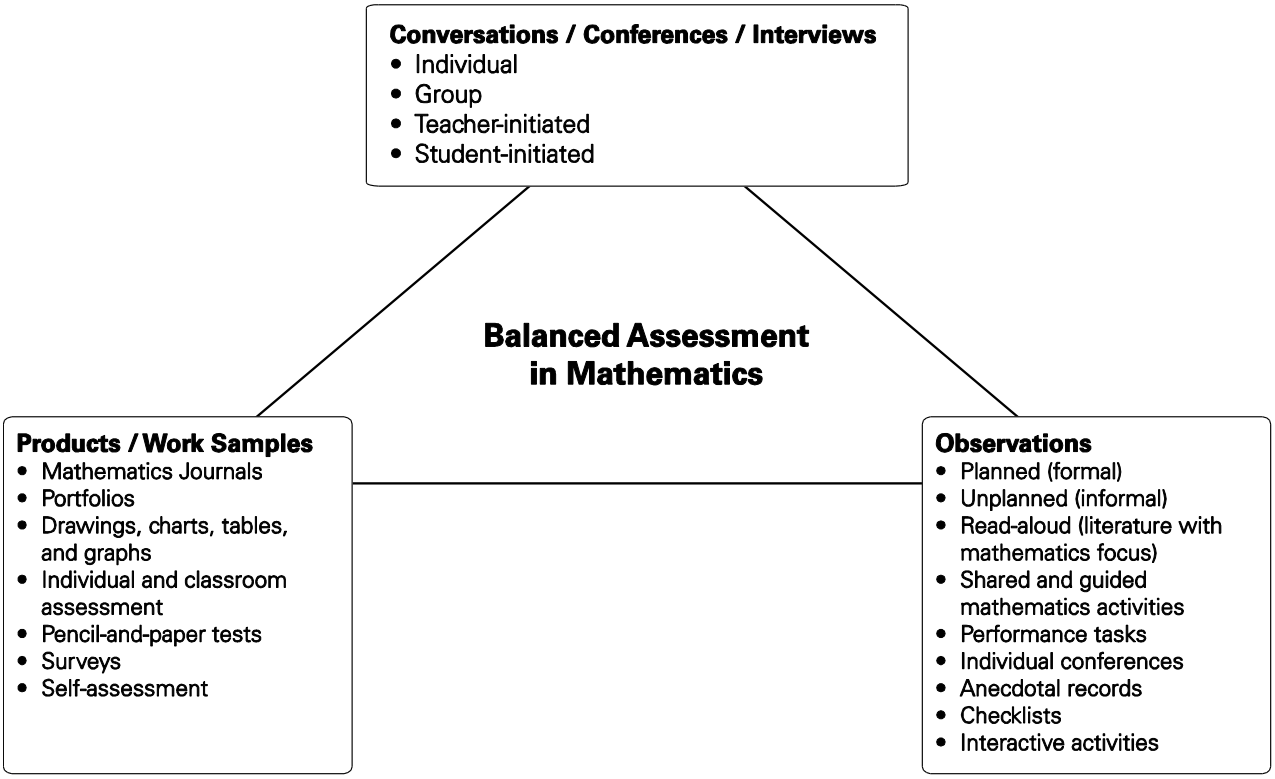
- providing clear goals, targets, and learning outcomes
- using exemplars, rubrics, and models to help clarify outcomes and identify important features of the work
- monitoring progress towards outcomes and providing feedback as necessary
- encouraging self-assessment
- fostering a classroom environment where conversations about learning take place, where students can check their thinking and performance and develop a deeper understanding of their learning

(Davies 2000)

Assessment for learning practices act as the scaffolding for learning, which only then can be measured through assessment of learning (summative assessment). Assessment of learning tracks student progress, informs instructional programming, and aids in decision making. Both forms of assessment are necessary to guide teaching, stimulate learning, and produce achievement gains.

Assessment of student learning should

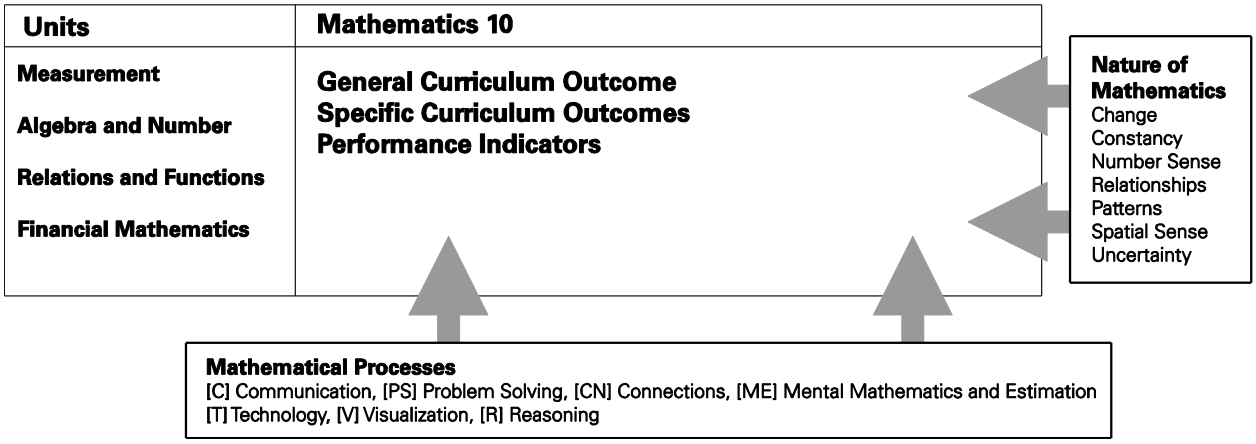
- align with curriculum outcomes
- clearly define criteria for success
- make explicit the expectations for students' performance
- use a wide variety of assessment strategies and tools
- yield useful information to inform instruction



Outcomes

Conceptual Framework for Mathematics 10–12

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



(Adapted with permission from Western and Northern Canadian Protocol, *The Common Curriculum Framework for K–9 Mathematics*, p. 5. All rights reserved.)

Structure of the Mathematics 10 Curriculum

Units

Mathematics 10 comprises four units:

- Measurement (M) (50–55 hours)
- Algebra and Number (AN) (50–55 hours)
- Relations and Functions (RF) (70–75 hours)
- Financial Mathematics (FM) (40–45 hours)

Outcomes and Performance Indicators

The Nova Scotia curriculum is stated in terms of general curriculum outcomes, specific curriculum outcomes, and performance indicators.

General Curriculum Outcomes (GCOs)

General curriculum outcomes are overarching statements about what students are expected to learn in each strand/sub-strand. The GCO for each strand/sub-strand is the same throughout the pathway.

Measurement (M)

Students will be expected to develop spatial sense and proportional reasoning.

Algebra and Number (AN)

Students will be expected to develop algebraic reasoning and number sense.

Relations and Functions (RF)

Students will be expected to develop algebraic and graphical reasoning through the study of relations.

Financial Mathematics (FM)

Students will be expected to demonstrate number sense and critical thinking skills.

Specific Curriculum Outcomes (SCOs) and Performance Indicators

Specific curriculum outcomes are statements that identify specific concepts and related skills underpinned by the understanding and knowledge attained by students as expected for a given grade.

Performance indicators are samples of how students may demonstrate their performance of the goals of a specific curriculum outcome. The range of samples provided is meant to reflect the scope of the SCO. In the SCOs, the word **including** indicates that any ensuing items *must* be addressed to fully achieve the learning outcome. The phrase **such as** indicates that the ensuing items are provided for clarification only and are *not* requirements that must be addressed to fully achieve the learning outcome. The word **and** used in an outcome indicates that both ideas must be addressed to achieve the learning outcome, although not necessarily at the same time or in the same question.

MEASUREMENT (M)

M01 Students will be expected to solve problems that involve linear measurement, using SI and imperial units of measure, estimation strategies, and measurement strategies.

Performance Indicators

M01.01 Provide referents for linear measurements, including millimetre, centimetre, metre, kilometre, inch, foot, yard, and mile, and explain the choices.
M01.02 Compare SI and imperial units, using referents.
M01.03 Estimate a linear measure, using a referent, and explain the process used.
M01.04 Justify the choice of units used for determining a measurement in a problem-solving context.
M01.05 Solve problems that involve linear measure, using instruments such as rulers, calipers, or tape measures.
M01.06 Describe and explain a personal strategy used to determine a linear measurement (e.g., circumference of a bottle, length of a curve, and perimeter of the base of an irregular 3-D object).

M02 Students will be expected to apply proportional reasoning to problems that involve conversions between SI and imperial units of measure.

Performance Indicators

M02.01 Explain how proportional reasoning can be used to convert a measurement within or between SI and imperial systems.

M02.02 Solve a problem that involves the conversion of units within or between SI and imperial systems.

M02.03 Verify, using unit analysis, a conversion within or between SI and imperial systems, and explain the conversion.

M02.04 Justify, using mental mathematics, the reasonableness of a solution to a conversion problem.

M03 Students will be expected to solve problems, using SI and imperial units, that involve the surface area and volume of 3-D objects, including right cones, right cylinders, right prisms, right pyramids, and spheres.

Performance Indicators

M03.01 Sketch a diagram to represent a problem that involves surface area or volume.

M03.02 Determine the surface area of a right cone, right cylinder, right prism, right pyramid, or sphere, using an object or its labelled diagram.

M03.03 Determine the volume of a right cone, right cylinder, right prism, right pyramid, or sphere, using an object or its labelled diagram.

M03.04 Determine an unknown dimension of a right cone, right cylinder, right prism, right pyramid, or sphere, given the object's surface area or volume and the remaining dimensions.

M03.05 Solve a problem that involves surface area or volume, given a diagram of a composite 3-D object.

M03.06 Describe the relationship between the volumes of right cones and right cylinders with the same base and height, and right pyramids and right prisms with the same base and height.

M04 Students will be expected to develop and apply the primary trigonometric ratios (sine, cosine, tangent) to solve problems that involve right triangles.

Performance Indicators

M04.01 Explain the relationships between similar right triangles and the definitions of the primary trigonometric ratios.

M04.02 Identify the hypotenuse of a right triangle and the opposite and adjacent sides for a given acute angle in the triangle.

M04.03 Solve right triangles, with or without technology.

M04.04 Solve a problem that involves one or more right triangles by applying the primary trigonometric ratios or the Pythagorean theorem.

M04.05 Solve a problem that involves indirect and direct measurement, using the trigonometric ratios, the Pythagorean theorem, and measurement instruments such as a clinometer or metre stick.

ALGEBRA AND NUMBER (AN)

AN01 Students will be expected to demonstrate an understanding of factors of whole numbers by determining the prime factors, greatest common factor, least common multiple, square root, and cube root.

Performance Indicators

AN01.01 Determine the prime factors of a whole number.
AN01.02 Explain why the numbers 0 and 1 have no prime factors.
AN01.03 Determine, using a variety of strategies, the greatest common factor or least common multiple of a set of whole numbers, and explain the process.
AN01.04 Determine, concretely, whether a given whole number is a perfect square, a perfect cube, or neither.
AN01.05 Determine, using a variety of strategies, the square root of a perfect square, and explain the process.
AN01.06 Determine, using a variety of strategies, the cube root of a perfect cube, and explain the process.
AN01.07 Solve problems that involve prime factors, greatest common factors, least common multiples, square roots, or cube roots.

AN02 Students will be expected to demonstrate an understanding of irrational numbers by representing, identifying, simplifying, and ordering irrational numbers.

Performance Indicators

AN02.01 Sort a set of numbers into rational and irrational numbers.
AN02.02 Determine an approximate value of a given irrational number.
AN02.03 Approximate the locations of irrational numbers on a number line, using a variety of strategies, and explain the reasoning.
AN02.04 Order a set of irrational numbers on a number line.
AN02.05 Express a radical as a mixed radical in simplest form (limited to numerical radicands).
AN02.06 Express a mixed radical as an entire radical (limited to numerical radicands).
AN02.07 Explain, using examples, the meaning of the index of a radical.
AN02.08 Represent, using a graphic organizer, the relationship among the subsets of the real numbers (natural, whole, integer, rational, irrational).

AN03 Students will be expected to demonstrate an understanding of powers with integral and rational exponents.

Performance Indicators

AN03.01 Explain, using patterns, why $a^{-n} = \frac{1}{a^n}$, $a \neq 0$.

AN03.02 Explain, using patterns, why $a^{\frac{1}{n}} = \sqrt[n]{a}$, $n > 0$.

AN03.03 Apply the following exponent laws to expressions with rational and variable bases and integral and rational exponents, and explain the reasoning.

- $(a^m)(a^n) = a^{m+n}$
- $a^m \div a^n = a^{m-n}$, $a \neq 0$.

- $(am)^n = a^{mn}$
- $(ab)^m = a^m b^m$
- $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$

AN03.04 Express powers with rational exponents as radicals and vice versa, when *m* and *n* are natural numbers, and *x* is a rational number.

$$x^{\frac{m}{n}} = \left(x^{\frac{1}{n}}\right)^m = \sqrt[n]{x}^{\,m} \text{ and } x^{\frac{m}{n}} = x^{m\,\frac{1}{n}} = \sqrt[n]{x^m}$$

AN03.05 Solve a problem that involves exponent laws or radicals.

AN03.06 Identify and correct errors in a simplification of an expression that involves powers.

AN04 Students will be expected to demonstrate an understanding of the multiplication of polynomial expressions (limited to monomials, binomials, and trinomials), concretely, pictorially, and symbolically.

Performance Indicators

AN04.01 Model the multiplication of two given binomials, concretely or pictorially, and record the process symbolically.

AN04.02 Relate the multiplication of two binomial expressions to an area model.

AN04.03 Explain, using examples, the relationship between the multiplication of binomials and the multiplication of two-digit numbers.

AN04.04 Verify a polynomial product by substituting numbers for the variables.

AN04.05 Multiply two polynomials symbolically, and combine like terms in the product.

AN04.06 Generalize and explain a strategy for multiplication of polynomials.

AN04.07 Identify and explain errors in a solution for a polynomial multiplication.

AN05 Students will be expected to demonstrate an understanding of common factors and trinomial factoring, concretely, pictorially, and symbolically.

Performance Indicators

AN05.01 Determine the common factors in the terms of a polynomial, and express the polynomial in factored form.

AN05.02 Model the factoring of a trinomial, concretely or pictorially, and record the process symbolically.

AN05.03 Factor a polynomial that is a difference of squares, and explain why it is a special case of trinomial factoring where $b = 0$.

AN05.04 Identify and explain errors in a polynomial factorization.

AN05.05 Factor a polynomial, and verify by multiplying the factors.

AN05.06 Explain, using examples, the relationship between multiplication and factoring of polynomials.

AN05.07 Generalize and explain strategies used to factor a trinomial.

AN05.08 Express a polynomial as a product of its factors.

RELATIONS AND FUNCTIONS (RF)

RF01 Students will be expected to interpret and explain the relationships among data, graphs, and situations.

Performance Indicators

RF01.01 Graph, with or without technology, a set of data, and determine the restrictions on the domain and range.
RF01.02 Explain why data points should or should not be connected on the graph for a situation.
RF01.03 Describe a possible situation for a given graph.
RF01.04 Sketch a possible graph for a given situation.
RF01.05 Determine, and express in a variety of ways, the domain and range of a graph, a set of ordered pairs, or a table of values.

RF02 Students will be expected to demonstrate an understanding of relations and functions.

Performance Indicators

RF02.01 Explain, using examples, why some relations are not functions, but all functions, are relations.
RF02.02 Determine if a set of ordered pairs represents a function.
RF02.03 Sort a set of graphs as functions or non-functions.
RF02.04 Generalize and explain rules for determining whether graphs and sets of ordered pairs represent functions.

RF03 Students will be expected to demonstrate an understanding of slope with respect to rise and run, line segments and lines, rate of change, parallel lines, and perpendicular lines.

Performance Indicators

RF03.01 Determine the slope of a line segment by measuring or calculating the rise and run.
RF03.02 Classify lines in a given set as having positive or negative slopes.
RF03.03 Explain the meaning of the slope of a horizontal or vertical line.
RF03.04 Explain why the slope of a line can be determined by using any two points on that line.
RF03.05 Explain, using examples, slope as a rate of change.
RF03.06 Draw a line, given its slope and a point on the line.
RF03.07 Determine another point on a line, given the slope and a point on the line.
RF03.08 Generalize and apply a rule for determining whether two lines are parallel or perpendicular.
RF03.09 Solve a contextual problem involving slope.

RF04 Students will be expected to describe and represent linear relations, using words, ordered pairs, tables of values, graphs, and equations.

Performance Indicators

RF04.01 Identify independent and dependent variables in a given context.
RF04.02 Determine whether a situation represents a linear relation, and explain why or why not.
RF04.03 Determine whether a graph represents a linear relation, and explain why or why not.
RF04.04 Determine whether a table of values or a set of ordered pairs represents a linear relation, and explain why or why not.

RF04.05 Draw a graph from a set of ordered pairs within a given situation, and determine whether the relationship between the variables is linear.
RF04.06 Determine whether an equation represents a linear relation, and explain why or why not.
RF04.07 Match corresponding representations of linear relations.

RF05 Students will be expected to determine the characteristics of the graphs of linear relations, including the intercepts, slope, domain, and range.

Performance Indicators
RF05.01 Determine the intercepts of the graph of a linear relation, and state the intercepts as values or ordered pairs.
RF05.02 Determine the slope of the graph of a linear relation.
RF05.03 Determine the domain and range of the graph of a linear relation.
RF05.04 Sketch a linear relation that has one intercept, two intercepts, or an infinite number of intercepts.
RF05.05 Identify the graph that corresponds to a given slope and y-intercept.
RF05.06 Identify the slope and y-intercept that correspond to a given graph.
RF05.07 Solve a contextual problem that involves intercepts, slope, domain, or range of a linear relation.

RF06 Students will be expected to relate linear relations to their graphs, expressed in
- slope-intercept form ($y = mx + b$)
- general form ($Ax + By + C = 0$)
- slope-point form ($y – y1) = m(x – x1$)

Performance Indicators
RF06.01 Express a linear relation in different forms, and compare the graphs.
RF06.02 Rewrite a linear relation in either slope-intercept or general form.
RF06.03 Generalize and explain strategies for graphing a linear relation in slope-intercept, general, or slope-point form.
RF06.04 Graph, with and without technology, a linear relation given in slope-intercept, general, or slope-point form, and explain the strategy used to create the graph.
RF06.05 Identify equivalent linear relations from a set of linear relations.
RF06.06 Match a set of linear relations to their graphs.

RF07 Students will be expected to determine the equation of a linear relation to solve problems, given a graph, a point and the slope, two points, and a point and the equation of a parallel or perpendicular line.

Performance Indicators
RF07.01 Determine the slope and y-intercept of a given linear relation from its graph, and write the equation in the form $y = mx + b$.
RF07.02 Write the equation of a linear relation, given its slope and the coordinates of a point on the line, and explain the reasoning.
RF07.03 Write the equation of a linear relation, given the coordinates of two points on the line, and explain the reasoning.

RF07.04 Write the equation of a linear relation, given the coordinates of a point on the line and the equation of a parallel or perpendicular line, and explain the reasoning.
RF07.05 Graph linear data generated from a context, and write the equation of the resulting line.
RF07.06 Determine the equation of the line of best fit from a scatterplot using technology and determine the correlation.
RF07.07 Solve a problem, using the equation of a linear relation.

RF08 Students will be expected to solve problems that involve the distance between two points and the midpoint of a line segment.

Performance Indicators
RF08.01 Determine the distance between two points on a Cartesian plane using a variety of strategies.
RF08.02 Determine the midpoint of a line segment, given the endpoints of the segment, using a variety of strategies.
RF08.03 Determine and endpoint of a line segment, given the other endpoint and the midpoint, using a variety of strategies.
RF08.04 Solve a contextual problem involving the distance between two points or midpoint of a line segment.

RF09 Students will be expected to represent a linear function, using function notation.

Performance Indicators
RF09.01 Express the equation of a linear function in two variables, using function notation.
RF09.02 Express an equation given in function notation as a linear function in two variables.
RF09.03 Determine the related range value, given a domain value for a linear function.
RF09.04 Determine the related domain value, given a range value for a linear function.
RF09.05 Sketch the graph of a linear function expressed in function notation.

RF10 Students will be expected to solve problems that involve systems of linear equations in two variables, graphically and algebraically.

Performance Indicators
RF10.01 Model a situation, using a system of linear equations.
RF10.02 Relate a system of linear equations to the context of a problem.
RF10.03 Determine and verify the solution of a system of linear equations graphically, with and without technology.
RF10.04 Explain the meaning of the point of intersection of a system of linear equations.
RF10.05 Determine and verify the solution of a system of linear equations algebraically.
RF10.06 Explain, using examples, why a system of equations may have no solution, one solution, or an infinite number of solutions.
RF10.07 Explain a strategy to solve a system of linear equations.
RF10.08 Solve a problem that involves a system of linear equations.

FINANCIAL MATHEMATICS (FM)

FM01 Students will be expected to solve problems that involve unit pricing and currency exchange, using proportional reasoning.

Performance Indicators

FM01.01 Compare the unit price of two or more given items.
FM01.02 Solve problems that involve determining the best buy, and explain the choice in terms of the cost as well as other factors, such as quality and quantity.
FM01.03 Compare, using examples, different sales promotion techniques.
FM01.04 Determine the percent increase or decrease for a given original and new price.
FM01.05 Solve, using proportional reasoning, a contextual problem that involves currency exchange.
FM01.06 Explain the difference between the selling rate and purchasing rate for currency exchange.
FM01.07 Explain how to estimate the cost of items in Canadian currency while in a foreign country, and explain why this may be important.
FM01.08 Convert between Canadian currency and foreign currencies, using formulas, charts, or tables.

FM02 Students will be expected to demonstrate an understanding of income to calculate gross pay and net pay, including wages, salary, contracts, commissions, and piecework.

Performance Indicators

FM02.01 Describe, using examples, various methods of earning income.
FM02.02 Identify and list jobs that commonly use different methods of earning income (e.g., hourly wage, wage and tips, salary, commission, contract, bonus, shift premiums).
FM02.03 Determine in decimal form, from a time schedule, the total time worked in hours and minutes, including time and a half and/or double time.
FM02.04 Determine gross pay from given or calculated hours worked when given
- the base hourly wage, with and without tips
- the base hourly wage, plus overtime (time and a half, double time)

FM02.05 Determine gross pay for earnings acquired by
- base wage, plus commission
- single commission rate

FM02.06 Explain why gross pay and net pay are not the same.
FM02.07 Determine the Canadian Pension Plan (CPP), Employment Insurance (EI), and income tax deductions for a given gross pay.
FM02.08 Determine net pay when given deductions (e.g., health plans, uniforms, union dues, charitable donations, payroll tax).
FM02.09 Investigate, with technology, “what if …” questions related to changes in income (e.g., What if there is a change in the rate of pay?)

FM03 Students will be expected to investigate personal budgets.

Performance Indicators

FM03.01 Identify income and expenses that should be included in a personal budget.
FM03.02 Explain considerations that must be made when developing a budget (e.g., prioritizing, and recurring and unexpected expenses).
FM03.03 Create a personal budget based on given income and expense data.

FM03.04 Collect income and expense data, and create a budget.
FM03.05 Modify a budget to achieve a set of personal goals.
FM03.06 Investigate and analyze, with or without technology, “what if …” questions related to personal budgets.

FM04 Students will be expected to explore and give a presentation on an area of interest that involves financial mathematics.

Performance Indicators
FM04.01 Collect primary or secondary data (statistical or informational) related to the topic.
FM04.02 Organize and present a project.
FM04.03 Create and solve a contextual problem that is related to the project.
FM04.04 Make informed decisions and plans related to the project.
FM04.05 Compare advantages and disadvantages as part of the project.

Mathematical Processes

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics.

Students are expected to
- communicate in order to learn and express their understanding of mathematics (Communication [C])
- connect mathematical ideas to other concepts in mathematics, to everyday experiences, and to other disciplines (Connections [CN])
- demonstrate fluency with mental mathematics and estimation (Mental Mathematics and Estimation [ME])
- develop and apply new mathematical knowledge through problem solving (Problem Solving [PS])
- develop mathematical reasoning (Reasoning [R])
- select and use technologies as tools for learning and solving problems (Technology [T])
- develop visualization skills to assist in processing information, making connections, and solving problems (Visualization [V])

The Nova Scotia curriculum incorporates these seven interrelated mathematical processes that are intended to permeate teaching and learning. The key to these process standards is presented in a box, as shown below, with each specific outcome within the units.

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics. Communication is important in clarifying, reinforcing, and modifying ideas, knowledge, attitudes, and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics.

Students also need to communicate their learning using mathematical terminology. Communication can help students make connections between and among the different representational modes—contextual, concrete, pictorial, linguistic/verbal, written and symbolic—of mathematical ideas. Students must communicate *daily* about their mathematics learning. This enables them to reflect, to validate, and to clarify their thinking and provides teachers with insight into students' interpretations of mathematical meanings and ideas.

Emerging technologies enable students to engage in communication beyond the traditional classroom to gather data and share mathematical ideas.

Problem Solving [PS]

Problem solving is one of the key processes and foundations within the field of mathematics. Learning through problem solving should be the focus of mathematics at all grade levels. Students develop a true understanding of mathematical concepts and procedures when they solve problems in meaningful contexts.

When students encounter new situations and respond to questions of the type, How would you...? or How could you ...?, the problem-solving approach is being modeled. Students develop their own problem-solving strategies by listening to, discussing, and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. Students should not know the answer immediately. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement. Students will be engaged if the problems relate to their lives, cultures, interests, families, or current events.

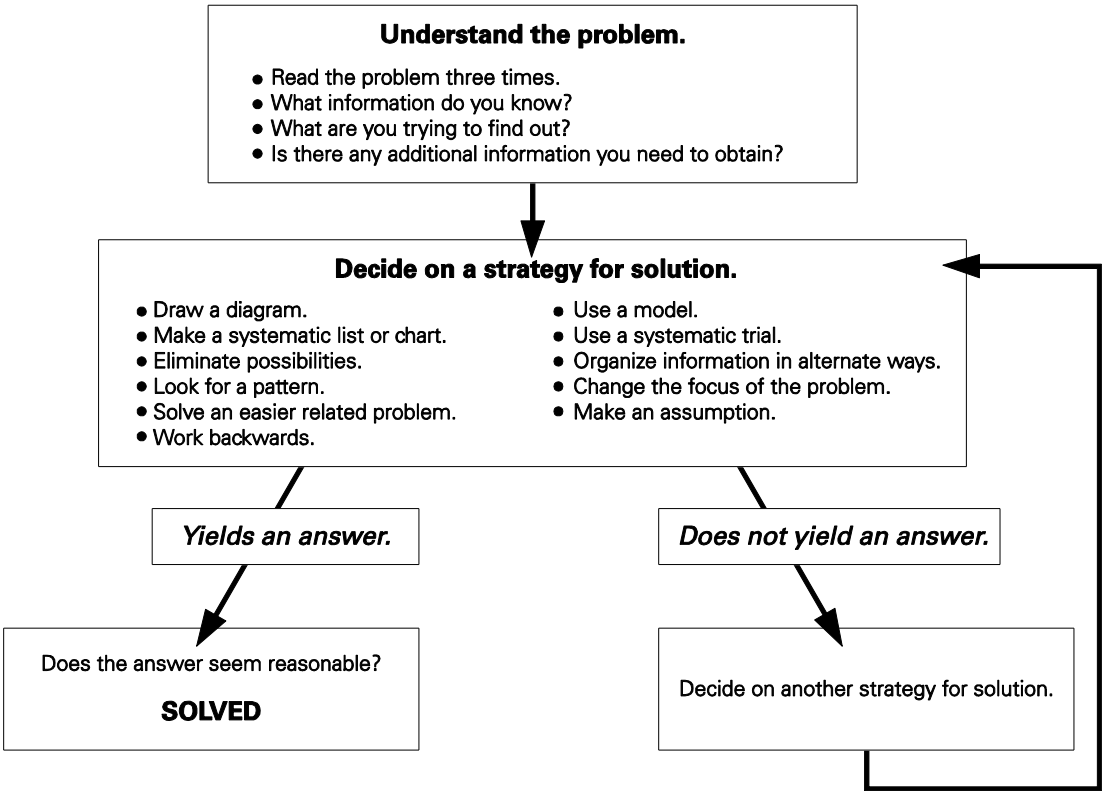
Both conceptual understanding and student engagement are fundamental in molding students' willingness to persevere in future problem-solving tasks. Problems are not just simple computations embedded in a story, nor are they contrived. They are tasks that are rich and open-ended, so there may be more than one way of arriving at a solution or there may be multiple answers. Good problems should allow for every student in the class to demonstrate their knowledge, skill, or understanding. Problem solving can vary from being an individual activity to a class (or beyond) undertaking.

In a mathematics class, there are two distinct types of problem solving: solving contextual problems outside of mathematics and solving mathematical problems. Finding the maximum profit given manufacturing constraints is an example of a contextual problem, while seeking and developing a general formula to solve a quadratic equation is an example of a mathematical problem.

Problem solving can also be considered in terms of engaging students in both inductive- and deductive-reasoning strategies. As students make sense of the problem, they will be creating conjectures and looking for patterns that they may be able to generalize. This part of the problem-solving process often involves inductive reasoning. As students use approaches to solving the problem, they often move into mathematical reasoning that is deductive in nature. It is crucial that students be encouraged to engage in both types of reasoning and be given the opportunity to consider the approaches and strategies used by others in solving similar problems.

Problem solving is a powerful teaching tool that fosters multiple, creative, and innovative solutions. Creating an environment where students openly look for, and engage in finding, a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive mathematical risk-takers.

A possible flow chart to share with students is as follows:



Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to one another or to real-world phenomena, students can begin to view mathematics as useful, relevant, and integrated. Learning mathematics within contexts and making connections relevant to learners can validate past experiences and increase student willingness to participate and be actively engaged. The brain is constantly looking for and making connections.

"Because the learner is constantly searching for connections on many levels, educators need to *orchestrate the experiences* from which learners extract understanding. … *Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching*." (Caine and Caine 1991, p. 5).

Mathematics should be viewed as an integrated whole rather than as the study of separate strands or units. Connections must also be made between and among the different representational modes—contextual, concrete, pictorial, linguistic/verbal, and symbolic. The process of making connections, in turn, facilitates learning. Concepts and skills should also be connected to everyday situations and other curricular areas. For example, when developing literacy skills students learn to make text-to-world, text-to-text, and text-to-self connections. Students can also make connections to make mathematics come alive through math-to-world, math-to-math, and math-to-self connections.

Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally without the use of external aids.

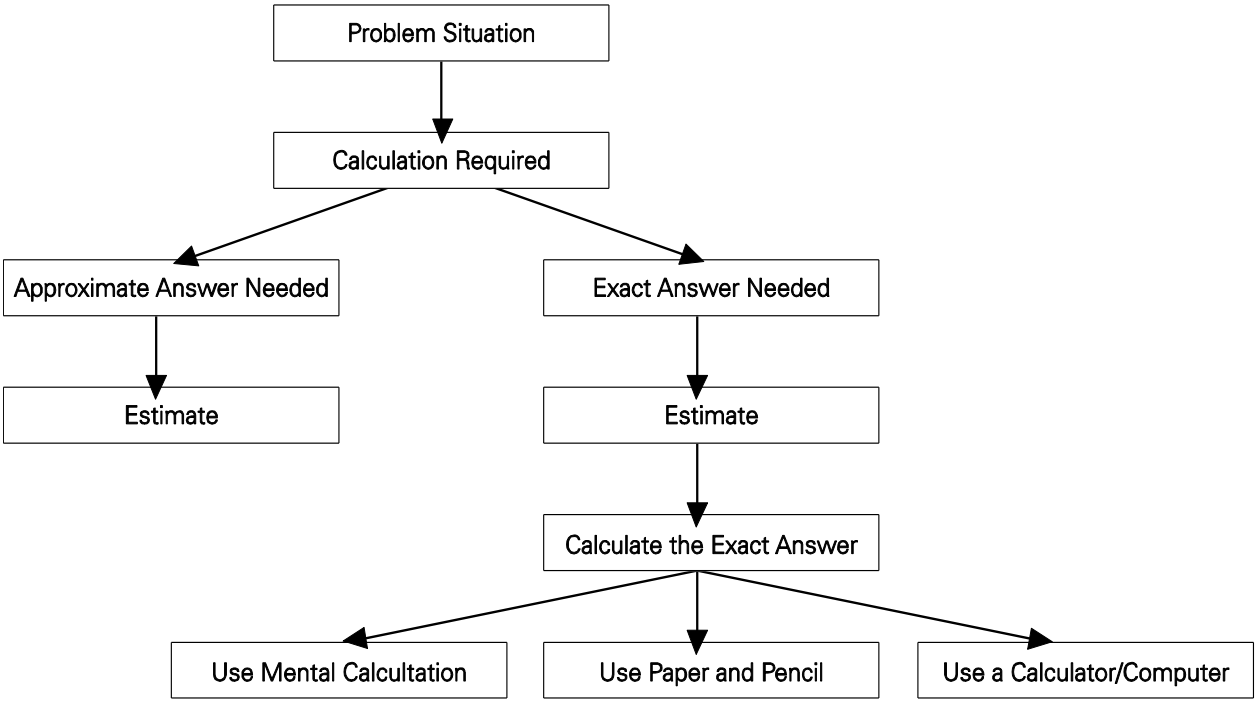
Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy and flexibility.

"Even more important than performing computational procedures or using calculators is the greater facility that students need—more than ever before—with estimation and mental math." (National Council of Teachers of Mathematics, May 2005).

Students proficient with mental mathematics "become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers, and are more able to use multiple approaches to problem solving." (Rubenstein 2001) Mental mathematics "provides a cornerstone for all estimation processes, offering a variety of alternative algorithms and nonstandard techniques for finding answers." (Hope 1988, p. v)

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when, and what strategy to use when estimating. Estimation is used to make mathematical judgments and develop useful, efficient strategies for dealing with situations in daily life. When estimating, students need to learn which strategy to use and how to use it.

Students need to develop both mental mathematics and estimation skills through context and not in isolation so they are able to apply them to solve problems. Whenever a problem requires a calculation, students should follow the decision-making process as illustrated below.



The skill of estimation requires a sound knowledge of mental mathematics. Both are necessary to many everyday experiences, and students should be provided with frequent opportunities to practise these skills.

Technology [T]

Technology can be effectively used to contribute to and support the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures, and solve problems.

Calculators, computers, and other technologies can be used to

- explore and represent mathematical relationships and patterns in a variety of ways
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- increase the focus on conceptual understanding by decreasing the time spent on repetitive procedures
- reinforce the learning of foundational concepts
- develop personal procedures for mathematical operations
- simulate situations
- develop number and spatial sense
- generate and test inductive conjectures

Technology contributes to a learning environment in which the curiosity of students can lead to rich mathematical discoveries at all grade levels. The use of technology should not replace mathematical understanding. Instead, technology should be used as one of a variety of approaches and tools for creating mathematical understanding.

Visualization [V]

Visualization "involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world." (Armstrong 1993, p. 10). The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number, spatial, and measurement sense. Number visualization occurs when students create mental representations of numbers. Being able to create, interpret and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and spatial reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure, when to estimate and to know several estimation strategies. (Shaw and Cliatt 1989, p. 150)

Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations. It is through visualization that abstract concepts can be understood by the student. Visualization is a foundation to the development of abstract understanding, confidence, and fluency.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking.
Questions that challenge students to think, analyze, and synthesize help them develop an understanding of mathematics. All students need to be challenged to answer questions such as, Why do you believe that's true/correct? or What would happen if ….

Mathematical experiences provide opportunities for students to engage in inductive and deductive reasoning. Students use inductive reasoning when they explore and record results, analyze observations, make generalizations from patterns, and test these generalizations. Students use deductive reasoning when they reach new conclusions based upon the application of what is already known or assumed to be true.

Mathematics reasoning involves informal thinking, conjecturing, and validating—these help students understand that mathematics makes sense. Students are encouraged to justify, in a variety of ways, their solutions, thinking processes, and hypotheses. In fact, good reasoning is as important as finding correct answers. The thinking skills developed by focusing on reasoning can be used in daily life in a wide variety of contexts and disciplines.

Nature of Mathematics

Mathematics is one way of trying to understand, interpret, and describe our world. There are a number of components that define the nature of mathematics, and these are woven throughout this document. These components include change, constancy, number sense, relationships, patterns, spatial sense, and uncertainty.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics. Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, … can be described as

- skip counting by 2s, starting from 4
- an arithmetic sequence, with first term 4 and a common difference of 2
- a linear function with a discrete domain

(Steen 1990, p. 184).

Students need to learn that new concepts of mathematics as well as changes to previously learned concepts arise from a need to describe and understand something new. Integers, decimals, fractions, irrational numbers, and complex numbers emerge as students engage in exploring new situations that cannot be effectively described or analyzed using whole numbers.

Students best experience change to their understanding of mathematical concepts as a result of mathematical play.

Constancy

Different aspects of constancy are described by the terms **stability**, **conservation**, **equilibrium**, **steady state** and **symmetry** (AAAS–Benchmarks 1993, p. 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include the following:

- The area of a rectangular region is the same regardless of the methods used to determine the solution.
- The sum of the interior angles of any triangle is 180°.
- The theoretical probability of flipping a coin and getting heads is 0.5.
- Lines with constant slope.

Number Sense

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy. (British Columbia Ministry of Education, 2000, p. 146) Continuing to foster number sense is fundamental to growth of mathematical understanding.

A true sense of number goes well beyond the skills of simply counting, memorizing facts, and the situational rote use of algorithms. Students with strong number sense are able to judge the reasonableness of a solution, describe relationships between different types of numbers, compare quantities, and work with different representations of the same number to develop a deeper conceptual understanding of mathematics.

Number sense develops when students connect numbers to real-life experiences and use benchmarks and referents. This results in students who are computationally fluent, flexible with numbers, and have intuition about numbers. The evolving number sense typically comes as a by-product of learning rather

than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections.

Relationships

Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects, variables, and concepts. The search for possible relationships involves collecting and analyzing data, analyzing patterns and describing possible relationships visually, symbolically, orally or in written form.

Patterns

Mathematics is about recognizing, describing, and working with numerical and non-numerical patterns. Patterns exist in all of the mathematical topics, and it is through the study of patterns that students can make strong connections between concepts in the same and different topics.

Working with patterns also enables students to make connections beyond mathematics. The ability to analyze patterns contributes to how students understand their environment. Patterns may be represented in concrete, visual, auditory, or symbolic form. Students should develop fluency in moving from one representation to another.

Students need to learn to recognize, extend, create, and apply mathematical patterns. This understanding of patterns allows students to make predictions and justify their reasoning when solving problems. Learning to work with patterns helps develop students' algebraic thinking, which is foundational for working with more abstract mathematics.

Spatial Sense

Spatial sense involves the representation and manipulation of 3-D objects and 2-D shapes. It enables students to reason and interpret among 3-D and 2-D representations.

Spatial sense is developed through a variety of experiences with visual and concrete models. It offers a way to interpret and reflect on the physical environment and its 3-D or 2-D representations.

Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to make predictions about the results of changing these dimensions.

Spatial sense is also critical in students' understanding of the relationship between the equations and graphs of functions and, ultimately, in understanding how both equations and graphs can be used to represent physical situations.

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty. Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty. The quality of the interpretation is directly related to the quality of the data. An

awareness of uncertainty allows students to assess the reliability of data and data interpretation. Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately. This language must be used effectively and correctly to convey valuable messages.

Curriculum Document Format

This guide presents the mathematics curriculum so that a teacher may readily view the scope of the outcomes that students are expected to achieve during that year. Teachers are encouraged, however, to examine what comes before and what follows after, to better understand how students' learning at a particular grade level is part of a bigger picture of concept and skill development.

The order of presentation in no way assumes or prescribes a preferred order of presentation in the classroom, but simply lays out the specific curriculum outcomes in relation to the overarching general curriculum outcomes.

When a specific curriculum outcome is introduced, it is followed by the mathematical processes and performance indicators for that outcome. A scope and sequence is then provided, which relates the SCO to previous and next grade SCOs. Also for each SCO, there are background information, assessment strategies, suggested instructional strategies, and suggested models and manipulatives, mathematical vocabulary, and resource notes. For each section, the guiding questions should be used to help with unit and lesson preparation.

SCO

Mathematical Processes

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections
[ME] Mental Mathematics and Estimation		
[T] Technology	**[V]** Visualization	**[R]** Reasoning

Performance Indicators

Describes observable indicators of whether students have met the specific outcome.

Scope and Sequence

Previous grade or course SCOs	Current course SCO	Following grade or course SCOs

Background

Describes the "big ideas" to be learned and how they relate to work in previous grade and work in subsequent courses.

Assessment, Teaching, and Learning

Assessment Strategies

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Assessing Prior Knowledge

Sample tasks that can be used to determine students' prior knowledge.

Whole-Class/Group/Individual Assessment Tasks

Some suggestions for specific activities and questions that can be used for both instruction and assessment.

Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcome and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

Suggested Learning Tasks

Suggestions for general approaches and strategies suggested for teaching this outcome.

Guiding Questions

- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

Suggested Models and Manipulatives

Mathematical Vocabulary

Resources/Notes

Contexts for Learning and Teaching

Beliefs about Students and Mathematics Learning

"Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge." (National Council of Teachers of Mathematics 2000, p. 20).

- The Nova Scotia mathematics curriculum is based upon several key assumptions or beliefs about mathematics learning that have grown out of research and practice. These beliefs include the following:
- Mathematics learning is an active and constructive process.
- Learning is most effective when standards of expectation are made clear with ongoing assessment and feedback.
- Learners are individuals who bring a wide range of prior knowledge and experiences and who learn via various styles and at different rates.
- Leaning is most effective when standards of expectation are made clear with ongoing assessment and feedback.

Students are curious, active learners with individual interests, abilities, and needs. They come to classrooms with varying knowledge, life experiences, and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Students construct their understanding of mathematics by developing meaning based on a variety of learning experiences. This meaning is best constructed when learners encounter mathematical experiences that proceed from simple to complex and from the concrete to the abstract. The use of manipulatives, visuals, and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students. At all levels of understanding, students benefit from working with a variety of materials, tools, and contexts when constructing meaning about new mathematical ideas. Meaningful student discussions also provide essential links among concrete, pictorial, and symbolic representations of mathematics. The learning environment should value, respect, and address all students' experiences and ways of thinking so that students are comfortable taking intellectual risks, asking questions, and posing conjectures. Students need to explore mathematics through solving problems in order to continue developing personal strategies and mathematical literacy. It is important to realize that it is acceptable to solve problems in different ways and that solutions may vary depending upon how the problem is understood.

Goals of Mathematics Education

The main goals of mathematics education are to prepare students to

- use mathematics confidently to solve problems
- communicate and reason mathematically
- appreciate and value mathematics
- make connections between mathematics and its applications
- become mathematically literate adults, using mathematics to contribute to society
- commit themselves to lifelong learning

Students who have met these goals will

- gain understanding and appreciation of the contributions of mathematics as a science, a philosophy, and an art
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical tasks and projects
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity about mathematics and situations involving mathematics

In order to assist students in attaining these goals, teachers are encouraged to develop a classroom atmosphere that fosters conceptual understanding. Students should be encouraged to

- take risks
- think and reflect independently
- share and communicate mathematical understanding
- solve problems in individual and group projects
- pursue greater understanding of mathematics
- appreciate the value of mathematics throughout history

Opportunities for Success

A positive attitude has a profound effect on learning. Environments that create a sense of belonging, encourage risk taking, and provide opportunities for student success help develop and maintain positive attitudes and self-confidence. Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, participate willingly in classroom activities, persist in challenging situations, and engage in reflective practices.

To experience success, students must be taught to set achievable goals and assess their progress as they work toward these goals. Striving toward success and becoming autonomous and responsible learners are ongoing, reflective processes that involve revisiting the setting and assessing of personal goals.

Engaging All Learners

*"*No matter how engagement is defined or which dimension is considered, research confirms this truism of education: *The more engaged you are, the more you will learn."* (Hume 2011, 6)

Student engagement is at the core of learning. This is critical for teachers to take into account when planning and implementing instruction. Effective instruction engages, embraces, and supports all learners through a range of learning experiences. This curriculum is designed to provide learning opportunities that reflect culturally proficient instructional and assessment practices and are equitable, accessible, and inclusive of the multiple facets of diversity represented in today's classrooms.

Engagement in learning occurs when students are provided with opportunities to become more invested in their learning. When teachers know their students as individual learners and as individual people, their students are more likely to be motivated to learn, participate in classroom activities, persist in challenging situations, and engage in reflective practices. Students often become more engaged when teachers demonstrate a genuine belief in each student's potential to learn.

SUPPORTIVE LEARNING ENVIRONMENTS

A supportive and positive learning environment has a profound effect on students' learning. In classrooms where students feel a sense of belonging, are encouraged to actively participate, are challenged without being frustrated, and feel safe and supported to take risks with their learning, students are more likely to experience success. It is realized that not all students will progress at the same pace or be equally positioned in terms of their prior knowledge of and skill with particular concepts and outcomes. Teachers provide all students with equitable access to learning by integrating a variety of instructional approaches and assessment activities that consider all learners and align with the following key principles:

- Instruction must be flexible and offer multiple means of representation.
- Students must have opportunities to express their knowledge and understanding in multiple ways.
- Teachers must provide options for students to engage in learning through multiple ways.

Teachers who know their students well become aware of individual learning differences and infuse this understanding into planned instructional and assessment decisions. They organize learning experiences to accommodate the many ways in which students learn, create meaning, and demonstrate their knowledge and understanding. Teachers use a variety of effective teaching approaches that may include

- providing all students with equitable access to appropriate learning strategies, resources, and technology
- offering a range of ways students can access their prior knowledge to connect with new concepts
- scaffolding instruction and assignments so that individual or groups of students are supported as needed throughout the process of learning
- verbalizing their thinking to model comprehension strategies and new learning
- balancing individual, small-group, and whole-class approaches to learning activities
- involving students in the co-creation of criteria for assessment and evaluation
- providing students with choice in how they demonstrate their understanding according to learning styles and preferences, building on individual strengths, and including a range of difficulty and challenge
- providing frequent and meaningful feedback to students throughout their learning experiences

LEARNING STYLES AND PREFERENCES

The ways in which students make sense of, receive, and process information, demonstrate learning, and interact with peers and their environment both indicate and shape learning preferences, which may vary widely from student to student. Learning preferences are influenced also by the learning context and purpose and by the type and form of information presented or requested. Most students tend to favour one learning style and may have greater success if instruction is designed to provide for multiple learning styles, thus creating more opportunities for all students to access learning. The three most commonly referenced learning styles are

- auditory (such as listening to teacher-presented lessons or discussing with peers)
- kinesthetic (such as using manipulatives or recording print or graphic/visual text)
- visual (such as interpreting information with text and graphics or viewing videos)

While students can be expected to work using all modalities, it is recognized that one or some of these modalities may be more natural to individual students than the others.

A Gender-Inclusive Curriculum

It is important that the curriculum respects the experiences and values of all students and that learning resources and instructional practices are not gender-biased. Teachers promote gender equity and inclusion in their classrooms when they

- articulate equally high expectations for all students
- provide equal opportunity for input and response from all students
- model gender-fair language and respectful listening in their interactions with students

Valuing Diversity: Teaching with Cultural Proficiency

Teachers understand that students represent diverse life and cultural experiences, with individual students bringing different prior knowledge to their learning. Therefore, teachers build upon their knowledge of their students as individuals and respond by using a variety of culturally-proficient instruction and assessment strategies. "Instruction that is embedded in socially meaningful contexts, and tasks that are meaningful and relevant to the lives of students, will engage students in high-level problem-solving and reasoning and enhance students' engagement (Frankenstein 1995; Gutstein 2003; Ladson-Billings 1997; Tate 1995)." (Herzig 2005)

Students with Language, Communication, and Learning Challenges

Today's classrooms include students who have diverse backgrounds, abilities, levels of development, and learning challenges. By observing and interacting with students as they work on assigned activities, teachers can identify areas where students may need additional support to achieve their learning goals. Teachers can then respond with a range of effective instructional strategies. Students who have English as an Additional Language (EAL) may require curriculum outcomes at different levels, or temporary individualized outcomes, particularly in language-based subject areas, while they become more proficient in their English language skills. For students who are experiencing difficulties, it is important that teachers distinguish between students for whom curriculum content is challenging and students for whom language-based issues are at the root of apparent academic difficulties.

Students who Demonstrate Gifted and Talented Behaviours

Some students are academically gifted and talented with specific skill sets or in specific subject areas. Most students who are gifted and talented thrive when challenged by problem-centred, inquiry-based learning and open-ended activities. Teachers may challenge students who are gifted and talented by adjusting the breadth, the depth, and/or the pace of instruction. Learning experiences may be enriched by providing greater choice among activities and offering a range of resources that require increased cognitive demand and higher-level thinking at different levels of complexity and abstraction. For additional information, refer to *Gifted Education and Talent Development* (Nova Scotia Department of Education 2010).

Connections across the Curriculum

The teacher should take advantage of the various opportunities available to integrate mathematics and other subjects. This integration not only serves to show students how mathematics is used in daily life, but it helps strengthen the students' understanding of mathematical concepts and provides them with opportunities to practise mathematical skills. There are many possibilities for integrating mathematics in career education, literacy, music, physical education, science, social studies, technology education, and visual arts.

Measurement
50–55 hours

GCO: Students will be expected to develop spatial sense and proportional reasoning.

Specific Curriculum Outcomes

Process Standards Key

[**C**] Communication	[**PS**] Problem Solving	[**CN**] Connections	[**ME**] Mental Mathematics and Estimation
[**T**] Technology	[**V**] Visualization	[**R**] Reasoning	

M01 Students will be expected to solve problems that involve linear measurement, using SI and imperial units of measure, estimation strategies, and measurement strategies. [ME, PS, V]

M02 Students will be expected to apply proportional reasoning to problems that involve conversions between SI and imperial units of measure. [C, ME, PS]

M03 Students will be expected to solve problems, using SI and imperial units, that involve the surface area and volume of 3-D objects, including right cones, right cylinders, right prisms, right pyramids, and spheres. [CN, PS, R, V]

M04 Students will be expected to develop and apply the primary trigonometric ratios (sine, cosine, tangent) to solve problems that involve right triangles. [C, CN, PS, R, T, V]

SCO M01 Students will be expected to solve problems that involve linear measurement, using SI and imperial units of measure, estimation strategies, and measurement strategies. [ME, PS, V]			
[**C**] Communication	[**PS**] Problem Solving	[**CN**] Connections	[**ME**] Mental Mathematics and Estimation
[**T**] Technology	[**V**] Visualization	[**R**] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have met the corresponding specific curriculum outcome.

M01.01 Provide referents for linear measurements, including millimetre, centimetre, metre, kilometre, inch, foot, yard, and mile, and explain the choices.
M01.02 Compare SI and imperial units, using referents.
M01.03 Estimate a linear measure, using a referent, and explain the process used.
M01.04 Justify the choice of units used for determining a measurement in a problem-solving context.
M01.05 Solve problems that involve linear measure, using instruments such as rulers, calipers, or tape measures.
M01.06 Describe and explain a personal strategy used to determine a linear measurement (e.g., circumference of a bottle, length of a curve, and perimeter of the base of an irregular 3-D object).

Scope and Sequence

Mathematics 9	**Mathematics 10**	**Grade 11 Mathematics Courses**
—	**M01** Students will be expected to solve problems that involve linear measurement, using SI and imperial units of measure, estimation strategies, and measurement strategies.	—

Background

Students are familiar with the metric system (the SI system) as a standard for measurement. Students were introduced to the basic SI units in Mathematics 3 and have extended their knowledge since that time to other units and to conversions within the SI system. Students should be able to identify commonly used SI units such as centimetres, metres, millilitres, litres, grams, and kilograms.

A referent is an object that can be used to help estimate a measurement. From the earliest introduction to metric units, students have had experience relating non-standard and standard units of measurement. They have used referents to estimate the length of an object in centimetres, metres, and millimetres.

While students have used the metric system in previous grades, this will be their first opportunity to study the imperial system. Students may be familiar with the imperial system for measurements such as distance in miles, height in feet and inches, weight in pounds, and capacity in gallons. For this outcome, students are responsible only for working with measures of length.

International System of Units, officially called the *Système International d'Unités* and abbreviated to SI, is based on the metric system. It is the primary system of measurement used throughout the world and in science. This system is convenient and logical. In the SI system, the basic unit of length is the metre. Other linear units of SI measurement, both larger and smaller than the metre, use prefixes that indicate powers of 10 (1 kilometre = 10^3 metres; 1 millimere = 10^{-3} metres).

The imperial system is a collection of units that were developed at different times to meet different needs. Because of this fact, the imperial system is not a decimal system, as is the SI system. Since imperial measures are based on traditional measurements rather than a base-ten system, sometimes imperial measures are written in fractional form. For example, inches on a measuring tape are divided into $\frac{1}{2}$ inch, $\frac{1}{16}$ inch, etc.

Students should recognize the terminology and abbreviations associated with the imperial measures of length: foot (ft. or '), inch (in. or "), yard (yd.) and mile (mi.).

Students will convert between various units in the SI system. Students have used and will continue to use conversions in science.

There are other instances where the imperial system is commonly used. For example, in the construction industry, lengths are measured in inches (2 in. × 4 in.). Sports, such as football and golf, measure distance using yards, and marathons are sometimes measured in miles.

There are also cases in which both systems are used. For example, wrenches come in imperial and metric sizes. Snowmobiles made in the United States require imperial-sized wrenches, whereas snowmobiles made elsewhere would use metric-sized wrenches.

The information that follows is given to support the introduction of the imperial system and a review of the SI system in terms of historical context. (Students should not be assessed on this historical information.)

HISTORICAL CONTEXT

The metric system was formally developed by France in the 1700s. The system is based on the linear measure of a metre. Originally intended to be one ten-millionth of the distance from the Earth's equator to the North Pole (at sea level), its definition has been periodically refined to reflect growing knowledge of metrology. Since 1983, it has been defined as “the length of the path travelled by light in a vacuum during a time interval of 1/299 792 458 of a second.”

In Canada in 1970, with rapidly advancing technology and expanding worldwide trade, the Canadian government adopted a policy for a single, coherent measurement system based on the Système International d'Unités (SI), the latest evolution of the metric system.

Although the measurements in the metric system are derived from scientific principles, the English units for measurements (and the subsequent American and imperial measurements) are based on nature and everyday activities. For example, a league is based on the distance that can be walked in one hour. Sailors (in previous times) would drop a weighted rope into the water, lowering it by lengths (where each length was measured by holding the rope between their outstretched hands) until the weight at the end of the rope touched the seabed. This led to the definition of the fathom as the distance from

the fingertips of one hand to the fingertips of the other, when the hands are held straight out to the sides. A grain (used to measure small quantities of precious metals) is the weight of a grain of wheat or barleycorn.

Such natural measures were well suited in a simple agricultural society. However, as trade and commerce grew, it was necessary to have more consistent measures (after all, not all grains of wheat have the same weight and not all sailors have the same arm length). Consequently, metal weights and lengths were produced to represent exact measures. These metal representations were then used to produce official scales and measurements to ensure that trade was based on standard quantities. For larger measures, such as a mile, it was impractical to build a metal equivalent; this type of measurement was therefore redefined to be multiples of the smaller measures. It is for this reason that the mile was changed in 1595 under Queen Elizabeth I's reign from the Roman standard of 5000 feet to 5280 feet (which is eight furlongs, each furlong equal to 10 chains, each chain equal to 22 yards, and each yard equal to three feet).

Despite the development and standardization of the English units of measure, their roots in ancient agriculture and trade have resulted in a diverse and relatively complex set of measurements. The various trades each developed their own measures, so in many cases the measure would depend on what it was being used for—a barrel of oil is not the same size as a cranberry barrel (there are, in fact, eight different barrel sizes). Likewise, there are fluid ounces and weight ounces, as well as different types of weight ounces that depend on what is being weighed. This complexity was not eliminated when the English system evolved into the imperial and American systems. The result is that these systems have approximately 300 different units of measurement. In comparison, the metric system has only seven basic units of measurement, and these can be increased or decreased in multiples of 10 to make larger or smaller units, or combined to make more complex units. The imperial system is not so straightforward.

For many years, the United States of America (USA) and the United Kingdom (UK) each continued to develop its own measurement standards with little to no co-ordination between the two. As a result, differences in the units evolved, and the two systems became increasingly different despite both having English units as a common ancestor. For example, the US pint has 16 ounces whereas the imperial pint has 20 ounces. In addition, there are a number of units that exist in one system but not in the other. Prior to the adoption of SI units, in 1970 Canada used a mixure of UK imperial units (the gallon) and US imperial units (the pint).

Since the mid-20th century, the US and UK standards organizations have worked together to bring the two systems closer together. This is in part due to the impact of globalization and the resulting need to have common measurements. It is also due to the growing acceptance of the metric system as the international standard for measurement. An example of both influences can be seen by the decision in 1958 (implemented in 1959) for the US yard and the UK yard to be jointly redefined as equal to 0.9144 metres.

Beginning in the mid-20th century, most countries using the imperial system have been gradually replacing the imperial system with the metric system. This is a complex transition involving not only a re-education of the population, but also considerable changes to machinery and production equipment.

Currently, the three countries that retain a non-metric system are Burma, Liberia, and the USA.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Student tasks such as the following could be completed to assist in determining students' prior knowledge.

- Ask students to make up a fictional animal, describing it in terms of height, mass, maximum speed, and body temperature. (It is likely that some of these descriptions will be in metric and some in imperial measure.)

- Activate prior knowledge regarding referents of measurement, which students have previously developed (1 metre is the approximate distance from the floor to a doorknob, 1 litre of milk, room temperature 21°C, 2 lb. of sugar, 1 kg of salt).

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

- Explain the difference between SI and imperial units.

- If you were to measure someone's height, would you use SI or imperial units? Why?

- Explain why yards are used instead of metres in football and golf.

- Explore the historical origin of various SI units and imperial units, such as the metre and the yard.

- Define the prefixes used in the SI system. Include mega-, giga-, tera- and others. In what other contexts do you see these prefixes being used?

- A relatively new billionaire, Isabel dos Santos, has a net worth of $2 billion. If she converted all her assets to hundred dollar bills how high would the stack reach if piled in a single stack?

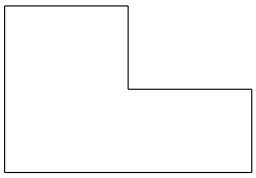
- Jason was asked how many cell phone charging cords it would take to go around the perimeter of the classroom. He measured the length of the cord to be 75 cm. Knowing 100 cm was in a metre, he did a conversion resulting in 75 cm = 7500 m.
  (a) Is Jason's answer correct? Why or why not?

(b) If the classroom measures 6 m × 5.5 m, how many cords would it take to go around the classroom?

- Measure the length of various objects using different measurement tools (metre sticks, measuring tapes, calipers). Convert the readings to other units that might be appropriate. For example, you might measure the thickness of a sheet of cardboard with calipers to illustrate the usefulness of mm rather than cm or m.

- List household objects that are approximately
  (a) 2 ft. long (b) 4 in. thick (c) 12 cm wide

- Using an appropriate SI unit, estimate the perimeter of the figure shown below.



- If all the angles in the figure shown above are right angles, is it necessary to measure all sides of the figure in order to determine its perimeter? Its area? Explain your reasoning.

- A hockey net is 6 ft. wide. Explain how you could use a referent to mark off a width of approximately 6 ft.?

- What referent could you use to estimate how much snow fell after a snowstorm? Explain your choice.

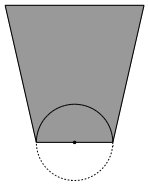
- Estimate the length of the following objects using a referent and explain how you determined the answer.
  (a) height of a door
  (b) width of a whiteboard
  (c) length of a keyboard
  (d) height of a light switch
  (e) height of an electrical outlet

- Participate in a school scavenger hunt to locate the following:
  (a) an object 2 dimes thick
  (b) an object 3 inches long
  (c) an object 4 sandwiches long

- Estimate (by using a referent) and then measure (by using a measuring tool) the following. For each explain your choice of measuring tool.
  (a) Within your school, how far must you walk from the library to the gymnasium?
  (b) What are the dimensions of the cafeteria?
  (c) What is the width of a computer screen?
  (d) What is the width of the staircase?
  (e) What is the perimeter of an MP3 player?
  (f) What is the perimeter of the base of the recycling bin?

- Estimate the total distance you would walk during a typical day at school. In groups of two or three, use referents to explore your individual routes throughout the building for a particular day in your school cycle. Once completed, share your results with the others in your class and discuss the strategies used to obtain your distances.

- Describe how you would determine the circumference of the largest part of a basketball. State the referent, the unit, and the measuring instrument used.



- Explain a strategy that you can use to determine the perimeter of the basketball key represented by the shaded region.

- List real-life examples of how the imperial system is used.

- Estimate each measure.
  (a) The height of a horse in feet.
  (b) The length of a hockey rink in metres.
  (c) The width of a cell phone in centimetres.
  (d) The length of a new pencil in inches.

- Why is the imperial system still used in Canada even though it is not the official system?

- Use print advertisements or the Internet to investigate products from building supply stores that show the use of imperial units for measurements. Examine what material or objects are measured in the imperial system and which ones are measured in the SI system. Record your findings and identify each measurement as to whether it is for length, area, volume, capacity, mass, or temperature.

- Using a measuring tape with inches and feet, measure 10 objects around the classroom to the closest $\frac{1}{8}$ of an inch.

FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should be used to inform instruction.

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SUGGESTED LEARNING TASKS

Effective instruction should consist of various strategies.

Guiding Question

- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

Consider the following sample instructional strategies when planning lessons.

- Students should be given an opportunity to reflect on and discuss the following:
  - What is measurement?
  - How do we measure things?
  - What are some commonly used units of measurement?

- Have students develop a set of referents, for both the metric and imperial systems of measurement, such as metre, gram, inch, and mile. Students should then use these referents to estimate the length of an unknown object. Some examples are shown below.
  - *Metre/Yard:* Height of a door knob from the floor
  - *Millimetre:* The thickness of a dime
  - *Centimetre/Inch:* Width/length between first and second knuckles of a pinky finger
  - *Kilometre:* Distance you could walk comfortably in 12 minutes
  - *Foot:* Slightly more than the length of a piece of loose-leaf
  - *Inch:* Diameter of a quarter
  - *Mile:* Distance you can walk comfortably in 20 minutes

Some common referents for linear measurement include the following:

1 mm	▪ Thickness of a dime. ▪ Thickness of a fingernail.
1 cm	▪ Width of a fingernail. ▪ Width of black keys on a standard piano. ▪ Width of a crayon. ▪ Width of a paper clip.
1 m	▪ Distance from a door knob to the floor.
1 km	▪ Distance you can walk comfortably in 12 minutes.
1 in.	▪ Thickness of a hockey puck. ▪ Length from end of thumb to first knuckle.
1 ft.	▪ Length of a standard floor tile.
1 yd.	▪ Distance from the tip of the nose to the outstretched fingers. ▪ Average length of a guitar.
1 mi.	▪ Distance you can walk comfortably in 20 minutes.

- Have students measure, in imperial units and then in metric units, their hand span, foot length, finger length, and stride length. Use these as referents to measure the length of the classroom, the width of a desk, and other items or spaces.

- The thumb can also be used as a measurement device. A person's measurement could be determined by measuring the distance around his or her thumb using string. Twice around the thumb is equal to once around the wrist. Twice around the wrist is once around the neck. Twice around the neck is once around the waist. Students should be encouraged to experiment with this.

- Each referent is a suggestion. Encourage students to select as their own referent something that makes sense to them. Students may, for example, use the distance from the floor to their waist as a referent for one metre. If they determine the height of the seat of a chair to be approximately half of their waist height, then the seat of the chair is 0.5 metres high. If students are determining the length of a room, for example, they could count the tiles on the floor since the length of a standard tile is 1 foot in length. Students should be given an opportunity to use their referents to provide estimations for various items and justify their choice of unit.

- Estimation activities help students focus on the attribute being measured and help develop familiarity with the measuring unit. Using referents, students should explore both SI and imperial units to measure objects.

- Students should use instruments such as rulers, calipers, and tape measures to determine lengths of objects using both SI and imperial units. Incorporating a variety of measuring devices and using shapes that interest students will provide more meaningful learning experiences. Ask students to give some examples of measuring instruments that are commonly used to measure distance in the home or workplace. Students can explain or demonstrate how one of these instruments works. They could brainstorm a list of objects, use their referents to estimate each length, and then actually measure the items to check their accuracy. The information could be displayed in the form of a poster or a pamphlet.

- Students should be given the opportunity to explore the environment to develop these measurement skills. Consider the following examples:
  (a) What is the length of the parking lot?
  (b) How far do you have to walk from the bus to the school's entrance?

(c) How much fencing is needed or used to enclose the school grounds?
(d) What is the circumference of a school bus tire?
(e) What is the width of a window?
(f) What is the height of a locker?

- Students should also be exposed to strategies when estimating the length of a curve or measuring irregularly shaped objects like a computer mouse, a horseshoe magnet, or a badminton racquet. Using string and rulers, encourage students to measure the circumference of a circular clock, for example. Students may determine the circumference by laying the string around the clock and then measuring the length of the string. This would be a good opportunity to extend this activity to estimate the length of a curve or a portion of the circular object without the use of string. Using a ruler, students could measure the distance between the endpoints of the curve. Since the shortest distance between two points is a straight line, their estimate of the length of the curve would be greater than this distance. Remind students to describe the process used to determine their answer.

Suggested Models and Manipulatives

- calipers
- metre and yard sticks
- rulers and measuring tapes
- string

Mathematical Vocabulary

Students need to be comfortable using the following vocabulary.

- new
  - foot (ft. or ')
  - inch (in. or ")
  - mile (mi.)
  - referent
  - yard (yd.)
- previous
  - centimetre (cm)
  - kilometre (km)
  - metre (m)
  - millimetre (mm)

Resources/Notes

Internet

- *Mathematics Learning Commons 7–9, Nova Scotia Virtual School* (Nova Scotia Department of Education and Early Childhood Development 2013):
  - Proportional Reasoning PowerPoint
    http://lrt.ednet.ns.ca/PD/math8support_11/prop_reasoning/01D_proportional_reasoning_ratios.ppt
  - Proportional Reasoning Problems
    http://lrt.ednet.ns.ca/PD/math8support_11/prop_reasoning/01K_question_bank.doc
  - Proportional Reasoning articles:
    http://lrt.ednet.ns.ca/PD/math8support_11/prop_reasoning/01L_problems_encourage_prop_sense.pdf
    http://lrt.ednet.ns.ca/PD/math8support_11/prop_reasoning/01M_multiple_ways_to_solve_proportions.pdf

- Professional Learning, K–12, Newfoundland and Labrador (Professional Learning NL 2013)
  www.k12pl.nl.ca
  The classroom clip demonstrates students estimating linear measurements using referents.
- FPSi, Specialist in French property, Metric Chart (Metric Table) (French Property, Services and Information Ltd. 2008)
  www.france-property-and-information.com/table-of-metric-and-imperial-units.htm?phpMyAdmin=24f3a0e02619b794a6db9c79d8b89c4e
  A list of the most common measures and their relationships to each other.

Print

- *Foundations and Pre-calculus Mathematics 10* (Burglind et al., Pearson 2010)
  - Student Book
    - Chapter 1, Sections 1, 2, and 3, pp. 2–25
  - Teacher Technology DVD
    - Teacher Resource
    - Blackline Masters
    - Smart Lessons
    - Animations
    - Dynamic Activities

Notes

SCO M02 Students will be expected to apply proportional reasoning to problems that involve conversions between SI and imperial units of measure. [C, ME, PS]			
[**C**] Communication [**T**] Technology	[**PS**] Problem Solving [**V**] Visualization	[**CN**] Connections [**R**] Reasoning	[**ME**] Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have met the corresponding specific curriculum outcome.

M02.01 Explain how proportional reasoning can be used to convert a measurement within or between SI and imperial systems.
M02.02 Solve a problem that involves the conversion of units within or between SI and imperial systems.
M02.03 Verify, using unit analysis, a conversion within or between SI and imperial systems, and explain the conversion.
M02.04 Justify, using mental mathematics, the reasonableness of a solution to a conversion problem.

Scope and Sequence

Mathematics 9	**Mathematics 10**	**Mathematics 11**
SS04 Students will be expected to draw and interpret scale diagrams of 2-D shapes.	**M02** Students will be expected to apply proportional reasoning to problems that involve conversions between SI and imperial units of measure.	**M01** Students will be expected to solve problems that involve scale diagrams, using proportional reasoning.

Background

Students will use proportions and proportional reasoning to solve problems that require conversions. A proportion is a statement where two ratios are equivalent.
For example, $\frac{100\text{ cm}}{1\text{ m}} = \frac{350\text{ cm}}{3.5\text{ m}}$.
Students were introduced to proportional reasoning in Mathematics 8 (8N05) and in Mathematics 9 continued to work on proportional reasoning through the study of similar polygons. Proper development of proportional reasoning in students means they must become multiplicative thinkers and be able to see and use the multiplicative relationships found within and between the ratios in a problem.

A portion of an SI unit is written in decimal form. For example one-fourth of a metre is written 0.25 m. A decimal system is used for several reasons. Using the decimal system makes it immediately clear when comparing numbers which value is greatest; additionally, doing calculations involving decimal numbers is straightforward.

Contrastingly, a portion of an imperial unit is generally written in fractional form. Many rulers, for example, are marked with imperial units showing one inch divided into eighths or sixteenths. Students sometimes have difficulty working with fractions. For example, students often have difficulty multiplying

a fraction by a whole number. Although they have worked with operations involving fractions in intermediate grades (7N05, 8N06, 9N03), this unit would be a good opportunity to review these concepts prior to completing any conversions.

Note the following equivalence charts:

1 foot	12 inches
3 feet	1 yard
1 mile	5280 feet
1 mile	1760 yards

Conversions need not be memorized, as they are easily accessible. Using the imperial system will provide practice using fractions.

To convert from one measurement system to another, students need to understand the relationship between the units of length in each system.

Unit analysis, also known as dimensional analysis, can often be used to assist students in their calculations. For example,

If converting from yards to inches

$$\frac{1\text{ yd.}}{3\text{ ft.}} \cdot \frac{1\text{ ft.}}{12\text{ in.}} = \frac{1\text{ yd.}}{36\text{ in.}}$$

Therefore, 1 yd. = 36 in.

If converting from feet to metres

$$\frac{1\text{ ft.}}{12\text{ in.}} \cdot \frac{1\text{ in.}}{2.54\text{ cm}} \cdot \frac{100\text{ cm}}{1\text{ m}} = \frac{100\text{ ft.}}{30.48\text{ m}}$$

Therefore, 100 ft. = 30.48 m or 1 ft. = 0.3048 m

Depending on the context of the problem, students will determine when conversions should be exact or when it might be better to use an approximate conversion. For example, 1 foot is exactly 12 inches, while 1 inch is approximately 2.54 cm. **Note:** Conversions between SI and imperial units should be limited to commonly used linear units of measure.

Consider the following:

cm ↔ in. m ↔ ft. km ↔ mi.

1 inch	$\doteq$ 2.54 centimetres
1 yard	$\doteq$ 0.9144 metres
1 mile	$\doteq$ 1.6093 kilometres

Unusual conversions such as **miles ↔ mm** should be avoided.

Students are expected to use proportional reasoning when converting between units. They should be able to see and use relationships within a ratio and between ratios to solve problems.

Consider the following examples:

Convert 200 metres to kilometres

$$\frac{200\,m}{x\,km}=\frac{1000\,m}{1\,km}$$

$$1000x=200$$

$$x=0.2\text{ km}$$

Therefore, 200 m = 0.2 km

Convert 180 inches to feet

$$\frac{180\,in.}{x\,ft.}=\frac{12\,in.}{1\,ft.}$$

$$12x=180$$

$$x=15\text{ ft.}$$

Therefore, 180 in. = 15 ft.

Convert 6 feet to centimetres

Step 1: Convert 6 ft. to in.

$\frac{6\text{ ft.}}{x\text{ in.}}=\frac{1\text{ ft.}}{12\text{ in.}}$; 6 ft. = 72 in.

Step 2: Convert 72 in. to cm

$$\frac{72\text{ in.}}{x\text{ cm}}=\frac{1\text{ in.}}{2.54\text{ cm}}$$

$$x=182.88\text{ cm}$$

Therefore, 6 ft. = 183 cm

It is important for students to notice that, while the units have changed, the actual distance has not. The distance of 200 m is the same as 0.200 km, 180 in. is the same as 15 ft., and 6 ft. is approximately the same as 183 cm.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

Guiding Questions

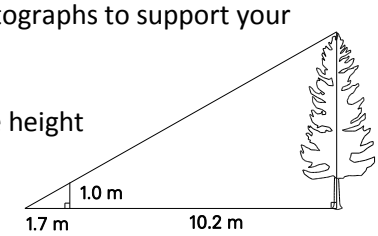
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Student tasks such as the following could be completed to assist in determining students' prior knowledge.

- A photograph measuring 12.5 cm by 17.5 cm needs to be enlarged by a factor of 1.5. What will be the new dimensions of the photograph? Draw a diagram of both photographs to support your reasoning.

- Given that the two triangles in the diagram are similar, determine the height of the tree.



WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

- How many toothpicks would you need to use to determine the perimeter of Nova Scotia?

- Fill in the blanks:
  (a) 130 cm = _____ m
  (b) _____ g = 150 mg
  (c) 60 L = _____ mL
  (d) 3.25 km = _____ cm
  (e) _____ g = 0.68 kg
  (f) 4 m^2 = _____ cm^2
  (g) 3 cm^2 = _____ mm^2

- Serena used proportional reasoning to do the conversion: 0.78 kg = _____ mg

  She wrote: $\frac{10\ 000\text{ mg}}{1\text{ kg}} = \frac{?\text{ mg}}{0.78\text{ kg}}$

  Where did she make an error? Complete the conversion correctly.

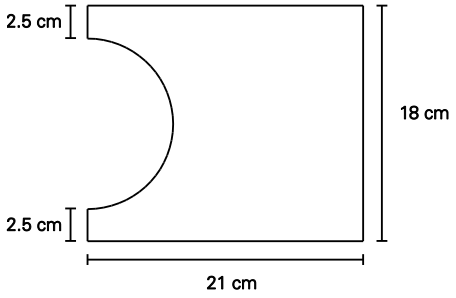
- Convert the following measurements.
  (a) 2.5 m = _____ km
  (b) 8 in. = _____ ft.
  (c) 7 m = _____ in.

- A road sign says to turn left in 1000 feet. Approximately how far is this distance in kilometres?

- Fill in the blanks:
  (a) 36 in. = _____ ft.
  (b) 6" = _____'
  (c) _____ in. = 2 ft.
  (d) 1 $yd.^2$ = _____ $ft.^2$

- A newborn baby weighs 7 pounds 8 ounces. The birth announcement in the newspaper said the baby weighed 7.5 lb. Given this, how many ounces are in a pound?

- Calculate the perimeter of the following diagram. Express your answer to the nearest tenth of a centimetre.



- Which distance below is the longest?
  0.7 in. 1000 yd. 1 km 910 m

- Explain when it might be appropriate to use an approximate conversion. Give an example.

- Isaac bought a second-hand treadmill online. It would only register in miles. Describe a conversion factor that could be used to estimate a conversion from miles to kilometres or vice versa.

- A jet is flying at the height of 28 000 ft. How many metres is this?

- The GPS in your car is set in miles. It estimates the distance to your destination to be 188 mi.
  (a) How many kilometres are you from your destination?
  (b) If the speed you are travelling is registered as 45 mph, how many km/h is this?
  (c) Given this information, what is the estimated time of arrival (ETA) if the current time is 10:20 a.m. and you maintain your average speed?

- Connie is building a wall in her basement using 2" × 4" studs.
  A 2" × 4" stud actually measures $1\frac{1}{2}$" × $3\frac{1}{2}$".
  To build the interior wall, Connie needs to secure the stud to the ceiling and the floor. She then must nail drywall on to both sides of the stud. She has placed the studs so that the narrower sides (the sides to which she will secure the drywall) face out.
  How thick is the wall if the drywall measures $\frac{5}{8}$".

- A carpenter would like to place trim around a rectangular window that measures 41 in. by 27 in. If the trim costs $1.92/ft., what is the approximate cost of the trim for the window before taxes? Verify the conversions using unit analysis.

- Convert 6 yd. = _____ cm, showing unit analysis.

- Madeleine and her friends go for a 15-minute walk. At the end of their walk they wonder how far they travelled in feet, metres, kilometres, and miles. Explain how you can estimate this distance, and show how to express the answer in the various units.

- Your family has decided to have wall-to-wall carpet installed in the den. You call up a local store that supplies flooring to get a price, and they tell you that the carpet will be $22.50/yd.^{2}, the underlay will be $6.50/yd.^{2}, and the installation is a flat fee of $100 per room.
  (a) When you measure your room to determine the cost of this job what units should you use?
  (b) If your friend measures it and tells you that it is a rectangle 15 ft. wide and 18 ft. long, determine the cost of carpeting this room. Don't forget to add the sales tax.

- Create two contextual problems that could be solved by a classmate.

FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should be used to inform instruction.

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
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- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SUGGESTED LEARNING TASKS

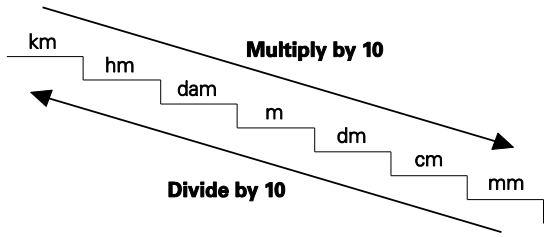
Effective instruction should consist of various strategies.

Guiding Question
- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

Consider the following sample instructional strategies when planning lessons.

- It could be useful for teachers to emphasize the ease of conversion between metric units given that it is developed on the base-ten model. This makes conversions easier with the SI system than the imperial system.

- For length measurements (not area or volume), the following step model may help students visualize converting within the SI measurement system. Each step represents multiplication or division by a factor of 10.



- Prompt students to discuss the value of the ratio $\frac{1\text{ km}}{1000\text{ m}}$.

  Since 1 km = 1000 m, the ratio $\frac{1\text{ km}}{1000\text{ m}} = 1$.

  Students should recognize that when a numerical value is multiplied by 1, the value remains the same. This is the basis of unit conversion.

- Many display tags on items in a store show the SI and imperial measurements. Students could make a display tag for a rectangular picture having dimensions 32 in. × 50 in. illustrating the dimensions of the picture in inches and in centimetres.

- Students should be encouraged to check the reasonableness of their answers when performing conversions. Does the answer make sense? Did the student overestimate or underestimate? For example, students should realize when converting 200 m to km, the answer should be less than 200.

- Reactivate proportional reasoning skills, covered in SCO N01, as a way to complete conversions within the SI system. The following examples illustrate the proportional relationships.

Within

Convert 7.5 cm to inches

$$1\text{ in.} \doteq 2.5\text{ cm}$$

$$\frac{2.5\text{ cm}}{7.5\text{ cm}} = \frac{1\text{ in.}}{x\text{ in.}}$$

$$\times 3 \quad \frac{2.5\text{ cm}}{7.5\text{ cm}} = \frac{1\text{ in.}}{x\text{ in.}} \quad \times 3$$

since 2.5 × 3 7.5

then 1 × 3 = 3

$\therefore$ 7.5 cm ≐ 3 in.

Between

Convert 5 kg to pounds

$$\frac{1\text{ kg}}{5\text{ kg}} = \frac{2.2\text{ lb.}}{x\text{ lb.}}$$

$$\times 2.2$$

$$\frac{1\text{ kg}}{5\text{ kg}} = \frac{2.2\text{ lb.}}{x\text{ lb.}}$$

$$\times 2.2$$

$x = 5 \times 2.2$

$x = 11.0$ lb.

- Hands-on activities allow students to be more engaged for this outcome, as well as developing skills for using measurement tools such as a measuring tape.

- Use of fractions can be demonstrated through cooking or construction. For example, cutting a recipe in half or doubling a recipe as well as converting inches to feet (or feet to inches) for construction purposes are excellent means of connecting measurement conversion to real-world situations.

Suggested Models and Manipulatives

- rulers and measuring tapes with imperial units
- rulers and measuring tapes with metric units

Mathematical Vocabulary

Students need to be comfortable using the following vocabulary.

- unit analysis

Resources/Notes

Print

- *Foundations and Pre-calculus Mathematics 10* (Burglind et al., Pearson 2010)
  - Student Book
    - Chapter 1, Sections 1 and 3, pp. 2–25
  - Teacher Technology DVD
    - Teacher Resource
    - Blackline Masters
    - Smart Lessons
    - Animations
    - Dynamic Activities

Videos

- DVL Videos, Multimedia Ednet Web Station, Learning Resources and Technology Services (Province of Nova Scotia 2013)
  - *Grade 7—Assess Addition of Decimals Using Make-One Strategy*
    http://dvl.ednet.ns.ca/videos/grade-7-assess-addition-decimals-using-make-one-strategy
  - *Grade 7—Introduce Make-Zero Strategy for Integers*
    http://dvl.ednet.ns.ca/videos/grade-7-introduce-make-zero-strategy-integers
  - *Grade 8—Assess Halve/Double Strategy for Percentages*
    http://dvl.ednet.ns.ca/videos/grade-8-assess-halvedouble-strategy-percentages
  - *Grade 8—Introduce Addition of Fractions Using Make-One Strategy*
    http://dvl.ednet.ns.ca/videos/grade-8-introduce-addition-fractions-using-make-one-strategy
  - *Grade 8—Reinforce Addition and Subtraction of Fractions by Rearrangement*
    http://dvl.ednet.ns.ca/videos/grade-8-reinforce-addition-and-subtraction-fractions-rearrangement

Notes

SCO M03 Students will be expected to solve problems, using SI and imperial units, that involve the surface area and volume of 3-D objects, including right cones, right cylinders, right prisms, right pyramids, and spheres. [CN, PS, R, V]			
[**C**] Communication [**T**] Technology	[**PS**] Problem Solving [**V**] Visualization	[**CN**] Connections [**R**] Reasoning	[**ME**] Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have met the corresponding specific curriculum outcome.

M03.01 Sketch a diagram to represent a problem that involves surface area or volume.
M03.02 Determine the surface area of a right cone, right cylinder, right prism, right pyramid, or sphere, using an object or its labelled diagram.
M03.03 Determine the volume of a right cone, right cylinder, right prism, right pyramid, or sphere, using an object or its labelled diagram.
M03.04 Determine an unknown dimension of a right cone, right cylinder, right prism, right pyramid, or sphere, given the object's surface area or volume and the remaining dimensions.
M03.05 Solve a problem that involves surface area or volume, given a diagram of a composite 3-D object.
M03.06 Describe the relationship between the volumes of right cones and right cylinders with the same base and height, and right pyramids and right prisms with the same base and height.

Scope and Sequence

Mathematics 9	**Mathematics 10**	**Mathematics 11**
G01 Students will be expected to determine the surface area of composite 3-D objects to solve problems.	**M03** Students will be expected to solve problems, using SI and imperial units, that involve the surface area and volume of 3-D objects, including right cones, right cylinders, right prisms, right pyramids, and spheres.	**M01** Students will be expected to demonstrate an understanding of the relationships among scale factors, area, surface areas, and volumes of similar 2-D shapes and 3-D objects.

Background

In Mathematics 7, students developed an understanding of 2-D measurement. Students calculated the area of triangles, circles, and parallelograms (7M02). In Mathematics 8, students determined the surface area and volume of 3-D objects limited to right rectangular prisms, right triangular prisms, and right circular cylinders (8M03, 8M04). The focus was on gaining a conceptual understanding of surface area through the use of nets rather than the use of formulae. Volume was calculated using
V = (base area) × height. Students also applied the Pythagorean theorem to determine the length of a side in a right triangle (8M01). In Mathematics 9, work with surface area was extended to include composite 3-D objects (9G01).

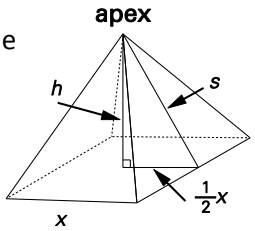
In this unit, students will extend the concepts of surface area and volume to include right pyramids, right circular cones, and spheres. To prevent mere memorization of formulas, it is important for students to understand how the surface area and volume formulas of 3-D objects are developed.

Understanding how to calculate surface area can be helpful in many real-world applications. For example, surface area can be used to estimate the amount of paint needed to paint a house or to know how much wrapping is needed to cover a container. Throughout this unit, students should be encouraged to draw diagrams to help them visualize the 3-D objects that are described.

A prism is a 3-D figure whose two end faces are congruent and parallel rectilinear figures, called bases, and whose sides are parallelograms. The shape of the base determines the name of the prism. A right prism has sides that are rectangular polygons with joining edges and faces that are perpendicular to the base faces.

A cylinder can be defined as a surface or solid bounded by two parallel planes and generated by a straight line moving parallel to the given planes and tracing a curve bounded by the planes and lying in a plane perpendicular or oblique to the given planes. However, for the purposes of this document, when the word **cylinder** is used, it will refer to the more common meaning, that of a right circular cylinder.

A pyramid is a 3-D figure with a polygon base. The shape of the base determines the name of the pyramid. Students should be exposed to a right pyramid with a triangular base, square base, and a rectangular base. Various websites display the nets of the different types of pyramids to give students a visual. When the base of a right pyramid is a regular polygon, the triangular faces are congruent and it is called a regular pyramid.



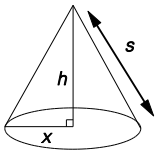
Students will have to differentiate between the height of a right pyramid and its slant height. The height refers to the perpendicular height from the apex to the base, *h*, and the slant height is the altitude of the triangular face, *s*. A common student misconception is relating the slant height to the edge of the pyramid.

The Pythagorean theorem will be used to calculate the slant height, given the perpendicular height and the appropriate base dimension, *x*, or given the edge length from the apex to the base and the appropriate base dimension.

A water demonstration of the Pythagorean theorem using the video clip accessed by www.youtube.com/watch?v=CAkMUdeB06o (2009) serves as an excellent springboard for review.

A cone can be defined as a solid whose surface is generated by a line passing through a fixed point and a fixed plane curve not containing the point, consisting of two equal sections joined at a vertex. Thus, using this definition, all pyramids are cones. However, for the purposes of this document when the word **cone** is used, it will refer to the more common meaning, that of a right circular cone.

The surface area of a cone consists of the area of the circular base and the curved surface. Students should be able to distinguish between the height and slant height of a cone. Although students were exposed to circles, circumference and arcs in Mathematics 9, the terms **sector** and **arc length** are new and should be defined.

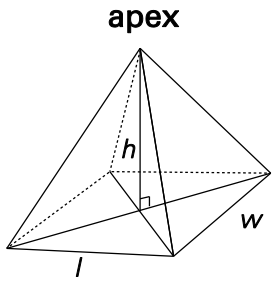


Students are familiar with the area formula of a circle, $A = \pi r^2$. They have not been exposed to the area of the curved surface of a cone. Students will use models to investigate the area of the curved surface. Once students have become fluent in determining the surface area of 3-D objects (prisms, cylinders, pyramids, and cones), they will then determine an unknown dimension that may not be the area. Students are expected to rearrange formulas at this level. They should first substitute the given information into the formula and then solve for the unknown. In the case where both the slant height and surface area are given, students will not be expected to find the radius of a right cone. Students will not be expected to solve for the radius of a cylinder when the height and surface area are given. These types of problems would require an application of the quadratic formula, which will not be introduced until Mathematics 11.

Students will use models to investigate the relationship between the volume of a right cone and a right cylinder, with the same base and height, and between the volume of a right pyramid and a right prism, with the same base and height. An investigation that may be used is found in the Suggested Learning Tasks.

Students will discover and then formalize that the volume of a right pyramid is found by calculating one-third of the volume of its related right prism. The same relationship exists between the right cone and its related right cylinder. Related 3-D objects have the same base and height.

The volume formula of a right cylinder ($V = \pi r^2 h$) was developed in Mathematics 8.



Volume of a right rectangular pyramid is given by

$$V = \frac{1}{3}\text{ (base area × height)}$$

$$V = \frac{1}{3}lwh$$

As students work through problems involving the volume of 3-D objects, they will also have to determine an unknown dimension. Students are expected to rearrange formulas at this level. They should substitute given information into the formula and then solve for the unknown. They can be expected to take the square root of both sides of the equation when solving for the radius of a cone or cylinder, but will not be expected to find the length of a cube given the volume since they will not be familiar with cube roots until they complete the material on roots and powers in this course. Discussions surrounding volume and missing measures, however, may be beneficial.

Note: Once students have completed the Number outcome related with roots and powers, teachers should revisit questions involving volume that would require students to use square and cube roots.

After students are comfortable working with the volume and surface area of prisms, cylinders, pyramids, and cones they are expected to explore the surface area and volume of spheres through an investigation. The investigation can be used to develop formulas for both the surface area and the volume of a sphere.

Students should be given opportunities to discuss real-life examples of spheres in their surroundings. Examples might include a tennis ball, a marble, a basketball, and a globe. Review the terms **radius** and

diameter as well as review the formula for the area of a circle. An orange, for example, approximates a sphere, and the area of its peel represents the surface area of the sphere.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

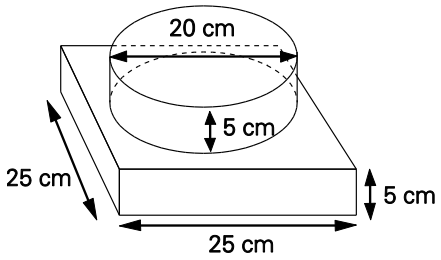
Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Student tasks such as the following could be completed to assist in assessing students' prior knowledge.

- Todd makes a two-layer carrot cake. The base layer is rectangular and the top layer is circular as shown. He puts pineapple slices between the layers instead of icing. He plans to cover the outside of the carrot cake with cream cheese icing. Describe how he can calculate the area that needs icing.



- If you have sheets of cardboard with dimensions of 27 cm × 43 cm, would you have a greater volume if you folded the sheets to make cylindrical containers with a height of 27 cm or with a height of 43 cm? (You plan on adding a circular base once the cardboard sheet is used for the sides.)

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

- How many golf balls will fill a suitcase?

- If you double the height of a right prism, do you double its surface area? Explain.

- A cone-shaped pile of road salt is covered with tarps to keep it dry. The radius of the base of the pile is 10.5 m and the height is 8.2 m.
  (a) Calculate the volume of the salt in the pile to the nearest cubic metre.

(b) Allowing 15% extra for overlap, what is the area of the tarps, to the nearest square metre?

- A toonie ($2 coin) has a 28 mm diameter. The outer ring is made of a nickel alloy. The inner core has a diameter of 16 mm and is made of a copper alloy. The thickness of the coin is 1.8 mm. Calculate the volume of nickel alloy in the coin
  (a) in cubic millimetres
  (b) in cubic centimetres

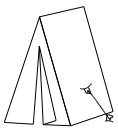
- The surface area of a softball is about 118 cm^2 greater than the surface area of a baseball. The radius of a baseball is 3.7 cm.
  (a) What is the radius of a softball to the nearest tenth of a centimetre?
  (b) How many times as great is the volume of a softball than the volume of a baseball to the nearest tenth?

- A container with a volume of 1000 cm^3 is required to hold a jigsaw puzzle. Find the dimensions for a container with the same volume if the container was a cube, a rectangular prism other than a cube, a cylinder, and, finally, a sphere. Which shape of container would take the least material to make? Why might your answer not be the best shape for the container?

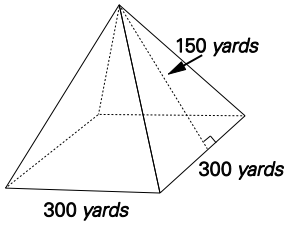
- Susan and her friend Alphonse were given the question, A water tank is in the shape of a right circular cylinder 30 ft. high and 8 ft. in diameter. How many square feet of sheet metal were used in its construction? Susan solved this question using an enclosed tank, while her friend Alphonse solved the question using an open-ended water tank. How much additional metal was required by the closed tank?

- A rectangular prism, measuring 12 cm × 8 cm × 5 cm, has a cube with side length 3 cm removed from its corner. Determine the prism's new surface area.

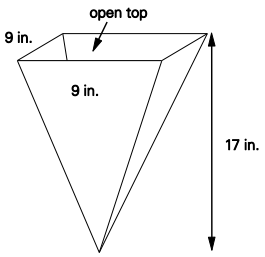
- A tent with a 3 yd. × 3 yd. square base and a height of 7.5 ft. needs a canvas cover. Determine the amount of canvas needed to cover the tent. The floor of the tent is not made of canvas.



- Determine the surface area of the object shown below.



- Alvaro hangs 6 flower pots, shaped as shown to the right, around his house. The flower pots need to be painted. How many square feet would Alvaro need to cover in order to paint the outside of the flower pots?



- Salima has made 10 conical party hats out of cardboard. How much cardboard was used in total if each hat has a radius of 14 cm and a slant height of 25 cm?

- Tyrone works at a local ice cream parlour making waffle cones. If a finished cone is 6 in. high and has a base diameter of 4 in., what is the surface area of the cone?

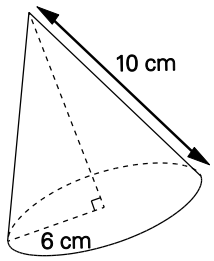
- A right cone has a surface area of 125 in.^{2} and a radius of 4.7 in. What is the slant height of the right cone?

- A cylinder has a surface area of 412 cm^2. The height is three times as great as the radius. Approximate the height of the cylinder.

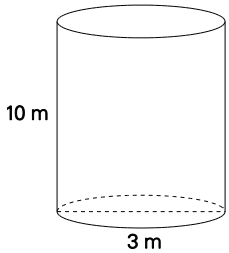
- A right square-based pyramid has a surface area of 154 cm^2. A right cone has a base radius of 3 cm. The cone and pyramid have equal surface areas. What is the height of the cone to the nearest tenth of a centimetre?

- Determine the volume of each of the following containers:

(a)



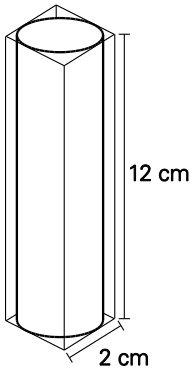
(b)



- A cone and a cylinder have the same height and the same base radius. If the volume of the cylinder is 81 cm^3, what is the volume of the cone in cubic centimetres? Explain.

- Find the volume of a square-based pyramid where the length of the sides of the base as well as the height all measure 2.7 ft.

- A closed cylindrical can is tightly packed inside a box. What is the volume of the empty space between the can and the box?



- A cone has a volume of 30 cm^3 and a base area of 15 cm^2. What is the height of the cone?

- A cylinder has a volume of 132.6 cm^3 and a height of 8.5 cm. What is the diameter of the cylinder?

- A cord of firewood occupies 128 ft.^{3}. Janesta has 3 storage bins for firewood that each measure 2 ft. × 3 ft. × 4 ft. Does she have enough storage space to hold a full cord of firewood? Explain.

- The Department of Transportation and Infrastructure Renewal would like to determine the volume of road salt they have in a stockpile near the Canso Causeway. What are some possible ways to determine the volume of the pile, assuming it is in the shape of a right cone?

- An official basketball has a radius of 12.3 cm and usually has a leather covering. In cm^2, approximately how much leather is required to cover 12 official basketballs?

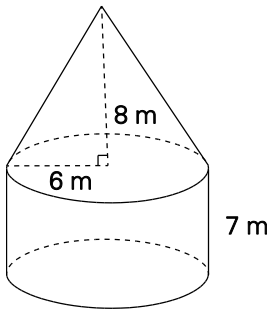
- Eight balls are put in a container. The radius of each ball is 10 cm. If the container is shaped like a square-based pyramid, approximately how much room will be left (volume not occupied by a ball) if each side of the base measures 40 cm and the height is 70 cm?

- A spherical ornament measures 12 cm in circumference. What is the approximate volume of the smallest cubed box that will hold this ornament?

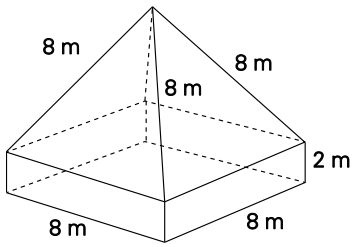
- A heavy sphere with a diameter of 20 cm is dropped into a right circular cylinder with a base radius of 10 cm and a height of 34 cm.
  (a) If the cylinder is half full of water, what is the total volume of the water and the sphere?
  (b) How high will the water rise once the sphere is completely under the water? (Once the sphere is dropped into the water, the water level will rise to a height that represents the volume of the water plus the volume of the sphere.)

- A sphere has a surface area of 80 in.^{2}. Determine the diameter.

- Determine how much paint is needed to paint the outside (all surfaces) of the following composite 3-D object. Usually, you can expect 1 gallon of paint to cover about 32.5 m^2.



- Calculate the volume of the following figure.



FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should be used to inform instruction.

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SUGGESTED LEARNING TASKS

Effective instruction should consist of various strategies.

Guiding Question

- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

Consider the following sample instructional strategies when planning lessons.

- A Word/Formula Wall and models of geometric shapes would be useful in this section.

- A problem-solving approach should be used to investigate surface area and volume. An emphasis should be placed on the development of the formulas rather than giving the students the formula and having them apply it. This can be facilitated by using nets, folding geometric shapes, or Polydrons.

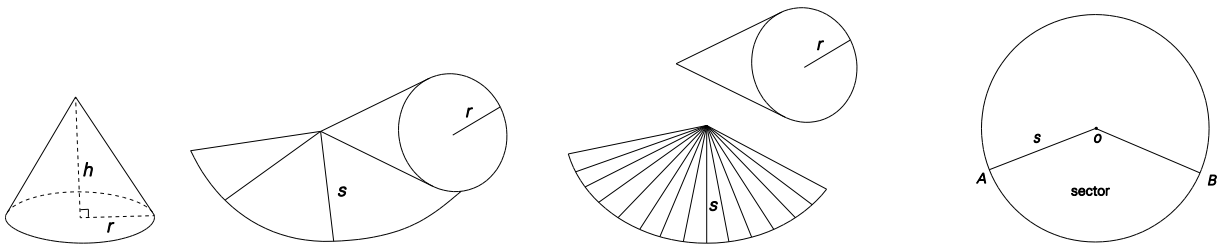
- Investigations to determine the volume of a cone and the volume of a pyramid:
  - Take a plastic prism and pyramid with the same height and base width. (**Note:** If you do not have these, you can use nets copied onto clear acetate, cut out, and taped carefully along the edges leaving a base open.) Using water, sand, or rice, experiment with filling the solids to determine relationships between their volumes. If the plastic solids are small, fill with water for a more precise approximation. Larger models are easier to work with and can be filled with various materials.

- Take a plastic cone and cylinder with the same height and base width. (**Note:** If you do not have these you can use nets copied onto clear acetate, cut out, and taped carefully along the edges leaving a base open.) Using water or rice, experiment with filling the solids to determine relationships among their volumes. If the plastic solids are small, fill with water for a more precise approximation. Larger models are easier to work with and can be filled with either material.

- In groups of two or three, students will explore the various nets of geometric prisms, form a 3-D object, and create a mobile. Ask students to determine the various surface area formulas by examining the individual faces of the net. Encourage students to calculate the surface area of a right pyramid using nets. They should recognize the surface area is the sum of the areas of the base and the four triangular faces. Have students write down the formula representing the area of each face prior to cutting out the net. Students can then form the 3-D object and develop the formula to represent the surface area; this will then become one piece of a mobile. Repeat for each solid. The mobile should include a right cylinder, right triangular prism, a right rectangular prism, a right triangular pyramid, and a right rectangular pyramid. Copyable pages for these nets are available in Appendices A.1–A.8.

- Students should be given an opportunity to apply the surface area of a right pyramid in real-world problems. For example, students can compare the surface area of two right square pyramids with different dimensions to determine which pyramid requires more glass to enclose its space.

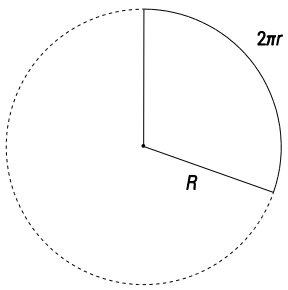
- Investigation for surface area of a cone:
  - Students lay a cone on its side on a flat surface. Rolling it one complete turn will form a sector of a circle. They should notice the area of the curved surface of the cone is equal to the area of the sector.



Length of arc *AB*
= circumference of base circle
= $2\pi r^2$

  - Students should be given an opportunity to discover the following :
    > What part of the cone would be the radius of the sector?
    > What is the relationship between the circumference of the base of the cone and the arc length of the sector?

- After students have had time to think about the relationships of arc length and circumference of the cone, they will be ready to determine the area of the curved surface of the cone. Proportional reasoning can be used to compare the length of the sector of the circle to the area of the complete circle containing that circle as shown below:
  - Let *r* represent the radius of the base circle.
  - Let *R* represent the radius of the circle containing the sector formed when the side of the cone is rolled on paper.
  - The fraction of the larger circle containing the sector could be determined as below.



$$\frac{\text{arc length of sector}}{\text{circumference of circle}} = \frac{2\pi r}{2\pi R} = \frac{r}{R}$$

  - The area of the side of the cone is the area of this sector. This would be a fraction, $\frac{r}{R}$, of the area of the circle with radius *R*. Thus the area of the side of the cone could be determined by

$$\frac{r}{R}(\pi R^2) = \frac{r\pi R^2}{R} = \pi r R$$

In most resources *R* is written as *s* to represent the slant height of the cone, for this reason the surface area of a cone is given by area of base + area of side or $A = \pi r^2 + \pi r s$.

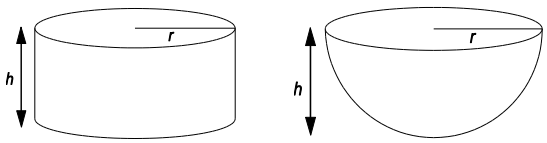
- Investigation for surface area of a sphere:
  - Use an orange to complete the following:
    - Find the greatest distance around an orange (around the centre of the orange) using string. Use this as the circumference of a cross section through the centre of the orange.
    - Using this measurement, find the radius of the sphere. ($C = 2\pi r$)
    - Use a compass to draw 6 circles with that radius.
    - Peel the orange and fill as many circles as completely as you can.
  - Students will discover that the surface area of a sphere is the area of the 4 circles. Therefore, for a sphere: $SA = 4\pi r^2$.

- Question students on how to find the surface area of a cone if the radius and the height are given, but the slant height is unknown. Students should first draw diagrams to help them organize their information and then apply the Pythagorean theorem. Remind students that problems involving multi-step calculations should be rounded only in the last step.

- Students should be given the opportunity to explore the surface area of objects in their everyday environment. Examples may include an ice cream cone, a water fountain cup, a can of soup, a hockey puck, or the packaging of a Toblerone bar.

- When determining surface area, encourage students to keep the context of the problem in mind. For example, to determine the surface area of a straw, the shape of which is a cylinder, the top and bottom would not be included.

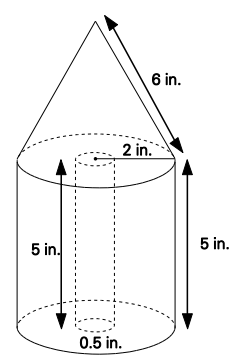
- Investigation for volume of a sphere:
  - When introducing the volume formula of a sphere, the following activity using a cylinder and a hemisphere having the same height and radius, would be beneficial.
    - > Cut a small, air-filled ball in half, creating a hemisphere.
    - > Obtain a cylinder having the same radius and height as the hemisphere.



  - Students will discover that when the hemisphere is fully filled with rice and placed in the cylinder, it will fill $\frac{2}{3}$ of the cylinder.

    The volume of a cylinder is $V = \pi r^2 h$. Therefore, the volume of the hemisphere would be $V = \frac{2}{3}\pi r^2 h = \frac{2}{3}\pi r^2 (r) = \frac{2}{3}\pi r^3$, since the height is also the radius of the hemisphere. The volume of a sphere is twice the volume of a hemisphere which results in $V = 2\left(\frac{2}{3}\pi r^3\right) = \frac{4}{3}\pi r^3$.

    Note: Once students understand the formula, they are better able to apply it in contextual problems. Students need to express the answer with the appropriate units.



- When decomposing a composite object, encourage students to look for component parts such as right cones, right cylinders, right prisms, right pyramids, and spheres. The focus here is for students to recognize that when calculating the surface area of a composite object, the area of overlap must be taken into consideration. The volume is determined by adding and subtracting the volumes independent of the overlap. In the diagram to the right, for example, the volume of the composite object can be determined by subtracting the volume of the inside cylinder from the total volume of the cone and the larger cylinder.

- Students should be given opportunities to explore the surface area and volume of composite objects in their everyday environment.

- Ask students to combine three or more shapes—including prisms, pyramids, and cylinders—to make an interesting sculpture. Use appropriate units, metric or imperial, to label its dimensions and find the total surface area and volume of the sculpture.

Suggested Models and Manipulatives

- nets of various prisms, pyramids, cylinders, and cones
- Polydrons
- Relational Geo-solids
- rice
- various cylinders, prisms, and spherical objects

MATHEMATICAL VOCABULARY

Students need to be comfortable using the following vocabulary.

- apex
- arc length
- cone
- cylinder
- height
- hemisphere
- rectangular-based prism
- regular prism
- sector
- slant height
- sphere
- square-based prism
- triangular-based prism

Resources/Notes

Internet

- Pythagorean theorem (YouTube 2009)
  www.youtube.com/watch?v=CAkMUdeB06o

Print

- *Foundations and Pre-calculus Mathematics 10* (Burglind et al., Pearson 2010)
  - Student Book
    - Chapter 1, Sections 4, 5, 6, and 7, pp. 26–61
  - Teacher Technology DVD
    - Teacher Resource
    - Blackline Masters
    - Smart Lessons
    - Animations
    - Dynamic Activities

Notes

SCO M04 Students will be expected to develop and apply the primary trigonometric ratios (sine, cosine, tangent) to solve problems that involve right triangles. [C, CN, PS, R, T, V]			
[**C**] Communication	[**PS**] Problem Solving	[**CN**] Connections	[**ME**] Mental Mathematics and Estimation
[**T**] Technology	[**V**] Visualization	[**R**] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have met the corresponding specific curriculum outcome.

M04.01 Explain the relationships between similar right triangles and the definitions of the primary trigonometric ratios.
M04.02 Identify the hypotenuse of a right triangle and the opposite and adjacent sides for a given acute angle in the triangle.
M04.03 Solve right triangles, with or without technology.
M04.04 Solve a problem that involves one or more right triangles by applying the primary trigonometric ratios or the Pythagorean theorem.
M04.05 Solve a problem that involves indirect and direct measurement, using the trigonometric ratios, the Pythagorean theorem, and measurement instruments such as a clinometer or metre stick.

Scope and Sequence

Mathematics 9	**Mathematics 10**	**Grade 11 Mathematics Courses**
G02 Students will be expected to demonstrate an understanding of similarity of polygons.	**M04** Students will be expected to develop and apply the primary trigonometric ratios (sine, cosine, tangent) to solve problems that involve right triangles.	**M01** Students will be expected to solve problems that involve the cosine law and the sine law, including the ambiguous case. (M11)* **T02** Students will be expected to solve problems, using the three primary trigonometric ratios for angles from 0° to 360° in standard position. (PC11)** **T03** Students will be expected to solve problems, using the cosine law and sine law, including the ambiguous case. (PC11)**

* M11—Mathematics 11
** PC11—Pre-calculus 11

Background

Trigonometry is the measurement of triangles. It is a branch of mathematics that deals with the relationship between the length of the sides of a triangle and the measure of its angles. An understanding of the Pythagorean theorem and similar triangles is essential to the study of right angle trigonometry. In Mathematics 8, students developed and applied the Pythagorean theorem to find the

length of missing sides of a right triangle (8M01). In Mathematics 9, students developed an understanding of similarity and used these properties to find the length of missing sides given a pair of similar triangles (9G02). In this unit, students will extend their knowledge to solve right triangles using the three primary trigonometric ratios.

Note: It is essential that students develop their understanding of the trigonometric ratios through the investigation of similar triangles. The teacher should not simply present them as formulas to be applied to problems.

In Mathematics 9, students solved equations of the form $a = \frac{b}{c}$ (9PR03).
A review of this skill may be necessary as students will, after developing their understanding of the trigonometric ratios, solve equations of this type.

For example, they will be exposed to equations such as
$\tan 30^\circ = \frac{x}{10}$ or $\tan 30^\circ = \frac{5}{x}$
when finding the length of the opposite side or the length of the adjacent side.

Students are familiar with the term **hypotenuse**. The terms **opposite** and **adjacent** are new. The terms could be discussed in real-world contexts, such as rooms in a hotel. It is common for people staying at a hotel to ask for adjacent rooms or rooms that are across the hall or opposite each other. Students should identify the hypotenuse, opposite, and adjacent sides in right triangles with a variety of sizes, labels, and orientations. Conventions for labelling triangles and angles should be discussed. Greek letters, such as θ, are often used to label acute angles, and lowercase letters corresponding to the vertices are used to label sides.

Students will complete an investigation and compile class results to observe that the ratio of the length of the side opposite and the length of the side adjacent to a given angle is a constant regardless of the size of the triangle. This investigation can be found in the suggested learning tasks. When this has been established, a discussion will have students identify this ratio. It is likely that some will observe that this ratio is the slope of the line or the rate of change of the line. You will tell them that this ratio has been named the “tangent.”

Many real-life applications dealing with measurement depend on the vertical and horizontal distances. Students could first explore the tangent ratio since it can be calculated using these distances. The sine and cosine ratios could then be developed.

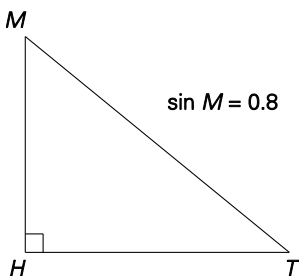
Similar to the tangent ratio, the sine and cosine ratio should be developed through investigation. These ratios can be discovered through paper-and-pencil activities or through the use of technology, such as Geometer’s Sketchpad. Students will specifically compare many similar right triangles to determine the ratio of the following lengths:

- opposite to hypotenuse
- adjacent to hypotenuse

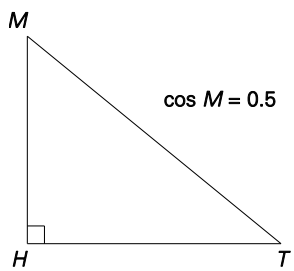
After students have observed that these ratios are constant regardless of the size of the triangles, these ratios can be defined as the sine (sin) and cosine (cos) ratios and are written as

- The sine ratio is defined as $\text{sine } \theta = \dfrac{\text{length of opposite side}}{\text{length of hypotenuse}}$.
- The cosine ratio is defined as $\text{cosine } \theta = \dfrac{\text{length of adjacent side}}{\text{length of hypotenuse}}$.

Unlike the tangent ratio, the sine ratio and cosine ratio both depend on the length of the hypotenuse. Students should also have an understanding of the value of sine and cosine ratios in terms of the lengths of the sides.



The length of *HT* is 0.8 times the length of *MT*.



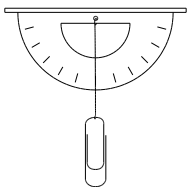
The length of *MH* is 0.5 times the length of *MT.*

This understanding will allow students to assess the reasonableness of their calculations involving the trigonometric ratios.

Emphasis will be on real-world applications using the trigonometric ratios. Students will first work with problems involving one right triangle. Only after they have been introduced to all three primary trigonometric ratios will problem-solving situations involving two or more right triangles be introduced.

The solutions to many problems require the measurement of line segments and angles. When a ruler or tape measure is used to determine the length of a segment, or a protractor is used to find the measure of an angle, we employ direct measurement. In many situations, however, it is inconvenient or impossible to make measurements directly. For example, it is difficult to directly measure the height of a tree or the width of a river. These measurements often need to be found by indirect methods. Starting with some known lengths of segments or angle measures, the tangent ratio can be applied to indirectly find the measurement in question.

A clinometer (a protractor-like device used to measure angles, shown on the right) can be used to gather data for the indirect measurement of the height of an object.

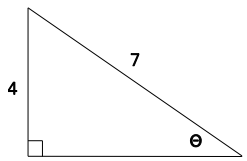


This would be a good opportunity to introduce the term **angle of elevation**. This concept will be revisited later when students are formally exposed to solving problems involving angle of elevation.

Using the clinometer, students can measure the angle between the horizontal and the line of sight to the top of the object. Measuring the horizontal distance from the observer to the object should provide the data needed to allow the objects height to be calculated using trigonometry. This activity is designed to provide students with an opportunity to perform a task and develop a sense of how practical mathematics can be. To link this to the workplace, invite a surveyor to visit the class and describe the requirements of his or her job, show the tools used on the job, and discuss and demonstrate how he or she would perform a similar task.

Students will also use the term **angle of inclination**. This is a new term and should be discussed in the context of problem solving. The angle of inclination is the acute angle between a horizontal line and a line segment. Examples include the inclination of a roof or staircase, the grade or inclination of a road, and the inclination of a ladder or ski lift. Motorists, carpenters, cyclists, and roofers, among others, should have an understanding of this concept.

There are situations where using the sine and cosine ratios rather than the tangent ratio may be considerably less work. Students should be encouraged to determine which trigonometric ratio can be used most efficiently in any given situation. Consider the following:



To use the tangent ratio, students would first have to determine the length of the adjacent side using the Pythagorean theorem and then calculate the missing angle. The sine ratio, however, could be used more efficiently to find the value of θ, as sine depends on the length of the opposite side and the hypotenuse, both of which are given.

To find the missing angle, students must set up the correct trigonometric ratio and use the corresponding trigonometric inverse function.

Students will solve problems involving angle of elevation (also known as angle of inclination) and angle of depression.

There is no explicit connection made between the angle of elevation and the angle of depression. Although the measure of the angle of elevation is equal to the measure of the angle of depression, this relationship will not be covered here as students have not yet been exposed to the parallel line theorem and alternate interior angles. Students could, however, explore this relationship by comparing the measures of the angles in a given problem.

Students will use their knowledge of the trigonometric ratios and the Pythagorean theorem to solve right triangle problems. They will determine the measures of all unknown sides and angles in the triangle. Students should be exposed to the following situations:

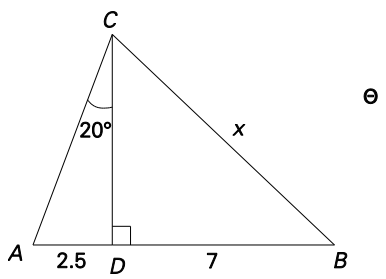
- Given a side and angle of a right triangle, find the remaining sides and angle.
- Given two sides of a right triangle, find the remaining side and the angles.

Students need to be cognizant of the variations in final answers that can arise when calculated values are used in subsequent calculations. Students should not be solving for an angle or a side length, approximating the measurement and then use that approximation in another calculation. This approximation of an angle or side length will lead to less accurate answers. Encourage students to use given information whenever possible.

Students are expected to solve problems involving two or more right triangles using a combination of trigonometric ratios and the Pythagorean theorem. They should be presented with problems that require multi-step solutions. A strategy for solving the problem must be developed before a solution is attempted.

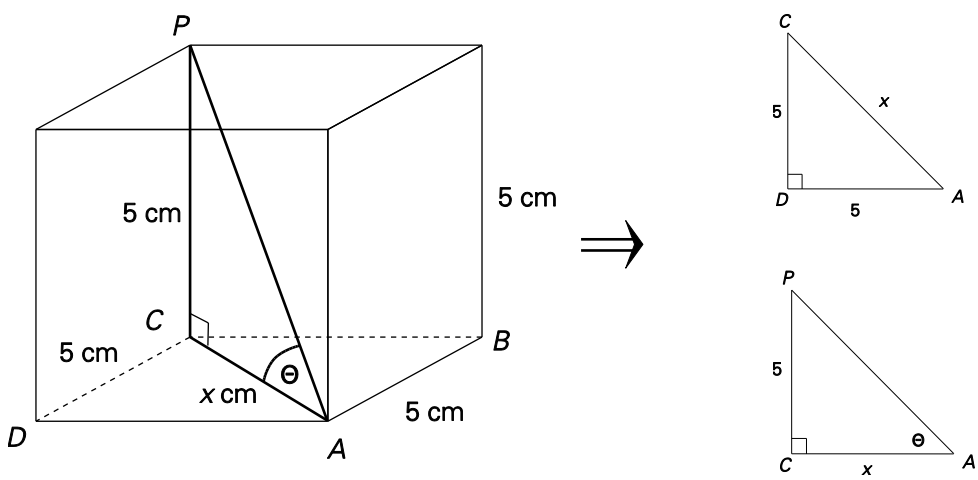
Consider the following:

To calculate the length of CB, more than one triangle will be used. Therefore, students have to decide which triangle to begin with.



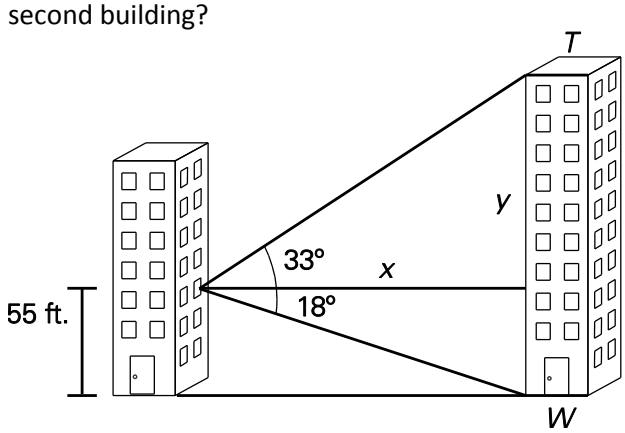
Students should recognize that Δ*ACD* must be used before Δ*BCD*.

When solving problems involving more than one right triangle, students should be exposed to three-dimensional problems. When solving a three-dimensional problem, it is important for students to be able to visualize right triangles within the diagram. Students can then redraw the right triangles in two dimensions and use the appropriate trigonometric ratio and/or apply the Pythagorean theorem to solve.



Using a 2-D diagram, the value of *x* can be determined in Δ*ACD* using the Pythagorean theorem. The tangent ratio can then be applied to find the measure of θ in Δ*APC*.

Students should solve problems with triangles in the same plane or in planes that are perpendicular to each other. For example, two buildings are separated by an alley. Joan is looking out of a window 55 feet above the ground in one building. She estimates that the measurement of the angle of depression to the base of the second building to be 18˚ and the angle of elevation to the top to be 33˚. How tall is the second building?



Students will need to work with the two triangles and use the tangent ratio to find a portion of the height of *TW*.

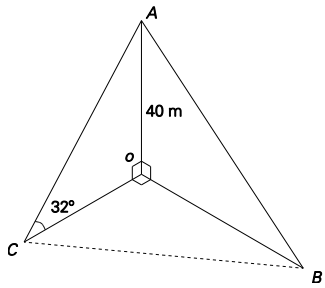
$$\tan 18^\circ = \frac{55}{x}$$

$$\tan 33^\circ = \frac{y}{x}$$

$TW = 55 + y$

Once students solve problems with triangles in the same plane, they can be exposed to using trigonometry to solve problems by considering triangles with a common side in different planes. In the diagram below, sides *AO*, *BO*, and *CO* are mutually perpendicular. Right triangle trigonometry can be used to find the length of *BC*.

For example, given the following diagram, find the length of *BC*.



Note: It is not an outcome of this course to solve problems involving right triangles that are not mutually perpendicular.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

Guiding Questions

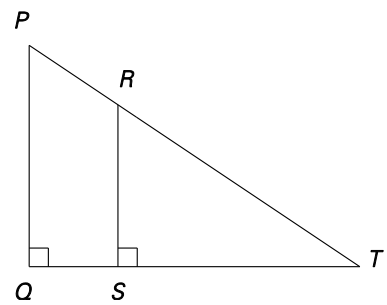
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Assessing Prior Knowledge

Students tasks such as the following could be completed to assist in determining students' prior knowledge.

- Given a set of triangles, sort the triangles and explain the means by which they were sorted. (Some should be similar, some right, some isosceles, some scalene). Discuss the various classifications. Sort triangles that are similar. What are the missing side lengths? How do you calculate these? Use the Pythagorean theorem to establish which of the triangles have right angles. See the following link for a PDF of 24 triangles (Bowles, Noble, and Wade, Inthinking 2013): www.teachmaths-inthinking.co.uk/files/teachmaths/files/Geometry/Similar%20Triangles/Similartriangles.pdf

- Given the diagram shown to the right,
  (a) Which triangles are similar? Why?
  (b) If PQ = 8.2 cm, QS = 5.3 cm, and ST = 7.3 cm, find RS.



Whole-Class/Group/Individual Assessment Tasks

Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

- Discuss the relationship between trigonometry and similar triangles. How is trigonometry developed from similar triangles? What are the advantages of trigonometry over similarity?

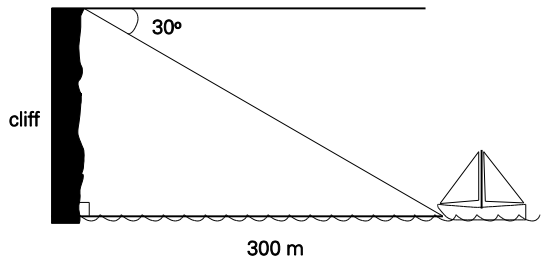
- Verify if the following two formulas are equivalent:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \text{ and } \text{hypotenuse} = \frac{\text{opposite}}{\sin\theta}$$

- Create a contextual problem that could be solved using the following formula:

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

- The angle of depression from the top of a cliff to a sailboat below is 30°. If the sailboat is 300 m from the cliff, what is the height of the cliff?



- Matteo has a summer job with a company that builds antenna towers. He needs to determine the length of cable needed to stabilize a 30 m tower. The cable must make a 65° angle with the ground. Draw a labelled diagram and calculate the length of the cable required.

- In ΔABC, where $\angle C = 90^\circ$, the hypotenuse is 13 cm and the side opposite to angle *A* is 12 cm.
  (a) What is the length of the side adjacent to angle *A*?
  (b) What is the length of the side opposite to angle *B*?
  (c) What is the measure of $\angle BAC$?

- Draw (to scale) two different right triangles where tan θ = 0.25.

- In a right triangle, the tangent of one of the acute angles is 1. Explain how the measures of the two legs are related.

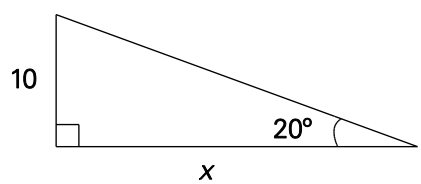
- Given a right triangle with acute angle θ and tan θ = 1.875, find the length of the side opposite angle θ if the side adjacent to angle θ is 2000 m.

- Find and correct the error in the following calculation.

$$\tan 20^\circ = \frac{x}{10}$$

$$10\tan 20^\circ = x\ 1$$

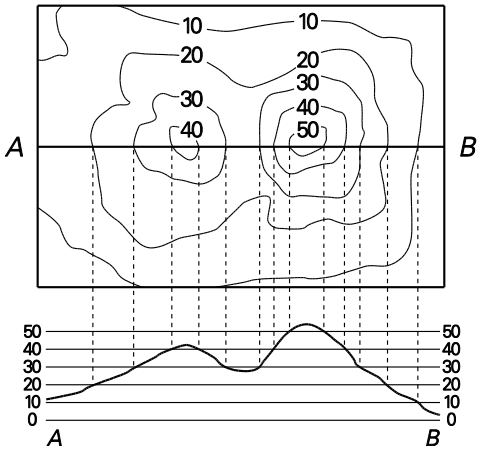
$$3.64 = x$$



- The dimensions of a rectangle are 20 cm × 30 cm, determine the measure of the angle created by the diagonal and the longer side.

- Two ships leave a common point. Ship *A* sails 3.4 km due west and Ship *B* sails 4.5 km due north. Describe how a speed boat leaving Ship *A* could get to Ship *B*'s position using the most direct route.

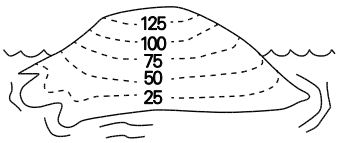
- Topographical maps are often used to describe the contour of land. Trigonometry can be used to determine the relative steepness of a trail or path up a mountain. Hikers generally consider that 15 degrees is a very steep trail and 30 degrees is a steep mountain slope.

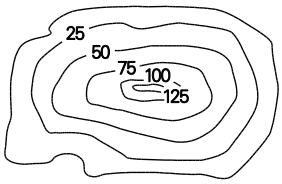


The top of this drawing is a contour map showing the hills that are illustrated at the bottom. On this map, the vertical distance between each contour line is 10 feet.

(a) Which is higher, hill *A* or hill *B*?
(b) Which is steeper, hill *A* or hill *B*?
(c) How many feet of elevation are there between contour lines
(d) How high is hill *A*?
(e) How high is hill *B*?
(f) Are the contour lines closer together on hill *A* or hill *B*?

- Calculate the angle of inclination to point *B* in the topographical map of an island. Note that the elevation change is 125 m and the distance from *A* to *B* is 850 m.





- Discuss the possible values of sine, cosine, and tangent for acute angles. How is tangent different than sine and cosine and how can this difference be explained?

- As an angle size increases from zero degrees to ninety degrees, the value of sine increases but the value of cosine decreases. Explain why this happens.

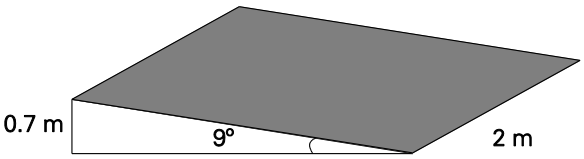
- Evaluate tan 30°, sin 30°, and cos 30°. What do you notice? Evaluate these expressions for other acute angles to confirm your conclusion.

- For homework, Bern was asked to construct any right triangle and to find the measure of the angles. He realizes that he has a ruler and his calculator, but he has left his protractor at school. How can he complete the homework assignment?

- A square is inscribed in a circle with a diameter of 6 cm. Determine the area of the square.

- A regular pentagon is inscribed in a circle with a radius of 5 m. Determine the area of the pentagon.

- Using a rectangular piece of plywood (shaded region), Camille and her friends build a ramp with the dimensions as shown. Approximate the area of the plywood to the nearest tenth of a square metre.

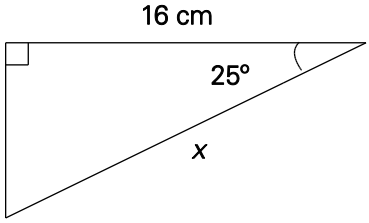


- Identify the error(s) in the following calculations. Determine the correct solution and explain how the person doing this question could have easily discovered that there was an error.
  Solve for x:

$$\sin 25^\circ = \frac{16}{x}$$

$x = 16(\sin 25°)$

$x = 6.8$ cm

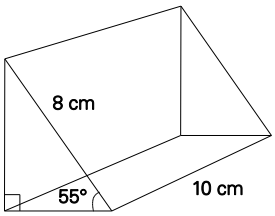


- Tiara argues there is no need for knowing trigonometry—if you want the side lengths of a triangle (or angle size) just measure them. Do you agree or disagree? Explain.

- A 13 m tall farmhouse is located 28.0 m away from a tree. The angle of elevation from the roof of the house to the top of the tree is 30°. What is the height of the tree?

- A pilot starts her takeoff and climbs steadily at an angle of 13°. Determine the horizontal distance the plane has travelled when it has climbed 5.6 mi. along its flight path. Express your answer to the nearest tenth of a mile.

- Calculate the volume and surface area of the right triangular prism shown on the right.

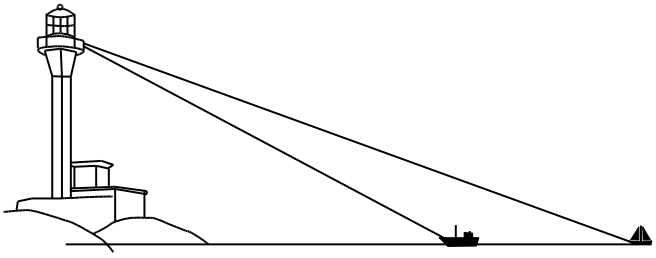


- The angle of depression from an airplane to the beginning of the runway is 28°. If the altitude of the airplane is 3000 m, determine the distance from the airplane to the beginning of the runway.

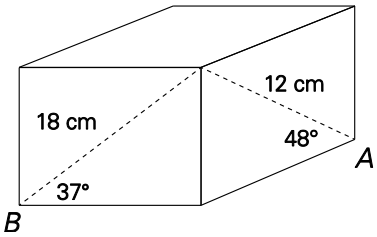
- From a hotel window 300 ft. above street level, Malik observes two buildings, one directly in front of the other. The angle of depression of the closer building is 36° while his angle of depression of the building further away is 20°. Determine the difference in their heights to the nearest metre.

- In ΔDEF, $\angle DEF = 90^\circ$ and $\angle EDF = 34^\circ$. If DE = 12 cm, determine the perimeter of ΔDEF.

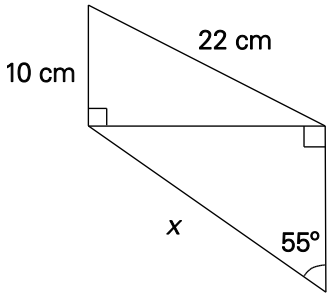
- A tourist at the Cape Forchu Lighthouse looks out and sees two boats in his line of site, a fishing boat at an angle of depression of 23° and a sailboat at an angle of depression of 9°. If the tourist is 33.5 m above the water, determine how far apart the two vessels are.

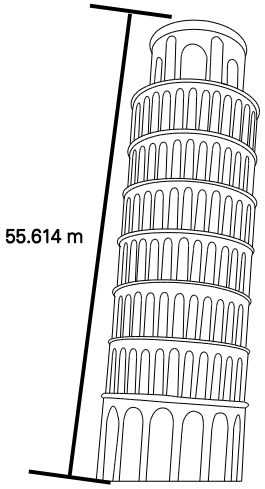


- Determine the shortest distance between A and B in the rectangular prism shown.



- Determine the value of *x* in the following diagram:

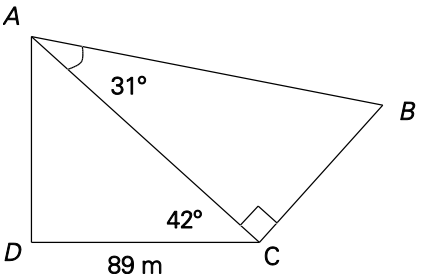




- The Leaning Tower of Pisa was 5.50° from vertical in the 1990s. The Italian government was afraid it would topple over and decided to straighten it a little to stabilize the structure. When work was completed in 2001, the tower leaned 3.99° from vertical. The tower is 55.614 m up the side. How much taller did the tower get?

- A lighthouse is located at the top of a cliff. From a point 150 m offshore, the angle of elevation of the foot of the lighthouse is 25° and the angle of elevation of the top of the lighthouse is 31°. Determine the height of the lighthouse to the nearest tenth of a metre.

- Find the length of *AB* to the nearest tenth of a metre.



- A forest ranger in a fire tower that is 100 m high sees a campsite at an angle of depression of 20°. He then turns 90° and sees a fire at an angle of depression of 12°. How far is the fire from the campsite?

FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should inform instruction.

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SUGGESTED LEARNING TASKS

Effective instruction should consist of various strategies.

Guiding Question

- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

Consider the following sample instructional strategies when planning lessons.

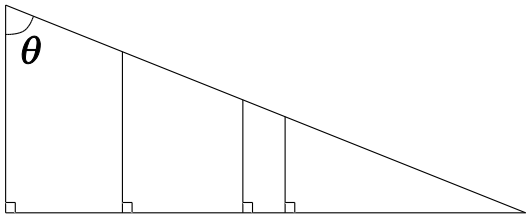
- In order to ensure that students understand the terms **theta**, **hypotenuse**, **adjacent**, and **opposite**, create five large right-angled triangles on the floor with masking tape then divide your class into five groups. As the students stand around the triangles each member of the group receives one of the following words or symbols: opposite, adjacent, hypotenuse, theta, θ (symbol for theta), and a 5 cm × 5 cm square.
  - Have students indicate the right angle with the square and then label the hypotenuse.
  - Have the person holding the symbol for theta place it in one of the other angles of the triangle. The word “theta” is then placed above it as reinforcement for the new terminology.
  - The student holding the opposite sign then goes to the theta sign and walks across the triangle to get to the “opposite” side and places his or her sign accordingly.
  - The adjacent sign is then placed on the side next to theta.
  - Finally, students are instructed to move theta to the other angle in the triangle. At that point the students responsible for opposite and adjacent must be relocated, but hypotenuse remains in place.

- Investigation to introduce students to the tangent ratio:
  - Have each of the students draw a right triangle containing a 60° angle and label the angle by writing 60° in the appropriate location. (You may need to illustrate how to use a protractor prior to having them draw a 60° angle.)
  - Once the triangles are constructed, ask students to exchange triangles and measure the marked angle as well as measure the two legs of the triangle.

– All the data should be collected and complied in the chart below:

Angle Size	Length of Side Opposite Angle	Length of Side Adjacent Angle (not hypotenuse)	Ratio of length of side opposite to length of side adjacent to the angle

– Pass out the copyable page in Appendix A.9 with four similar right triangles and have students measure the size of the angle θ, the length of the side opposite, and the length of the side adjacent to the angle θ for each of the four similar triangles.



– Students should place their measurements in the chart shown below and then calculate the ratio of the length of the side opposite θ to the length of the side adjacent to θ.

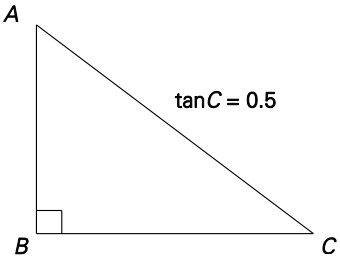
Angle Size	Length of Side Opposite Angle	Length of Side Adjacent Angle (not hypotenuse)	Ratio of Length of Side Opposite to Length of Side Adjacent to the Angle

(**Note:** Have students keep this information as it will be used later to determine the length of the hypotenuse as well as investigate the sine and cosine ratios.)

– Students should note that, while the side lengths are not the same in the four triangles, the ratio of the lengths is the same.

– Before giving this ratio a name (tangent), ask students to think about what this ratio represents.

The tangent ratio is defined as: $\text{tangent } \theta = \dfrac{\text{opposite}}{\text{adjacent}}$



- Students should have an understanding of the tangent ratio in terms of the lengths of the sides. For instance, in the diagram shown here, students should realize that the length of *AB* is half the length of *BC*. This understanding will allow students to assess the reasonableness of their calculations involving trigonometric ratios.

- Engage students in a discussion about the size of an angle when $\tan\theta = 1$. What can be said about the angles of a right triangle when the tangent ratio is larger than one or less than one?

- In any triangle, if measurements of two angles of the triangle are known, the measure of the third angle can be calculated since the sum of the measures of the angles must total 180°. In a right triangle, students will use the tangent ratio to determine the measure of the missing acute angle given only the lengths of the opposite and adjacent side. This will require the use of the inverse of the tangent ratio. Practice with the calculator is essential, and students should be aware of the need to work in degree mode.

- Students previously used the Pythagorean theorem in Mathematics 9 to find the length of the third side of a right triangle when given the lengths of two sides. Through the use of the tangent ratio, the length of a side can now be determined if only one leg length and one acute angle measurement are provided.

- Students should judge the reasonableness of their answers. Consider asking questions such as the ones that follow:
  - Given that the tangent is smaller than 1, should the opposite side be longer or shorter than the adjacent side?
  - Should the measure of the acute angle be greater than or less than 45°?

- Investigation to introduce students to the sine and cosine ratios:
  - Students record the data that they collected in the activity where they investigated the tangent ratio into the chart below and then calculate or measure the length of the hypotenuse and record it in the space provided.

Angle Size	Length of Opposite Side	Length of Adjacent Side	Length of Hypotenuse	Ratio of Lengths of Opposite Side and Hypotenuse	Ratio of Lengths of Adjacent Side and Hypotenuse	Ratio of Lengths of Opposite Side and Adjacent Side.

  - Students now calculate the ratio of the length of the opposite side to the length of the hypotenuse and the ratio of the length of the adjacent side to the length of the hypotenuse.

  - If the data from all students can be entered into an Excel file, the ratios can be automatically calculated, and students will see that their values are all very similar if not identical to those gathered by their classmates. (Discuss why the values may differ.)

  - After noting these constant ratios, students can use technology to calculate the sin θ and cos θ and see how these are the names given to the ratios of the length of the opposite to the length of the hypotenuse and the length of the adjacent to the length of the hypotenuse.

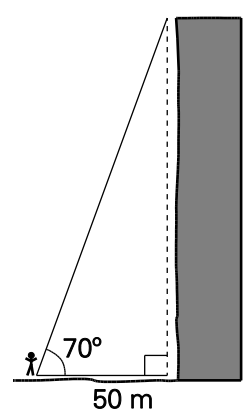
- The investigations of similar triangles leading to the naming of the three primary trigonometric ratios can lead to the discussion of the Trigonometric Table and how it can be used to determine what the value of theta is.

- For right angle triangles, investigate such questions as
  - Are the lengths of the legs always smaller than the length of the hypotenuse?
  - Is the shortest side always opposite the smallest angle?

- Use a selection of questions listed below to help students develop a deeper understanding of trigonometric ratios.
  - The cosine of an angle in a right triangle is close to 1. What are possible values for the two acute angles of the triangle? What might the side lengths be? Explain.
  - The cosine of an angle is greater than its sine. What do you know about the measure of the angle?
  - How are sine and cosine alike? How are they different?
  - What is the greatest possible value of sine? The least? Why?
  - What is the greatest possible value of cosine? The least? Why?
  - What is the greatest possible value of tangent? The least? Why?
  - The cosine of an angle in a right triangle, rounded to the nearest thousandth, is 0.707. What might the dimensions of the triangle be?
  - Amal says that the sine of acute $\angle A$ of a right triangle cannot be greater than the cosine of acute $\angle B$ in the same triangle. Do you agree? Explain.

- Demonstration using Geometer's Sketchpad (Key Curriculum 2013):
  - Use the Geometer's Sketchpad to show that, although we can change the lengths of the sides, the ratios of the side lengths will always remain the same.
  - Discuss how human measurement always contains error, which would explain the discrepancies in the calculations completed earlier. Using a computer program such as Geometer's Sketchpad allows us to complete the same calculations and measurement with a higher degree of accuracy.
  - Explain to students that the ancient Greeks noticed that these ratios did not change and so gave names to each of the ratios: sine, cosine, and tangent. They recorded the ratios for each different angle in tables (show students a trigonometric table). We used these tables until technology was invented that could store the tables and retrieve data for us.

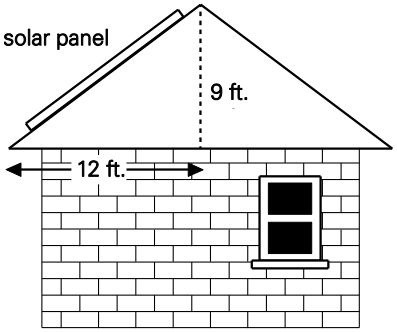
- Pose the following question: How could you use what you know about trigonometric ratios to solve the following problem?
  - Joe is stuck at the bottom of a cliff. How long of a rope will you need to fish him out?
  - Use the following questions to help guide students through the process of solving the problem.
    > Do you see the right triangle?
    > Where are the opposite, adjacent, and hypotenuse?
    > Of these three, which is the side length you know? Which is the side length you are trying to calculate?
    > Which trigonometric ratio uses these sides?



> Write the trigonometric ratio, filling in any of the information you know?
> Can you simplify using the trigonometric table in your calculator?
> Can you then solve the remaining algebraic problem?
> Run through this process once again, creating a set of notes that students can use for reference.

- Have students practise calculating the lengths of missing sides. Pair up students. Have each pair choose two problems to complete and post solutions in the classroom. Have students share their solutions with the class.
  - Follow up the activity with the debriefing questions listed below.
    > How did you determine which trigonometric ratio to use?
    > What is your strategy for finding the length of the missing side?
    > How do you know if you should multiply or divide to determine the solution?
    > How can you use your calculator so that you don't need to round until you have the final solution?

- Pose this question: How could you use what you know about trigonometric ratios to solve the following problem?
  - South-facing solar panels on a roof work best when the angle of inclination of the roof—that is, the angle between the roof and the horizontal line—is approximately equal to the latitude of the house. The latitude of Fort Smith, NWT, is approximately 60°. Determine whether this design for a solar panel is the best for Fort Smith. Justify your answer.
    > *Follow-up:* What would you have to change in the roof design in order to make the solar panel work properly? Justify your answer.

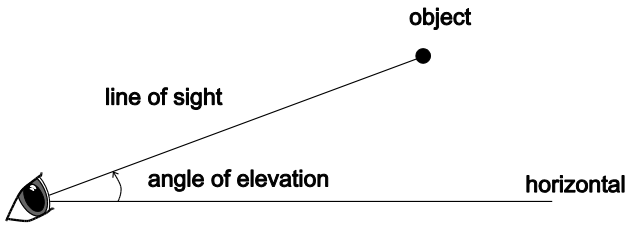


  - Use the following questions to help guide students through the process of solving the problem.
    > Do you see the right triangle?
    > Where are the opposite, adjacent, and hypotenuse?
    > What makes this question different from those we just did?
    > Of the three side lengths, which do you know?
    > Which trigonometric ratio uses these sides?
    > Can you write the trigonometric ratio, filling in any of the information you know?
    > How can you use your calculator to find the missing angle?
  - Run through the process just used once again, creating a set of notes that students can use for reference.

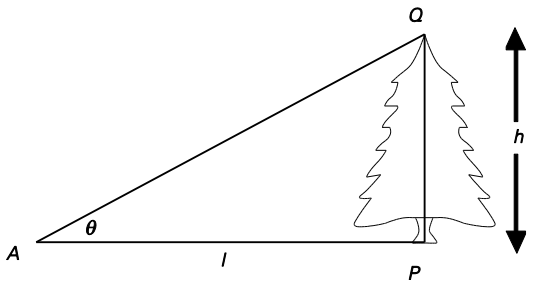
- Have students practise calculating the measures of missing angles. Pair up students. Have each pair choose two problems to complete, and post solutions in the classroom. Ask students to share their solutions with the class.

- Follow up the activity with the debriefing questions listed below.
  - How did you determine which trigonometric ratio to use?
  - What is your strategy for finding the measure of the missing angle?
  - How do you know if your answer is reasonable?

- Students can illustrate their understanding of angle of elevation by suggesting situations in the real world that involves the angle of elevation. Examples include a child in a tree fort looking up at a plane and a person on the ground looking up at a cat in the tree.

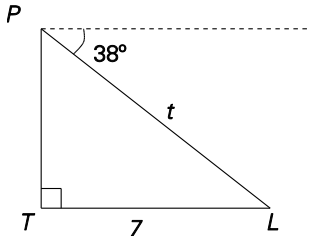


- Solving problems involving the angle of elevation requires transferring information regarding distances and angles to a right triangle. Consider the following:
  - At a distance of 22 ft. from a tree, the angle of elevation of the top of a tree is 35°. Find the height of the tree rounded to the nearest foot. Substituting θ = 35°and l = 22 ft., students would use the tangent ratio to find the height of the tree, *h*.



- A common mistake when working with angle of elevation is incorrectly transferring information to the corresponding right triangle. To help students avoid this, remind learners that angle of elevation is always formed with the horizontal and never with the vertical.

- Problems involving the angle of depression will also be developed when working with one triangle. The angle of depression is the angle between the horizontal and the line of sight to an object beneath the horizontal. The complement of the angle of depression will be used to help find the unknown measure in a right triangle. Students may not have been introduced to the terms **complement** or **complementary** in previous grades. It is, thus, important to define these terms. Consider the following:
  - Students will find the complement of 38° and use the sine ratio to find the unknown length *t*.



- Students may have difficulty deciding where to start when solving right triangles. The order in which unknown measurements are found will depend on the specific problem and each student's personal choice. A few guidelines, however, would be helpful. Students should
  - sketch, label, and place all known information correctly on the triangle
  - use the information to select the correct trigonometric ratio and/or apply the Pythagorean theorem
  - apply the knowledge that the sum of the angles in a triangle totals 180°

- Students should be encouraged to verify their work. If trigonometry was used to find the lengths of the missing sides, the Pythagorean theorem can be used to verify the results. When verifying angle measurements, encourage students to ensure that the sum of the angles totals 180°. Students should also check the reasonableness of their answers by ensuring that the smallest angle, for example, is opposite the shortest side.

- Use angle of elevation and angle of depression to help create contextual problems.

- Students could explore topographical maps of their own area to calculate the average angle of inclination between two points.

- Students could research a real-life application of right triangle trigonometry and then create a poster that illustrates this application.

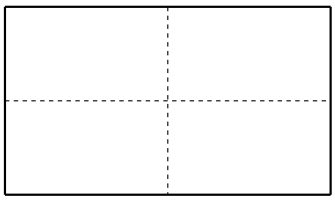
- Set up four stations and have groups of students find the height of various objects, using a clinometer, and record their results using trigonometry. Some objects may include the height of a basketball net, the height of a clock on the wall, the height of a gym wall, and the height of a door.

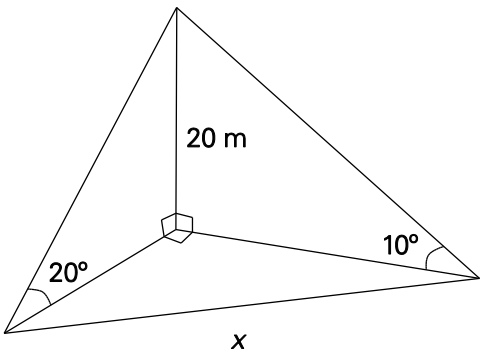
- Use clinometers and have students go outside to find the height of tall buildings or trees using trigonometric ratios.

- Work with students to create diagrams that are labelled correctly from word problems (a skill that many students struggle with).

- Students could make a graffiti wall. Each student draws a right triangle on a sticky note, labels the right angle, and marks one acute angle with a star. They also write a length value for any two sides. Have them post their notes on the wall. Students then choose a sticky note other than their own and determine which ratio can be identified. They then place it in the appropriate section of the board under the sine, cosine, or tangent. Students then choose another sticky note and solve for the indicated angle.

- When working in 3-D, students may find it helpful to create 3-D models. This can be done simply by using a piece of paper, card stock, or filing card.
  - Fold along the dotted lines, as shown. Next, cut along one of the dotted lines from the edge to the center.
  - Fold to create the corner of a box. The lengths and angles can then be placed in the appropriate locations such as shown below.





Suggested Models and Manipulatives

- clinometers (these can be constructed using cardboard, a straw, string, and a weight)
- index cards
- measuring tapes
- protractors
- rulers
- trigonometric tables

Mathematical Vocabulary

Students need to be comfortable using the following vocabulary.

- adjacent side
- angle of depression
- angle of elevation
- angle of inclination
- complement and complementary
- cosine θ; cos θ
- direct measurement
- hypotenuse
- opposite side
- similar triangles
- sine θ; sin θ
- tangent θ; tan θ
- theta (θ)
- topographical map

Resources/Notes

Internet

- Interactive Mathematics, Learn Math by Playing with It. "5. Signs of the Trigonometric Functions." (Bourne 2013)
  www.intmath.com/trigonometric-functions/5-signs-of-trigonometric-functions.php
  Students can refer to this website to explore the effect of changing angle sizes on the tangent function.
- Professional Learning K–12, Newfoundland and Labrador (Professional Learning NL 2013)
  www.k12pl.nl.ca
  The professional learning site provides a classroom clip of students using the clinometer to measure the height of various objects.
- Illuminations, Resources for Teaching Math (National Council of Teachers of Mathematics 2013)
  - *Building Height*
    http://illuminations.nctm.org/LessonDetail.aspx?id=L764
  - *Construction of Clinometer*
    http://illuminations.nctm.org/Lessons/BuildingHeight/BuildingHeight-OH.pdf
- Plaincode, Clinometer on iPhone/iPod Touch (Plaincode 2012)
  http://plaincode.com/products/clinometer
  Application for iPad and iPhone (Clinometer)
- Teachmathematics.net (Bowles, Noble, and Wade, Inthinking 2013)
  www.teachmaths-inthinking.co.uk/files/teachmaths/files/Geometry/Similar%20Triangles/Similartriangles.pdf
  PDF of 24 triangles.

Print

- *Foundations and Pre-calculus Mathematics 10* (Burglind et al., Pearson 2010)
  - Student Book
    - Chapter 2, Sections 1–7, pp. 68–121
  - Teacher Technology DVD
    - Teacher Resource
    - Blackline Masters
    - Smart Lessons
    - Animations
    - Dynamic Activities

Software

- Geometer's Sketchpad (Key Curriculum 2013)

Notes

Algebra and Number
55–60 hours

GCO: Students will be expected to develop algebraic and graphical reasoning and number sense.

Specific Curriculum Outcomes

Process Standards Key

[**C**] Communication	[**PS**] Problem Solving	[**CN**] Connections	[**ME**] Mental Mathematics and Estimation
[**T**] Technology	[**V**] Visualization	[**R**] Reasoning	

AN01 Students will be expected to demonstrate an understanding of factors of whole numbers by determining the prime factors, greatest common factor, least common multiple, square root, and cube root. [C, ME, R]

AN02 Students will be expected to demonstrate an understanding of irrational numbers by representing, identifying, simplifying, and ordering irrational numbers. [CN, ME, R, V]

AN03 Students will be expected to demonstrate an understanding of powers with integral and rational exponents. [C, CN, PS, R]

AN04 Students will be expected to demonstrate an understanding of the multiplication of polynomial expressions (limited to monomials, binomials, and trinomials), concretely, pictorially, and symbolically. [CN, V, R]

AN05 Students will be expected to demonstrate an understanding of common factors and trinomial factoring, concretely, pictorially, and symbolically. [C, CN, V, R]

SCO AN01 Students will be expected to demonstrate an understanding of factors of whole numbers by determining the prime factors, greatest common factor, least common multiple, square root, and cube root. [C, ME, R]			
[**C**] Communication [**T**] Technology	[**PS**] Problem Solving [**V**] Visualization	[**CN**] Connections [**R**] Reasoning	[**ME**] Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have met the corresponding specific curriculum outcome.

AN01.01 Determine the prime factors of a whole number.
AN01.02 Explain why the numbers 0 and 1 have no prime factors.
AN01.03 Determine, using a variety of strategies, the greatest common factor or least common multiple of a set of whole numbers, and explain the process.
AN01.04 Determine, concretely, whether a given whole number is a perfect square, a perfect cube, or neither.
AN01.05 Determine, using a variety of strategies, the square root of a perfect square, and explain the process.
AN01.06 Determine, using a variety of strategies, the cube root of a perfect cube, and explain the process.
AN01.07 Solve problems that involve prime factors, greatest common factors, least common multiples, square roots, or cube roots.

Scope and Sequence

Mathematics 8	**Mathematics 10**	**Grade 11 Mathematics Courses**
N1 Students will be expected to demonstrate an understanding of perfect squares and square roots, concretely, pictorially, and symbolically (limited to whole numbers). **Mathematics 9** **N05** Students will be expected to determine the square root of positive rational numbers that are perfect squares.	**AN01** Students will be expected to demonstrate an understanding of factors of whole numbers by determining the prime factors, greatest common factor, least common multiple, square root, and cube root.	**RF02** Students will be expected to Students will be expected to demonstrate an understanding of the characteristics of quadratic functions, including vertex, intercepts, domain and range, and axis of symmetry. (M11)* **RF01** Students will be expected to factor polynomial expressions of the following form where *a*, *b*, and *c* are rational numbers. ▪ $ax^2 + bx + c$, $a \neq 0$ ▪ $a^2x^2 - b^2y^2$, $a \neq 0$, $b \neq 0$ ▪ $a(f(x))^2 + b(f(x)) + c$, $a \neq 0$ ▪ $a^2(f(x))^2 - b^2(g(y))^2$, $a \neq 0$, $b \neq 0$ (PC11)** **RF05** Students will be expected to solve problems that involve quadratic equations. (PC11)**

* M11—Mathematics 11 ** PC11—Pre-calculus 11

Background

Students in Mathematics 8 investigated square roots of whole numbers up to $\sqrt{144}$, including perfect squares and estimates of non-perfect squares. They would have explored these relationships concretely, pictorially, and symbolically for whole numbers.

In Mathematics 9, the study of square roots was extended to finding the square root of positive rational numbers that are perfect squares—including whole numbers, fractions, and decimals—using perfect squares as benchmarks to help students estimate.

The Mathematics 10 curriculum focuses on the factoring of whole numbers using a variety of strategies. This SCO serves as an introduction to factoring, greatest common factors, and least common multiples. Students will use a variety of strategies, including factoring, to determine the roots of perfect squares and perfect cubes.

Factors are numbers that are multiplied to get a product. For example, the factors of 12 are 1, 2, 3, 4, 6, 12. To factor a number means to write (express) the number as a product of its factors. To factor 12, a student may write 1 × 12, 2 × 6 (6 × 2), 3 × 4 (4 × 3), 2 × 2 × 3. To fully factor a number or to give the prime factorization of a number is to write the number as a product of prime factors. The prime factorization of 12 is 2 x 2 x 3. A review of prime numbers, composite numbers, and prime factorization may be necessary before students are introduced to prime factors.

The numbers 0 and 1 have no prime factors. When 1 is divided by a prime number, the answer is never a whole number; therefore, 1 has no prime factors. Zero is divisible by all prime numbers ($0 \div 2 = 0$, $0 \div 3 = 0$, etc.). This would appear to indicate that zero has an infinite number of prime factors. However, if 2 is a factor of 0, then so is the number zero. Since division by 0 is undefined, 0 has no prime factors.

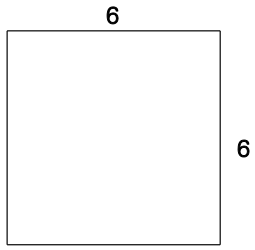
As an example of prime factorization, 24 can be expressed as a product of its prime factors: $24 = 2 \times 2 \times 2 \times 3$, or $24 = 2^3 \times 3$. To avoid confusion with the variable *x*, students should also be comfortable using a dot to indicate multiplication, as in $24 = 2 \bullet 2 \bullet 2 \bullet 3$.

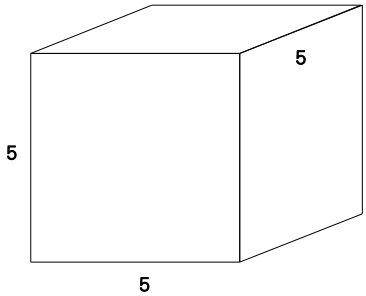
The product of two numbers of equal value is the square of those numbers. If the factors are whole numbers, the product is a perfect square. Conversely, two factors of equal value are the square roots of the square. For example, 25 is the square of 5 expressed symbolically as $5^2 = 25$ and the principle square root of 25 is 5 expressed symbolically as $\sqrt{25} = 5$.

Note: Since ($-5^2 = 25$) then there are actually two solutions for $x^2 = 25$. If $x^2 = 25$, then $x = \pm\sqrt{25} = \pm 5$.

Likewise, the product of three numbers of equal value is the cube of those numbers. If the factors are whole numbers, the product is a perfect cube. Conversely, three factors of equal value are the cube roots of the cube. For example, 27 is the cube of 3 expressed symbolically as $3^3 = 27$, and the cube root of 27 is 3 expressed symbolically as $\sqrt[3]{27} = 3$.

A perfect square can be represented by the area of a square. The length (measure) of each side of the square is the square root of the area. A perfect cube can be represented by the volume of a cube. The length (measure) of each side of the cube is the cube root of the volume.





Area = 36 square units
$\sqrt{36} = 6$
36 is a perfect square
6 is its square root

Volume = 125 cubic units
$\sqrt[3]{125} = 5$
125 is a perfect cube
5 is its cube root

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Student tasks such as the following could be completed to assist in determining students' prior knowledge.

- Is 8 the square root of 16? Why or why not?
- Without using the square root function on a calculator determine the value of $\sqrt{196}$.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

- Determine the first 10 prime numbers and explain your strategy for finding these numbers and for testing that they are prime.

- Draw a factor tree for 10, 60, and 120 to determine the prime factors.
  (For enrichment, have students work with larger numbers and encourage them to express the prime factors in simplified form. For example, the prime factors of 3300 are $2 \bullet 2 \bullet 3 \bullet 5 \bullet 5 \bullet 11$. These could be written as $2^2 \bullet 3 \bullet 5^2 \bullet 11$.)

- Explain the difference between listing the factors of a number and listing the multiples of a number.

- Play the following game.
  - Organize yourselves in pairs. Each of you will draw two playing cards (take out the face cards) to create a two-digit number. Player A determines the prime factors of the number and then adds up the prime factors to determine their score. Player B will continue the same procedure. The first player to reach 50 points wins.

- Play the following game.
  - Divide the class into two groups. Each team will appoint a spokesperson. Using a 10 × 10 grid as shown below, Team A will pick a number. (See Appendix A.10 for a copyable version of this hundred chart.) That number is then circled on the grid and cannot be chosen again by either team. Team A will be awarded the number of points equivalent to the number they picked. Team B will be awarded a point-value equivalent to the sum of all that number's factors, as identified by Team B and excluding the number itself, that have not already been counted. The numbers they are awarded are then circled on the grid and cannot be chosen again by either team. For example, if Team A picked 50, the number 50 would be circled on the grid and Team A would get 50 points and Team B would, ideally, identify the numbers 1, 2, 25, 5, and 10. If they identified all these numbers, none of which had already been selected, Team B would be awarded 1 + 2 + 25 + 5 + 10 or 43 points. In this case, Team A gained 7 points in that move. As the game progresses and more of the numbers have been claimed, strategies become more complex. The team with the most points when all numbers have been selected wins.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

- Express each of the following numbers as a product of its prime factors.
  (a) 12
  (b) 28
  (c) 63

- Determine the greatest common factor (GCF) of each pair of numbers.
  (a) 15, 20
  (b) 16, 24
  (c) 28, 42

- In Tuesday's class, Erin has been practising determining the GCF for two integers. Her friend Anja asks her if she can determine the GCF of each pair of monomials. Erin decides to give it a try. Explain how she might find the GCF for each of the following pairs.
  (a) $4a$, $6a$
  (b) $2x^2$, $3x$
  (c) $12abc$, $3abc$
  (d) $9mn^2$, $8mn$
  (e) $6x^2y^2$, $9xy$

- Write 729 as a product of prime factors. Determine whether it is a perfect square and/or a perfect cube through grouping of the prime factors.
  (**Note:** 729 = 3 • 3 • 3 • 3 • 3 • 3. The prime factors can be grouped as (3 • 3 • 3)(3 • 3 • 3) to give the square of 27, or they can be grouped as (3 • 3)(3 • 3)(3 • 3) to give the cube of 9. The number 729 is both a perfect square and a perfect cube.)

- Pencils come in packages of 10. Erasers come in packages of 12. Jason wants to purchase the smallest number of pencils and erasers so that he will have exactly the same number of pencils as he has erasers. How many packages of pencils and how many packages of erasers should Jason buy?

- One trip around a track is 440 metres. One runner can complete one lap in 8 minutes, and the other runner can complete it in 6 minutes. How long will it take for both runners to arrive at their starting point together if they start at the same time and maintain their respective speeds?

- What is the length of the side of a square plot of land if its area is 1764 m^2?

- If the volume of a cube is 125 cm^3, what is the length of each side?

- An aquarium that is shaped like a cube has a volume of 216 m^3. It has glass on the bottom and four sides, but it has no top. The edges are reinforced with angle iron. What is the area of glass required? What is the length of angle iron required?

- A right rectangular prism measures 9 in. × 8 in. × 24 in. What are the dimensions of a cube with the same volume?

- Determine the cube root of 3375 in a variety of ways. This could include the use of prime factorization, the use of benchmarks, and/or the use of a calculator.

- A cube has a volume of 2744 cm^3. What is the diagonal distance through the cube from one corner to the opposite corner?

- The lowest whole number that is both a perfect square and a perfect cube is the number 1. Determine the next lowest whole number that is both a perfect square and a perfect cube, and explain your strategy.

FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should be used to inform instruction.

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SUGGESTED LEARNING TASKS

Effective instruction should consist of various strategies.

Guiding Question
- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

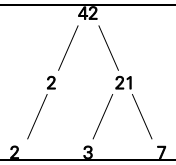
Consider the following sample instructional strategies when planning lessons.

- *Perfect Squares*
  - To review their current knowledge and ensure that students understand the relationship between a squared number and the shape of a square, use geo-boards and elastics or 1 cm graph paper and ask students if they can form a square that contains a given area. For example, ask students if they can draw a square on graph paper or construct a square on the geo-board with an area of 269 cm^2. Make sure to include area examples that are not perfect squares.

- *Perfect Cubes*
  - To review their current knowledge and ensure that students understand the relationship between a cubed number and the shape of a cube, use interlocking cubes and ask them if they can build a cube that contains a given volume. For example, ask students if they can construct a

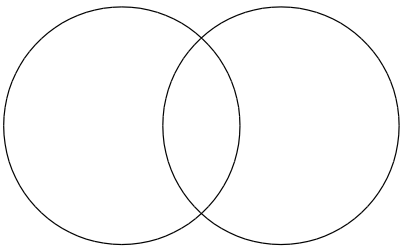
cube with the volume of 12 cm^3. Make sure to include volume examples that are not perfect cubes.

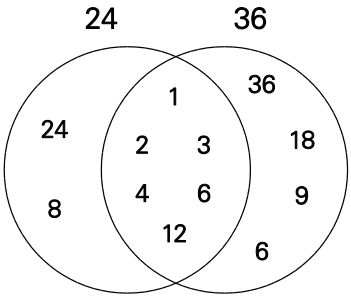
- *Factoring*
  - Students should become familiar with a variety of ways to find the prime factors of a whole number, including factor trees and repeated division by prime factors.
  - Students should be encouraged to use diagrams, manipulatives (such as counters), factor trees, and calculators to solve problems.

Factor Tree	**Manipulatives**
	Group 42 counters or interlocking cubes into 2 groups of 21 and then further group each group of 21 into 3 groups of 7. Since the groups can't be further broken down, the prime factorization would be 2 • 3 • 7.

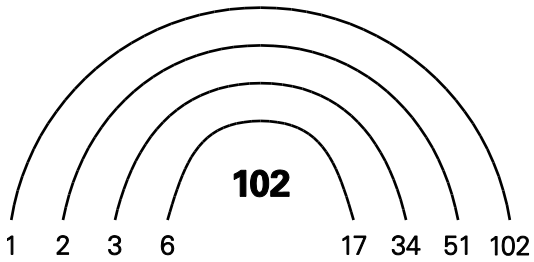
- *Greatest Common Factor (GCF)*
  - Students should explore a variety of ways to find the greatest common factor (GCF) of two or more numbers. For two numbers, one way to determine the GCF is to identify the prime factors common to both numbers, and then to take the product of these factors. For example, for 60 and 24, 60 = 2 • 2 • 3 • 5, and 24 = 2 • 2 • 2 • 3. Multiplying the factors in common gives 2 • 2 • 3 = 12, so 12 is the GCF for 60 and 24.

  - A Venn diagram can be used to have students determine the greatest common factor. For students to use a Venn Diagram, they would
    > write one number they wished to factor above the left-hand circle (e.g., students could write "24" above the circle)
    > write the second number they wished to factor above the right-hand circle (e.g., they could write "36" above the right-hand circle)
    > write all of the factors of 24 inside the circle labelled "24" and write all of the factors for 36 inside the circle labelled "36," placing the common factors of the two numbers inside the intersection of the two circles (In this example, 1, 2, 3, 4, 6, and 12 are common factors.)
    > Conclude that the GCF would be the product of these common numbers (In this case it would be 12.)





  - If students have difficulty factoring a number, you could try using division facts to determine all the factors of each number and record the factors as a rainbow, or as a list of factors.



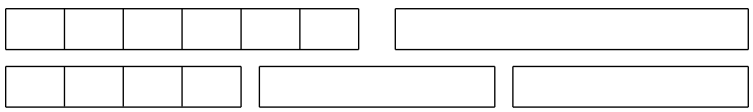
Factor 102
1 × 102
2 × 51
3 × 34
6 × 17

The factors of 102 are 1, 102, 2, 51, 3, 34, 6, and 17.

- *Least Common Multiple (LCM)*
  - Students should also be encouraged to explore a variety of ways to find the smallest multiple shared by two or more numbers (least common multiple or LCM). One method to determine the LCM is to compare the multiples of each number until a common multiple is found. For example, the multiples of 6 are 6, 12, 18, 24, 30, and the multiples of 10 are 10, 20, 30. The first multiple they have in common is 30, so this is the LCM.

  - Once students understand the concept of LCM using models and smaller numbers, have them extend this to finding the LCM of larger numbers using other methods, such as listing numbers and their multiples until the same multiple appears in all lists. For example, for the numbers 18, 20, and 30, the LCM is 180, which is the first multiple found in all three lists:
    Multiples of 18: 18, 36, 54, 72, 90, 108, 126, 144, 162, **180**, ...
    Multiples of 20: 20, 40, 60, 80, 100, 120, 140, 160, **180**, ...
    Multiples of 30: 30, 60, 90, 120, 150, **180**, ...

  - Have students use interlocking unit cubes to create multiples of two or more numbers for which they are determining the LCM. When the chains are of equal length, they will have found the LCM. For example, considering the numbers 6 and 4, two lengths of 6 is the same length as three lengths of 4. Therefore the LCM is 12. This will give students a visual image of the conceptual mechanics at work in determining the LCM.



- *Square and Cube Roots*
  - For large numbers, use prime factorization to determine square roots and cube roots. For example, after students determine the prime factors of the number, have them see if they can make equal groups of square roots or of cube roots as shown below.

    54: $2 \times 27 = 2 \times 3 \times 3 \times 3$

    $\sqrt{54} = \sqrt{2 \times 3 \times 3^2} = \sqrt{3^2}\sqrt{2 \times 3} = 3\sqrt{6}$

    54: $2 \times 27 = 2 \times 3 \times 3 \times 3$

    $\sqrt[3]{54} = \sqrt[3]{2 \times 3^3} = \sqrt[3]{3^3}\sqrt[3]{2} = 3\sqrt[3]{2}$

  - As an extension, have them do this with numbers that have 4th and 5th roots.

SUGGESTED MODELS AND MANIPULATIVES

- colour tiles
- counters
- geo-boards and elastics
- interlocking cubes
- Venn diagrams

MATHEMATICAL VOCABULARY

Students need to be comfortable using the following vocabulary.

- cube root
- greater common factor (GCF)
- least common factor (LCF)
- prime factors
- square root

Resources/Notes

Internet

- Funbrain, "Tic Tac Toe Squares" (Pearson 2013)
  www.funbrain.com/cgi-bin/ttt.cgi?A1=s&A2=17&A3=0
  Square Roots Tic Tac Toe
- Hotmath, "Number Cop." (Hotmath, Inc. 2013)
  http://hotmath.com/hotmath_help/games/numbercop/numbercop_hotmath.swf
  Prime Number Game
- "The Factor Game." (Burgis 2000)
  http://illuminations.nctm.org/tools/factor/index.html
  Factor Game
- Fun4theBrain, "Snowball Fight! - Least Common Multiple (LCM)" (Fun 4 The Brain 2013)
  http://fun4thebrain.com/beyondfacts/lcmsnowball.html
  LCM Game

Print

- *Foundations and Pre-calculus Mathematics 10* (Burglind et al., Pearson 2010)
  - Student Book
    - Chapter 3, Sections 1 and 2, pp. 132–149
    - Chapter 4, Section 1, pp. 204–206
  - Teacher Technology DVD
    - Teacher Resource
    - Blackline Masters
    - Smart Lessons
    - Animations
    - Dynamic Activities

Notes

SCO AN02 Students will be expected to demonstrate an understanding of irrational numbers by representing, identifying, simplifying, and ordering irrational numbers. [C, ME, R, V]			
[**C**] Communication	[**PS**] Problem Solving	[**CN**] Connections	[**ME**] Mental Mathematics and Estimation
[**T**] Technology	[**V**] Visualization	[**R**] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have met the corresponding specific curriculum outcome.

AN02.01 Sort a set of numbers into rational and irrational numbers.
AN02.02 Determine an approximate value of a given irrational number.
AN02.03 Approximate the locations of irrational numbers on a number line, using a variety of strategies, and explain the reasoning.
AN02.04 Order a set of irrational numbers on a number line.
AN02.05 Express a radical as a mixed radical in simplest form (limited to numerical radicands).
AN02.06 Express a mixed radical as an entire radical (limited to numerical radicands).
AN02.07 Explain, using examples, the meaning of the index of a radical.
AN02.08 Represent, using a graphic organizer, the relationship among the subsets of the real numbers (natural, whole, integer, rational, irrational).

Scope and Sequence

Mathematics 9	**Mathematics 10**	**Grade 11 Mathematics Courses**
N03 Students will be expected to demonstrate an understanding of rational numbers by comparing and ordering rational numbers and solving problems that involve arithmetic operations on rational numbers. **N05** Students will be expected to determine the square root of positive rational numbers that are perfect squares. **N06** Students will be expected to determine an approximate square root of positive rational numbers that are non-perfect squares.	**AN02** Students will be expected to demonstrate an understanding of irrational numbers by representing, identifying, simplifying, and ordering irrational numbers.	**RF02** Students will be expected to demonstrate an understanding of the characteristics of quadratic functions including vertex, intercepts, domain and range, and axis of symmetry. (M11)* **AN02** Students will be expected to solve problems that involve operations on radicals and radical expressions with numerical and variable radicands. (PC11)** **AN03** Students will be expected to solve problems that involve radical equations (limited to square roots). (PC11)**

* M11—Mathematics 11
** PC11—Pre-calculus 11

Background

Students had experience calculating ratios and working with integers, decimals, and fractions in middle school. In Mathematics 9, students were introduced to operations with negative fractions and the concept of rational numbers. Rational numbers are numbers that can be written as fractions, ratios, or repeating or terminating decimals.

In Mathematics 10, students will be introduced to the concept of irrational numbers, or numbers that cannot be written as a fraction. When expressed as a decimal, these numbers do not repeat or terminate. Students will determine the relationship between irrational numbers and natural, whole, integer, rational, and real numbers. They will then use a variety of strategies (excluding the use of a calculator) to estimate the values of irrational numbers and to locate them on a number line.

Natural (N), Whole (W), and Integer (Z or, less commonly, I) number sets are discrete and do not include fractions or decimals that cannot be reduced to obtain a Natural, Whole, or Integer.

- Natural numbers (1, 2, 3, 4, 5, ...)
- Whole numbers (0, 1, 2, 3, 4, ...)
- Integer numbers (... –3, –2, –1, 0, 1, 2, 3, ...)

The Rational (Q) number set includes all numbers that can be written as the quotient of two integers. This includes all numbers in the Natural, Whole, and Integer sets (see above), as well as numbers that can be written with repeating or terminating decimals.

Examples of rational numbers include

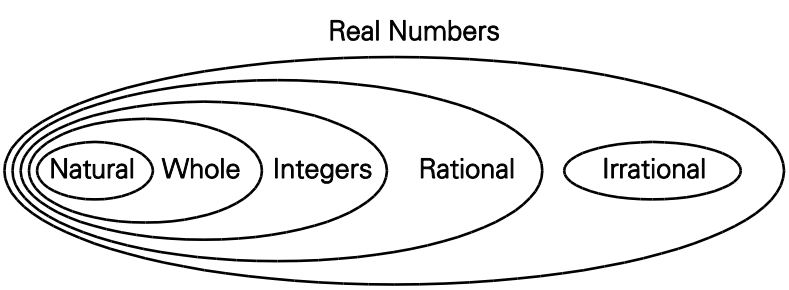
$$\sqrt[3]{27} = 3, \quad \frac{15}{3}, \quad \frac{-3}{1}, \quad \frac{1}{6} = 0.1\dot{6}, \quad 1\frac{1}{4} = 1.25, \quad \frac{2}{7} = 0.285714285714$$

The Irrational ($\overline{Q}$) number set is made up of numbers with non-repeating and non-terminating decimals.

Examples of irrational numbers include

$\pi = 3.1415926535897$ ...
$\sqrt{2} = 1.141421356237$ ...
the Golden Ratio = 1.16180339887 ...
$\sqrt{99} = 9.949874371066$ ...

The Real (R) number set includes all rational numbers and all irrational numbers.



When written as a radical, irrational numbers can be identified as those for which the radicand is not a perfect square, a perfect cube, or a perfect multiple with reference to the index. For example,

Irrational	Rational
$\sqrt{60} = \sqrt{2 \times 2 \times 3 \times 5} = 2\sqrt{15}$	$\sqrt{121} = \sqrt{11 \times 11} = 11$
$\sqrt{10} = \sqrt{2 \times 5}$	$\sqrt[3]{8} = \sqrt[3]{2 \times 2 \times 2} = 2$
$\sqrt[3]{9} = \sqrt[3]{3 \times 3}$	$\sqrt[4]{81} = \sqrt{3 \times 3 \times 3 \times 3} = 3$

For the radical $\sqrt[a]{b}$, *b* is the radicand and *a* is the index.

Entire radicals have a numerical coefficient of one. For example, $\sqrt[3]{16}$, $\sqrt{200}$.

Mixed radicals have a numerical coefficient other than one. For example, $4\sqrt{3}$, $3\sqrt[3]{2}$.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

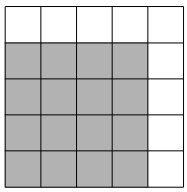
Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Student tasks such as the following could be completed to assist in determining students' prior knowledge.

- Order the following numbers on a number line: 35, $-0.66\overline{6}$, 0.5, –5, 8.
- Determine a rational number between 4.6 and 4.7 and another between 3.08 and 3.09.
- Mia has $\frac{2}{3}$ of her birthday cake left. If she cuts it into 4 equal-sized pieces, what fraction of the cake would each piece represent?
- Between what two whole numbers does $\sqrt{15}$ fall?
- Identify the errors that were made in each of the following situations:
  $\sqrt{16} = 8$ and $\sqrt{0.036} = 0.6$
- Determine what number and its square root can be represented by this grid if the whole square represents 1.

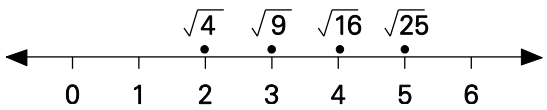


WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

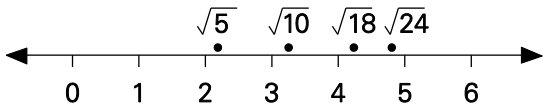
Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

- Lead students through a series of number line activities that develop their understanding of perfect squares, non-perfect squares, and entire and mixed radicals. This will help students see the relation between numerical values and radical expressions.

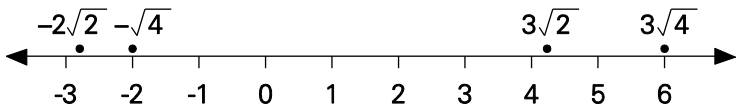
  Step 1

  Step 2

  Step 3

- Place a variety of rational and irrational numbers in three envelopes, ensuring that each envelope contains a variety of both rational and irrational numbers. Have students then place these values in order on a number line. The difficulty level should increase with each successive envelope.

  1st set: $\sqrt{25}$, -3, -1.5, $\frac{1}{4}$, $\sqrt{4}$, $-1\frac{1}{3}$

  2nd set: π, $\sqrt{\frac{1}{9}}$, $-\sqrt{25}$, 1.321698345 ..., $\frac{4}{7}$, $-\frac{3}{8}$, $-\sqrt{10}$

  3rd set: $\sqrt{18}$, $3\sqrt{2}$, $-1\frac{1}{7}$, $\sqrt[4]{81}$, -2.876143792 ...

- Create a Jeopardy game that challenges students to deal with all number subsets—natural, whole, integers, rational, real, and irrational.

- Explain the meaning of each of the following:

  (a) $\sqrt[3]{8}$ (b) $\sqrt[4]{81}$ (c) $\sqrt[5]{32}$

- Sungjoo has missed class and does not know how to determine an approximate value for an irrational number. Explain to Sungjoo, in your own words, how to do this.

- Dean was given a list of different radicals. He wants to determine if they can be represented by a rational or irrational number and then determine the approximate location of each on a number line. Describe a method he can use and justify your reasoning.

- Set up a clothesline across the whiteboard to represent a number line. As a class, establish several benchmarks. Each student will be given a card with an expression of an irrational value. He or she will pin the card along the number line, and should then be able to explain why he or she placed the card in that position.

- Neela simplified $\sqrt{200}$ to $2\sqrt{50}$, thinking that was the simplest radical form. Is Neela correct? Explain.

  Note: Guide students to

  $\sqrt{200} = \sqrt{4 \times 50} = 2\sqrt{50} = 2\sqrt{25 \times 2} = 2 \times 5\sqrt{2} = 10\sqrt{2}$

  or

  $\sqrt{100 \times 2} = \sqrt{10^2} \times \sqrt{2} = 10\sqrt{2}$

- Students could play the following games:
  - Students work in pairs. Give students a set of cards containing pairs that display equivalent mixed radicals and entire radicals. All cards should be placed face down on the table. The first student turns over two of the cards, looking for cards with numbers of equal value. If he or she gets a pair, he or she removes the cards and goes again. If the overturned cards do not form a pair, they are turned over and it is the other player's turn. The player with the most matches at the end of the game wins.

  - Students could work in groups of three or four. Each group will be given a set of cards. Each card will have a different mixed radical. The group will then work together to sort the cards from largest to smallest. The first group with the cards sorted in the correct order wins the competition.

FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should be used to inform instruction.

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SUGGESTED LEARNING TASKS

Effective instruction should consist of various strategies.

Guiding Question

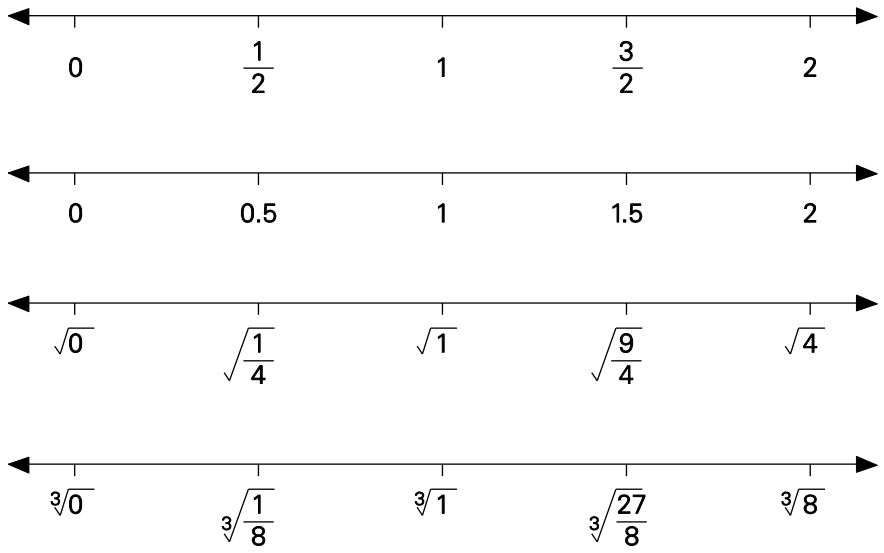
- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

Consider the following sample instructional strategies when planning lessons.

- Ask students to place all types of rational numbers (fractions, integers, decimals, whole numbers, and radicals with a perfect square) on a number line.

- Have students sort a mixture of rational and irrational numbers into their respective categories. Encourage students to convert decimals to fractions and vice versa in order to complete this task.

- Double, triple, or quadruple number lines can be very helpful in terms of equating various representations of numbers. See the following for an example.



- Have students place irrational numbers on a number line using benchmarks. For example, students should be able to place $\sqrt{9}$ easily, and $\sqrt{10}$ can be placed nearby.

- As a check of understanding, provide a list of irrational numbers already placed on a number line with some placed incorrectly. Ask students to find the errors and have them explain how they came to that conclusion. A good understanding of radicals will be indicated when students demonstrate the ability to switch back and forth between mixed radicals and entire radicals. Different methods can be used. For example,
  Mixed to Entire

  $4\sqrt{3} = \sqrt{4 \times 4 \times 3} = \sqrt{48}$

  $3\sqrt[3]{2} = \sqrt[3]{3 \bullet 3 \bullet 3} \bullet \sqrt[3]{2} = \sqrt[3]{27} \bullet \sqrt[3]{2} = \sqrt[3]{27 \bullet 2} = \sqrt[3]{54}$

  Entire to Mixed

  $\sqrt[3]{16} = \sqrt[3]{8 \bullet 2} = 2\sqrt[3]{2}$

  $\sqrt{200} = \sqrt{2 \bullet 2 \bullet 2 \bullet 5 \bullet 5} = \sqrt{(2 \bullet 2) \bullet 2 \bullet (5 \bullet 5)} = 2 \bullet 5\sqrt{2} = 10\sqrt{2}$

Suggested Models and Manipulatives

- number lines
- Venn diagrams

Mathematical Vocabulary

Students need to be comfortable using the following vocabulary.

- coefficient
- entire radical
- index
- irrational numbers
- mixed radical
- radicand
- rational numbers
- real numbers
- simplest radical form

Resources/Notes

Internet

- Pre-Algebra Review Topic: Practice with Rational and Irrational Numbers (Schultzkie 2012)
  www.regentsprep.org/Regents/math/ALGEBRA/AOP1/Prat.htm
  Multiple choice: Rational vs. Irrational Number.
- "Number System Muncher." June Reed (Reed 2002)
  http://staff.argyll.epsb.ca/jreed/math9/strand1/munchers.htm
  Game to play that considers various number systems.

Print

- *Foundations and Pre-calculus Mathematics 10* (Burglind et al., Pearson 2010)
  - Student Book
    - Chapter 4, Sections 1, 2, and 3, pp. 202–221
  - Teacher Technology DVD
    - Teacher Resource
    - Blackline Masters
    - Smart Lessons
    - Animations
    - Dynamic Activities

Notes

SCO AN03 Students will be expected to demonstrate an understanding of powers with integral and rational exponents. [C, CN, PS, R]			
[**C**] Communication	[**PS**] Problem Solving	[**CN**] Connections	[**ME**] Mental Mathematics and Estimation
[**T**] Technology	[**V**] Visualization	[**R**] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have met the corresponding specific curriculum outcome.

AN03.01 Explain, using patterns, why $a^{-n} = \frac{1}{a^n}$, $a \neq 0$.

AN03.02 Explain, using patterns, why $a^{\frac{1}{n}} = \sqrt[n]{a}$, $n > 0$.

AN03.03 Apply the following exponent laws to expressions with rational and variable bases and integral and rational exponents, and explain the reasoning.

- $(a^m)(a^n) = a^{m+n}$
- $a^m \div a^n = a^{m-n}$, $a \neq 0$.
- $(am)^n = a^{mn}$
- $(ab)^m = a^m b^m$
- $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$, $b \neq 0$

AN03.04 Express powers with rational exponents as radicals and vice versa, when *m* and *n* are natural numbers, and *x* is a rational number.

$$x^{\frac{m}{n}} = \left(x^{\frac{1}{n}}\right)^m = \sqrt[n]{x}^{\,m} \quad \text{and} \quad x^{\frac{m}{n}} = x^{m\,\frac{1}{n}} = \sqrt[n]{x^m}$$

AN03.05 Solve a problem that involves exponent laws or radicals.

AN03.06 Identify and correct errors in a simplification of an expression that involves powers.

Scope and Sequence

Mathematics 9	Mathematics 10	Pre-calculus 11
N01 Students will be expected to demonstrate an understanding of powers with integral bases (excluding base 0) and whole number exponents by ▪ representing repeated multiplication using powers ▪ using patterns to show that a power with an exponent of zero is equal to one ▪ solving problems involving powers **N02** Students will be expected to demonstrate an understanding of operations on powers with integral bases (excluding base 0) and whole number exponents.	**AN03** Students will be expected to demonstrate an understanding of powers with integral and rational exponents.	**AN02** Students will be expected to solve problems that involve operations on radicals and radical expressions with numerical and variable radicands. **AN03** Students will be expected to solve problems that involve radical equations (limited to square roots).

Background

In Mathematics 9, students investigated the concepts of exponents, bases, and powers. They also developed an understanding of powers with integral bases and whole number exponents. Students explored the generalized exponent laws using numerical components, emphasizing order of operations. Common misunderstandings were addressed, such as $6^5 + 6^2 \neq 6^7$, $(2^3)^2 \neq 2^5$, $(5^3 \times 5^4) \neq 5^{12}$.

For consistency and understanding, teachers are reminded to continue the practice established in Mathematics 9 of using “six to the exponent of four,” or “six to the fourth,” in lieu of “six to the power of four.”

It will likely be necessary to review the five exponent laws (9N02) before starting this outcome. Expose students to strings of questions that demonstrate the pattern behind the rule, as described in the Suggested Learning Tasks that follow.

In Mathematics 10, students will explore the meaning of integral exponents (such as 6^{-3}) and rational exponents (such as $8^{\frac{2}{3}}$), extending an exploration of patterns to explain negative and fraction bases and exponents (such as $(-8)^{\frac{1}{3}}$, $\left(\frac{1}{4}\right)^{-3}$).

Rational exponents should be restricted to those for which the numerator and denominator are Natural numbers ($x^{\frac{m}{n}}$, $m, n \in N$ (1, 2, 3, 4, 5, ...)). Relate rational exponents to radicals and vice versa. For example, $4^{\frac{1}{2}} = \sqrt{4}$, $27^{\frac{1}{3}} = \sqrt[3]{27}$, $3^{\frac{4}{2}} = (3^4)^{\frac{1}{2}} = \sqrt[2]{3^4} = \sqrt{3^4}$.
.

Students will establish a clear link between numerical bases and exponents (such as 2^4, 8^2) and literal bases and exponents (such as x^4, 8^n) in order to develop and then apply the exponent laws to literal bases and exponents.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Student tasks such as the following could be completed to assist in assessing students' prior knowledge.

- Explain why $(-2)^4 \neq -2^4$.
- Using a pattern, explain why $3^0 = 1$.
- Without using a calculator, determine the value of each of the following: 3^4, $(-2)^4$, 5^0, $2^3 \bullet 2^5$, $3^4 \bullet 9$.
- Morgan's calculator is broken and she wants to evaluate $\frac{16\,384}{32}$.

  Using the table of powers of 2, explain how Morgan can evaluate $\frac{16\,384}{32}$.

2^1	2
2^2	4
2^3	8
2^4	16
2^5	32
2^6	64
2^7	128

2^8	256
2^9	512
2^{10}	1024
2^{11}	2048
2^{12}	4096
2^{13}	8192
2^{14}	16384

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

- Describe the pattern and find the next two numbers in the sequence 1, 4, 27, 256, ...

- A science experiment shows that the numbers of bacteria in a Petri dish will double every hour. If there are 1000 bacteria after 8 hours, how many will there be after
  (a) 9 h (b) 11 h (c) 14 h

- Identify and explain the errors in the following:
  - First set (whole number exponents)
    (a) $4^3 + 4^2 = 4^5$
    (b) $\frac{x^6}{x^3} = x^2$
    (c) $(10^2)^5 = 10^7$
    (d) $\left(\frac{1}{4}\right)^2 = \frac{1}{8}$
    (e) $(x - y)^3 = 3x - 3y$
    (f) $3^5 \times 3^2 = 3^{10}$
    (g) $5^3 \div 5^4 = \frac{3}{4}$
  - Second set (integral exponents)
    (a) $a^4 \bullet a^{-2} = a^{-8}$
    (b) $b^{-10} \div b^5 = b^{-5}$
    (c) $(c^{-3})^2 = c^{-1}$
    (d) $\left(\frac{2}{3}\right)^{-2} = \frac{-4}{-6}$
  - Third set (rational exponents)
    (a) $2^{\frac{1}{2}} \bullet 2^{\frac{1}{2}} = 2^{\frac{1}{4}}$
    (b $3^{\frac{3}{4}} \div 3^{\frac{1}{4}} = 3^3$
    (c) $\left(4^{\frac{2}{5}}\right)^2 = 4^{2\frac{2}{5}}$
    (d) $\left(5^{\frac{1}{2}}\right)^2 = 5^{2\frac{1}{2}}$

    Note: First to third represents a progression of difficulty from easier to harder.

- Fill in the blanks to make a true statment.
  (a) $5^{-2} = \frac{\square}{\square}$
  (b) $6^{\square} = \frac{1}{6^2}$
  (c) $\square^{-6} = \frac{1}{10^6}$
  (d) $4^{-x} = \frac{1}{\square}$

  Note: Answer boxes should be placed in various positions to stretch understanding.

- Use the table of values to evaluate the following:

  (a) $\frac{1}{256} \times 16\,384$

  (b) $8192 \times \frac{1}{64}$

  (c) $\frac{1}{64} \times \frac{1}{32}$

  (d) $32 \div \frac{1}{8}$

  (e) $\dfrac{16\,384 \times \frac{1}{64}}{\frac{1}{8} \times 64}$

$2^{-10} = \frac{1}{1024}$	$2^{-5} = \frac{1}{32}$	$2^0 = 1$	$2^5 = 32$	$2^{10} = 1024$
$2^{-9} = \frac{1}{512}$	$2^{-4} = \frac{1}{16}$	$2^1 = 2$	$2^6 = 64$	$2^{11} = 2048$
$2^{-8} = \frac{1}{256}$	$2^{-3} = \frac{1}{8}$	$2^2 = 4$	$2^7 = 128$	$2^{12} = 4096$
$2^{-7} = \frac{1}{128}$	$2^{-2} = \frac{1}{4}$	$2^3 = 8$	$2^8 = 256$	$2^{13} = 8192$
$2^{-6} = \frac{1}{64}$	$2^{-1} = \frac{1}{2}$	$2^4 = 16$	$2^9 = 512$	$2^{14} = 16\,384$

Evaluate the following expressions by substituting the values given

(a) $5x^4 + 6xy$ if $x = 2$, $y = 3$

(b) $(2x)^2$ if $x = 4$

(c) $(t + s)^{-3}$ if $t = 2$, $s = 4$

- Indicate whether the following statements are always true, sometimes true, or never true. Justify your answer.
  - The value of a power with a negative exponent is less than 0.
  - The value of a power for which the base is a fraction is less than 1.
  - Two powers with the exponent 0 have the same value.

- During an exam, three students evaluate $2^{-2} \times 2^0$ as follows:

  Hana: $2^{-2} \times 2^0 = 4^{-2} = \frac{1}{4^2} = \frac{1}{16}$

  Zuri: $2^{-2} \times 2^0 = 2^0 = 1$

  Michel: $2^{-2} \times 2^0 = 4^0 = 1$

  - Identify the errors made by the students.
  - Determine the correct answer, and then justify your answer by explaining each step.

- In pairs, explain how to evaluate powers such as $(-3)^{-2}$ and -3^{-2}. Compare your answers with other groups, and then compare as a class.

- A cube-shaped storage container has a volume of 24.453 m^3. What are the dimensions of the container to the nearest tenth?

- The area (A) of a face of a cube is given by $A = V^{\frac{2}{3}}$ where V represents the volume of the cube. If $V = 64\ cm^3$, determine the value of A.

- The value of a car (V) is given by the equation, $V = 32\,000(0.85)^{\frac{T}{2}}$ where T represents the age of the vehicle. Estimate the value of the car after 5 years.

- Is $-32^{\frac{2}{5}}$ equal to $(-32^{\frac{2}{5}})$? Explain your reasoning.

- Create a board game that involves the use of exponent laws.

- Create a classroom poster or a paper foldable that outlines the exponent laws discussed in this unit.

- Create a unique bingo card for Exponent Bingo. Distribute a blank bingo card to each student. Teachers should predetermine various expressions involving exponent laws that they would like students to simplify. These expressions should be placed in a bag, with the simplified expressions written on the board. Ask students to write one of the simplified expressions in each square. The centre square should remain a "free" space. The teacher pulls an expression from a bag. Students then simplify the expression, find its value on their card, and cross it off. The first person with a straight line or four corners wins, or the first person with an X or a T on the bingo card could win.

FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should be used to inform instruction.

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SUGGESTED LEARNING TASKS

Effective instruction should consist of various strategies.

Guiding Question
- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

Consider the following sample instructional strategies when planning lessons.

- Have students develop the laws of exponents using an activity with counters. One counter represents the initial amount before doubling; therefore, it is zero times doubled or zero times tripled.

Number of counters after times doubled	0	1	2	3	4	5	…	Number of counters after times tripled	0	1	2	3	4	5	…
Number of counters	1	2	4					Number of counters	1	3	9				
Written as 2^n	2^0	2^1	2^2					Written as 3^n	3^0	3^1	3^2				

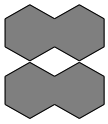
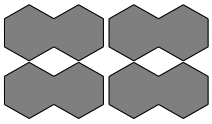
This can be extended to multiplying by some other factor.

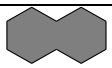
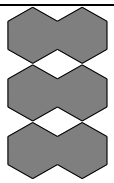
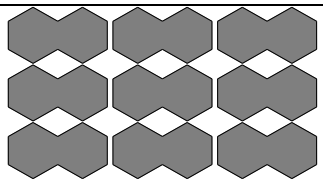
- Have students use fraction blocks and/or fraction strips to develop the result when the exponent is a negative integer.

  Note that students have to understand that the pattern block  represents 1.

  In the following chart, one double hexagon pattern block represents the initial amount before doubling, therefore it is zero times doubled or zero times tripled.

  Caution: For the representations that follow, it might be best to start with a chart that has only positive numbers in the top row. After students have explored and understand the positive exponents, then the negative exponents can be explored and added to the chart.

Number of times doubled	**–2**	**–1**	**0**	**1**	**2**
Pattern blocks after times doubled.					
Written as 2^n	$2^{-2} = \frac{1}{4}$	$2^{-1} = \frac{1}{2}$	$2^0 = 1$	$2^1 = 2$	$2^2 = 4$

Number of times tripled	**–2**	**–1**	**0**	**1**	**2**
Pattern blocks after times tripled.	Cannot be represented by pattern blocks without cutting the rhombus into thirds. 				
Written as 3^n	$3^{-2} = \frac{1}{9}$	$3^{-1} = \frac{1}{3}$	$3^0 = 1$	$3^1 = 3$	$3^2 = 9$

Note that students must understand that the fraction strip represents 1. In the following chart, the fraction strip represents the initial amount before doubling (it is zero times doubled)

Number of times doubled	**–2**	**–1**	**0**	**1**	**2**
Fraction strips after times doubled.					
Written as 2^n	$2^{-2} = \frac{1}{4}$	$2^{-1} = \frac{1}{2}$	$2^0 = 1$	$2^1 = 2$	$2^2 = 4$

Instead of using fraction strips, teachers could use another rectangular model that breaks into smaller rectangles to illustrate this idea. The original rectangle would represent 1. Students could break their model up to represent 2^{-1}, 2^{-2}, 2^{-3} or 3^{-1}, 3^{-2}, 3^{-3}, etc. Students could be challenged to determine all the expressions that can be represented by this model without breaking it into smaller pieces than the pre-determined squares. Students could also be asked what types of models would be good choices for this activity, and how they would make this decision.

After students have used manipulatives to explore negative exponents, the teacher can use patterns to introduce negative exponents. Students should follow the progression of the pattern first with integral bases. This can then be extended to a general rule in the exponent laws using literal expressions.

Review of Mathematics 9	$\frac{2^4}{2^2} = 2^2$	$\frac{2 \bullet 2 \bullet 2 \bullet 2}{2 \bullet 2} = 2^2$	$\frac{2}{2} \bullet \frac{2}{2} \bullet 2 \bullet 2 = 1 \bullet 1 \bullet 4$	$\frac{16}{4} = 4$
	$\frac{2^4}{2^3} = 2^1$	$\frac{2 \bullet 2 \bullet 2 \bullet 2}{2 \bullet 2 \bullet 2} = 2^1$	$\frac{2}{2} \bullet \frac{2}{2} \bullet \frac{2}{2} \bullet 2 = 1 \bullet 1 \bullet 1 \bullet 2$	$\frac{16}{8} = 2$
	$\frac{2^4}{2^4} = 2^0$	$\frac{2 \bullet 2 \bullet 2 \bullet 2}{2 \bullet 2 \bullet 2 \bullet 2} = 2^0$	$\frac{2}{2} \bullet \frac{2}{2} \bullet \frac{2}{2} \bullet \frac{2}{2} = 1 \bullet 1 \bullet 1 \bullet 1$	$\frac{16}{16} = 1$
New to Mathematics 10	$\frac{2^4}{2^5} = 2^{-1}$	$\frac{2 \bullet 2 \bullet 2 \bullet 2}{2 \bullet 2 \bullet 2 \bullet 2 \bullet 2} = 2^{-1}$	$\frac{2}{2} \bullet \frac{2}{2} \bullet \frac{2}{2} \bullet \frac{2}{2} \bullet \frac{1}{2} = 1 \bullet 1 \bullet 1 \bullet 1 \bullet \frac{1}{2}$	$\frac{16}{32} = \frac{1}{2}$

- Use tables of examples to demonstrate to students that they can use what they already know to find missing information. For example, $2^{-3} = \frac{1}{2^3} = \frac{1}{8} = 0.125$.

- As in the introduction to negative exponents, students should be given the opportunity to discover the pattern behind rational exponents. This should occur prior to direct instruction.

Review of Mathematics 9	$9^3 \bullet 9^3$	$= (9 \bullet 9 \bullet 9)\ (9 \bullet 9 \bullet 9)$	$= 9^{3+3}$	$= 9^6$	$= 729 \bullet 729$	$= 531441$
	$8^3 \bullet 8^3$	$= (8 \bullet 8 \bullet 8)\ (8 \bullet 8 \bullet 8)$	$= 8^{3+3}$	$= 8^6$	$= 512 \bullet 512$	$= 262144$
	$9^2 \bullet 9^2$	$= 9 \bullet 9 \bullet 9 \bullet 9$	$= 9^{2+2}$	$= 9^4$	$= 81 \bullet 81$	$= 6561$
	$8^2 \bullet 8^2$	$= 8 \bullet 8 \bullet 8 \bullet 8$	$= 8^{2+2}$	$= 8^4$	$= 64 \bullet 64$	$= 4096$
	$9^1 \bullet 9^1$	$= 9 \bullet 9$	$= 9^{1+1}$	$= 9^2$	$= 9 \bullet 9$	$= 81$
	$8^1 \bullet 8^1$	$= 8 \bullet 8$	$= 8^{1+1}$	$= 8^2$	$= 8 \bullet 8$	$= 64$
New to Mathematics 10	$9^{\frac{1}{2}} \bullet 9^{\frac{1}{2}}$	$= \sqrt{9} \bullet \sqrt{9}$	$= 9^{\frac{1}{2}+\frac{1}{2}}$	$= 9^1$	$= 3 \bullet 3$	$= 9$
	$8^{\frac{1}{3}} \bullet 8^{\frac{1}{3}} \bullet 8^{\frac{1}{3}}$	$= \sqrt{8} \bullet \sqrt{8} \bullet \sqrt{8}$	$= 8^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}}$	$= 8^1$	$= 2 \bullet 2 \bullet 2$	$= 8$

- Create centres in the classroom containing mathematical expressions that involve the exponent laws. Students will participate in a carousel activity where they are asked to move throughout the centres to identify and correct errors.

SUGGESTED MODELS AND MANIPULATIVES

- counters
- fraction strips
- pattern blocks

MATHEMATICAL VOCABULARY

Students need to be comfortable using the following vocabulary.

- integral exponents
- powers
- rational exponents

Resources/Notes

Internet

- Softschools.com, “Less than Greater than—Game.” (Softschools.com 2013)
  www.softschools.com/math/games/less_than_greater_practice.jsp
  Free math games.
- Math Interactives, “Exploring Laws of Exponents—Use It” (Alberta Education 2003)
  www.learnalberta.ca/content/mejhm/index.html?l=0&ID1=AB.MATH.JR.NUMB&ID2=AB.MATH.JR.NUMB.EXPO&lesson=html/object_interactives/exponent_laws/use_it.html
  This interactive mathematics resource uses the scenario of a paleontological dig to allow the user to explore the laws of exponents. The resource also includes print activities, solutions, and learning strategies.
- SuperTeacherTools, “Classroom Jeopardy.” (SuperTeacherTools 2013)
  www.superteachertools.com/jeopardy/usergames/Nov201044/game1288705963.php
  Jeopardy game

Print

- *Foundations and Pre-calculus Mathematics 10* (Burglind et al., Pearson 2010)
  - Student Book
    - Chapter 4, Sections 2, 3, 4, 5, and 6, pp. 207–249
  - Teacher Technology DVD
    - Teacher Resource
    - Blackline Masters
    - Smart Lessons
    - Animations
    - Dynamic Activities

Notes

SCO AN04 Students will be expected to demonstrate an understanding of the multiplication of polynomial expressions (limited to monomials, binomials, and trinomials), concretely, pictorially, and symbolically. [CN, R, V]			
[**C**] Communication [**T**] Technology	[**PS**] Problem Solving [**V**] Visualization	[**CN**] Connections [**R**] Reasoning	[**ME**] Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have met the corresponding specific curriculum outcome.

(It is intended that the emphasis of this outcome be on binomial by binomial multiplication, with extension to polynomial by polynomial to establish a general pattern for multiplication.)

AN04.01 Model the multiplication of two given binomials, concretely or pictorially, and record the process symbolically.
AN04.02 Relate the multiplication of two binomial expressions to an area model.
AN04.03 Explain, using examples, the relationship between the multiplication of binomials and the multiplication of two-digit numbers.
AN04.04 Verify a polynomial product by substituting numbers for the variables.
AN04.05 Multiply two polynomials symbolically, and combine like terms in the product.
AN04.06 Generalize and explain a strategy for multiplication of polynomials.
AN04.07 Identify and explain errors in a solution for a polynomial multiplication.

Scope and Sequence

Mathematics 9	**Mathematics 10**	**Pre-calculus 11**
PR05 Students will be expected to demonstrate an understanding of polynomials (limited to polynomials of degree less than or equal to 2). **PR06** Students will be expected to model, record, and explain the operations of addition and subtraction of polynomial expressions, concretely, pictorially, and symbolically (limited to polynomials of degree less than or equal to 2).	**AN04** Students will be expected to demonstrate an understanding of the multiplication of polynomial expressions (limited to monomials, binomials, and trinomials), concretely, pictorially, and symbolically.	**AN03** Students will be expected to solve problems that involve radical equations (limited to square roots). **AN04** Students will be expected to determine equivalent forms of rational expressions (limited to numerators and denominators that are monomials, binomials, or trinomials). **AN05** Students will be expected to perform operations on rational expressions (limited to numerators and denominators that are monomials, binomials, or trinomials).

Mathematics 9 (continued)	Mathematics 10 (continued)	Pre-calculus 11 (continued)
PR07 Students will be expected to model, record, and explain the operations of multiplication and division of polynomial expressions (limited to polynomials of degree less than or equal to 2) by monomials, concretely, pictorially, and symbolically.		**AN06** Students will be expected to solve problems that involve rational equations (limited to numerators and denominators that are monomials, binomials, or trinomials).

Background

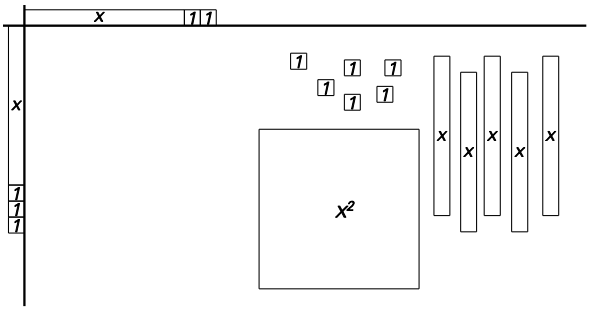
Terminology associated with polynomials was introduced in Mathematics 9. Teachers should continue to use these terms in context to allow students to integrate them into their vocabulary. This vocabulary includes **term**, **variable**, **constant**, **co-efficient**, **polynomial**, **degree of a term**, **degree of a polynomial**, **monomial**, **binomial**, and **trinomial**.

In Mathematics 9, polynomials, limited to degree 1 or 2, were added and subtracted, and polynomials were multiplied and divided—concretely, pictorially, and symbolically. Students multiplied powers with integer bases. They expressed polynomials with algebra tiles, with diagrams and pictures, and with symbols. Multiplication and division of polynomials were limited to polynomials with monomials.

In Mathematics 10, multiplication of polynomials is extended to polynomials by other polynomials. The goal of this outcome is for all students to become proficient in concrete, pictorial, and symbolic representations of the multiplication of polynomial expressions.

Algebra tiles and area models build an understanding of the concepts behind the symbols and **are not** considered optional for students who are able to master the more traditional symbolic models without these tools. Although there will be a greater reliance on symbolic representations as students progress into higher grades, proficiency with concrete and pictorial representations will help to build a deeper understanding of the concepts and their applications. Emphasis should be placed on the ability to switch between alternate representations, leading to a proficiency in symbolic representation.

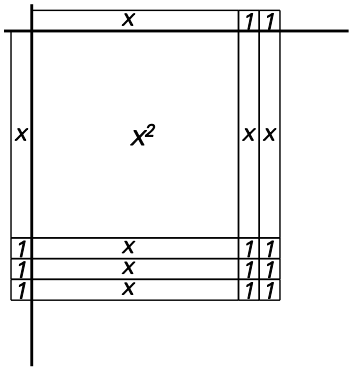
The following illustrates the multiplication of the polynomial: $(x + 2)(x + 3)$, expressed concretely, pictorially, and symbolically. Be sure to review the values of each tile and how those tiles fit together to create the area.

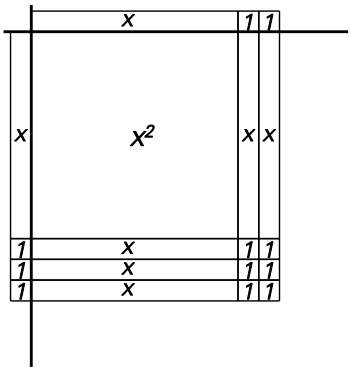


	x	2
x	$x \bullet x = x^2$	$x \bullet 2 = 2x$
3	$3 \bullet x = 3x$	$(2)(3) = 6$

$(x + 2)(x + 3)$
$= x^2 + 2x + 3x + 6$
$= x^2 + 5x + 6$

$(x + 2)(x + 3)$
$= x^2 + 3x + 2 + 6$
$= x^2 + 5x + 6$





Concrete (with algebra tiles) ⟷ Pictorial (on paper) ⟷ Symbolic

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

Guiding Questions

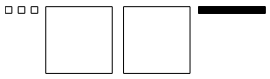
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Students tasks such as the following could be completed to assist in determining students' prior knowledge.

- Is $4x + 3x^2 + 2$ equivalent to $3x^2 + 4x + 2$? Explain using a concrete model.
- Describe a real-life situation that would model the binomial expression $2x + 3$.
- From the following expressions, identify those that are equivalent to $-2y^2 + y - 3$.

  (a) $y - 3 - 2y^2$

  (b) 

  (c) $y^2 - 1 + 4y - 3y^2 - 3y - 2$

  (d) $-y^2 - 3$

  (e) 

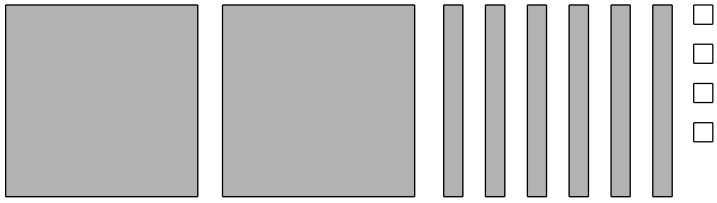
- Identify and correct the errors in the following:
  Step 1: $(2x^2 - 3x + 2) - (x^2 + x - 1)$
  Step 2: $2x^2 - 3x + 2 - x^2 + x - 1$
  Step 3: $x^2 - 2x - 1$

- Model how to determine the product or quotient for each of the following using algebra tiles or diagrams. Record the process symbolically
  (a) $3(2x - 1)$
  (b) $\dfrac{3x^2 - 6x}{-3x}$

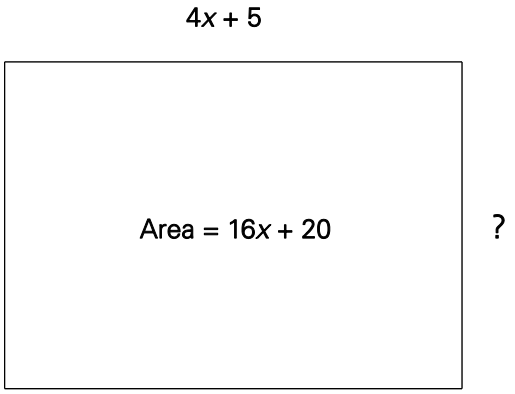
WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

- Record the trinomial represented by these algebra tiles.

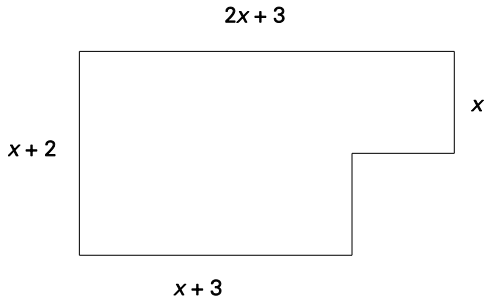


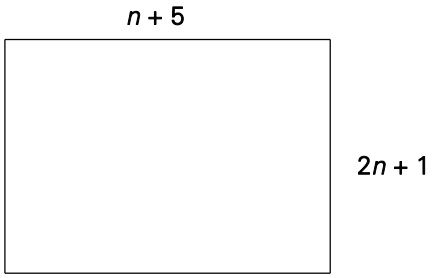
- Determine the value of the missing sides of the rectangle.



- Use the distributive property to multiply the following polynomials concretely with algebra tiles, pictorially and symbolically. Verify the product by substituting the variable with a number value.
  (a) $2(y + 3)$
  (b) $3b(4 + 2b)$
  (c) $2(x^2 + 5x + 4)$
  (d) $(x + 4)(x + 3)$
  (e) $(3x + 2)(x)$
  (f) $(5 + x)(2x + 1)$
  (g) $(6 + 2y)(1 + y)$
  (h) $(2x + 3)(x + 9)$

- Find an expression for the area of each of the following (diagrams are not drawn to scale):





- A classmate has missed the lesson on multiplying binomials. How would you explain to him the process used to determine the product of two binomials?

- Use a rectangle model to illustrate the multiplication of the following two digit numbers:
  (a) 17×14 (b) 16×11 (c) 21×12

- Use the rectangle model to multiply the following:
  (a) $(4x + 6)(3x - 5)$
  (b) $(2m + 2)(5m - 4)$
  (c) $(-3x + 2)(2x - 7)$

- Use algebra tiles to model the binomial products and record your answers.
  (a) $(x + 3)(x + 5)$
  (b) $(x + 1)(x - 4)$
  (c) $(x - 2)(x - 5)$

- Use the distributive property to find the product of the following binomials:
  (a) $(x + 5)(x + 4)$
  (b) $(d – 1)(d + 14)$
  (c) $(2 + f)(f – 7)$
  (d) $(9 – w)(5 – w)$
  (e) $(k + 10)(k – 40)$

- Complete the following:
  (a) How many terms are created when $(x + 1)(x + 2)$ is multiplied?
  (b) When the product of $(x + 1)(x + 2)$ is written as $x^2 + 2x + 1x + 2$, two terms can be combined. Will this always be the case when two binomials are multiplied together?
  (c) How many terms are created when determining the product of $(x + 1)(x^2 + 4x + 2)$? How many pairs of like terms can be combined? Will this always be the case when a binomial is multiplied by a trinomial?
  (d) How many terms are created when determining the product of $(x + 1)(x^3 + x^2 + 4x + 2)$? How many sets of like terms can be combined? Will this always be the case when a binomial is multiplied by a fourth-degree polynomial?
  (e) What pattern can you find in the above answers?

- Rima solved the following multiplication problem:
  $(3x + 4)(x + 3)$
  $= 3x + 9x + 4x + 12$
  $= 16x + 12$
  (a) Is Rima's answer correct?
  (b) Rima checked her work by substituting $x = 1$. Was this a good choice to verify the multiplication?
  (c) When verifying work, what numbers should be avoided? Why?

- Complete the following:
  (a) Why is $(x + 4)(x^2 – 3x + 2) = (x^2 – 3x + 2)(x + 4)$?
  (b) How can you check that $(b – 1)(b – 2)(b – 3) = b^3 – 6b^2 + 11b – 6$?
  (c) Find a shortcut for multiplying $(x + 5)^2$. Why does this work? Will the same type of shortcut work for multiplying $(x + 5)^3$?

FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should inform instruction.

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions
- Does the lesson fit into my yearly/unit plan?

- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

Note: Caution should be exercised when teaching students to multiply two binomials. Instructors may have previously used the FOIL (**F**irsts, **O**utsides, **I**nsides, **L**asts) acronym. This should be avoided. Students need to understand that, while having an organized plan to manage binomial multiplication is important, the order of multiplication is *not* important. They need to note that each term from the first expression must be multiplied by each and every term from the second expression. Some students may generalize the FOIL acronym to a binomial by a trinomial and miss multiplying some of the terms.

SUGGESTED LEARNING TASKS

Effective instruction should consist of various strategies.

Guiding Question

- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

Consider the following sample instructional strategies when planning lessons.

- Present students with a selection of binomial multiplied by binomial expressions with the product included. Ask students to find the pattern or the relationship between the questions and their product. Have students discover strategies on their own by showing them repeated examples until they identify the pattern. Once each student has come up with a strategy, have him or her test it to make sure it applies to a multitude of examples.

- After students have completed the binomial multiplication, have them verify their solutions through substitution. For example, for $(x + 2)(x + 3) = x^2 + 5x + 6$

  Substitute $x = 3$ to verify

left side	**right side**
$(x + 2)(x + 3)$	$x^2 + 5x + 6$
$= (3 + 2)(3 + 3)$	$= 3^2 + 5(3) + 6$
$= (5)(6)$	$= 9 + 15 + 6$
$= 30$	$= 30$

- Have students explore the multiplication of various types of binomials, and ask them to develop a strategy for multiplying binomials of this type.
  (a) $(x - a)(x + a)$
  (b) $(x + a)^2$
  (c) $(x - a)^2$
  (d) $(ax - b)(ax + b)$
  (e) $(ax + b)^2$
  (f) $(ax - b)^2$

SUGGESTED MODELS AND MANIPULATIVES

- algebra tiles

MATHEMATICAL VOCABULARY

Students need to be comfortable using the following vocabulary.

- binomial
- co-efficient
- constant
- degree of a polynomial
- degree of a term
- monomials
- multiply polynomials
- polynomial
- substitution
- term
- trinomials
- variable

Resources/Notes

Internet

- Illuminations: Resources for Teaching Math, "Algebra Tiles." (National Council of Teachers of Mathematics 2013)
  http://illuminations.nctm.org/ActivityDetail.aspx?ID=216

Print

- *Foundations and Pre-calculus Mathematics 10* (Burglind et al., Pearson 2010)
  - Student Book
    - Chapter 3, Sections 4, 5, 6, and 7, pp. 157–187
  - Teacher Technology DVD
    - Teacher Resource
    - Blackline Masters
    - Smart Lessons
    - Animations
    - Dynamic Activities

Notes

SCO AN05 Students will be expected to demonstrate an understanding of common factors and trinomial factoring, concretely, pictorially, and symbolically. [C, CN, R, V]			
[**C**] Communication	[**PS**] Problem Solving	[**CN**] Connections	[**ME**] Mental Mathematics and Estimation
[**T**] Technology	[**V**] Visualization	[**R**] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have met the corresponding specific curriculum outcome.

AN05.01 Determine the common factors in the terms of a polynomial, and express the polynomial in factored form.
AN05.02 Model the factoring of a trinomial, concretely or pictorially, and record the process symbolically.
AN05.03 Factor a polynomial that is a difference of squares, and explain why it is a special case of trinomial factoring where $b = 0$.
AN05.04 Identify and explain errors in a polynomial factorization.
AN05.05 Factor a polynomial, and verify by multiplying the factors.
AN05.06 Explain, using examples, the relationship between multiplication and factoring of polynomials.
AN05.07 Generalize and explain strategies used to factor a trinomial.
AN05.08 Express a polynomial as a product of its factors.

Scope and Sequence

Mathematics 9	**Mathematics 10**	**Grade 11 Mathematics Courses**
PR05 Students will be expected to demonstrate an understanding of polynomials (limited to polynomials of degree less than or equal to 2). **PR06** Students will be expected to model, record, and explain the operations of addition and subtraction of polynomial expressions, concretely, pictorially, and symbolically (limited to polynomials of degree less than or equal to 2). **PR07** Students will be expected to model, record, and explain the operations of multiplication and division of polynomial expressions (limited to polynomials of degree less than or equal to 2) by monomials, concretely, pictorially, and symbolically.	**AN05** Students will be expected to demonstrate an understanding of common factors and trinomial factoring, concretely, pictorially, and symbolically.	**RF02** Students will be expected to demonstrate an understanding of the characteristics of quadratic functions, including vertex, intercepts, domain and range, and axis of symmetry. (M11)* **AN04** Students will be expected to determine equivalent forms of rational expressions (limited to numerators and denominators that are monomials, binomials, or trinomials). (PC11)** **AN05** Students will be expected to perform operations on rational expressions (limited to numerators and denominators that are monomials, binomials, or trinomials. (PC11)**

Mathematics 9 (continued)	Mathematics 10 (continued)	Grade 11 Mathematics Courses (continued)
		AN06 Students will be expected to solve problems that involve rational equations (limited to numerators and denominators that are monomials, binomials, or trinomials. (PC11)** **RF01** Students will be expected to factor polynomial expressions of the following form where *a*, *b* and *c* are rational numbers. ▪ $ax^2 + bx + c$, $a \neq 0$ ▪ $a^2x^2 - b^2y^2$, $a \neq 0$, $b \neq 0$ ▪ $a(f(x))^2 + b(f(x)) + c$, $a \neq 0$ ▪ $a^2(f(x))^2 - b^2(g(y))^2$, $a \neq 0$, $b \neq 0$ (PC11)** **RF05** Students will be expected to solve problems that involve quadratic equations. (PC11)**

* M11—Mathematics 11
** PC11—Pre-calculus 11

Background

The concept of factoring whole numbers was introduced earlier in this unit in AN01, and the multiplication of binomials and trinomials was addressed in AN04.

For this outcome, students will develop an understanding of common factors and trinomial factoring in the form $ax^2 + bx + c$ where *b* and *c* are integers. For purposes of differentiation, $a = 1$, or $a > 1$. Trinomial factoring will include perfect squares and the difference of squares.

Exploration of common factors and trinomial factoring should begin with factoring as the inverse of multiplication; the introduction should employ algebra tiles and the area model. There should be a progression from concrete to pictorial to symbolic representations to build an understanding of the concepts behind the symbols. Mastering concrete and pictorial representations *should not be considered optional* for students who are able to master symbolic models without these tools.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

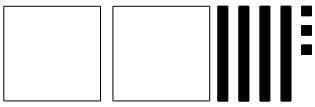
Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Students tasks such as the following could be completed to assist in determining students' prior knowledge.

- Given a polynomial, identify the terms **degree**, **variables**, **coefficients**, and **constants** in each. Use a model to represent the polynomial. Use a chart (provided by the teacher) similar to the one below and insert any polynomial expression or model into that chart, thus completing the remaining cells.

Expression	# of terms	Degree	Variables	Coefficient(s)	Constant	Model
3	1	0	none	none	3	
$x^2 + 2x - 3$	3	2	x	1, 2	–3	
$4x - 2x^2 + 3$	3	2	x	4, –2	3	

- Explain how the model shown below can be used to determine the answer to $\frac{3x+3}{3}$.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample student tasks and activities (that can be adapted) for either assessment for learning or assessment of learning.

- Fill in the blanks to make a true statement:
  (a) $12x + 18y = (\square)(2x + 3y)$
  (b) $3x^2 - 5x = (\square)(3x - 5)$
  (c) $4ab + 3ac = (\square)(4b + 3c)$
  (d) $3y^2 + 18y = 3y(y + \square)$
  (e) $14a - 12b = 2(\square - 6b)$
  (f) $x^2 + 6x + \square = (x + \square)(x + \square)$
  (g) $x^2 + \square x + 12 = (x + \square)(x + \square)$
  (h) $x^2 - \square x + 5 = (x + \square)(x + \square)$
  (i) $x^2 - \square x - 12 = (x - \square)(x + \square)$
  (j) $2x^2 - x - 6 = (2x + \square)(x - \square)$
  (k) $\square x^2 + \square x + \square = (3x + 1)(2x + 5)$

Note: Questions (a) to (k) represent increasing levels of difficulty.

- Determine two different trinomials that both have $(x + 3)$ as a factor. Verify your answer by substituting a specific value, such as $x = 2$, for the variable.

  Note: Repeat this activity with a variety of factors.

- Factor the following expressions. Verify your answer by substituting a specific value for the variable.
  (a) $x^2 - 9$
  (b) $12x^2 - 12x + 3$
  (c) $9x^2 - 4y^2$
  (d) $y^2 - 16$
  (e) $1 - 64t^2$
  (f) $x^2 + 6x + 9$
  (g) $15 - x - 2x^2$
  (h) $4m^2 - 25$

- Ask students to use algebra tiles to factor the following expressions and explain why some expressions cannot be factored.
  (a) $5m^2 - 35m$
  (b) $3x^4 + x^2$
  (c) $15e^2g^5 - 20e^7g^2$

- Explain why $x^2 + 3x + 4$ cannot be factored.

- How many integer values are there for k for which $x^2 + kx + 24$ is factorable?

- Given $s^2 - 3s - 10$, find and correct the mistake in the factoring below.

  Sum = –3, product = –10
  numbers are –2 and 5
  $(s - 2)(s + 5)$

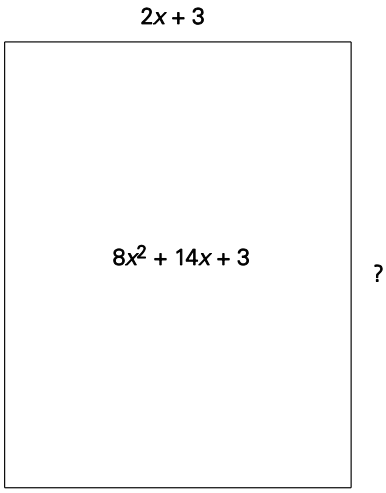
- Given $2x^2 + 6x - 40$, find and correct the mistakes in the factoring below.

  $2x^2 + 6x - 40$
  $= 2(x^2 + 3x - 40)$
  $= 2(x - 8)(x + 5)$

- A rectangle has a product given by the expression $15x^2 - 7x - 30$. If the product represents the area of the rectangle in square centimetres, complete the following:
  (a) Find an expression for the length and width of the rectangle.
  (b) Find the length of the rectangle given that its width is 4 cm.

- If $6x^2 + kx - 7$ is factorable, how many different integer values are there for k? Explain.

- Give an example of a polynomial that is factored completely and an example of one that is not. Explain the differences between them.



- The area of a rectangle is represented by the product $8x^2 + 14x + 3$ square units. If the length of one side is $(2x + 3)$ units, ask students to determine the width of the other side. How did knowing one of the factors help the students determine the other factor?

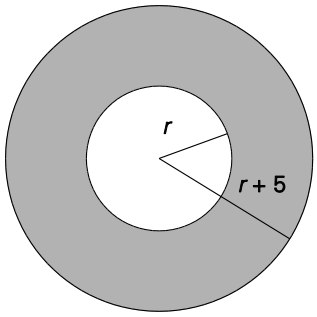
- Factor the trinomial represented by these algebra tiles and record the result (shaded represents positive).

- Using the binomial factor that you have been given, find one or more trinomials, displayed around the classroom, that has your binomial as a factor. Then, find the person who has the other factor for one of "your" posted trinomials. Both of you should stand next to this trinomial.

  Note: To do this activity, create some cards with factors for the students and some larger cards with trinomials that can be posted around the room.

- In each trinomial, how is the coefficient of the linear term related to the product of the coefficient of the squared term and the constant term?
  (a) $4x^2 - 12x + 9$
  (b) $25x^2 - 20x + 4$
  (c) $16x^2 + 24x + 9$

- The diagram shows two concentric circles with radii r and $r + 5$. Write an expression for the area of the shaded region and factor the expression completely. If $r = 4$ cm, calculate the area of the shaded region to the nearest tenth of a square centimetre.



- Determine two values of n that allow the polynomial $25b^2 + nb + 49$ to be a perfect square trinomial. Use them both to factor the trinomial.

- Explain why $x^2 - 81$ can be factored but $x^2 + 81$ cannot be factored.

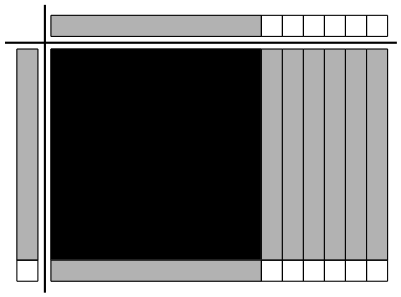
- Find a shortcut for multiplying $(x + 5)(x - 5)$. Explain why this works.

- Explain why $(x + 3)^2 \neq x^2 + 9$.

- Now that you have factored the different types of polynomials, which type of factoring do you find easier and why? Which type of factoring do you find the most difficult and why?

- *Factoring Bingo:* You have a blank bingo card and 24 polynomial expressions in factored form. Randomly write one expression in each square on your card. Allow for a free space. Show a polynomial (in expanded form) and match it to the corresponding factored form on your bingo card and cross it off. Traditional bingo alignments win—horizontal, vertical, and diagonal. Options such as four corners may also be used. (**Note:** This game can be varied such that students are provided with only the polynomial in expanded form or with a mixture of polynomials in factored and expanded form.)

- Explain how the model shown below can be used to determine the answer for $\frac{x^2 + 7x + 6}{x + 1}$.



FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should inform instruction.

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SUGGESTED LEARNING TASKS

Effective instruction should consist of various strategies.

Guiding Question

- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

Consider the following sample instructional strategies when planning lessons.

- To encourage deep understanding, it is recommended that there is a progression from concrete to pictorial to symbolic representations. Complexity should be gradually increased. For example, begin with binomials with positive values, extend to binomials with negative values, then mixed values, and finally, use trinomials with coefficients greater than 1 as the highest level of understanding.

- Avoid teaching factoring as an algorithm. Take the time to develop an understanding and have students discover patterns and rules rather than presenting a process.

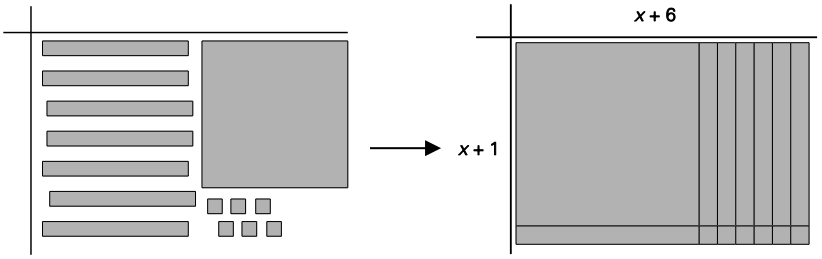
- As opposed to explicitly teaching students that factoring is the opposite of the distributive property, encourage students to come up with their own strategies through the use of repeated examples. Be sure to have students test their strategies to ensure they work with various examples. When introducing a difference of two squares, again, have students discover the rule as opposed to explicitly teaching it.

- Algebra placemats can be laminated and provided as individual workspaces for students when working with algebra tiles. Alternatively (or additionally), laminated cardstock can be used with dry-erase markers to visually represent work.

- Factoring can be introduced as the inverse of multiplication using algebra tiles and the area model. Starting with the product to be factored, algebra tiles can be arranged as a rectangle, the dimensions of which will be the factors.

- To factor a polynomial using an algebra placemat, students will assemble the tiles into a rectangular shape. The factors are the dimensions of the rectangle. When there is more than one solution, this rectangle could be arranged differently. For example, to factor $4x + 2$, arrange tiles on the placemat as a rectangle of dimensions 2 and $2x + 1$. The sides of the rectangle are its factors, so $4x + 2 = 2(2x + 1)$.

- For example, to factor $x^2 + 7x + 6$, arrange tiles on the placemat as a rectangle of dimensions $(x + 1)$ and $(x + 6)$. The sides of the rectangle are its factors; therefore



$x^2 + 7x + 6 = (x + 1)(x + 6)$

Suggested Models and Manipulatives

- algebra tiles

Mathematical Vocabulary

Students need to be comfortable using the following vocabulary.

- difference of squares
- factored form
- factoring
- perfect square trinomial

Resources/Notes

Internet

- Mangahigh.com. "The Wrecks Factor" (Blue Duck Education 2013)
  www.mangahigh.com/en_us/games/wrecksfactor
  Factoring Game
- Polynomial Bingo (activity), Institute for Math and Science Education UA Fort Smith (Roberta Parks, n.d.)
  http://makingmathfun.wikispaces.com/file/view/Polynomial+Factoring+Bingo.pdf
  Factoring Bingo

Print

- *Foundations and Pre-calculus Mathematics 10* (Burglind et al., Pearson 2010)
  - Student Book
    - Chapter 3, Sections 3, 4, 5, 6, and 8, pp. 157–181, 188–197
  - Teacher Technology DVD
    - Teacher Resource
    - Blackline Masters
    - Smart Lessons
    - Animations
    - Dynamic Activities

Notes

Relations and Functions
70–75 hours

GCO: Students will be expected to develop algebraic and graphical reasoning through the study of relations.

Specific Curriculum Outcomes

Process Standards Key

[**C**] Communication	[**PS**] Problem Solving	[**CN**] Connections	[**ME**] Mental Mathematics and Estimation
[**T**] Technology	[**V**] Visualization	[**R**] Reasoning	

RF01 Students will be expected to interpret and explain the relationships among data, graphs, and situations. [C, CN, R, T, V]

RF02 Students will be expected to demonstrate an understanding of relations and functions. [C, R, V]

RF03 Students will be expected to demonstrate an understanding of slope with respect to rise and run, line segments and lines, rate of change, parallel lines, and perpendicular lines. [PS, R, V]

RF04 Students will be expected to describe and represent linear relations, using words, ordered pairs, tables of values, graphs, and equations. [C, CN, R, V]

RF05 Students will be expected to determine the characteristics of the graphs of linear relations, including the intercepts, slope, domain, and range. [CN, PS, R, V]

RF06 Students will be expected to relate linear relations to their graphs, expressed in

- slope-intercept form ($y = mx + b$)
- general form ($Ax + By + C = 0$)
- slope-point form ($y - y_1) = m(x - x_1$)

[CN, R, T, V]

RF07 Students will be expected to determine the equation of a linear relation to solve problems, given a graph, a point and the slope, two points, and a point and the equation of a parallel or perpendicular line. [CN, PS, R, V]

RF08 Students will be expected to solve problems that involve the distance between two points and the midpoint of a line segment. [C, CN, PS, T, V]

RF09 Students will be expected to represent a linear function, using function notation. [CN, ME, V]

RF10 Students will be expected to solve problems that involve systems of linear equations in two variables, graphically and algebraically. [CN, PS, R, T, V]

SCO RF01 Students will be expected to interpret and explain the relationships among data, graphs, and situations. [C, CN, R, T, V]			
[**C**] Communication [**T**] Technology	[**PS**] Problem Solving [**V**] Visualization	[**CN**] Connections [**R**] Reasoning	[**ME**] Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have met the corresponding specific curriculum outcome.

RF01.01 Graph, with or without technology, a set of data, and determine the restrictions on the domain and range.
RF01.02 Explain why data points should or should not be connected on the graph for a situation.
RF01.03 Describe a possible situation for a given graph.
RF01.04 Sketch a possible graph for a given situation.
RF01.05 Determine, and express in a variety of ways, the domain and range of a graph, a set of ordered pairs, or a table of values.

Scope and Sequence

Mathematics 9	**Mathematics 10**	**Grade 11 Mathematics Courses**
PR02 Students will be expected to graph linear relations, analyze the graph, and interpolate or extrapolate to solve problems.	**RF01** Students will be expected to interpret and explain the relationships among data, graphs and situations.	**RF01** Students will be expected to model and solve problems that involve systems of linear inequalities in two variables. (M11)* **RF02** Students will be expected to demonstrate an understanding of the characteristics of quadratic functions, including vertex, intercepts, domain and range, and axis of symmetry. (M11) **RF02** Students will be expected to graph and analyze absolute value functions (limited to linear and quadratic functions) to solve problems. (PC11)** **RF04** Students will be expected to analyze quadratic functions of the form $y = ax^2 + bx + c$ to identify characteristics of the corresponding graph, including vertex, domain and range, direction of opening, axis of symmetry, *x*- and *y*-intercepts and to solve problems. (PC11)

* M11—Mathematics 11 ** PC11—Pre-calculus 11

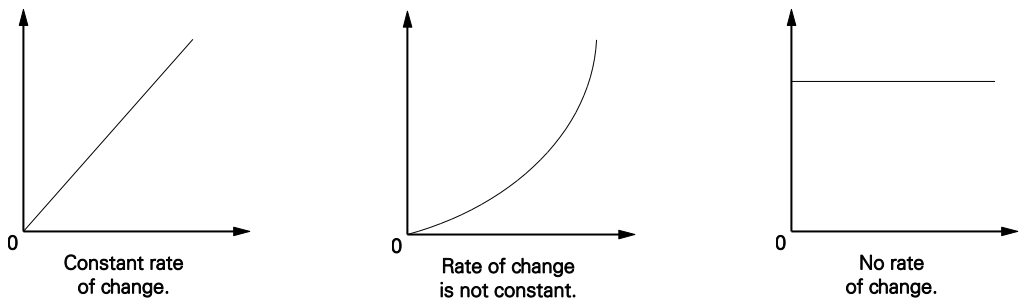
Background

Throughout the middle grades, students investigated plotting points on graphs and on a Cartesian plane, as well as interpolating and extrapolating from a given graph. In Mathematics 9, students used a table of values to create linear graphs of real-life situations and used those graphs to extrapolate and interpolate data. In Mathematics 10, they will describe a possible situation for a given graph and sketch a graph for a given situation. They will analyze a variety of graphs, including non-linear types. For example, students will be exposed to distance-time graphs and speed-time graphs.

Students need to develop an awareness of the following concepts:

- A graph is an effective way to show the relationship between two quantities.
- A constant rate of change is represented graphically by a straight line, and the steepness of the line indicates the rate at which one quantity is changing in relation to the other.

Not all relationships are represented by straight lines. It is, therefore, essential that students realize that a curve shows that the rate of change is not constant. A horizontal line means that there is no rate of change, since every value on the horizontal axis is related to the same value on the vertical axis.



Students will interpret data given to them in various forms such as table of values, or real-life situations. They will be able to sketch a graph from a set of data or for a given situation, and conversely will be able to describe a situation given a graph.

Discrete Data: Data are discrete when values have a finite or limited number of possible values, such as the number of students in a class, number of tickets sold, hourly wage, or number of items that were purchased. The plotted points are not joined together.

Continuous Data: Data are continuous for an interval when there are an infinite number of possible data points within that specific interval, such as temperature or time. The graphical representation is represented by connected points.

In Mathematics 7, students studied the concept of central tendency and were introduced to range in the context of statistics (one variable). In Mathematics 9, students solved and graphed linear inequalities (9PR04). They are familiar with the inequality signs, >, ≥, ≤, <. In Mathematics 10, this outcome now introduces the concepts of the domain and range. **Note:** Range is now introduced in the context of functions (two variables). For a given relationship, students will determine restrictions on the domain (the set of all independent variables or *x*-values) and the range (the set of all dependent variables or *y*-values).

Students will now use the inequality signs to express the domain and range given the different representations of a function. Students will become comfortable expressing domain and range using words, set notation, and interval notation. This is the first time that students have been exposed to set notation and interval notation. Students will continue to use set notation and interval notation in Mathematics 11.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

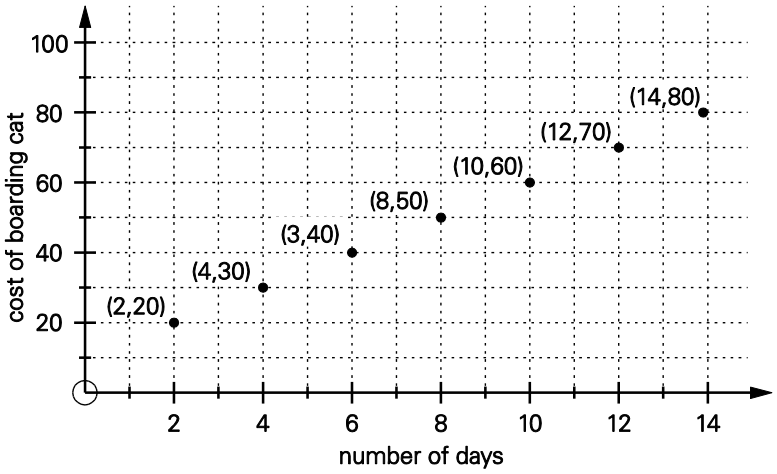
Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

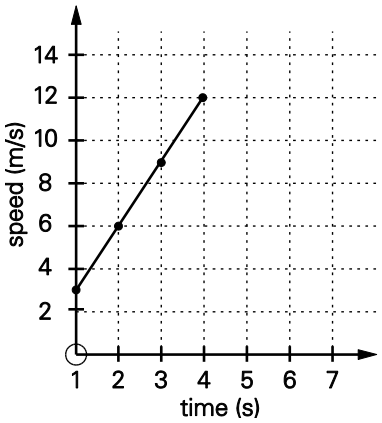
Student tasks such as the following could be completed to assist in determining students' prior knowledge.

- Cassia decides to pay a neighbour to board her cat, Sir Fluff, while she is away visiting relatives. She represents the cost on the graph shown below.



(a) What is the cost, per day, for Cassia to have her neighbour take care of Sir Fluff?
(b) If she extends her visit to three weeks (21 days), how much is Cassia going to pay her neighbour for taking care of Sir Fluff?

- Given the following graph, describe the pattern. Describe a situation that could result in the graph.



- Determine if the values in the following table satisfy the corresponding inequality.

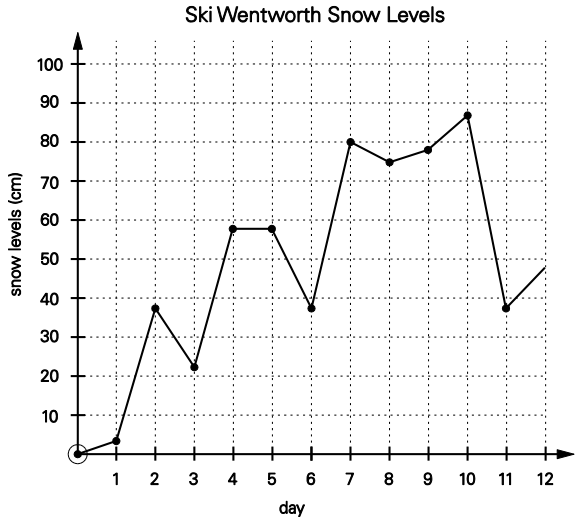
Inequality	**Values**
$x > 3$	5, 7, 9, 10
$-3x + 12 < 36$	–9, –10, –15.2
$\frac{x}{4} + 6 \geq -2$	–10, 15, $\frac{2}{3}$, 7

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

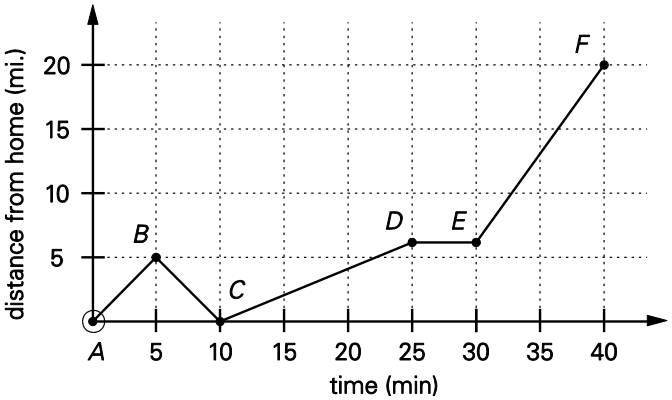
- A photographer charges a sitting fee of $20 and $1.50 for every photograph ordered.
  (a) Graph the situation using at least 5 points.
  (b) Explain why the points are not connected.
  (c) Is it possible that a person would be charged $30? Justify your answer.

- Write a paragraph interpreting the graph shown below.



- For each of the following situations, do a rough sketch of the relation and include a reasonable domain and range.
  (a) When you turn on a hot water faucet, the temperature of the water depends on how many seconds the water has been running. Sketch a graph of temperature versus time.
  (b) Until the end of adolescence, your height depends on your age. Sketch a graph of height versus age.
  (c) The number of holiday cards sold depends on the time of year. Sketch a graph of the number of cards sold versus the month of the year using holidays your family or community members celebrate. (These may include days such as Easter, Ramadan, Diwali, Hannukah, Mabon, Buddha Day, and Chinese New Year. If you cannot think of any, or if you do not celebrate any holidays, use birth dates as a data set.)
  (d) You put some ice cubes in a glass and fill it with cold water on a summer day. Sketch a graph of the temperature of the water versus the time it is sitting on a table.
  (e) The time of sunset depends on the time of the year. Sketch a graph of the time of sunset versus time of the year.

- The following graph shows Duane leaving home at point *A* and going to a party at point *F*.



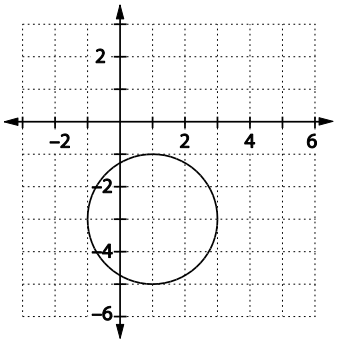
  (a) What was the slowest speed? What was the maximum speed?
  (b) What could segment *BC* represent? Explain your reasoning.
  (c) What could segment *DE* represent? Explain your reasoning.
  (d) Describe a scenario to represent the graph.

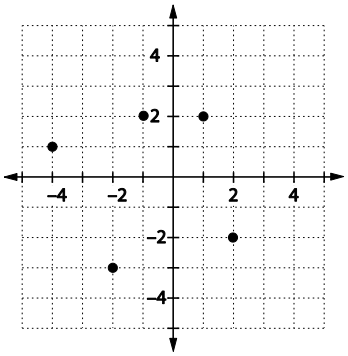
- The following data represents the sales of a song on iTunes in Nova Scotia during a seven-day period.

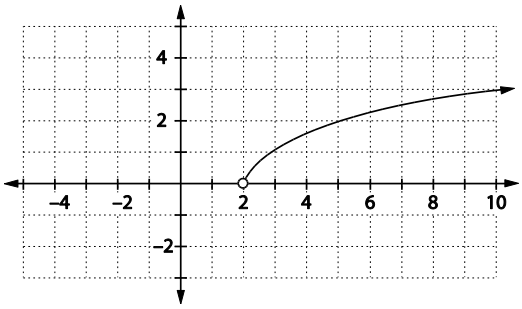
Day	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Number sold in hundreds	5	7	9	11	13	15	17

  (a) Explain why the relation is a function.
  (b) Is the data continuous or discrete? Explain why.
  (c) Draw a graph to represent the data.
  (d) Write the domain and range of the function.

- Write the domain and range for each of the following, using, where applicable,
  - words
  - a list
  - set notation
  - interval notation

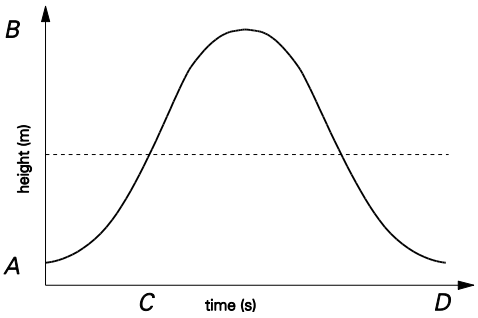






- Alexis is running in a marathon. She runs at a consistent rate. The distance, d, in km, that she is from the finish line in terms of time, t, in hours, since she began the race can be described by the equation $d = 20 – 2.5t$. If the domain of the relation is all real numbers between 0 and 8, determine the range. Explain the significance of the domain and range in terms of the context of this question.

- A Ferris wheel has a diameter of 30 m, with the centre 18 m above the ground. It makes one complete rotation every 60 seconds. The graph to the right shows the height of one of the chairs on the Ferris wheel starting at the lowest point.
  (a) What are the values of *A*, *B*, *C,* and *D*. What do they represent?
  (b) What are the domain and range of the graph?



- This table shows the population of four Nova Scotia communities in 2012.

Community	Population (2011)
Halifax	390 100
Truro	45 900
New Glasgow	35 800
Kentville	26 400

(a) Describe the relation in words.
(b) Represent this relation as a set of ordered pairs.
(c) Represent this relation as an arrow diagram.

FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should inform instruction.

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SUGGESTED LEARNING TASKS

Effective instruction should consist of various strategies.

Guiding Question

- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

Consider the following sample instructional strategies when planning lessons.

- Ensure students can complete the following tasks in order to demonstrate an understanding of how graphs and data are related to life:
  - Explain the situation that a graph represents.
  - Graph the data when given an explanation of a particular situation.
  - Create a graph with the axes labeled *x* and *y* and then exchange their graph with someone else and have them develop a scenario for the given graph.
  - Explain situations to their peers to show their understanding.

- Rather than using *x* and *y*, emphasis should be given to labelling the axes of the graph to represent the given situation. This can be emphasized using a distance/time graph. If distance represents the distance from home, for example, interpreting the graph would be different than if distance represents the distance from school. Encourage students to use labels that clearly describe what is being represented by the graph and to define what each variable represents.

- To help students develop their understanding of various relationships, they should use technology as well as paper and pencil. Graphing data using technology, students can use a graphing calculator or a variety of software, such as spreadsheets, Autograph (Eastmond Publishing Ltd. 2013), Smart Notebook Math Tools (SMART Technologies 2013), MimioStudio's Mimio Math Tools (2013), or Geometer's Sketchpad (Key Curriculum 2013). This would be a good opportunity to review whether

or not data points should be connected in a given scenario. Arrows on graphs indicate that the graph continues; students should be encouraged to use them as needed.

- Some students have difficulty expressing a relation in words. Have each student choose one ordered pair in the relation and then write a sentence involving the two elements. For example, (dime, 0.10) could be interpreted as “A dime has a value of $0.10.” or “Ten cents is the value of one dime.”

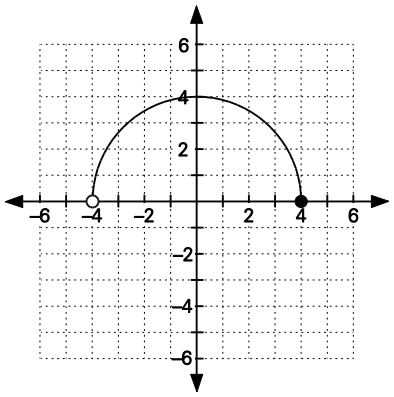
- On distance-time or speed-time graphs, some students will interpret a line segment that rises from left to right as a person travelling up a hill. Remind them that a line segment that rises from the left to the right indicates that both variables are increasing. Therefore, on a distance-time graph, a line segment like this indicates that the distance travelled is increasing as time increases. On a speed-time graph, it shows that the speed is increasing as time increases.

- It is important to provide students with a variety of problems that use both discrete and continuous data. Students should be given a variety of opportunities to develop their critical-thinking skills as they determine which numbers are reasonable in a given context, including examples of discrete and continuous data. Remind them that a list of all data points is only possible if the data is discrete.

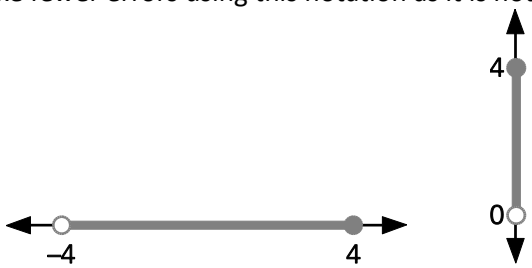
- Domain and range must be explored through data, graphs, and given situations. Students should understand which numbers are “reasonable” in any given context. For example negative values for a measurement of width would not make sense.

- As students study functions and their graphs, it is important that they understand that certain properties of a graph can provide information about a given situation. The general shape of the graph, the scale, and the starting and ending points are important features. Whether a line segment is horizontal, slopes upward to the right, or slopes downward to the right should also be addressed. Curves are used, for example, when the change in the independent and dependent variable is not constant. Students should be given an opportunity to reflect on and discuss the following:
  (a) Why do some graphs pass through the origin but other graphs do not?
  (b) What does a horizontal line represent on a speed-time graph?
  (c) What does a horizontal line represent on a distance-time graph?
  (d) What does a segment sloped upward to the right represent on a distance-time graph?
  (e) When should the data points be connected? How do we determine if the data is continuous or discrete?

- Students often have difficulty determining the domain and range of various graphs. Before students use set notation and interval notation, they may find it helpful to orally explain the domain and range and then proceed to translate their answer into words. Consider the following graph:



- Shading the relevant sections of the horizontal and vertical axis to visualize the restrictions on the independent and dependent variables can be a very powerful tool. This would be a good lead into interval notation. Interval notation uses different brackets to indicate an interval. Students often make fewer errors using this notation as it is not dependent on inequality signs.



- Students can then be exposed to set notation. A common student error occurs when the incorrect inequality sign is used. Set notation should indicate whether the data is continuous or discrete. The inequality $x < 2$, for example, may not indicate if the data is continuous or discrete unless the symbol for the number set is specified. A review of number systems may be necessary here.

- The following table illustrates the different ways of expressing the domain and range.

	Domain	**Range**
Words	Set of all real numbers between –4 and 4, not including –4 but including 4	Set of all real numbers between 0 and 4, inclusive
Set Notation	$\{x \mid -4 < x \leq 4,\ x \in \mathbf{R}\}$ **This is usually read as** *x* such that *x* is greater than –4 and less than or equal to 4, and is a member of the Real number system (**R**)	$\{y \mid 0 \leq y \leq 4, y \in R\}$ **This is usually read as** *y* such that *y* is greater than or equal to 0 and less than or equal to 4, and is a member of the Real number system (**R**)
List	No list (continuous data)	No list (continuous data)
Interval Notation	(−4, 4]	[0, 4]
Alternate Interval Notation (used in IB)	]−4, 4]	[0, 4]

SUGGESTED MODELS AND MANIPULATIVES

- graph paper

MATHEMATICAL VOCABULARY

Students need to be comfortable using the following vocabulary.

- continuous data
- discrete data
- domain
- interval notation
- range
- set notation

Resources/Notes

Print

- *Foundations and Pre-calculus Mathematics 10* (Burglind et al., Pearson 2010)
  - Student Book
    - Chapter 5, Sections 1, 2, 3, 4, and 5, pp. 254–299
  - Teacher Technology DVD
    - Teacher Resource
    - Blackline Masters
    - Smart Lessons
    - Animations
    - Dynamic Activities

Software

- Autograph (Eastmond Publishing Ltd. 2013)
- Geometer's Sketchpad (Key Curriculum 2013)
- MimioStudio (Mimio 2013)
- Smart Notebook (SMART Technologies 2013)
- Spreadsheet software

Notes

SCO RF02 Students will be expected to demonstrate an understanding of relations and functions. [C, R, V]			
[**C**] Communication	[**PS**] Problem Solving	[**CN**] Connections	[**ME**] Mental Mathematics and Estimation
[**T**] Technology	[**V**] Visualization	[**R**] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have met the corresponding specific curriculum outcome.

RF02.01 Explain, using examples, why some relations are not functions but all functions are relations.
RF02.02 Determine if a set of ordered pairs represents a function.
RF02.03 Sort a set of graphs as functions or non-functions.
RF02.04 Generalize and explain rules for determining whether graphs and sets of ordered pairs represent functions.

Scope and Sequence

Mathematics 9	**Mathematics 10**	**Grade 11 Mathematics Courses**
PR01 Students will be expected to generalize a pattern arising from a problem-solving context using linear equations and verify by substitution. **PR02** Students will be expected to graph linear relations, analyze the graph, and interpolate or extrapolate to solve problems.	**RF02** Students will be expected to demonstrate an understanding of relations and functions.	**RF02** Students will be expected to demonstrate an understanding of the characteristics of quadratic functions, including vertex, intercepts, domain and range, and axis of symmetry. (M11)* **RF02** Students will be expected to graph and analyze absolute value functions (limited to linear and quadratic functions) to solve problems. (PC11)** **RF03** Students will be expected to analyze quadratic functions of the form $y = a(x - p)^2 + q$ and determine the vertex, domain and range, direction of opening, axis of symmetry, x- and y-intercepts. (PC11) **RF04** Students will be expected to analyze quadratic functions of the form $y = ax^2 + bx + c$ to identify characteristics of the corresponding graph, including vertex, domain and range, direction of opening, axis of symmetry, *x*- and *y*-intercepts and to solve problems. (PC11)

* M11—Mathematics 11
** PC11—Pre-calculus 11

Background

In Mathematics 8, students examined the various ways to describe a relation (8PR01). When given a linear relation, they represented the relation using ordered pairs, tables of values, and graphs. In Mathematics 9, the focus was on writing an expression or equation given the pictorial, oral, or written form of the relation. Students graphed linear relations and used interpolation and extrapolation to solve problems. They were exposed to discrete and continuous data (9PR02). In Mathematics 10, students learn that functions are a specific type of relation. They are also introduced to the terms **domain** and **range** in the context of a graph. This is the first time the concept of a function is introduced.

As students work with patterns, tables, and graphs, they should realize that a relation can be represented in a variety of ways and that each representational form is a viable way to explore a problem. A relation can be described using the following:

- arrow diagrams
- equations
- graphs
- ordered pairs
- table of values
- word

This will be an introduction to the concept of relations and functions. Given a graph or a table of values, students should be able to determine and explain the difference between a relation and a function.

Relation: There is a relationship that exists between *x* (the independent variable) and *y* (the dependent variable) where for every value of *x* in a relation there is *at least* one corresponding value of *y*.

Function: There is a relationship that exists between *x* (the independent variable) and *y* (the dependent variable) where for every value of *x* there is *only one* corresponding value of *y*.

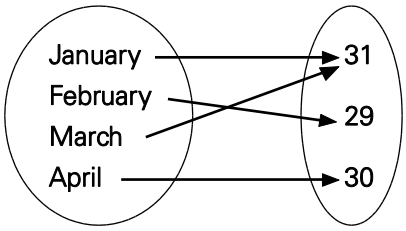
Relations can be represented in various forms as shown below.

Examples of functions:

Table
Points on a graph.

x	***y***
2	4
3	6
4	10
5	10
6	10

Mapping or Arrow Diagram
Number of days in the month.

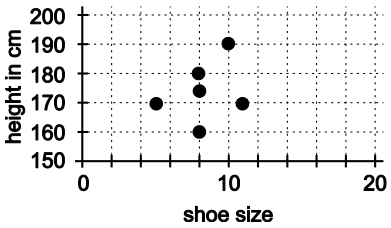


Ordered Pairs
Vehicle that has this number of wheels.

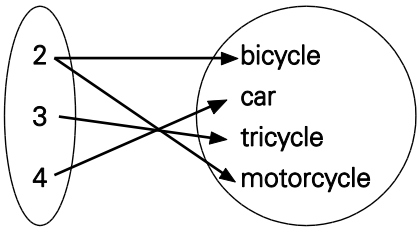
{(unicycle, 1), (bicycle, 2), (motorcycle, 2), (tricycle, 3), (car, 4)}

Examples that are not functions:

Graph
Shoe size versus height.



Mapping or Arrow Diagram
Number of wheels on a vehicle.



Ordered Pairs
Name and home of students at a workshop.

{(Marie, Ottawa), (Cheng, Toronto), (Matthew, Halifax), (Saadia, Bathurst), (Mathieu, Rivière-du-Loup)}

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

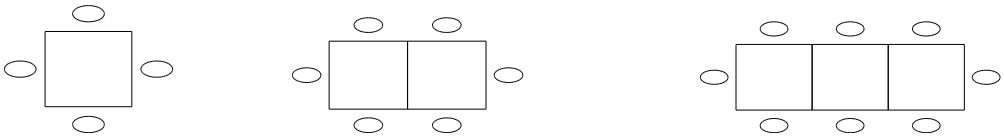
Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Student tasks such as the following could be completed to assist in determining students' prior knowledge.

- When placing chairs around a table, Amad sees that he can place 4 chairs around one table; if he pushes two tables together, Amad can place 6 chairs around the two tables; if he pushes 3 tables together, he can place 8 chairs as shown in the diagram below.



  (a) Describe in words how to determine the number of chairs if you know the number of tables by completing the sentence, The number of chairs can be found by _____.
  (b) How many chairs will Amad need if 8 tables are pushed together to form a row at a banquet?
  (c) Write an equation that describes the number of chairs needed in terms of the number of tables that have been pushed together.

- Your class is planning a trip to the Shubenacadie Wildlife Park. The school will have to pay $200 for the bus plus $5 per student. Explain how you would find the cost for 42 students.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

- Below are two groups of tables. The first group lists four data sets of relations that are functions and the second group four data sets of relations that are not functions. Express each of the relations as a graph, as an arrow diagram, and as a set of ordered pairs. Describe to a partner how you would determine whether each representation is a function or not.

Relations that are functions.

x	*y*
2	1
4	2
6	7
8	3
10	4

x	*y*
1	2
2	4
3	6
4	8
5	10

x	*y*
0	–2
1	4
2	–3
3	5
4	6

x	*y*
1	7
2	–1
3	2
4	6
5	4

Relations that are not functions.

x	*y*
1	2
2	4
3	6
4	8
1	10

x	*y*
2	1
4	2
2	7
8	3
10	4

x	*y*
3	–2
1	4
2	–3
3	5
4	6

x	*y*
1	7
5	–1
3	2
4	6
5	4

- Using real-life examples, create two relations to share with a partner, each with a different format (table, arrow diagram, graph, set of ordered pairs). One must be a function and one a non-function. Your partner must then explain which relation is the function and which is not.

- Determine if the following sets of ordered pairs represent functions.
  (a) (2, 4), (3, 6), (4, 6), (5, 6), (6, 12), (7, 14)
  (b) (−3, 7), (0, 10), (3, 13), (3, –5), (6, 16), (9, 19)

- Give an example of a graph or set of ordered pairs that represents a function. Use the definition of a function to support your answer.

- Give an example of a graph or set of ordered pairs that does not represent a function. Use the definition of a function to support your answer.

FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should inform instruction.

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?

- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SUGGESTED LEARNING TASKS

Effective instruction should consist of various strategies.

Guiding Question

- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

Consider the following sample instructional strategies when planning lessons.

- Begin by asking students to think about situations where there may be more than one answer for a corresponding question. Examples would be, What number, when squared, gives you 25? and What is the length of songs that sell for 99 cents on iTunes?

- Then ask students to think about situations where there is only one possible answer for a question. Examples would be, What number, when doubled, gives you 22? or How much tax do you pay when you buy a $100 pair of sneakers? These type of relationships are called functions.

- Students should develop the idea that all functions are relations but not all relations are functions.

- Students will use a variety of personal strategies to determine if a relation is a function.
  - It is important that you don't tell the students directly how to determine if a relation is a function
  - Using the definition of a function, students can determine if a set of ordered pairs is a function by looking for repeated values of the independent variable.
  - As students graph relations, they should be encouraged to develop a visual representation of what is and isn't a function.
  - Once students have constructed their various strategies, they will be ready to discover that a graph is not a function when any vertical line intersects the graph at more than one point, indicating that one input value has more than one output value. The vertical line test can then be used to sort a set of graphs as functions or non-functions.
  - When using different strategies, encourage students to explain their reasoning as to why a relation does or does not represent a function.

- Tables and graphs of functions or non-functions can be photocopied and distributed among students to discuss and determine whether they are functions or non-functions.

Suggested Models and Manipulatives

- graph paper
- measuring tapes

Mathematical Vocabulary

Students need to be comfortable using the following vocabulary.

- extrapolate
- function
- interpolate
- relation
- vertical line test

Resources/Notes

Internet

- Professional Learning K–12, Newfoundland and Labrador (Professional Learning NL 2013)
  www.k12pl.nl.ca
  The professional learning site provides a classroom clip of students finding the different representations of a linear relation and completing a puzzle.

Print

- *Foundations and Pre-calculus Mathematics 10* (Burglind et al., Pearson 2010)
  - Student Book
    - Chapter 5, Sections 1 and 2, pp. 256–275
    - Chapter 5, Section 5, pp. 287–297
  - Teacher Technology DVD
    - Teacher Resource
    - Blackline Masters
    - Smart Lessons
    - Animations
    - Dynamic Activities

Notes

SCO RF03 Students will be expected to demonstrate an understanding of slope with respect to rise and run, line segments and lines, rate of change, parallel lines, and perpendicular lines. [PS, R, V]			
[**C**] Communication	[**PS**] Problem Solving	[**CN**] Connections	[**ME**] Mental Mathematics and Estimation
[**T**] Technology	[**V**] Visualization	[**R**] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have met the corresponding specific curriculum outcome.

RF03.01 Determine the slope of a line segment by measuring or calculating the rise and run.
RF03.02 Classify lines in a given set as having positive or negative slopes.
RF03.03 Explain the meaning of the slope of a horizontal or vertical line.
RF03.04 Explain why the slope of a line can be determined by using any two points on that line.
RF03.05 Explain, using examples, slope as a rate of change.
RF03.06 Draw a line, given its slope and a point on the line.
RF03.07 Determine another point on a line, given the slope and a point on the line.
RF03.08 Generalize and apply a rule for determining whether two lines are parallel or perpendicular.
RF03.09 Solve a contextual problem involving slope.

Scope and Sequence

Mathematics 9	**Mathematics 10**	**Mathematics 11**
PR02 Students will be expected to graph linear relations, analyze the graph, and interpolate or extrapolate to solve problems.	**RF03** Students will be expected to demonstrate an understanding of slope with respect to rise and run, line segments and lines, rate of change, parallel lines, and perpendicular lines.	**M01** Students will be expected to solve problems that involve the application of rates.

Background

In previous grades, students explored linear relations. While they have sketched graphs of linear relations using a table of values (8PR01, 9PR02), this is the first formal opportunity students have to study slope in a mathematics course. However, slope or steepness is a common day-to-day concept, and students may also have been introduced to slope in science class.

Note: It is important not to introduce students to the equation of a line by talking about $y = mx + b$. This rote use of a formula tends to promote memorization rather than understanding.

In this unit, students will connect the concept of slope to the idea of measuring the rate of change. They will determine the slope of line segments and explore the slopes of parallel and perpendicular lines.

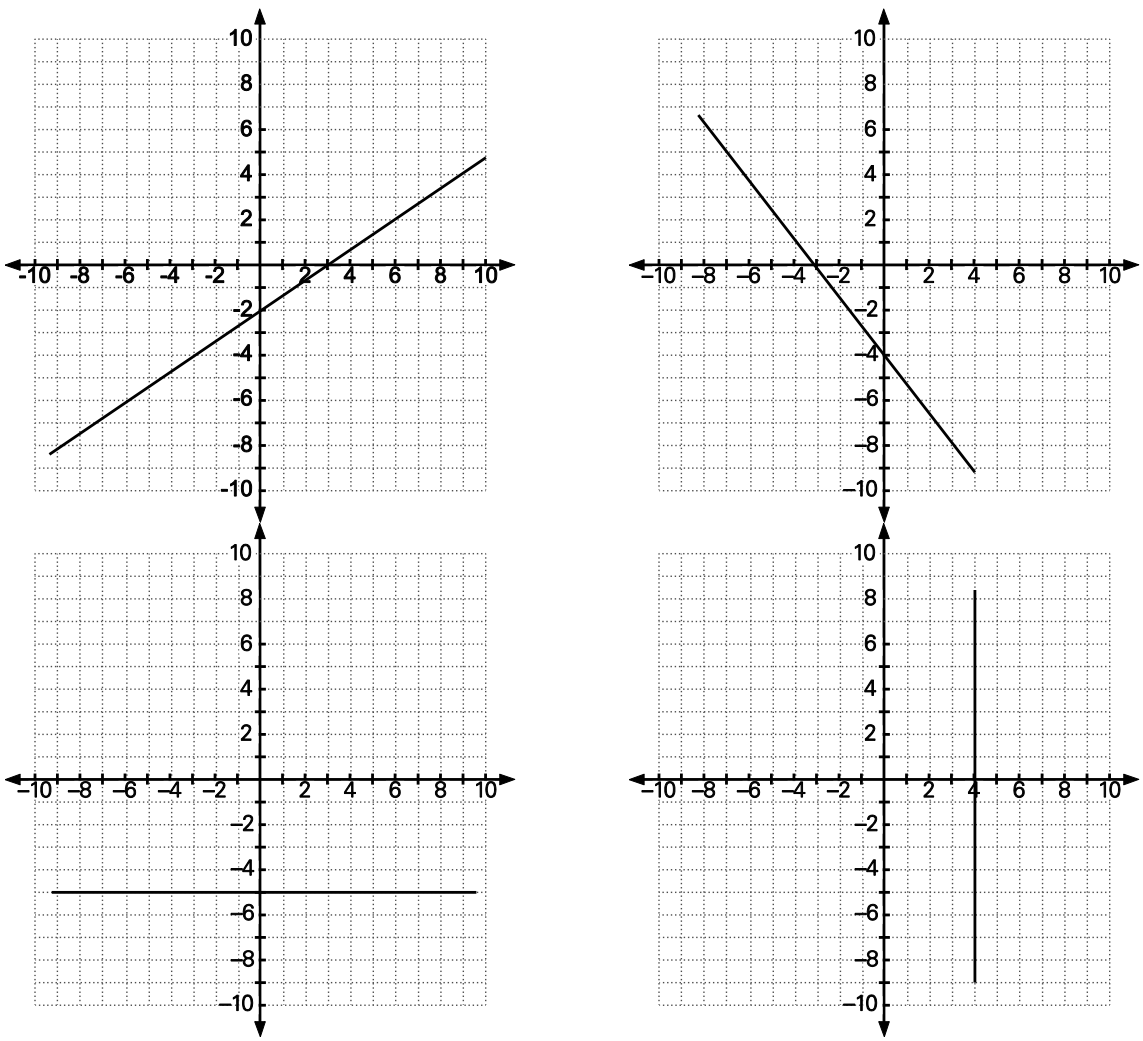
For this outcome, students should understand and be proficient using different methods for finding slope of a line.

On a graph, slope can be determined by finding the $\frac{\text{rise}}{\text{run}}$ or the $\frac{\text{vertical change}}{\text{horizontal change}}$.

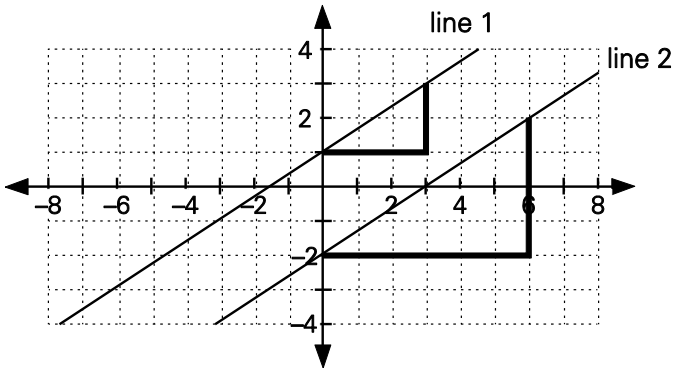
As a rate of change slope can be expressed as $\frac{\text{change in } y}{\text{change in } x}$ or $\frac{\Delta y}{\Delta x}$.

As an algorithm slope can be determined using $\frac{y_2 - y_1}{x_2 - x_1}$ or as $\frac{y_1 - y_2}{x_1 - x_2}$.

Students were exposed to vertical and horizontal lines in Mathematics 9 (9PR02). They recognized the equations of a vertical line ($x = a$) and horizontal line ($y = b$), as well as the corresponding graphs. This will now be extended to include the slope of vertical and horizontal lines. Students will be expected to determine the slope of a line in cases in which there is no change in x or there is no change in y.
On graphs, students should quickly be able to identify slopes as positive, negative, zero, or undefined.



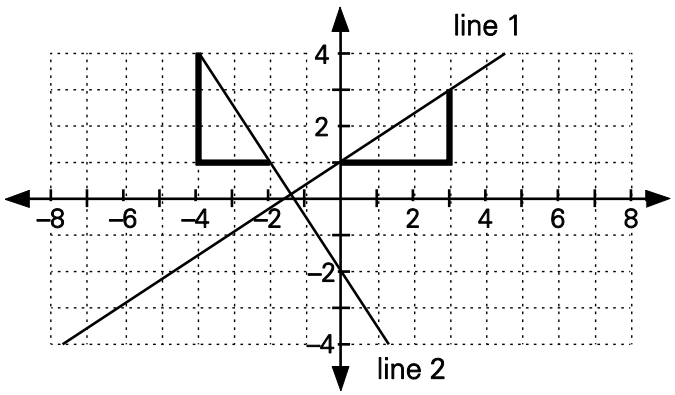
In this unit, students will discover that the slopes of parallel lines are equal and that the slopes of perpendicular lines are negative reciprocals of each other. For example,



Line 1 has a slope of $\frac{2}{3}$.

Line 2 has a slope of $\frac{4}{6}$.

Since $\frac{2}{3} = \frac{4}{6}$ these two lines are parallel.



Line 1 has a slope of $\frac{2}{3}$.

Line 2 has a slope of $-\frac{3}{2}$.

Since $\frac{2}{3}$ is the negative reciprocal of $-\frac{3}{2}$, these two lines are perpendicular.

Students will relate the change in the independent variable, *x* (run), and the change in the dependent variable, *y* (rise), to slope.

Students can determine the rate of change from a graph by determining the changes in the independent and dependent variables. The rate of change should be linked to the steepness and direction of the line. They will then make the connection that the rate of change is actually the slope of a line. It provides information about how one quantity changes with respect to a second quantity.

Students will be able to draw a line when given its slope and one point of the line. They will be able to use slope and a point in conjunction with interpolation or extrapolation to determine a second point on the line.

A solid understanding of slope as a concept rather than just a calculation is necessary as a foundation for the discussions of slope that are part of further mathematical study.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

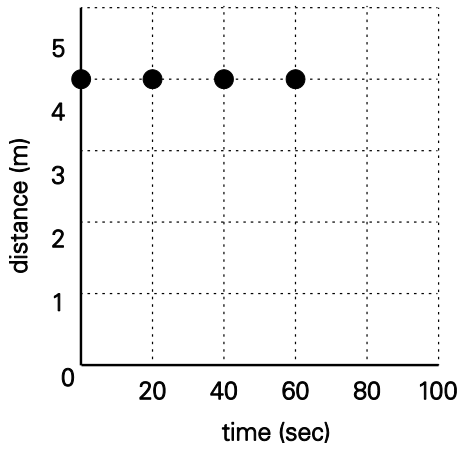
Student tasks such as the following could be completed to assist in determining students' prior knowledge.

- A taxi cab charges the rates shown in the following table.

Length of trip (km)	5	10	15
Total cost ($)	9.25	15.50	21.75

(a) Plot these points on a coordinate grid.
(b) Discuss if these points should be joined.
(c) Explain why the graph does not start at the origin.
(d) From the graph, find the length of a trip that costs $25.
(e) From the graph, find the cost of a 12-km trip.

- Give students the following graph and have them complete the activities below.

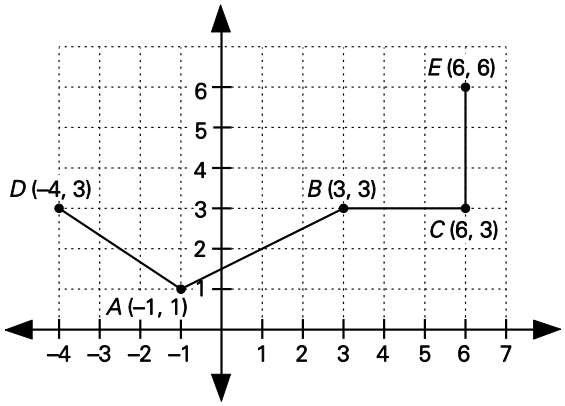


(a) Create a table of values.
(b) Describe the pattern found in the graph.
(c) Describe a situation that the graph might represent.
(d) Write an equation that represents this situation.

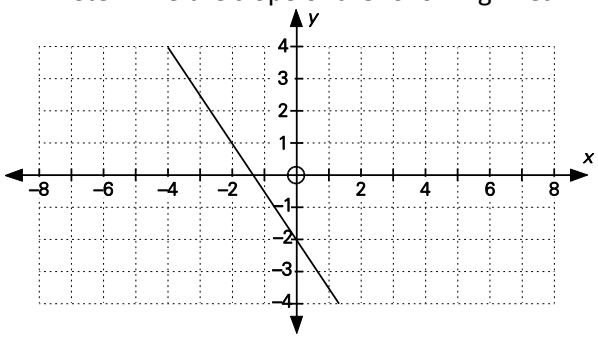
WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

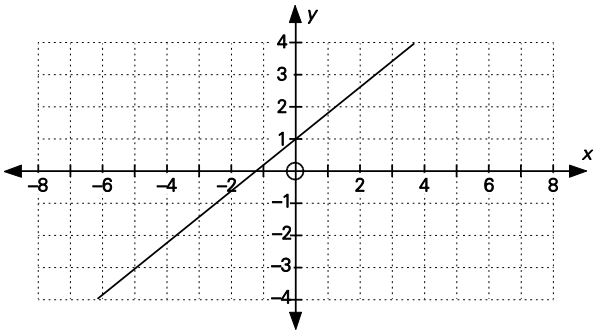
Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

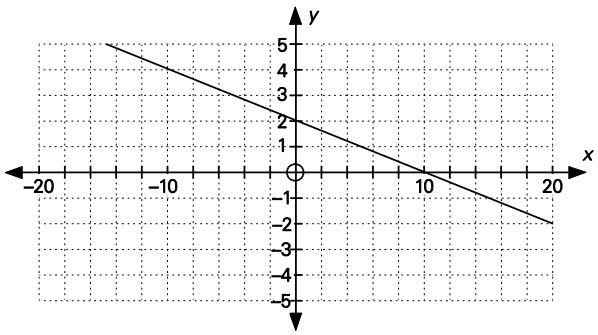
- For the diagram shown below, have students identify which segment has a positive slope, a negative slope, a slope of 0, or a slope that is undefined.

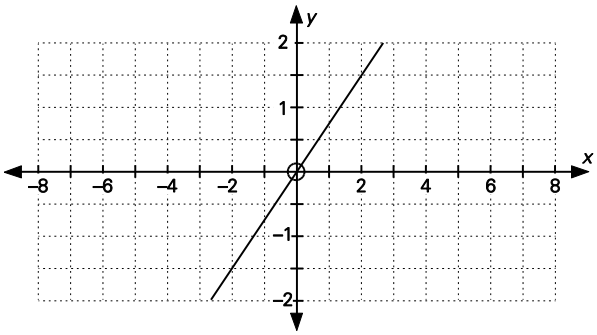


- Determine the slope of the following lines.



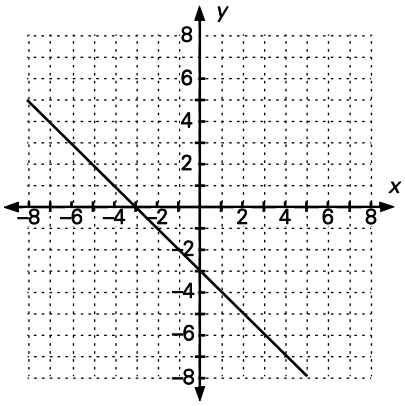


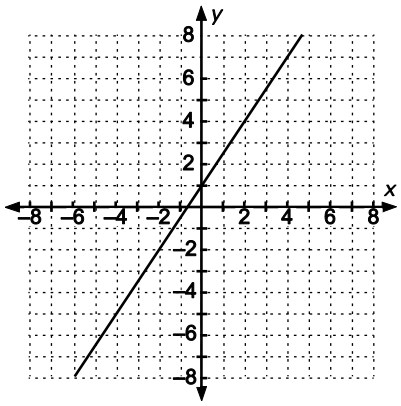




- Create a collage of everyday objects that involve the concept of slope (possible examples include mountains, wheelchair ramps, highway ramps, and so on).

- Determine if each of the following lines has a positive or negative slope.





- Create a brochure, mural, poster, drawing, slideshow, flash animation, or comic that does the following:
  - Outlines the process for determining the slope of a line and classifies lines as having positive and negative slopes.
  - Illustrates your understanding of slope with respect to rise and run and your understanding of positive and negative slopes.

- Explain why the slope of a horizontal line is 0 and why the slope of a vertical line is undefined.

- Draw a line, given the following information, and explain how to use the slope to locate two additional points on each line.
  (a) Passing through (–2, 5) and having a slope of 2.
  (b) Passing through (1, 4) and having a slope of $-\frac{4}{5}$.
  (c) Passing through (–2, –5) and having a slope of $\frac{2}{3}$.
- If $A(-3, -1)$, $B(0, y)$, and $C(3, -9)$ are collinear (lie on the same line), find the value of y.

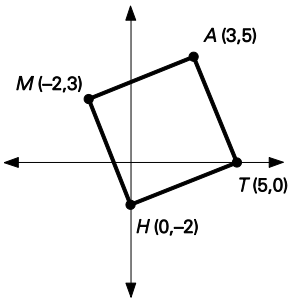
- Determine the slope of the segment joining the following points:
  (a) (–1, 6) and (4, 8) (b) (–6, –4) and (–4, 6)

- Marine Park in Pouch Cove, Newfoundland, is the home of two giant waterslides that measure 40 ft. high by 200 ft. long. Determine the slope of the slides.

- The slope of AB is $-\frac{4}{5}$. The slope of CD is $\frac{w}{35}$. Given $AB \parallel CD$, determine the value of w.

- A line with slope –2 passes through the points $(9, 3x)$ and $(5, 2x)$. Find the value of x.

- Prove that MH and AT are parallel in the quadrilateral shown to the right.



FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should inform instruction.

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SUGGESTED LEARNING TASKS

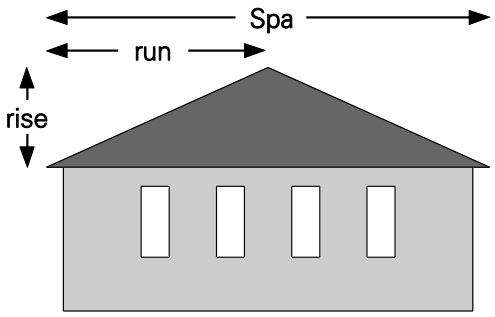
Effective instruction should consist of various strategies.

Guiding Question

- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

Consider the following sample instructional strategies when planning lessons.

- Provide photos of slopes in everyday life. Possible examples include the grade of land, ski hill, roof of a building, wheelchair ramp, or stairway. Ask students why slope is important. The relationship of rise to vertical and run to horizontal can be explored here leading to the overall concept of slope being $\frac{\text{rise}}{\text{run}}$.



- Provide a grid with various slopes labelled. Have students calculate the individual slopes and discuss the concept of positive, negative, and undefined slopes.

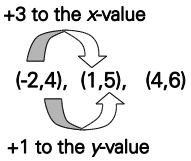
- Encourage students to explore real-life applications involving the slope of a line or line segment. For example, students could investigate the steepness of a roof, the incline on a treadmill, or a wheelchair ramp at their school. Proficiency in determining the slope of an oblique line is recommended prior to discussing the slopes of vertical and horizontal lines.

- When determining slope, a common student error occurs when scales are ignored and students simply count the blocks on a grid. Students should first identify the scale on each axis prior to determining the slope. Another error occurs when students interpret the slope as a ratio of run to rise. This would be a good opportunity to discuss the reasonableness of their answer. Slope represents the steepness of a line. When comparing more than one line, the steeper line should have the greater slope; if students are using $\frac{\text{rise}}{\text{run}}$, this will not be the case.

- Have students draw a line given a specific slope. Compare between students to demonstrate that not all lines with a common slope will be in the same position on the graph. Students should recognize that only one line segment can be drawn with a given slope and point.

- Encourage students to explain why the slope of a horizontal line is zero and the slope of a vertical line is undefined. They should reason that for a horizontal line the rise is 0. Since the slope is rise over run, the value of the ratio is 0 to change in *y*. They should also reason that for a vertical line, the run is 0 since there is no change in *x*. The slope of a vertical line is undefined since division by zero is undefined.

- If students have been given the slope and a point on the line and are using that information to determine another point on a line, they can use patterning or a graph. For example, when drawing a line that has a slope of $\frac{1}{3}$ passing through the point (–2, 4), students may approach this problem using patterns.

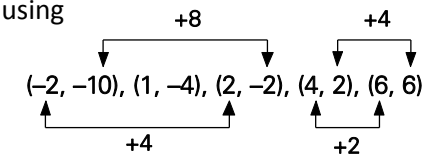


  Students may consider plotting the given point on a coordinate grid and using the slope to locate other points. In the above example, they would plot the point (–2, 4) and locate other points by moving up 1 unit and right 3 units. It is important for students to recognize that a slope of $\frac{1}{3}$ is equivalent to $\frac{-1}{-3}$. Therefore, they could also locate other points by moving down 1 unit and left 3 units. This is particularly important when students explore systems of equations.

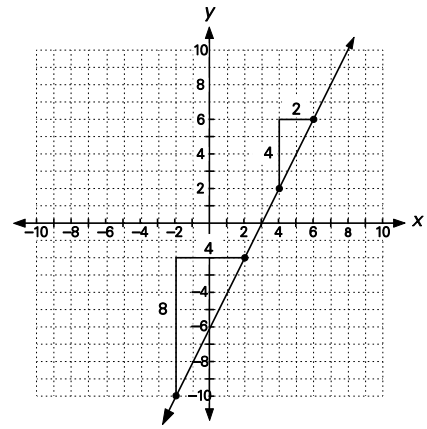
  Note: It is important to use the words **left** or **right** and *not* use **over** when describing horizontal motion. To say to move over three units, for example, does not tell what direction the student should be moving.

- Drawing lines with a given slope would be a good opportunity to encourage students to verify the slope value and the direction of the line. A positive slope, for example, indicates an increase in *y* as *x* increases, resulting in a line slanting upward from left to right. A common error occurs when dealing with negative slopes; students often interpret $-\frac{1}{3}$ as $\frac{-1}{-3}$, in which the negative applies to both the rise and the run.

Students should be encouraged to determine the slope of a line using several pairs of non-consecutive points. Students can compare slopes using ordered pairs or they can use a graph. Consider the examples shown on the right:



Using both examples, students should conclude that the slope is constant (the same), regardless of which two points on the line they examine. Therefore, the slope of the line is the slope of any segment of the line.



- Once students have defined the slope of a line as the ratio of the change in *y* to the change in *x*, this definition will be extended to derive the slope formula. To find the vertical and horizontal distances between two points, students can use a graph or they can subtract the coordinates of the two points, namely (x_1, y_1) and (x_2, y_2). Students will be exposed to the change in the *y*-value as the difference between the *y*-coordinates. Similarly, the change in the *x*-value is the difference between the *x*-coordinates. This will lead them to the slope formula:

$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$

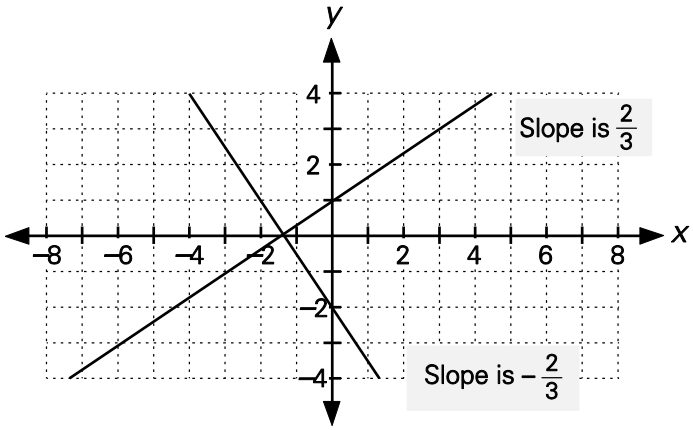
- Students often question which point should be labelled (x_1, y_1). The subscripts indicate that you have two points to work with. Students decide which point to label as point 1 and which to label as point 2. When using the slope formula, the important thing is to find the difference in the *x*-value and the difference in the *y*-value in the same order for each subtraction. For this reason, it is important that you vary which point you decide to use as (x_1, y_1).

- Students will use slope to determine whether two lines are parallel, perpendicular, or neither. It is important to explore and make the connection that parallel lines have equal slopes and perpendicular lines have negative reciprocal slopes. An activity that will work here is to have students draw three lines that are parallel. They should then find and compare the slope of each given the formula.
- To investigate the slopes of perpendicular lines, provide three sets of lines that are perpendicular, or have students construct sets of perpendicular lines and then find and compare the slope of each set.

- When determining the rate of change, encourage students to use units in the calculation. This helps students understand what the rate of change represents. This understanding is critical for further studies of slope.

  Example:
  - When a casserole is placed in the oven, the temperature of the casserole increases as time goes by, thus the rate of change could be “degrees Celsius/minute.”

- If additional workers are added to a shift at a factory making strawberry boxes, the production rate of change could be “number of boxes/worker.”
- When a person is climbing a hilly trail, the distance travelled might be compared to the height gained so that the rate of change would be “m/metre” or “km/metre.”
- When working with perpendicular lines, students could also verify that the product of the slopes is –1. Consider the following example.

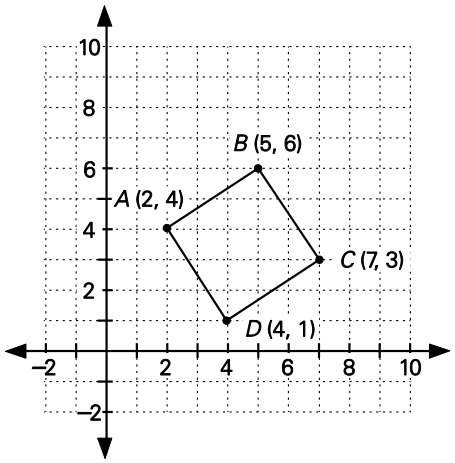


The product of $\frac{2}{3}$ and $-\frac{3}{2}$ is –1.

- Have students extend this concept of perpendicular lines to include vertical and horizontal lines.

- Have students work in groups of two or more for the following activities.
  - Each group should be given 12 cards with graphs on them. Students work together to determine if the graphs have a positive or negative slope.
  - Working in pairs, take a deck of 26 cards—13 cards with a graph and 13 cards with a given slope written on them. The dealer shuffles all the cards and deals them out. Students match the graph with the correct slope. Remove the matches and place them face up on the table. Next, players draw a card from their partner. They should locate the matching graph/slope and add it to their pairs. Students take turns drawing cards from their partner’s hands. The game continues until all matches are made.
  - Each student is given a card with a graph on it. Students should locate a classmate who has a line that is parallel to theirs, and then locate a classmate who has a line that is perpendicular to theirs.

- Problems involving slope can be extended to classify geometric shapes. Students can, for example, determine if a quadrilateral is a rectangle by proving that the opposite sides are parallel and that it also contains a right angle.



Suggested Models and Manipulatives

- graph paper
- protractor
- rulers

MATHEMATICAL VOCABULARY

Students need to be comfortable using the following vocabulary.

- collinear
- constant
- delta notation
- horizontal change
- negative reciprocals
- parallel
- perpendicular
- slope
- vertical change

Resources/Notes

Print

- *Foundations and Pre-calculus Mathematics 10* (Burglind et al., Pearson 2010)
  - Student Book
    - Chapter 6, Sections 1 and 2, pp. 330–353
    - Chapter 5, Sections 6 and 7, pp. 300–323
  - Teacher Technology DVD
    - Teacher Resource
    - Blackline Masters
    - Smart Lessons
    - Animations
    - Dynamic Activities

Notes

SCO RF04 Students will be expected to describe and represent linear relations, using words, ordered pairs, tables of values, graphs, and equations. [C, CN, R, V]			
[**C**] Communication	[**PS**] Problem Solving	[**CN**] Connections	[**ME**] Mental Mathematics and Estimation
[**T**] Technology	[**V**] Visualization	[**R**] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have met the corresponding specific curriculum outcome.

RF04.01 Identify independent and dependent variables in a given context.
RF04.02 Determine whether a situation represents a linear relation, and explain why or why not.
RF04.03 Determine whether a graph represents a linear relation, and explain why or why not.
RF04.04 Determine whether a table of values or a set of ordered pairs represents a linear relation, and explain why or why not.
RF04.05 Draw a graph from a set of ordered pairs within a given situation, and determine whether the relationship between the variables is linear.
RF04.06 Determine whether an equation represents a linear relation, and explain why or why not.
RF04.07 Match corresponding representations of linear relations.

Scope and Sequence

Mathematics 9	**Mathematics 10**	**Grade 11 Mathematics Courses**
PR01 Students will be expected to generalize a pattern arising from a problem-solving context using linear equations and verify by substitution. **PR02** Students will be expected to graph linear relations, analyze the graph, and interpolate or extrapolate to solve problems.	**RF04** Students will be expected to describe and represent linear relations, using words, ordered pairs, tables of values, graphs, and equations.	**RF02** Students will be expected to demonstrate an understanding of the characteristics of quadratic functions, including vertex, intercepts, domain and range, and axis of symmetry. (M11)* **RF02** Students will be expected to graph and analyze absolute value functions (limited to linear and quadratic functions) to solve problems. (PC11)**

* M11—Mathematics 11
** PC11—Pre-calculus 11

Background

Since Mathematics 7, students have solved contextual problems using linear equations. Students have also represented linear data in a variety of forms, such as tables, graphs, equations, or situations.

In Mathematics 9, students used patterns in the tables of values and graphs to recognize that a linear relation occurs when there is a constant change in the independent and dependent variable (9R01). Examples were limited to increments of 1 in the independent variable. This will now be extended to include increments other than 1.

Students previously represented relations in a variety of ways. In this outcome, students will focus on linear relations and will move interchangeably among the various representations.

In Mathematics 10, students will determine if a relationship is linear or not by using a variety of methods: common differences, graph shape, degree of the equation, and creating a table of values. They will learn to identify which variable is independent and which is dependent. Students will find the slope in each of the various forms studied and recognize that, in any linear relationship, the slope is constant, no matter which representation is being used.

Students will determine whether a relation is linear or non-linear. Using a table of values or ordered pairs, they will check to see if the changes in the independent and dependent variable are constant. Linear relations have graphs that are straight lines. Comparison of a table and its graph enables students to recognize that the constant change in the independent variable represents the horizontal change in the graph. Likewise, the constant change in the dependent variable represents the vertical change in the graph. This constant change will be introduced to students as rate of change.

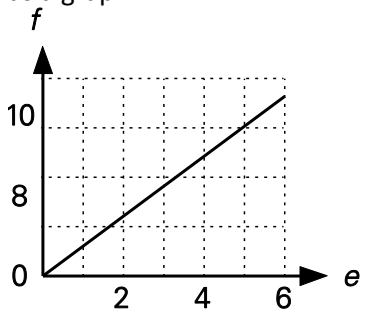
As students work with the various representations of relations, they should develop an understanding of the connections between them. Alternate representations can strengthen students' awareness of symbolic expressions and equations.

For example, exploring one relationship using a variety of representations:

- Suppose that data were gathered comparing the length of an ear compared to the length of a person's face. It was determined that the length of the face was approximately triple that of the ear. This could be expressed
  - *in words*: Three times the length of your ear, *e*, is equal to the length of your face, *f*, from chin to hairline.
  - *as an equation*: $f = 3e$
  - *as a set of ordered pairs*: (4, 12), (4.5, 13.5), (5, 15), (5.5, 16.5), (6, 18), (6.5, 19.5)
  - as a Table of Values:

Ear Length, *e* (cm)	**Face Length, *f* (cm)**
4.0	12.0
4.5	13.5
5.0	15.0
5.5	16.5
6.0	18.0
6.5	19.5

  - as a graph:



In Mathematics 9, students graphed linear relations represented as equations by creating a table of values. For a linear relation, they recognized that a constant change in the independent variable resulted in a constant change in the dependent variable. Encourage students to distinguish between linear and non-linear equations without graphing. Students can also explore whether equations are linear, for example, by observing the degree of the equation.

Students will draw the conclusion that there are a number of ways to determine whether a relation is a linear relation or a non-linear relation.

- Linear relations have graphs that are straight lines.
- In the table of values of a linear relation, values of *y* increase or decrease by a constant amount as values of *x* increase or decrease by a constant amount.
- When a linear relation is written as an equation, it will contain one or two variables, and there will be no term with a degree higher than one.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Student tasks such as the following could be completed to assist in determining students' prior knowledge.

- Given the equation, $y = 2x + 5$, describe this relation in words. Make up a problem that could be solved using this equation.

- Write a linear equation to represent the pattern in the given table of values.

Time (s)	0	1	2	3	4
Height (m)	2	3.5	5	6.5	8

(a) Describe a context for the equation.
(b) How would you describe the slope in this situation?

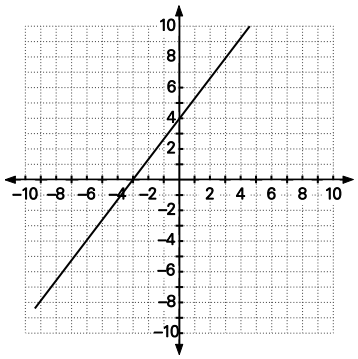
WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

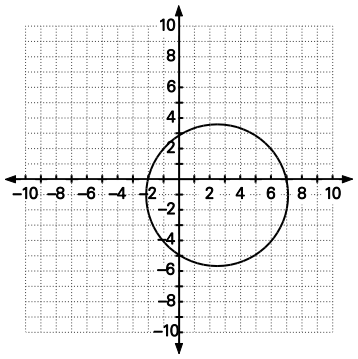
Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

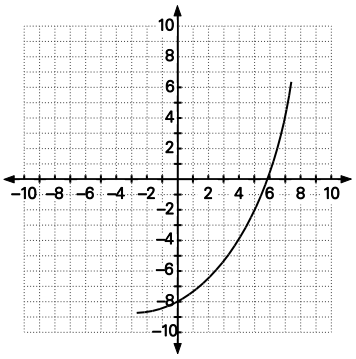
- Using the stocks page from the newspaper (either that the teacher has posted on the wall or by using the Internet to access the TSX), pick a stock company of your choice and then record and graph the value of the stock over a one-month period. Use your graph to determine
  - linear and/or non-linear sections
  - the slope of the line of best fit for the linear sections (determined by eye)

  Discuss with your classmates what factors might affect the value of the stock and relate this to current events. (Alternatively, students could look at the history of that stock over the previous month and extrapolate data to determine if they should invest in this stock by studying slopes).

  Which of the following relations are linear?







Time (months)	Height (inches)
0	22
6	23
12	25
18	27
24	32

Price ($)	Taxes ($)
10	1.50
60	9.00
110	16.50
160	24.00
210	31.50

Area (sq. ft.)	Cost ($)
100	600
150	900
200	1200
225	1350
250	1500

- Which of the following relations are linear?
  (a) {(2, 10), (4, 15), (6, 20), (8, 25), (10, 30), 12, 35)}
  (b) {(0, 1), (20, 2), (40, 4), (60, 8), (80, 1), (100, 32)}
  (c) $x^2 - 5x + 3 = y$
  (d) $x + 5 = 13$
  (e) $y = 23$
  (f) $5 + x^3 = 2x + 1$
  (g) $y = 3x + 12$

- Create a table of values for each of the following functions, graph the data, and determine if the function represents a linear relation.
  (a) $y = -4x + 7$
  (b) $d = t^2 + t - 2$
  (c) $g = 0.5t + 8$

- Determine if the following situations represent a linear relation and explain your reasoning.
  (a) A local taxi company charges a flat rate of $3.75 plus $0.75 per kilometre.
  (b) Paola is using blocks to build a tower. She starts with three blocks and adds two blocks to get each successive tower.
  (c) An investment increases in value by 10% each year.

FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should inform instruction.

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SUGGESTED LEARNING TASKS

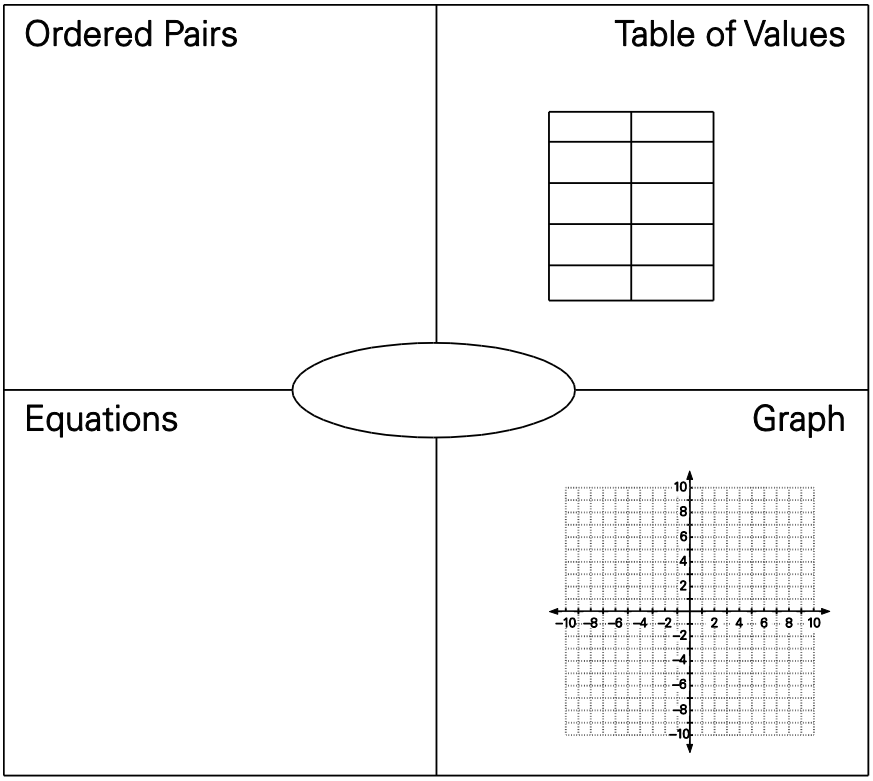
Effective instruction should consist of various strategies.

Guiding Question
- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

Consider the following sample instructional strategies when planning lessons.

- Provide students with data from various real-life situations and have them determine which relations are linear and which are non-linear. Have them explain what this means.

- Provide the students with one of the following: an equation, a set of ordered pairs or table of values, a graph, or a word description of a situation. Have the students generate the other three representations. For example,
  - given the equation $y = 3x - 5$, create a table of values or a set of ordered pairs, a graph, and a word description of a situation described by the equation
  - An organizer such as the one shown below could be used. (See Appendix B for a copyable version.)



- Give students a list of situations and relationships, providing time for them to work together to determine if the situation would be a linear or non-linear relation. For example, the relationship between step length and distance travelled is linear, but the relationship between length of sides and area of a square is non-linear.

- Have students compare linear equations and non-linear equations. Provide linear and non-linear equations and have students complete a table of values as well as a graph for each. For example, equations could include

  $y = x + 2$ $\quad$ $y = 6x - 7$ $\quad$ $y = x$ $\quad$ $y = x^2 + 2x + 1$ $\quad$ $y = x^2$

  Technology may also be used to illustrate the graphs.

Suggested Models and Manipulatives

- graph paper

Mathematical Vocabulary

Students need to be comfortable using the following vocabulary.

- common difference
- degree
- dependent variable
- independent variable
- linear

Resources/Notes

Print

- *Foundations and Pre-calculus Mathematics 10* (Burglind et al., Pearson 2010)
  - Student Book
    - Chapter 5, Sections 2, 5, and 6, pp. 264–273, 287–310
  - Teacher Technology DVD
    - Teacher Resource
    - Blackline Masters
    - Smart Lessons
    - Animations
    - Dynamic Activities

Internet

- EducationWorld, Connecting to Math in Real Life (Petti 2013)
  www.educationworld.com/a_curr/mathchat/mathchat019.shtml
  Data sets.

Notes

SCO RF05 Students will be expected to determine the characteristics of the graphs of linear relations, including the intercepts, slope, domain, and range. [CN, PS, R, V]			
[**C**] Communication [**T**] Technology	[**PS**] Problem Solving [**V**] Visualization	[**CN**] Connections [**R**] Reasoning	[**ME**] Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have met the corresponding specific curriculum outcome.

RF05.01 Determine the intercepts of the graph of a linear relation, and state the intercepts as values or ordered pairs.
RF05.02 Determine the slope of the graph of a linear relation.
RF05.03 Determine the domain and range of the graph of a linear relation.
RF05.04 Sketch a linear relation that has one intercept, two intercepts, or an infinite number of intercepts.
RF05.05 Identify the graph that corresponds to a given slope and y-intercept.
RF05.06 Identify the slope and *y*-intercept that correspond to a given graph.
RF05.07 Solve a contextual problem that involves intercepts, slope, domain, or range of a linear relation.

Scope and Sequence

Mathematics 9	**Mathematics 10**	**Grade 11 Mathematics Courses**
PR02 Students will be expected to graph linear relations, analyze the graph, and interpolate or extrapolate to solve problems.	**RF05** Students will be expected to determine the characteristics of the graphs of linear relations, including the intercepts, slope, domain, and range.	**RF02** Students will be expected to demonstrate an understanding of the characteristics of quadratic functions, including vertex, intercepts, domain and range, and axis of symmetry. (M11)* **RF03** Students will be expected to analyze quadratic functions of the form $y = a(x - p)^2 + q$ and determine the vertex, domain and range, direction of opening, axis of symmetry, *x*- and y-intercepts. (PC11)** **RF04** Students will be expected to analyze quadratic functions of the form $y = ax^2 + bx + c$ to identify characteristics of the corresponding graph, including vertex, domain and range, direction of opening, axis of symmetry, and *x*- and *y*-intercepts and to solve problems. (PC11)

Mathematics 9	Mathematics 10	Grade 11 Mathematics Courses
		RF05 Students will be expected to solve problems that involve quadratic equations. (PC11) **M01** Students will be expected to solve problems that involve the application of rates. (M11)

* M11—Mathematics 11
** PC11—Pre-calculus 11

Background

Students had experience interpolating or extrapolating information to solve problems using linear relations in Mathematics 9.

For this outcome, students will identify and express intercepts as values or ordered pairs (for example, either *y*–intercept of 2 or (0, 2) could be used to express the same intercept). Students will also explain what the intercepts represent when given the graph. Students will graph linear relations when given one intercept and slope, or when given both intercepts.

Students should work with situations wherein the rate of change is positive, negative, or zero.

Students can use graphs to determine the coordinates of the *x*- and *y*-intercepts. Ensure that students represent the horizontal and vertical intercepts as ordered pairs, (x, 0) and (0, y) respectively. Encourage students to explain what a horizontal and vertical intercept means in a contextual problem. For example, the vertical intercept may be the initial value in a context and the horizontal intercept may be the end value in a context.

Students will apply their mathematical learning about relations to determine the characteristics of a linear relation. They will revisit domain and range as it applies to linear graphs. Students will also use rate of change and vertical intercepts to identify graphs and vice versa. This work should help strengthen the connections between the various representations and the rate of change and vertical intercepts. Students should recognize, for example, that if the rate of change is positive, the line segment will slant upward to the right.

Students have now been exposed to rate of change and the vertical intercept of a linear relation. This would be a good opportunity to make a connection between a linear graph and its equation within the context of a problem. Students will not be exposed to the slope-intercept form until the next section.

Note: It is important not to introduce students to the equation of a line by talking about $y = mx + b$. This rote use of a formula tends to promote memorization rather than understanding.

A sketch of a linear relation may have one, two, or an infinite number of intercepts. Encourage students to explore the three possibilities by sketching graphs and discussing their findings. A line that lies on an axis, for example, has an infinite number of intercepts with that axis. A horizontal or vertical line that does not lie on an axis has only one intercept. An oblique line has two intercepts (one *x*-intercept and

one *y*-intercept). When students are provided with a visual representation of the situations, students gain a better understanding of the concept.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Student tasks such as the following could be completed to assist in determining students' prior knowledge.

- You have just purchased a new cell phone. The phone plan costs $10 per month and $0.10 per minute. Create a graph to represent the situation. Using the graph, estimate the cost of sending 100 text messages.

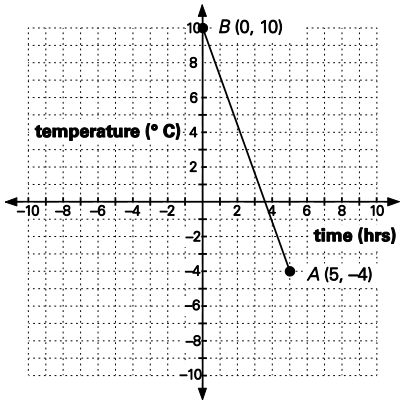
- When Nelson is driving his all-terrain vehicle (ATV) on a long distance off-road trek, he observes the amount of gas remaining in his tank over a period of time.

Distance travelled (km)	50	100	150	200	250
Amount of gas remaining in tank (L)	18	14	10	6	2

  (a) Determine the amount of gas in the tank when Nelson began his trek.
  (b) How far can Nelson travel before he runs out of gas?

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.



- The graph to the right represents the temperature of sea water placed in a freezer. Explain the meaning of the slope, what Point *A* represents, what the *x*- and *y*-intercepts represent, and the domain and range.

  Note: The slope represents the rate at which the temperature is decreasing. The *y*-intercept represents the temperature of the water when it is placed in the freezer. The *x*-intercept indicates the

temperature after 4 hours. Point *A* represents the time and temperature at which the water begins to freeze. (Sea water freezes at –4°C.)

- Jian has a monthly cell phone plan represented by the equation $C(n) = 0.15n + 25$ where C is the total charge and n represents the number of text messages sent and received.
  (a) Explain why the equation represents a linear relation.
  (b) State the rate of change. What does it represent?

- Parvana wants to sell her car. The cost to place an advertisement in the newspaper is $15.30. This includes three lines of text and a picture. Each additional line of text would cost $2.50. Write an equation to represent the linear function that would represent this situation.

- Many students participate in the 5 km race for charity. Adrienne donates $30 of her own money. She also collects $20/km run in pledges.
  (a) Determine an appropriate domain and range for this situation, and use a table of values to graph the function.
  (b) Do either the *x*-intercept or *y*-intercept have any meaning in this situation? Explain.
  (c) Write the function as a linear equation in two variables.

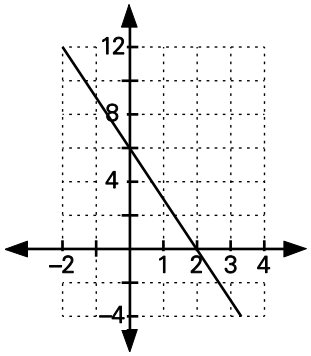
- Determine the *x*- and *y*-intercepts for each of the following:
  (a) $2x - 3y = 12$
  (b) $3x - 24 = 8y$
  (c) $5x - 2y + 12 = 0$

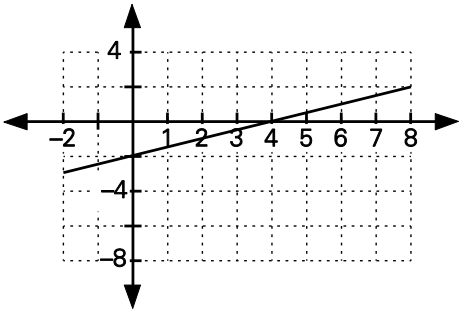
- When Marie is driving her truck on a long distance off-road trek, she observes the amount of gas remaining in her tank over a period of time.

Distance travelled (km)	50	100	150	200	250
Amount of gas remaining in tank (L)	35	30	25	20	15

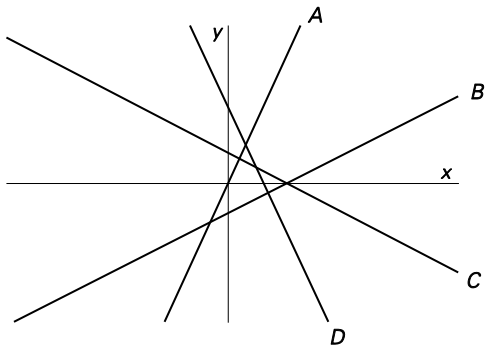
  (a) Determine the *x*-intercept for this data and explain its meaning.
  (b) Determine the *y*-intercept for this data and explain its meaning.

- State the slope and intercepts for each of the following:

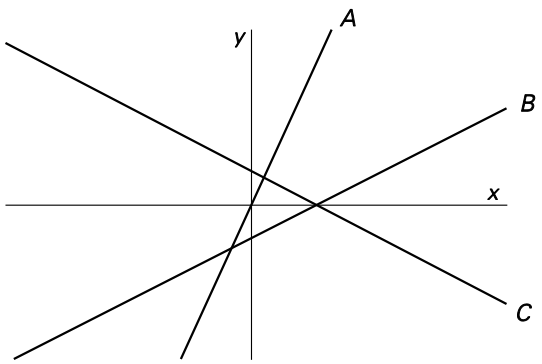




- The graphs shown below have slopes of 2, $\frac{1}{2}$, $-\frac{1}{2}$, and –2. Match the slope to each of the graphs.



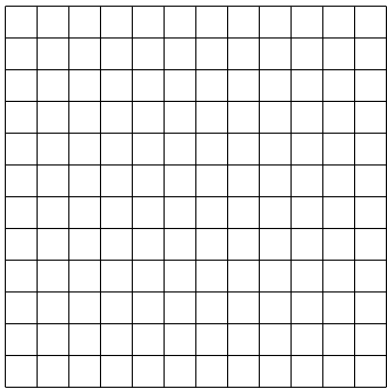
- The graphs shown below have *y*-intercepts of 2, 0, and –2. Match the *y*-intercept to each of the graphs.



- A hot tub contains 1600 L of water and is emptied at a constant rate of 20 L per minute.

(a) Complete the table of values and sketch a graph to represent the relation.
(b) Determine the *y*-intercept. What does this value represent in this context?
(c) Determine the *x*-intercept. What does this value represent in this context?
(d) Determine the slope. What does this value represent in this context?
(e) State the domain and range and explain their importance in terms of the context of situation.

Time (min)	Volume (L)
0	
20	
40	
60	
80	
100	



FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should inform instruction.

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?

- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SUGGESTED LEARNING TASKS

Effective instruction should consist of various strategies.

Guiding Question

- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

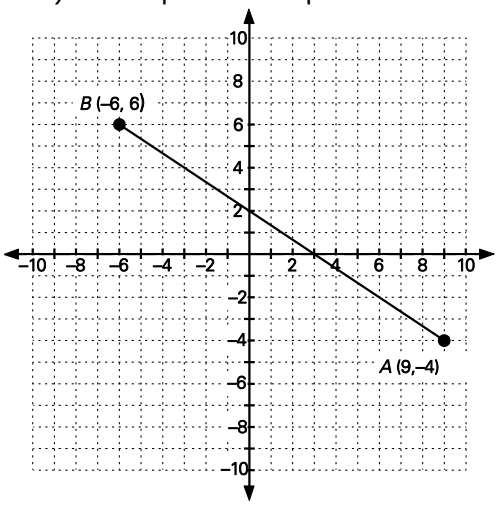
Consider the following sample instructional strategies when planning lessons.

- A quick review of the various ways to determine slope would be advisable. Students could be presented with graphs of linear equations and be required to state the slopes, intercepts, and the domain and range. When expressing domain and range that contain end points, students should express domain and range in either set and/or interval notation:

  Domain: $\{x \mid -6 \leq x \leq 9, x \in \mathrm{R}\}$ and $[-6, 9]$

  Range: $\{y \mid -4 \leq y \leq 6, y \in \mathrm{R}\}$ and $[-4, 6]$

  The x-intercept can be expressed as stating, x-intercept at 3 or (3, 0).
  The y-intercept can be expressed as stating, y-intercept at 2 or (0, 2).



- Students sometimes mistakenly identify the vertical intercept as that on the *x*-axis. Reinforce with students that vertical axis is the *y*-axis and horizontal axis is the *x*-axis.

- Students should be provided with examples in which characteristics of linear relations are applied to real-life situations. When students see a connection between their mathematical learning and real life, their understanding is enhanced. Furthermore, they should see examples in which both intercepts are meaningful and other examples where only one of the intercepts makes any sense in context.

SUGGESTED MODELS AND MANIPULATIVES

- graph paper

MATHEMATICAL VOCABULARY

Students need to be comfortable using the following vocabulary.

- intercept
- *x*-intercept
- *y*-intercept

Resources/Notes

Print

- *Foundations and Pre-calculus Mathematics 10* (Burglind et al., Pearson 2010)
  - Student Book
    - Chapter 5, Section 7, pp. 311–323
    - Chapter 6, Sections 1, 4, 5, and 6, pp. 330–343, 357–387
  - Teacher Technology DVD
    - Teacher Resource
    - Blackline Masters
    - Smart Lessons
    - Animations
    - Dynamic Activities

Notes

SCO RF06 Students will be expected to relate linear relations to their graphs, expressed in ▪ slope-intercept form ($y = mx + b$) ▪ general form ($Ax + By + C = 0$) ▪ slope-point form ($y - y_1) = m(x - x_1$) [CN, R, T, V]			
[**C**] Communication [**T**] Technology	[**PS**] Problem Solving [**V**] Visualization	[**CN**] Connections [**R**] Reasoning	[**ME**] Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have met the corresponding specific curriculum outcome.

RF06.01 Express a linear relation in different forms, and compare the graphs.
RF06.02 Rewrite a linear relation in either slope-intercept or general form.
RF06.03 Generalize and explain strategies for graphing a linear relation in slope-intercept, general or slope-point form.
RF06.04 Graph, with and without technology, a linear relation given in slope-intercept, general, or slope-point form, and explain the strategy used to create the graph.
RF06.05 Identify equivalent linear relations from a set of linear relations.
RF06.06 Match a set of linear relations to their graphs.

Scope and Sequence

Mathematics 9	**Mathematics 10**	**Pre-calculus 11**
PR02 Students will be expected to graph linear relations, analyze the graph, and interpolate or extrapolate to solve problems. **PR03** Students will be expected to model and solve problems where a, b, c, d, e, and f are rational numbers, using linear equations of the form ▪ $ax = b$ ▪ $\frac{x}{a} = b, a \neq 0$ ▪ $ax + b = c$ ▪ $\frac{x}{a} + b = c, a \neq 0$ ▪ $ax = b + cx$ ▪ $a(x + b) = c$ ▪ $ax + b = cx + d$ ▪ $a(bx + c) = d(ex + f)$ ▪ $\frac{a}{x} = b, x \neq 0$	**RF06** Students will be expected to to relate linear relations to their graphs, expressed in ▪ slope-intercept form ($y = mx + b$) ▪ general form ($Ax + By + C = 0$) ▪ slope-point form ($y - y_1) = m(x - x_1$)	**RF06** Students will be expected to solve, algebraically, and graphically, problems that involve systems of linear-quadratic and quadratic-quadratic equations in two variables. **RF07** Students will be expected to solve problems that involve linear and quadratic inequalities in two variables.

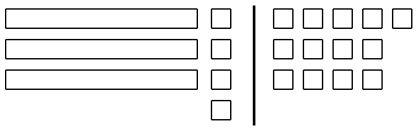
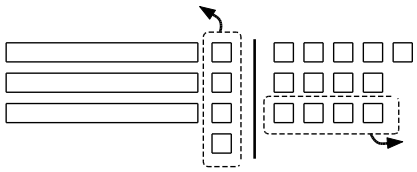
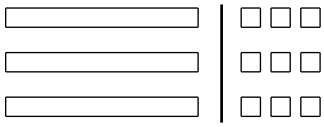
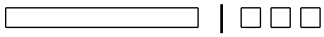
Background

In Mathematics 8, students had experience solving one and two-step equations in the form of

- $ax = b$
- $\frac{x}{a} = b$, $a \neq 0$
- $ax + b = c$
- $\frac{x}{a} + b = c$, $a \neq 0$
- $a(x + b) = c$

In Mathematics 9, students continued to solve equations including integers and rational numbers and model the solution using algebra tiles. These equations were ones where the "unknown" is found on both sides of the equal sign or where the unknown is found in the denominator. In addition, these equations sometimes required more than two steps in order to solve for the unknown.

Models such as algebra tiles should be used to help students develop understanding of the process of solving equations. Students should be used to modelling the solution to a linear equation using algebra tiles and recording the solution symbolically as shown here.

Algebra Tiles	**Algebraic Symbols**
	$3x + 4 = 13$
	$3x + 4 - 4 = 13 - 4$
	$3x = 9$
	$\frac{3x}{3} = \frac{9}{3}$
	$x = 3$

A review of the various methods to solve equations developed in Mathematics 7 and 8 may be necessary prior to teaching this section. These methods include the use of algebra tiles, inspection, and systematic trials (guess and test). In problem-solving situations, it was emphasized that once a solution has been obtained, it should be checked for accuracy by substitution into the original equation.

Proper use of vocabulary should be also modelled. Terms that teachers should use and understand include the following: **relationship**, **equality**, **algebraic equation**, **distributive property**, **like terms**, **balancing**, **the zero principle**, **the elimination process**, **isolating variables**, **coefficient, constant**, and **equation versus expression**.

Linear equations can be rewritten into several different forms. This outcome provides an introduction to three forms of equations for linear relations. Students will learn to identify all three forms of the equation and to change from one form to another.

Students will graph linear relations from each of the three forms of equations. The three forms should not be taught as a process; rather, students should be given the opportunity to discover these concepts and strategies through investigations.

The slope-intercept form for a linear relation is $y = mx + b$ where m is the slope of the line and b is the y-intercept. Vertical lines, having undefined slope, are not represented by this form.

When given the slope-intercept form, the linear relation can be graphed using the slope (m) and the y-intercept (b).

Example: $y = 2x + 6$

- The y-intercept is 6, or (0, 6).
- The slope is $2 = \frac{2}{1} = \frac{6}{3}$.
- To find a second point, start at the intercept (0, 6), go right 1 unit and up 2 units to (1, 8) *or* go right 3 units and up 6 units to (3, 12).

Draw a line through (0, 6) and (1, 8) or (3, 12) for the graph of the linear function.

When using the general form, the linear relation can be graphed using the x-intercept and the y-intercept.

Example: $2x + 4y + 6 = 0$

- The x-intercept can be found by letting $y = 0$ and solving for x:

  $2x + 4(0) + 6 = 0$
  $2x + 6 = 0$
  $2x = -6$
  $x = -3$
  Therefore, the x-intercept is –3 or (–3, 0).

- The y-intercept can be found by letting $x = 0$ and solving for y:

  $2(0) + 4y + 6 = 0$
  $4y + 6 = 0$
  $4y = -6$
  $y = -\frac{3}{2}$
  Therefore, the y-intercept is $-\frac{3}{2}$ or $(0, -\frac{3}{2})$.

Draw a line through (–3, 0) and (0, –1.5) for the graph of the linear relation.

When graphing the linear relation using an intercept and one point $2x + 4y + 6 = 0$

- the x-intercept is (–3, 0)
- the slope is $\frac{-1.5-0}{0-(-3)} = \frac{-1.5}{3} = -\frac{1}{2}$ *or* $-\frac{A}{B} = -\frac{2}{4}$ so start at the intercept and go right 4 (add 4 to x) and down 2 (subtract 2 from y) to (1, –2)

Draw a line through (–3, 0) and (1, –2) for the graph of the linear relation.

The slope-point form for a linear equation is $(y - y_1) = m(x - x_1)$ where m is the slope of the line and (x_1, y_1) is any point on the line. This form comes directly from the slope formula. The point-slope form expresses the fact that the difference in the y-coordinate between two points on a line $(y - y_1)$ is proportional to the difference in the x-coordinate $(x - x_1)$. The proportionality constant is m (the slope of the line). Using the slope-point form, the linear relation is drawn by using (x_1, y_1) and the slope (m).

For example: is $(y - 6) = 2(x - 3)$

- When drawing the graph of the linear relation by joining two points
  - one point on the line is (x_1, y_1) or (3, 6)
  - the slope is $2 = \frac{2}{1}$, so to find a second point, start at (3, 6) and go right 1 (add 1 to x_1), and up 2 (add 2 to y_1) to (7, 5)
- Draw a line through (6, 3) and (7, 5) for the graph of the linear relation.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Student tasks such as the following could be completed to assist in determining students' prior knowledge.

- Leah solved the following equation. Check for any errors. If any were made, indicate where and make the necessary changes to correct them.

  $\frac{1}{3}(x - 2) = 5(x + 6)$
  $3(x - 2) = 5(x + 6)$
  $3x - 6 = 5x + 30$
  $3x - 6 + 6 = 5x + 30 + 6$
  $3x - 5x = 5x - 5x + 36$
  $-2x = 36$
  $\frac{-2x}{-2} = \frac{36}{-2}$
  $x = -18$

- Solve and verify.

  (a) $\frac{-5}{x} = -2$

  (b) $\frac{x}{2} - 3 = 1\frac{1}{6}$

  (c) $\frac{m}{3} - \frac{3m}{4} = 10$

  (d) $\frac{1}{2}k - 5 = 4 - k$

- Using tiles, solve the following and record each step algebraically.
  $-6x + 2 = 8x - 12$

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

- Three equations are given:

  $y = 2x + 3$ $\quad$ $2x - y + 3 = 0$ $\quad$ $(y - 7) = 2(x - 2)$

  Without rearranging these equations, graph them. What do you notice?

- Abdul missed the class in which they learned how to graph a linear equation in slope-intercept form. He is trying to graph $y = -0.25x + 6$. Explain how this can be done.

- Explain your strategy for graphing a linear relation such as $y - 4 = 3(x - 7)$.

- Identify the slope and a point on the line for each of the following equations and then graph the resulting line.

  (a) $y = 2x + 1$

  (b) $y = -\frac{1}{3}x + 2$

(c) $y = 4$
(d) $y - 3 = 2(x - 1)$
(e) $2x - 3y + 12 = 0$
(f) $y + 6 = -\frac{1}{4}(x - 5)$
(g) $-3x + 4y + 24 = 0$

- Rewrite each of the following equations in slope-intercept form.
  (a) $(y - 4) = -2(x + 1)$
  (b) $-3x + 6y + 18 = 0$

- Rewrite each of the following equations in general form:
  (a) $y + 1 = \frac{1}{2}(x - 6)$
  (b) $y = 4x - 1$

- The lines $nx + 12y - 2 = 0$ and $3x + ny + 6 = 0$ are parallel. What are the possible values of n?

FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should inform instruction.

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SUGGESTED LEARNING TASKS

Effective instruction should consist of various strategies.

Guiding Question
- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

Consider the following sample instructional strategies when planning lessons.

- Provide students with an opportunity to compare the graph and the linear equation. Initiate questions concerning the changes in m and b and how it affects the graph. The focus is for the students to make the connection that m represents the slope or rate of change and b is the vertical or y-intercept.

- Students will be expected to graph a linear relation given its equation in slope-intercept form and explain the strategy they used to create the graph. Encourage them to plot the y-intercept and then use the slope to generate additional points. Ensure that students make predictions about the appearance of the graph before they actually graph it. For example, if the slope is negative, the graph should slant down to the right. Students should be exposed to technology, such as graphing calculators and other software, when drawing a linear relation.

- Before introducing students to the slope-point or general forms of a linear equation, ensure that students are able to graph a linear equation when given it in slope-intercept form.

- Once students have become comfortable and proficient working with the slope-intercept form of the equation, the slope-point form of a linear equation should be developed by using the slope formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ or $\frac{y - y_1}{x - x_1}$ and multiplying both sides of the equation by $(x - x_1)$ to introduce students to slope-point form of a linear equation, $y - y_1 = m(x - x_1)$.

- Students need to observe that, when graphing a linear relation given its equation in slope-point form, the slope of the line, m, and one point on the line, (x_1, y_1), can be determined directly from the equation. Similar to graphing lines in slope-intercept form, students will plot the point and then use the slope to determine another point on the line. A common student error occurs when a point is incorrectly identified. For example, when given the equation $y - 5 = -2(x + 1)$, students may interpret the given point as $(1, -5)$ or $(5, -1)$. This would be a good opportunity to discuss the operation of subtraction in the slope-point form. Provide students with examples where they match a graph to its corresponding equation and have them justify their choice.

- Before introducing students to the general form of a linear equation, ensure that students are able to graph a linear equation when given it in slope-intercept or in slope-point form.

- Once students have become both comfortable and proficient working with equations of a linear relation using slope-intercept form and slope-point form, you will be extending their understanding to encompass the general form $Ax + By + C = 0$, in which A is a whole number and B and C are integers. This form is particularly important in Mathematics 11 when students are exposed to quadratic equations.

- Have the students explore the relationship between the x-intercept, the y–intercept, and the slope of the graph of a linear relation in its general form. The students may observe that the general form of a linear relation provides an efficient method for determining the x- and y-intercepts, but it is more important that the students understand how to obtain the x-intercept, the y-intercept, and the slope without these generalized formulas.

- Students should be encouraged to further explore the general form $Ax + By + C = 0$. They could explore the effects of changing parameters on a graph of $Ax + By + C = 0$ through the use of a graphing calculator, Autograph (Eastmond Publishing Ltd. 2013), TI-SmartView (Texas Instruments 2013), or FX Draw (Efofex Software 2013). For particular values of A, B, and C, for example, the lines can be vertical, horizontal, or oblique.

- Once students have been introduced to the equations of the three forms of linear relations, encourage them to rewrite the equations interchangeably in the various forms. Through this work, students should make the connections between the general form and the value of $-\frac{A}{B}$ for slope, as well as $-\frac{C}{B}$ for the y-intercept.
- Encourage students to use technology—such as graphing calculators, Autograph (Eastmond Publishing Ltd. 2013), FX Draw (Efofex Software 2013), Smart Notebook Math Tools (SMART Technologies 2013), and other software—to graph a linear relation. If students choose to use the graphing calculator, they must first isolate the y-variable. In Mathematics 9, students solved linear equations (9PR03) using inverse operations; therefore, rearranging equations is not new. Reinforce with students that isolating the y-variable, when written in slope-point form or in general form, does not mean rearranging the equation completely in slope-intercept form.

- Have students graph the same linear equation in all three forms by finding two points from the equations given. This will show the same graph can be expressed using three different types of equations. Have students discuss and determine which form of the equation they find to be easiest in creating the graph.

- Students should discuss the strategies used when graphing lines in the three different forms and question which form of a linear equation is preferred when graphing.
  - In general formula, it is most convenient to find the intercepts and use them to graph the line.
  - In slope-point form, any point on a line can be used when determining the equation of the line.
  - In slope-intercept form, the y-intercept must be used to graph the line.

- At this point students should be able to graph a linear equation without changing forms, given its equation in slope-point, slope-intercept, or general forms. They may prefer to use one form, but should recognize that there are advantages to each of the forms.

- In a group of three people, identify one person as a “slope-intercept form,” one as “slope-point form,” and one as “general form.” Each person in the group creates an equation in his or her form and then draws the graph of that equation. They then pass their equation and graph to the person on their right. That person will now write the equation in their form. Pass this one more time to complete all three forms for each equation. Check each others as a group to ensure it has been done correctly.

- Switch each person’s responsibility to another equation form twice more and repeat the process, until everyone has had a chance to practise each form. Check answers as a group, and pass in the results.

- Equivalent equations can be obtained from an existing equation by adding or subtracting the same term to both sides of the equation or multiplying or dividing the entire equation by the same value. There are many strategies that can be used to identify equivalent linear relations.

  - As a first approach, students may choose to rewrite all equations in one particular form to check whether the equations are the same. If the equations are already written in general form, for example, students could check to see if the equations are multiples of each other.
  - Students could use two test points and determine if they satisfy multiple equations. Discuss with students why using at least two test points is important. This concept will be important for work with solving systems of linear equations later in the unit (RF9).
  - Students may want to compare the slope and *y*-intercept of the equations. This can be done graphically or algebraically.
  - Exposing students to the alternate methods not only strengthens their awareness of the concept, but also gives them a choice when identifying equivalent linear relations from a set of linear relations.

- Ask students to work in groups. Each group should be given a deck of cards with a variety of linear equations on them. Students should work together to identify equivalent linear relations from their deck.

SUGGESTED MODELS AND MANIPULATIVES

- graph paper

MATHEMATICAL VOCABULARY

Students need to be comfortable using the following vocabulary.

- elimination process
- equality
- equivalent equations
- general form
- slope-intercept form
- slope-point form
- zero principle

Resources/Notes

Print

- *Foundations and Pre-calculus Mathematics 10* (Burglind et al., Pearson 2010)
  - Student Book
    > Chapter 6, Sections 3, 4, 5, and 6, pp. 354–387
  - Teacher Technology DVD
    > Teacher Resource
    > Blackline Masters
    > Smart Lessons
    > Animations
    > Dynamic Activities

Software

- Autograph (Eastmond Publishing Ltd. 2013)
- FX Draw (Efofex Software 2013)
- Smart Notebook Math Tools (SMART Technologies 2013)
- TI-SmartView (Texas Instruments 2013)

Notes

SCO RF07 Students will be expected to determine the equation of a linear relation to solve problems, given a graph, a point and the slope, two points, and a point and the equation of a parallel or perpendicular line. [CN, PS, R, V]			
[**C**] Communication [**T**] Technology	[**PS**] Problem Solving [**V**] Visualization	[**CN**] Connections [**R**] Reasoning	[**ME**] Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have met the corresponding specific curriculum outcome.

RF07.01 Determine the slope and y-intercept of a given linear relation from its graph, and write the equation in the form $y = mx + b$.

RF07.02 Write the equation of a linear relation, given its slope and the coordinates of a point on the line, and explain the reasoning.

RF07.03 Write the equation of a linear relation, given the coordinates of two points on the line, and explain the reasoning.

RF07.04 Write the equation of a linear relation, given the coordinates of a point on the line and the equation of a parallel or perpendicular line, and explain the reasoning.

RF07.05 Graph linear data generated from a context, and write the equation of the resulting line.

RF07.06 Determine the equation of the line of best fit from a scatter plot using technology and determine the correlation.

RF07.07 Solve a problem, using the equation of a linear relation.

Scope and Sequence

Mathematics 9	**Mathematics 10**	**Pre-calculus 11**
PR01 Students will be expected to generalize a pattern arising from a problem-solving context using linear equations and verify by substitution.	**RF07** Students will be expected to determine the equation of a linear relation to solve problems, given a graph, a point and the slope, two points, and a point and the equation of a parallel or perpendicular line.	**RF03** Students will be expected analyze quadratic functions of the form $y = a(x - p)^2 + q$ and determine the vertex, domain and range, direction of opening, axis of symmetry, and x- and y-intercepts. **RF04** Students will be expected to analyze quadratic functions of the form $y = ax^2 + bx + c$ to identify characteristics of the corresponding graph, including vertex, domain and range, direction of opening, axis of symmetry, and x- and y-intercepts, and to solve problems.

Background

Students have been exposed to patterns through interpretation of graphs of linear relations in earlier grades. From a pictorial pattern, students should be able to identify and write the pattern rule and create a table of values in order to write an expression that represents the situation. When an oral or written pattern is given, students should be able to write an expression directly from that pattern.

Students have learned to identify and match linear equations with their respective graphs and have also learned to find slope and points from graphs. This outcome introduces students to writing equations using the slope-point form of linear equations when given a graph from which they will identify a point on the line and the slope, points on the graph, and/or the slope directly.

For a scatter plot, the line of best fit will be the linear relation that most closely describes the data. The equation of this line and its correlation should be determined using technology (such as the graphing calculator or other technology). Within this context, students should discuss and explore the concept of correlation and how it varies.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

Guiding Questions

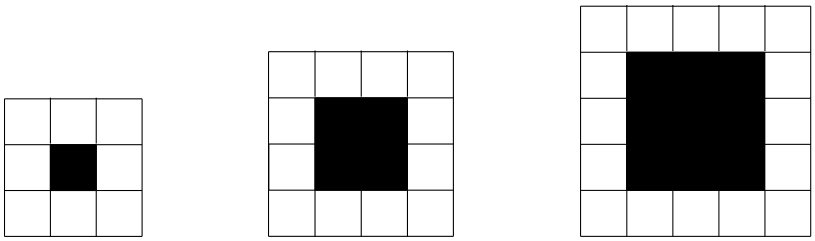
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

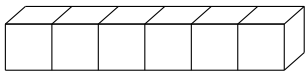
Student tasks such as the following could be completed to assist in determining students' prior knowledge.

- Write a sentence that explains the pattern that connects the number of bricks, *b*, to the side length, *s*.

- Write an expression representing the number of bricks, *b*, around a square fire-pit with side length, *s*.



Determine an expression describing the surface area, *A*, obtained when a number of cubes, *n*, are linked together to form a train such as the one shown below.



WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

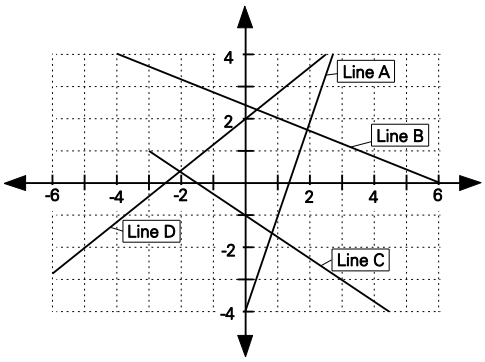
- Kuniko wanted to go on a student exchange to Mexico, but she only had $56 dollars in her bank account. She got a job walking dogs in her neighbourhood for $50 a week. If she saves every penny she makes, write the equation to best represent the total amount of money she will have in her bank account after a certain number of weeks. Using this equation, determine how many weeks it will take her to save $2000.

  Extension: Kuniko's friend Carmen started with $250 and was paid the same amount. Calculate the number of weeks it would take Carmen to save $2000.

  (**Note:** This is a good place to discuss discrete and continuous data and whether a linear relation could represent both.)

- Create a design using a combination of horizontal, vertical, and oblique lines. Transfer this design to a coordinate grid and determine the equations of the lines needed to generate the design.

- State equations for each of the following graphs:



- Determine the equation of a line passing through (2, 6) and parallel to the line $2x + 3y = 12$

- Determine the equation of a line passing through (–1, –4) and perpendicular to $y = 3x - 1$.

- Students from Ingonish are planning a trip to Kejimkujik National Park. If 50 students go, it will cost $1200. If 80 students go, the cost will be $1500. Ask students to graph this data and write the equation of the resulting line. If 76 students plan on taking the trip, ask students how much it will cost.

- The cost of a taxi ride is given by $C = 3.20 + 1.75\,k$, where C is the cost in \$ and k is the number of kilometres. Ask students to find the cost to travel 12 kilometres.

- While surfing the Internet, you find a site that claims to offer "the most popular source for the cheapest DVDs anywhere." Unfortunately, the website is not clear about how much they charge for each DVD and how much is charged for shipping and handling, but it does give you the following information:

Number of DVDs Ordered	1	2	3
Total Cost (includes S&H)	\$15	\$24	\$33

  (a) Plot the data shown in the table and write an equation for the line.
  (b) Your friend says that he can get a dozen DVDs from this website for \$90. Is he correct? Explain.
  (c) How much would it cost to order 50 DVDs from this website?

- Trevor is not sure whether to write a linear equation using slope-intercept form or slope-point form. Depending on the information given, is one more efficient than the other? Explain your reasoning.

- The following chart shows the average prices for a loaf of bread and a bag of flour at a local convenience store for a period of three months. It seems reasonable that a higher price of flour would result in a higher cost for a loaf of bread. Determine the line of best fit to describe the price of a loaf of bread in terms of the price of flour.

Month	**Bag of Flour**	**Loaf of Bread**
1st month	\$3.50	\$2.89
2nd month	\$3.55	\$2.92
3rd month	\$3.75	\$3.03

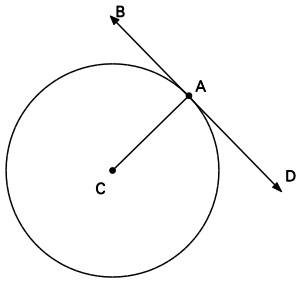
- Write the equation of the line given the following information:
  - the line has a slope of $\frac{1}{2}$ and passes through (–3, 6).
  - the line passes through (–2, 6) and (4, –8).
  - the line passes through (4, 10) and is parallel to the line $y = 2x - 4$.
  - the line passes through (2, 5) and is perpendicular to $y = -\frac{1}{4}x + 9$.

- Julie has played the game Flow Free and has logged her personal best results as shown in the chart below.

Time (minutes)	$\frac{1}{2}$	1	2	4
Number of puzzles solved	5	9	16	30

  - When determining a line of best fit, will you expect the correlation coefficient to be positive or negative? Explain.
  - Determine the line of best fit and use it to predict how many puzzles Julie could solve in 3 minutes.

- The line *BD* is tangent to the circle at point *A* (2, 4). If the centre of the circle is *C* (–1, 1), write the equation of the tangent line *BD*.

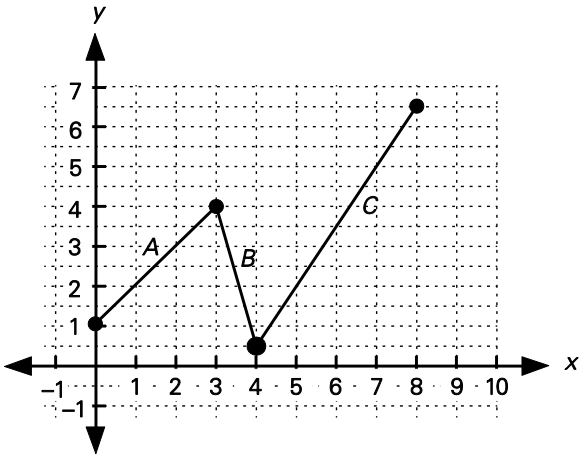


(RF7.2, RF7.3, RF7.6)

- Find the equation of a line with the same *x*-intercept as the line $2x + 3y + 6 = 0$ and is parallel to the line $2x – 5y – 7 = 0$.

- Find the equation of a line with the same *x*-intercept as the line $x – 2y = 5$ and the same *y*-intercept as the line $5x – 2y = 8$.

- The graph shown below is made up of linear segments *A*, *B*, and *C*. Write an equation in slope-point form for the line that contains each segment.



FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should inform instruction.

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SUGGESTED LEARNING TASKS

Effective instruction should consist of various strategies.

Guiding Question

- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

Consider the following sample instructional strategies when planning lessons.

- Give the students several graphs (one with a positive slope, one with a negative slope, one parallel to the *x*-axis, and one parallel to the *y*-axis). Working with a partner, have students determine the slope-intercept, the slope-point, and the general forms for each of the graphs.

- When given the slope and the coordinates of a point on the line, students will have to find the *y*-intercept in order to write the equation in slope-intercept form. Students have a choice of strategies they can use when they are asked to find the *y*-intercept. One approach involves graphing. Students can plot the given point and use the slope to find the *y*-intercept. This is an efficient method when the *y*-intercept is an integral value. Students should be exposed to the algebraic method of determining the *y*-intercept through the use of substitution. The graphing method provides students with a nice visual, but the algebraic method is more effective in finding any *y*-intercept.

- Students will progress from writing the equation of a linear relation in slope-intercept form, given the slope and a point on the line, to writing the equation given the coordinates of two points on a line.

- It is important for students to recognize that the equation written in slope-intercept form is dependent on the slope. Students will determine the slope using a graph or the slope formula. Once the slope is determined, students can then find the *y*-intercept using the graphical or algebraic methods developed earlier. Prompt students to have a discussion about how much information is necessary in order to determine the equation of a line in slope-intercept form and then demonstrate why it does not matter which point is substituted into the equation to find the *y*-intercept.

- Students should practise various ways of finding a point and the slope, depending on the information given, in order to determine the equation of a linear relation.

- To determine a point on the line you could do one of the following:
  - Select a point from the graph.
  - The student is given the *x*- and *y*-coordinates of a point on the line.
  - An *x*- or *y*-intercept is determined from a given equation by substituting either $x = 0$ (for the *y*-intercept) or $y = 0$ (for the *x*-intercept).
  - A table of values for the line is provided to the student from which any point can be selected.
- To determine the slope of the line you could do one of the following:
  - A value for slope is given directly.
  - Any two points on the line are selected and the rise and run between the points is determined to give the slope.
  - The *x*- and *y*-coordinates for two points are given, and the slope formula is used to calculate the slope.
  - A parallel line is given, and the slope is determined from this line.
  - A perpendicular line is given, and its slope determined. The negative reciprocal is the slope of the line.
  - The line is horizontal, and therefore, the slope is zero.
  - The line is vertical, and therefore, the slope is undefined.

- Depending on the information given, students can express the equation of the linear relation in slope-intercept, general, or slope-point form. Students should be encouraged to determine which form best suits a given situation.

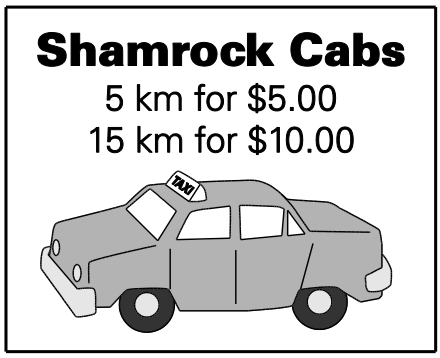
- Encourage students to verify their equation is correct by selecting a point that is on the line and then checking to see if it satisfies the equation. Students should examine various graphs, including horizontal and vertical lines.

- Students should recognize that, when given the slope and a point on the line (other than the *y*-intercept), writing equations in slope-point form requires less algebraic steps than writing equations in slope-intercept form. Encourage students to choose the more appropriate form of an equation when specific information is given. For example, the slope-intercept form indicates the slope of the line and its *y*-intercept; the slope-point form indicates the slope and its coordinates of a point on the line. Although both equations look different, they still represent the same function. Students should be exposed to examples wherein the equation of a line written in slope-point form is rearranged and expressed in slope-intercept form.

- Ask students to participate in the activity Relation Match. For this activity, students work in groups of two or more. Each group should be given a deck of cards containing 13 cards with an equation in slope-intercept form and 13 cards with a graph. The dealer shuffles all the cards and deals them out. Students match the graph and the equation. Remove the matches and place them face up on the table. Next, a player draws a card from their partner. They should locate the matching equation/graph and add it to their pairs. Students take turns drawing cards from their partner's hands. The game continues until all matches are made.

- To participate in the game Slope Relay, students should work in teams. Draw or display two coordinate grids on the white boards. Read two points and have students race to plot the points, draw the correct line, and determine its equation.

- Students should be exposed to situations in which two points are generated from a given context. Once students have generated the equation of a linear relation, they can use it to solve a variety of problems. Consider the advertisement, to the right, posted on a community bulletin board.



  Students will determine the equation that represents this linear relationship between the distance travelled and the cost of the cab ride. Encourage them to use the equation to then solve a particular problem. For example, if Paola travels 82 km, how much will she have to pay?

- Students can model a real-life situation using a linear relation in slope-point form. An understanding of what the slope represents, and making a connection between the data and the context of the problem, makes the mathematics more meaningful for students. When asked to use the function developed to solve a particular problem, students can substitute the value directly into the equation and solve for the unknown variable.

  For example, students could be given the following situation: The temperature in a sauna is rising at a constant rate of 2°F every 5 minutes. Ten minutes after Bevan enters the sauna, the temperature is 80°F. Determine when the temperature will reach 120°F. It is simplest to find the slope-point form of the equation since the slope and a point have been given.

  $T - 80 = \frac{2}{5}(m - 10)$

  $120 - 80 = \frac{2}{5}(m - 10)$

  $40 = \frac{2}{5}(m - 10)$

  $100 = m - 10$

  $110 = m$

  Therefore, without rearranging the equation to slope-intercept form, students are able to determine the answer to the question.

- Scatter plots are one of three topics covered in the Pearson Canada WNCP *Nova Scotia Curriculum Companion*, available as a free download in the Mathematics 10 folder on the Grade 10 Learning Commons moodle.

- When exploring scatter plots, the following site provides an interactive program that allows students to quickly plot points and explore the effect on the *r* value, as they plot points closer to and further from the line of best fit. Multiple Linear Regression activity: www.shodor.org/interactivate/activities/Regression

Suggested Models and Manipulatives

- graph paper
- linking cubes

MATHEMATICAL VOCABULARY

Students need to be comfortable using the following vocabulary.

- correlation (positive and negative)
- scatter plot

Resources/Notes

Internet

- Math Playground, "Save the Zogs" (MathPlayground.com 2013)
  www.mathplayground.com/SaveTheZogs/SaveTheZogs.html
  Linear equations game
- The Computational Science Reference Desk (CSERD 2013)
  www.shodor.org/interactivate/activities/Regression
  Multiple Linear Regression activity

Print

- *Foundations and Pre-calculus Mathematics 10* (Burglind et al., Pearson 2010)
  - Student Book
    - Chapter 6, Sections 4, 5, and 6, pp. 357–387
  - Teacher Technology DVD
    - Teacher Resource
    - Blackline Masters
    - Smart Lessons
    - Animations
    - Dynamic Activities

- *Nova Scotia Curriculum Companion* document found on Moodle (Available only to Nova Scotia teachers)

Notes

SCO RF08 Students will be expected to solve problems that involve the distance between two points and the midpoint of a line segment. [C, CN, PS, T, V]			
[**C**] Communication [**T**] Technology	[**PS**] Problem Solving [**V**] Visualization	[**CN**] Connections [**R**] Reasoning	[**ME**] Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have met the corresponding specific curriculum outcome.

RF08.01 Determine the distance between two points on a Cartesian plane using a variety of strategies.
RF08.02 Determine the midpoint of a line segment, given the endpoints of the segment, using a variety of strategies.
RF08.03 Determine and endpoint of a line segment, given the other endpoint and the midpoint, using a variety of strategies.
RF08.04 Solve a contextual problem involving the distance between two points or midpoint of a line segment.

Scope and Sequence

Mathematics 9	**Mathematics 10**	**Grade 11 Mathematics Courses**
M01 Students will be expected to solve problems and justify the solution strategy using circle properties including the following: ▪ The perpendicular from the centre of a circle to a chord bisects the chord. ▪ The measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc. ▪ The inscribed angles subtended by the same arc are congruent. ▪ A tangent to a circle is perpendicular to the radius at the point of tangency.	**RF08** Students will be expected to solve problems that involve the distance between two points and the midpoint of a line segment.	—

Background

Students will develop both the distance formula and midpoint formula by building on their former knowledge and understanding of *xy*-coordinates on a Cartesian plane, to which they were introduced in Mathematics 6. They will also build on their understanding of the Pythagorean theorem, which they used to solve problems in Mathematics 8 and 9.

Teachers should take care to develop a clear understanding of the distance formula and the midpoint formula as distinct concepts. Experience has shown that students frequently confuse these two formulas.

Developing an understanding of the distance formula as a means of determining the distance between two points will build on students' understanding of the Pythagorean theorem.

Explorations can begin with determining the length of the special cases of horizontal or vertical lines on a Cartesian plane in which the distance is the difference between the two *x*-values (horizontal) or the two *y*-values (vertical). The distance formula is the general form of a rearrangement of the Pythagorean formula for finding the length of the hypotenuse. The formula for the distance between two points *A* and *B* is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\ .$$

The development of an understanding of the midpoint formula is based on a different concept—that of the mean of two values. On a Cartesian plane, the midpoint of a line is found by determining the mean of the *x*- and the *y*-coordinates of the two end points of the line. If the endpoints are $P(x_1, y_1)$ and $Q(x_2, y_2)$, the coordinates for the midpoint will be the mean between the two *x*-values and the mean between the two *y*-values, or

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

In Mathematics 9, students worked with circles and their various properties. They know that the perpendicular from the centre of a circle to a chord bisects the chord, and that the tangent to a circle is perpendicular to the radius at the point of tangency. Given this information, they are able to extend their use of the distance and midpoint formulas to work with circle diameters and chord lengths and to solve related problems.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Assessing Prior Knowledge

Student tasks such as the following could be completed to assist in determining students' prior knowledge.

- How much horizontal distance is necessary in order for straight ramp 8 m long to be built to a height of 1.5 m?

- In a computer catalogue, a computer monitor is listed as being 22 inches. This distance is the diagonal distance across the screen. If the screen measures 12 inches in height, what is the actual width of the screen to the nearest inch?

- Plot the points *A* (3, 5); *B* (–2, 4); *C* (–1, –3); and *D* (4, –2) on graph paper. What type of quadrilateral is *ABCD*?

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

- The line segment *AB* is formed by joining the ordered pairs *A* (–4, 3) and *B* (2, –3).
  (a) Determine the length of the line segment *AB*.
  (b) Find the coordinates of the midpoint *AB*.

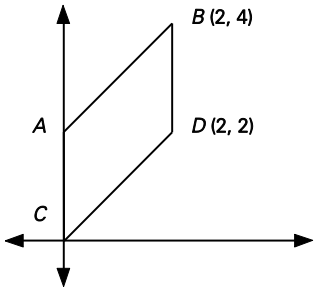
- A triangle has the vertices *A* (–3, 1), *B* (1, 7), and *C* (5, 1).
  (a) Find the perimeter.
  (b) Classify the triangle as scalene, isosceles, or equilateral.

- Show that points *P* (5, –1), *Q* (2, 8), and *R* (–2, 0) lie on a circle with a centre of *C* (2, 3).

- One endpoint of a line segment is (–4, 3). The midpoint is (–3, 6). Find the other endpoint.

- Given two points, *A* and *B*, find the point $\frac{1}{3}$ of the way from *A* to *B*.

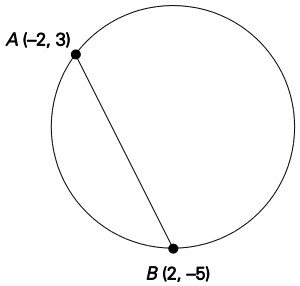
- What is the distance between *B* and the midpoint $\overline{CD}$?



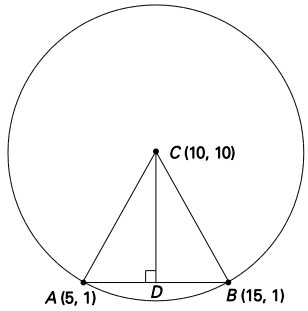
- Given that *AB* has the midpoint, *M* (2, –3) and an endpoint, *B* (–5, 1), what are the coordinates of *A*?

- Points *A* (3, 9) and *B* (12, 15) are joined to form *AB*. What is the midpoint of *AB*?

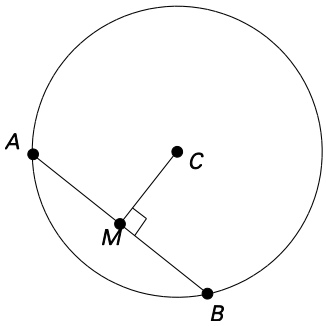
- Given the diagram shown to the right, find the equation of the perpendicular bisector of the chord.



- Given the following diagram, in which *C* is the centre of the circle and *D* is the midpoint of the chord *AB*, determine the length of *CD*.



- Given that *C* is the centre of the circle and *AB* is a chord, if *AB* is 24 cm and *CM* is 5 cm, what is the length of the diameter of the circle?



- If *AB* and *CD* are perpendicular bisectors of each other, and *CD* has endpoints *C* (5, –2) and *D* (–13, 12), what are the coordinates of the point of intersection of *AB* and *CD*?

FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should inform instruction.

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?

- How will the diverse learning needs of students be met?

SUGGESTED LEARNING TASKS

Effective instruction should consist of various strategies.

Guiding Question

- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

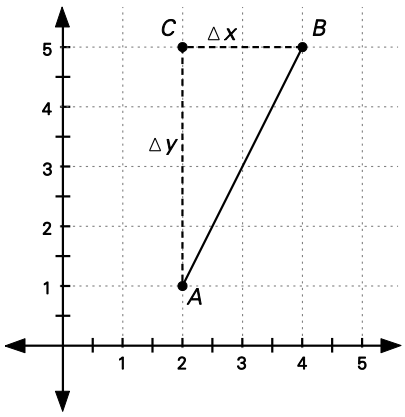
Consider the following sample instructional strategies when planning lessons.

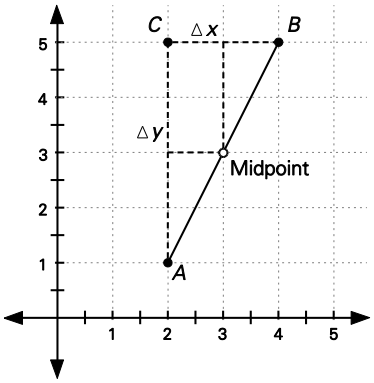
- The length and midpoint of a line are two of three topics covered in the Pearson Canada WNCP *Nova Scotia Curriculum Companion*, available in the Mathematics 10 folder on the Grade 10 Learning Commons moodle.
- Students should be given a chance to develop formulae rather than receiving them at the beginning of the process. The following activity can set the stage for this understanding.

- You are planning a trip across Canada, you have to travel through all 10 provinces and 3 territories.
  (a) Find the total distance between the start and finish that you will travel.
  (b) Split your trip up into 8 days, find the distance you will travel each day. Try to pick locations you would want to visit.
  (c) You must make a pit stop every day for gas and food. You have to stop exactly half-way every day. Find your stopping town (your midpoint).
  (See Appendix A.11 for a copyable version of this map.)

- With their knowledge of Pythagorean theorem, students should be able to determine the length of a line segment on graph paper (see below). Section 4.2, *Mathematical Modeling*, Book 3 (Barry et al. 2002), can be used as a supplemental resource for this unit.

- A similar diagram can be used to develop an understanding of the midpoint formula in a specific case and then for the general case, using the mean of *x*-coordinates and the mean of the *y*-coordinates.





$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Suggested Models and Manipulatives

- graph paper
- map of Canada (Appendix A.10)
- rulers

Mathematical Vocabulary

Students need to be comfortable using the following vocabulary.

- chord
- midpoint
- perpendicular bisector
- tangent line to circle

Resources/Notes

Print

- *Nova Scotia Curriculum Companion* document found on Moodle
- *Mathematical Modeling, Book 3,* Section 4.2 (Barry et al. 2002)

Notes

SCO RF09 Students will be expected to represent a linear function, using function notation. [CN, ME, V]			
[**C**] Communication	[**PS**] Problem Solving	[**CN**] Connections	[**ME**] Mental Mathematics and Estimation
[**T**] Technology	[**V**] Visualization	[**R**] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have met the corresponding specific curriculum outcome.

RF09.01 Express the equation of a linear function in two variables, using function notation.
RF09.02 Express an equation given in function notation as a linear function in two variables.
RF09.03 Determine the related range value, given a domain value for a linear function.
RF09.04 Determine the related domain value, given a range value for a linear function.
RF09.05 Sketch the graph of a linear function expressed in function notation.

Scope and Sequence

Mathematics 9	**Mathematics 10**	**Grade 11 Mathematics Courses**
PR01 Students will be expected to generalize a pattern arising from a problem-solving context using linear equations and verify by substitution.	**RF09** Students will be expected to represent a linear function, using function notation.	**RF outcomes** Function notation required for all RF outcomes. (M11*, PC11)**

* M11—Mathematics 11
** PC11–Pre-calculus 11

Background

In Mathematics 8, students solved linear equations of the form $Ax + B = C$ (8PR02). They will use this skill when they are given the value of the dependent variable and are asked to solve for the independent variable.

This outcome provides students with an introduction to function notation for linear functions A function gives each input value (x) a unique corresponding output value (y). The $f(x)$ notation can be thought of as another way of representing the y-value.

Students should make a connection between the input and output values and ordered pairs as visualized on a graph. For example, the notation $f(2) = 5$ indicates that the point with coordinates (2, 5) lies on the graph of $f(x)$.

In function notation such as $f(x)$, the f is an arbitrary name, but g and h are commonly used, as in $g(x)$ and $h(x)$. The letter in the parentheses indicates the independent variable used when the function is represented by an equation. For example, writing $A(r) = \pi r^2$ indicates A is the name of the function and r is the independent variable.

Students will be expected to express an equation in two variables using function notation. For example, the equation $y = 4x – 1$ can be written as $f(x) = 4x – 1$. Conversely, they will express an equation in

function notation as a linear function in two variables. For example, $h(t) = -3t + 1$ can be written as $h = -3t + 1$. Often the function name is related to a context as in this example, in which the function name is $h(t)$, for a problem that involves the height (h) of an object at a certain time (t).

Students will determine the range value given the domain value. For example, if students are given $f(x) = 5x - 7$, they should be able to determine $f(1)$. Conversely, students will determine the domain value given the range value. For example, for the function $f(x) = 4x - 1$, where $f(x) = 3$, students will solve for x.

While students will not study the composition of functions at this time, the input for a function can be another function. For example, students could be asked to determine the simplified expression for $f(x + 1)$ if $f(x) = 3x - 5$.

Throughout this unit, students have sketched graphs using a variety of methods. Students will now be exposed to graphing linear functions with an emphasis on function notation. They will graph linear functions using the following:

- The horizontal and vertical intercepts.
- Rate of change and vertical intercept.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Student tasks such as the following could be completed to assist in determining students' prior knowledge.

- Given the equation $h = 2t + 5$, describe this relation in words. Make up a problem that could be solved using this equation.

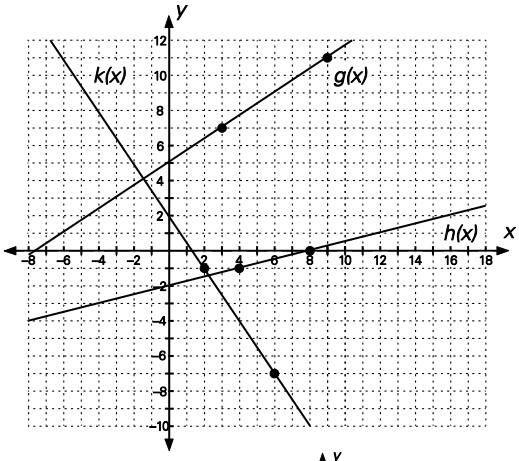
- Given that the cost of having a yard raked can be described in terms of time (as shown in the chart below), determine an equation that will describe that cost in terms of time.

Time (hours)	2	3	3.5	4
Cost ($)	50	70	80	90

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

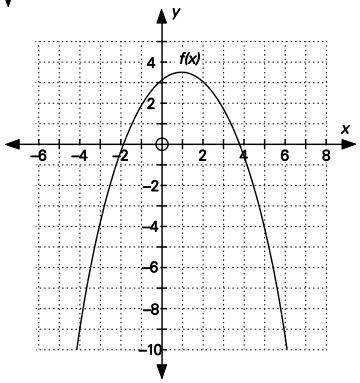
Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

- Given $g(x) = 2x - 3$, $h(x) = 7 - 3x$, and $k(x) = x + 1$, calculate the following
  (a) $g(15)$
  (b) $h(28)$
  (c) x when $k(x) = 17$
  (d) $g(x + 2)$
  (e) $h(5x)$
  (f) $k(2a + 1)$
  (g) x when $g(x) = k(x)$
  (h) x when $h(2x) = k(3x - 2)$



- Use the graphs shown to the right in order to determine
  (a) the values of $k(2)$, $h(4)$, and $g(0)$
  (b) where $h(x) = 0$

- Evaluate the following:
  (a) $d(t) = 3t + 4$, determine $d(3)$
  (b) $f(x) = x^2 - 2x - 24$, determine $f(-2)$
  (c) $h(t) = 4t^2 - 3t$, determine $h(1) + h(-2)$.
  (d) $f(x) = 5x - 11$, find the value of x that makes $f(x) = 9$.
  (e) $g(x) = -2x + 5$, find the value of x that makes $g(x) = -7$.



- Given the function $f(x)$, shown to the right, find
  (a) $f(-2)$
  (b) $f(2)$
  (c) x such that $f(x) = -9$

- The perimeter of a rectangle is $P = 2l + 2w$. If it is known that the length must be 6 ft., then the perimeter is a function of the width. Write this function using function notation.

- The volume of a cylinder is given by the equation $V = \pi r^2 h$. If it is known that the radius is 5 cm, then the volume is a function of the height of the cylinder. Write this function using functional notation.

- If $C(x)$ is linear and $C(5) = 11$ and $C(12) = 25$, sketch the graph of $C(x)$.

- For a circle, $A(r) = \pi r^2$ and $C(r) = 2\pi r$, determine the values of $A(3)$ and $C(3)$ and state what these values represent.

- For a cube, $V(x) = x^3$ and $A(x) = 6x^2$, determine x when
  (a) $V(x) = 512$
  (b) $A(x) = 864$
  (c) Explain what your answers represent.

FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should inform instruction.

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SUGGESTED LEARNING TASKS

Effective instruction should consist of various strategies.

Guiding Question

- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

Consider the following sample instructional strategies when planning lessons.

- Many students seem puzzled by function notation well after it has been introduced. They often ask, Why can't we just write $y = 2x + 1$ instead of $f(x) = 2x + 1$? To motivate students to use function notation and improve understanding, try using multi-variable functions instead of single-variable functions in introducing this notation.

- You can pose the following type of question: Suppose you are texting your friend Alek and ask him for the dimensions of a bookshelf that he has offered to give you. He replies with a list of three numbers: 12 in., 20 in., 24 in. Do you know the specific height, width, and depth of the bookshelf?
  - What is missing in the above scenario is the "function definition." The function definition tells you what the function does, how many parameters (or "arguments") the function requires as inputs, what those parameters are, and in what order they occur. In this example, the following information is probably what you wanted to know: Dimension (depth, height, width).

- Later in mathematical studies, functions will often be described in terms of multiple variables. Functional notation thus provides the clearest, most concise method possible to communicate information. For example, the volume of a cylinder might be described as a function $V(r,h) = \pi r^2 h$ and you could thus concisely ask for $V(3, 4)$ when you wanted to find the volume of a cylinder with a radius of 3 and a height of 4.

- Another view of functional notation that may assist students in seeing its value as a concise way of communication is to describe three equations.
  - Line 1: $y = 2x + 3$
  - Line 2: $y = 5 – 4x$
  - Line 3: $y = 1.5x + 6$

  Next, ask a student to find the y-value when $x = 1$. They would also need to know what equation was to be used. To clarify, you would need to say, "Find the y-value in the equation $y = 5 – 4x$ when $x = 1$."

  Using functional notation, however, simplifies the problem.
  - Line 1: $f(x) = 2x + 3$
  - Line 2: $g(x) = 5 – 4x$
  - Line 3: $h(x) = 1.5x + 6$

  Asking students to find $g(1)$ requires no further clarification.

  Functional notation also has the added benefit of crossing language barriers, since $g(1) = ?$ does not depend on the understanding of any language other than mathematics.

- A virtual function machine can also be used to clarify the idea of functional notation. Alternately, you can create function machines simply. Just collect a few containers: a coffee can, a mason jar, and a box would serve the purpose. Label each of these containers with an equation, such as $C(n) = 2n + 1$, $M(n) = n + 5$ and $B(n) = n^2$. Ensure that the students are able to describe what each function machine does.
  - The coffee can would serve the function of doubling a number and adding 1.
  - The mason jar would add 5 to the number.
  - The box would square the number.

  At this point, you take a piece of paper and put a number such as 7 on three separate pieces of paper. Drop one of the pieces of paper in the coffee can and stir, shake, or agitate it. Ask the students what number will result when the 7 has had time to be transformed by the coffee can function. (Make sure you have placed the answer 15 in the coffee can prior to this lesson). Then describe how the function machine is processing the number 7 (doubling to 14 and then adding 1 to 15) and finally draw the result out of the tin. Repeat this process with the mason jar (yielding a 12) and the Box (yielding a 49).

  You may wish to plan for another input number as well. If you are able to manage the sleight of hand necessary, you can present this as a bit of magic and have some fun with functional notation. This process effectively emphasizes functional notation and provides a visual connection that students are likely to remember.

- A common student error occurs when the parentheses are mistakenly used as multiplication in function notation. For example, $f(4) = 8$ does not mean $4f = 8$. It is important for students to make the connection that this is a place holder which represents the domain value.

- Another error occurs when students are given a function such as $h(t) = 4t - 3$ where $h(t) = 18$ and are asked to determine t. Students often substitute the given value for the independent variable rather than the dependent variable. This would be a good opportunity to reiterate the purpose and meaning of function notation.

- A review of rearranging equations would be helpful since function notation requires the linear relation to be solved for the y-variable.

- It is important that to ensure that students make the connection between the input and output values and ordered pairs. For example, the notation $f(2) = 5$ indicates that the point with coordinates (2, 5) lies on the graph of $f(x)$. In this way, students visually recognize both the input and output of the function.

- At least two points are necessary to draw a line. Students will be exposed to finding the vertical intercept by evaluating $f(0)$ and finding the horizontal intercept by solving $f(x) = 0$. As an alternative, students can graph a line using the rate of change from the equation and the vertical intercept. Examples of linear functions with restricted domain and range should also be included.

Suggested Models and Manipulatives

- three empty containers

Mathematical Vocabulary

Students need to be comfortable using the following vocabulary.

- function notation
- input value
- output value

Resources/Notes

Internet

- The Computational Science Education Reference Desk, "Number Cruncher" (CSERD 2013)
  http://shodor.org/interactivate/activities/NumberCruncher
  Function Machine Virtual Manipulative

Print

- *Foundations and Pre-calculus Mathematics 10* (Burglind et al., Pearson 2010)
  - Student Book
    - Chapter 5, Section 2, pp. 264–273; Section 5, pp. 287–297; Section 7, pp. 311–323
  - Teacher Technology DVD
    - Teacher Resource
    - Blackline Masters
    - Smart Lessons
    - Animations
    - Dynamic Activities

Notes

SCO RF10 Students will be expected to solve problems that involve systems of linear equations in two variables, graphically and algebraically. [CN, PS, R, T, V]			
[**C**] Communication [**T**] Technology	[**PS**] Problem Solving [**V**] Visualization	[**CN**] Connections [**R**] Reasoning	[**ME**] Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have met the corresponding specific curriculum outcome.

RF10.01 Model a situation, using a system of linear equations.
RF10.02 Relate a system of linear equations to the context of a problem.
RF10.03 Determine and verify the solution of a system of linear equations graphically, with and without technology.
RF10.04 Explain the meaning of the point of intersection of a system of linear equations.
RF10.05 Determine and verify the solution of a system of linear equations algebraically.
RF10.06 Explain, using examples, why a system of equations may have no solution, one solution, or an infinite number of solutions.
RF10.07 Explain a strategy to solve a system of linear equations.
RF10.08 Solve a problem that involves a system of linear equations.

Scope and Sequence

Mathematics 9	**Mathematics 10**	**Grade 11 Mathematics Courses**
PR02 Students will be expected to graph linear relations, analyze the graph, and interpolate or extrapolate to solve problems. **PR03** Students will be expected to model and solve problems where a, b, c, d, e, and f are rational numbers, using linear equations of the form ▪ $ax = b$ ▪ $\frac{x}{a} = b, a \neq 0$ ▪ $ax + b = c$ ▪ $\frac{x}{a} + b = c, a \neq 0$ ▪ $ax = b + cx$ ▪ $a(x + b) = c$ ▪ $ax + b = cx + d$ ▪ $a(bx + c) = d(ex + f)$ ▪ $\frac{a}{x} = b, x \neq 0$	**RF10** Students will be expected to solve problems that involve systems of linear equations in two variables, graphically and algebraically.	**RF01** Students will be expected to model and solve problems that involve systems of linear inequalities in two variables. (M11)* **RF06** Students will be expected to solve, algebraically and graphically, problems that involve systems of linear-quadratic and quadratic-quadratic equations in two variables. (PC11)** **RF07** Students will be expected to solve problems that involve linear and quadratic inequalities in two variables. (PC11)

* M11—Mathematics 11
** PC11—Pre-calculus 11

Background

In Mathematics 9, students modelled and solved problems using linear equations with one variable (9PR03). In this unit, they are introduced to systems of equations with two variables. Students will initially create a linear system to model a situation; they will also write a description of a situation that might be modelled by a given linear system.

Solving a linear system graphically is now extended to solving systems algebraically using substitution and elimination. In Mathematics 9, students were exposed to solving linear equations with fractional coefficients (9PR03). In the previous unit, they eliminated fractional coefficients in a linear equation by multiplying by the LCM of the denominators (RF06). Students will now solve linear systems graphically, with and without technology, and progress to solving linear systems symbolically, using substitution and elimination.

Students will model situations using a system of linear equations. Ensure they understand and define the variables that are being used to represent the unknown quantity. This would be a good opportunity to discuss that in order to solve a system, the number of unknowns must match the number of equations.

Students will also describe possible situations that could be modelled by a given linear system. Encourage students to share their responses so they can be exposed to a variety of real-life contexts.

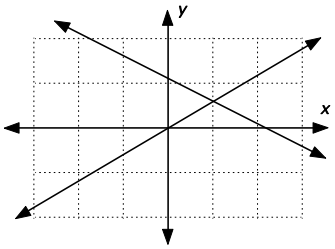
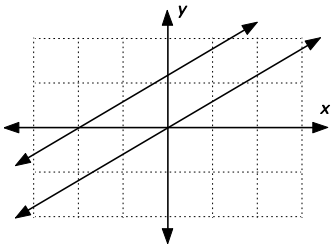
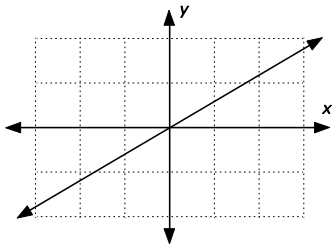
When two lines intersect, the coordinates of the point of intersection is the solution of the linear system. Students will model situations using a system of linear equations and, conversely, describe a possible situation that is modelled by a given linear system. They will learn both graphical and algebraic methods for solving systems of linear equations.

Students will translate a word problem into a system of linear equations and will solve the problem by graphing, thereby verifying the solution. They should be comfortable solving systems of linear equations graphically, both with and without technology. Emphasis should be placed on real-life applications like cell phone providers or cab companies.

In previous outcomes, students graphed linear equations using the slope-intercept method, the slope-point form, and using the x- and y-intercept method. Identifying the form of the equation will help students determine which method they should choose when graphing the lines. However, there are limitations to solving a linear system by graphing. For example, non-integral intersection points may be difficult to determine exactly and the solution will be an estimate of the coordinates of the point of intersection. For exact answers, algebraic methods can be used instead.

Students will solve systems of linear equations algebraically using substitution and elimination. After determining the solution to the linear system, they will verify the solution by direct substitution or by graphing.

Systems of linear equations can have different numbers of solutions. By graphing the linear system, students can determine if the system of linear equations has one solution (intersecting lines), no solution (parallel lines), or an infinite number of solutions (coincident lines).

Intersecting Lines	Parallel Lines	Coincident Lines
One solution  different slopes *y*-intercepts can be the same or different	No solution  same slope different *y*- intercepts	An infinite number of solutions  same slope same *y*-intercepts

Students can also use the slope and *y*-intercept of each equation to determine the number of solutions of the linear system. When the slopes of the lines are different, the lines intersect at one point and the system has one solution. When the slopes of the lines are the same but the *y*-intercept is different, the two lines will never intersect and the system has no solution. When the slopes of the lines and the *y*-intercept is the same, the two lines are identical and the system has an infinite number of solutions.

The elimination method will also indicate the number of solutions of a linear system. When a unique value for *x* and *y* can be determined, the system of equations has one solution. When adding equations eliminates the *x*-variable resulting in a false statement, such as $0x + 0y = 27$, there are no solutions and the lines must be parallel. When the result is $0x + 0y = 0$ for any value of *x* or *y*, this statement is true, and there are an infinite number of solutions of the linear system, indicating that the lines are coincident.

Linear systems can be solved either graphically or algebraically. It is important that students be able to identify which method is most efficient.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Student tasks such as the following could be completed to assist in determining students' prior knowledge.

- Using graph paper, graph the equations $y = 2x + 1$ and $2x + 3y = 11$ on the same set of axes. Where do these lines intersect?

- Given the equation $y = 4x – 7$, determine the value of y if you replace x with $2y$.

- Given the equation $3x + 2y = 14$, determine the value of x if you replace y with $x – 2$.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

- For the following situations create two equations, and use these equations to solve the problems algebraically.
  (a) At a high school hockey game, students paid admission of $4 per ticket and adults paid $6. The number of students who attended was 300 more than the number of adults. If the total of all ticket sales was $2400, how many of the attendees were students and how many were adults?
  (b) Discount Taxi charges $2 per kilometre with an initial fee of $4. In the same community Swift Taxi charges $2 per kilometre with an initial fee of $5. Create equations for each taxi company and graph both on the same graph. When will Swift Taxi be the more economical to use? Explain.
  (c) Kerstin has a beautiful farm near Lake Ainslie, on which she raises emus and donkeys. On her farm, there are 64 legs and 20 heads. How many of each animal does she have?

- Create a situation relating to coins that can be modelled by the following linear system and explain the meaning of each variable: $x + y = 24$; $0.25x + 0.05y = 4.50$.

- Explain and demonstrate the meaning of having systems of equations that are parallel, by creating and solving your own problem.

- Describe or model a situation that cannot be modelled by a linear system and explain why a linear system is unsuitable.

- Tyreese bought 8 books. Some books cost $13 each and the rest of the books cost $24 each. He spent a total of $209. Write a system of linear equations that could represent this situation.

- Jorge solved the linear system $2x + 3y = 6$ and $x – 2y = –6$. His solution was (2, 4) Verify whether Jorge's solution is correct. Explain how Jorge's results can be illustrated on a graph.

- Jill and Tony are both carpenters. Jill earns $40 per day plus $10 per hour. Tony earns $50 per day plus $5 per hour. Graphically represent the linear system relating Jill's earnings and Tony's earnings. Identify the solution to the linear system and explain what it represents.

- Describe a real-life situation in which the graphs of the lines in the linear system are *not* parallel but the linear system has no solution. This could involve situations where the domain is restricted. For example, the cellular phone plans modelled in the table below would never have the same cost.

Plan	Base Monthly Fee ($)	Cost per Minute (¢)
A	30	5
B	10	2

- Write a linear system that has an infinite number of solutions. Explain what happens when you try to solve the system using elimination.

- Explain which system you would prefer to solve without using technology.

$$\left.\begin{aligned} y &= \frac{9}{2}x - \frac{23}{2} \\ y &= \frac{2}{11}x - \frac{16}{11} \end{aligned}\right\} \qquad \left.\begin{aligned} y &= -\frac{2}{5}x - \frac{2}{5} \\ y &= -\frac{3}{7}x - \frac{33}{70} \end{aligned}\right\}$$

- Explain to another student how you would solve the following system of linear equations. Justify the method you chose.

$$\left.\begin{aligned} 4x - 7y &= -39 \\ 3x + 5y &= -19 \end{aligned}\right\}$$

- A test has 20 questions worth a total of 100 points. The test consists of selected response questions worth 3 points each and constructed response questions worth 11 points each. How many multiple choice questions are on the test?

- The cost of a buffet dinner for a family of six was $48.50 ($11.75 per adult, $6.25 per child). How many adults and how many children were in the family?

- Create a linear system that could be solved more efficiently through
  (a) substitution rather than using elimination or graphing (Solve the system.)
  (b) elimination rather than using substitution or graphing (Solve the system.)

- Research the cost of renting a car in Sydney for a single day.
  (a) How would the study of linear relations and linear systems help you decide which company to rent from?
  (b) What is the effect of distance travelled?
  (c) What other variable(s) effect the cost of renting a car?

- Solve the linear system using elimination.

$$\left.\begin{aligned} 2x - 5y &= 10 \\ 4x - 10y &= 20 \end{aligned}\right\}$$

  (a) What does the solution tell you about the nature of the lines of the equations?
  (b) Convert the equations to slope-intercept form to confirm this conclusion.

FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should inform instruction.

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SUGGESTED LEARNING TASKS

Effective instruction should consist of various strategies.

Guiding Question

- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

Consider the following sample instructional strategies when planning lessons.

- Begin the solving of systems of equations by having students compare two companies that provide the same service or product. They should create an equation to represent each service and then graph, by hand, the relationships for both companies on the same graph. Include examples in which the systems of equations do not intersect, and are parallel.

- Students will be working with the graphs of linear systems and how to find their solutions graphically. There are many different ways to graph linear equations without using technology. Previously in this unit, students were exposed to graphing linear equations using slope-intercept method, slope-point form, and using the *x*- and *y*-intercept method. Identifying the form of the equation will help students decide which method they should choose when graphing the lines.

- Students should discuss which service would be best when considering various situations—such as cell phone plans, taxi rides, or driveway ploughing—and, at the point at which lines intersect, discuss whether the advantages of one plan over another change. For example, a comparison of cell phone plans and different usage can be made, considering that one person may text mostly and only use

his or her phone sporadically, while another person may use his or her phone continuously and seldom text.

- If equations do not intersect, have students discuss what this means in terms of what service is less expensive. For example, two taxi companies may have different base rates, but may charge the same amount per kilometre travelled. The company with the lower base rate would always be less expensive. Are there other factors that might influence the choice of a taxi company in a scenario like this?

- Once students are comfortable solving a system of equations graphically without technology, introduce the use of technology to solve this same system. To solve a system of linear equations using graphing technology, students may need to rearrange equations to functional form. Also, it may be necessary to review domain and range appropriate to the context to help students to determine appropriate settings for the viewing window. Ensure that students have some exposure to systems whose intersection point is difficult to read or obtain exactly, such as $2x + 3y = 11$ and $x – 6y = 14$. This will set the stage for the solution of a system of equations using algebraic methods.

- Encourage students to continue to eliminate fractional coefficients before proceeding to solve systems algebraically. You may need to remind students how to do this efficiently.

- After determining the solution to the linear system, ensure that students verify the solution of the system either by direct substitution or by graphing. It is important to reiterate that manual graphing may only give an approximate solution, especially when solutions involve fractions.

- While the development of symbolic representation to obtain the solution to a linear system is important, emphasis should continue to be on the development of the linear system from a problem-solving context.

- Although linear systems can be solved graphically or algebraically, it is important that students identify which approach is most efficient. Although students can make the connection between the solution of a linear system and the point of intersection of the graphs, manual graphing may only provide an approximation of the solution.

- Provide students an opportunity to decide which algebraic method is more efficient when solving a linear system by focusing on the coefficients of like variables. If necessary, rearrange the equations so that like variables appear in the same position in both equations. Substitution may be more efficient if the coefficient of a term is 1 or if one of the equations is already arranged so that one of the variables and its coefficient are isolated. Elimination, on the other hand, may be more efficient if the variable in both equations have the same magnitude coefficient or if the two equations are already arranged so that they are formatted in the same way.

Substitution Works Well	**Elimination Works Well**
$2x – 5y = 12$ and $x = y + 2$	$2x – 5y = 12$ and $x – y = 2$
$2x – 7y + 10 = 0$ and $7y = x +1$	$2x – 7y + 10 = 0$ and $–x + 7y = 1$

- Discuss why systems of linear equations can have different numbers of solutions. Knowing that parallel lines do not intersect, for example, students can predict the system would have no solution. Students can then identify the number of solutions of a linear system using different methods. By graphing the linear system, students can determine if the system of linear equations has one

solution (intersecting lines), no solution (parallel lines), or an infinite number of solutions (coincident lines). As an alternative to graphing, students can use the slope and *y*-intercept of each equation to determine the number of solutions of the linear system. When the slopes of the lines are different, the lines intersect at one point and the system has one solution. When the slopes of the lines are the same but the *y*-intercept is different, the two lines will never intersect, and the system has no solution. When the slopes and the *y*-intercepts of the lines are the same, the two lines are identical and the system has an infinite number of solutions. Students can also apply the elimination method to determine the number of solutions of a linear system. When a system of equations has one solution, a unique value for *x* and *y* can be determined. However, this is not always the case. Consider the following example:

Solve: $\left.\begin{array}{l} -2x - 3y = 12 \\ 2x + 3y = 15 \end{array}\right\}$

Adding the equations eliminates the *x* variable resulting in $0x + 0y = 27$. For any value of *x* or *y*, this is a false statement. This indicates there are no solutions and the lines must be parallel. Students can also encounter a situation where $0x + 0y = 0$. For any value of *x* or *y*, the statement is true. There are an infinite number of solutions of the linear system and the lines are coincident. This particular possibility is easier to determine by inspection. If students reduce the equations to lowest terms (dividing each term by the GCF), they can identify whether the equations are equivalent. If the equations are equivalent, the graphs overlap, resulting in an infinite number of intersection points.

- *Chutes and Ladders:* Each group of four students is given a game board, 1 die, 1 pack of system of equations cards, and 4 disks to move along the board. Ask students to draw a card and roll the die. If the roll is a 1, the student will solve the system by graphing. If they roll a 2 or 5, they solve the system by substitution. If they roll a 3 or 4, they solve the system by elimination. If the roll is a 6, they solve by a method of their choice. If the system is solved correctly, the student moves the number of spaces on the board that corresponds to the roll on the die. If the answer is incorrect, the person on the left has the opportunity to answer the question and move on the board.

SUGGESTED MODELS AND MANIPULATIVES

- graph paper
- rulers

MATHEMATICAL VOCABULARY

Students need to be comfortable using the following vocabulary.

- coincident lines
- intersection lines
- intersection point
- parallel lines
- substitution

Resources/Notes

Print

- *Foundations and Pre-calculus Mathematics 10* (Burglind et al., Pearson 2010)
  - Student Book
    - Chapter 7, Sections 1–6, pp. 392–455
  - Teacher Technology DVD
    - Teacher Resource
    - Blackline Masters
    - Smart Lessons
    - Animations
    - Dynamic Activities

Notes

Financial Mathematics
40–45 hours

GCO: Students will be expected to demonstrate number sense and critical thinking skills.

Specific Curriculum Outcomes

Process Standards Key

[**C**] Communication	[**PS**] Problem Solving	[**CN**] Connections	[**ME**] Mental Mathematics and Estimation
[**T**] Technology	[**V**] Visualization	[**R**] Reasoning	

FM01 Students will be expected to solve problems that involve unit pricing and currency exchange, using proportional reasoning. [CN, ME, PS, R]

FM02 Students will be expected to demonstrate an understanding of income to calculate gross pay and net pay, including wages, salary, contracts, commissions, and piecework. [C, CN, R, T]

FM03 Students will be expected to investigate personal budgets. [C, PS, R, T]

FM04 Students will be expected to explore and give a presentation on an area of interest that involves financial mathematics. [C, CN, ME, PS, R, T, V]

SCO FM01 Students will be expected to solve problems that involve unit pricing and currency exchange, using proportional reasoning. [CN, ME, PS, R]			
[**C**] Communication [**T**] Technology	[**PS**] Problem Solving [**V**] Visualization	[**CN**] Connections [**R**] Reasoning	[**ME**] Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have met the corresponding specific curriculum outcome.

FM01.01 Compare the unit price of two or more given items.
FM01.02 Solve problems that involve determining the best buy, and explain the choice in terms of the cost as well as other factors, such as quality and quantity.
FM01.03 Compare, using examples, different sales promotion techniques.
FM01.04 Determine the percent increase or decrease for a given original and new price.
FM01.05 Solve, using proportional reasoning, a contextual problem that involves currency exchange.
FM01.06 Explain the difference between the selling rate and purchasing rate for currency exchange.
FM01.07 Explain how to estimate the cost of items in Canadian currency while in a foreign country, and explain why this may be important.
FM01.08 Convert between Canadian currency and foreign currencies, using formulas, charts, or tables.

Scope and Sequence

Mathematics 9	**Mathematics 10**	**Mathematics 11**
N03 Students will be expected to demonstrate an understanding of rational numbers by comparing and ordering rational numbers and solving problems that involve arithmetic operations on rational numbers. **G01** Students will be expected to demonstrate an understanding of similarity of polygons.	**FM01** Students will be expected to solve problems that involve unit pricing and currency exchange, using proportional reasoning.	**M01** Students will be expected to solve problems that involve the application of rates.

Background

Students are introduced to proportional reasoning in Mathematics 8 (8N05), and in Mathematics 9 they continue to work on proportional reasoning through the study of similar polygons. For students to experience proper development of proportional reasoning, they must become multiplicative thinkers and be able to see and use the multiplicative relationships found within and between the ratios in the problem.

Students will extend their skills with proportional reasoning to everyday situations such as shopping, calculating taxes, and currency exchange. Teachers should work from simpler to more complex examples as students increase their proficiency.

Proportional reasoning will be used in estimating and calculating the unit price. Estimation and proportional reasoning are skills that have been identified as weaknesses in our adult population, but they are critical components of financial mathematics.

For students to become financially knowledgeable consumers, they must be able to estimate and/or calculate total cost, taking into account discounts and additional costs such as taxes and shipping. The students must also take into account other factors such as ethical implications, product quality, and practicality before making a purchase.

On a more global level, this topic will allow students to explore the use of ratio to estimate or calculate a currency value based on fluctuating currency rates.

Selling rate and purchasing (buying) rate are terms related to currency exchange. Selling rate is the rate at which a bank sells money to the consumer. The purchasing rate is the rate at which a bank buys money from the consumer. It should be noted that the selling and purchasing rates are not the same and can change at any time.

Consider the following situation: Filipe decides to travel to Japan. He converts C$500 to yen and receives 44030 yen in cash. Just minutes after completing this conversion, Filipe finds out that his trip has been cancelled and he returns to the bank to change the 44030 yen back into Canadian dollars. He receives C$462.84. This transaction cost Filipe C$37.16. A bank has two rates for exchanging cash—a buying rate (in this case 0.010512) and a selling rate (in this case 0.011356). If Filipe had bought non-cash, such as traveller's cheques, the rates he would have received would have been more favourable. His C$500 would have purchased 44185 yen, and returning his 44185 yen would have him receiving C$471.32 (Cost to him would have been C$28.68).

A bank explains this difference in cash or non-cash rates as follows: "Exchange rates applied to cash transactions include shipping and handling charges, making the exchange rate for cash less favourable than the non-cash rate. Non-cash rates are applied to paper instruments such as cheques, drafts, and traveller's cheques. Non-cash rates are also applied to incoming and outgoing wire payments. These instruments are easier to manage and incur less time and cost for processing than cash transactions. Therefore, a more favourable rate is applied to non-cash instruments."

These specific rates can be found online. Students should understand that it saves money if they convert their Canadian dollars to the local currency of their travel destination before they leave Canada. Most banks, foreign exchange kiosks, and hotels in other countries charge commission or service charges for converting your Canadian dollars to their local currency.

The following table represents a currency unit when compared to the Canadian dollar. Since exchange rates frequently fluctuate, it is important that students understand how to read a table such as this one. The complete table can be found on the Royal Bank of Canada website at www.rbcroyalbank.com/cgi-bin/travel/currency-converter.pl.

Country	Currency	Bank Buy Rate	Bank Sell Rate
United States	US Dollars (USD)	0.9965	1.0535
European Union	Euros (EUR)	1.2956	1.4061
Great Britain	Pounds Sterling (GBP)	1.5008	1.6045
Switzerland	Swiss Francs (CHF)	1.0605	1.1513
Japan	Yen (JPY)	0.010512	0.011356
Australia	Australian Dollars (AUD)	0.9958	1.1245
New Zealand	New Zealand Dollars (NZD)	0.8110	0.9161
Denmark	Danish Kroners (DKK)	0.1735	0.1912
Norway	Norwegian Kroners (NOK)	0.1729	0.1913
Sweden	Swedish Kroners (SEK)	0.1524	0.1686
Bahrain	Bahraini Dinar (BHD)	2.4623	2.9518
Barbados	Barbados Dollars (BBD)	0.4712	0.5564
Belize	Belize Dollar (BZD)	0.4701	0.5613
Bermuda	Bermuda Dollars (BMD)	0.8481	1.0571
Brazil	Brazilian Real (BRL)	0.4764	0.5688

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

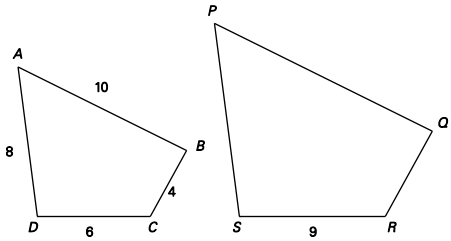
Student tasks such as the following could be completed to assist in determining students' prior knowledge.

- Order these rational numbers from least to greatest.

  $\frac{7}{5}$, –0.95, $\frac{3}{4}$, 1.52, 0.777..., $-\frac{11}{10}$

- A case of 12 cans of juice costs $4.80. Samuel wants to determine the cost of each can. Explain how Samuel can do this.

- If the polygons to the right are similar, determine the measure of all missing sides.



WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

- Felix can make 3 dozen tea biscuits with $2\frac{1}{4}$ cups of flour. How many cups of flour will he need to make 9 dozen tea biscuits?

- I travelled 40 km in 80 minutes. How far did I travel in one hour?

- Four grey garden tiles are used for every 3 red ones. If you use 210 tiles, how many are grey and how many are red?

- There are 3 oranges for every 1 apple in the bowl. How many oranges will there be if there are 5 apples? How many apples and oranges are there if the bowl has 20 pieces of fruit in it?

- When mixing paint, which combination will result in the bluer shade of green—2 parts blue with 3 parts yellow, or 3 parts blue with 5 parts yellow? Explain why.

- Mathieu ate $\frac{2}{3}$ of a box of chocolates. That left only 16 chocolates for his brother Michael. How many chocolates were in the box at the start?

- The regular price of a pair of shoes is $140. During a sale, the store gives a discount of 35%. What will the price of the shoes be?

- One store advertises a “buy one sweater, get one for half price” sale. A second store advertises a 20% off sale on all sweaters. Rachel decides to buy two $28 sweaters. What would be her cost per sweater at each of the two stores?

- A 12-oz. bottle of barbecue sauce costs $1.54. A 16-oz. bottle of barbecue sauce costs $1.99. Which is the better buy?

- Anya decides to adapt the following five-star recipe for holiday punch to make enough for 50 people. Morgan decides to use the same recipe to make enough for 15 people.
  (a) How much of each ingredient does Anya need to purchase?
  (b) How much of each ingredient does Morgan need to purchase?

  Original recipe makes 20 servings

4 cups cranberry juice cocktail	8 cups prepared lemonade
2 cups orange juice	1 (4 ounce) jar maraschino cherries
1 (2 L) bottle ginger ale	1 orange, sliced in rounds

- Seth went to the cafeteria at lunch to purchase milk. The cafeteria had two different sizes available. The 250-mL milk costs $0.45 while the 500-mL milk costs $0.80.
  (a) Which milk is the better buy?
  (b) What other factors may influence his decision?

- First estimate your answer, then solve for *x*. Compare the estimated and calculated answers to check if your answer is correct.
  (a) $\frac{268}{5 \text{ rolls}} = \frac{x}{1 \text{ roll}}$
  (b) $\frac{x}{300} = \frac{1}{1.56}$
  (c) A box of 12 pencils costs $1.69. How much does one pencil cost?

- Tires for All Inc.

  Regular $118
  Now 30% off!

  Best Value Tires Inc.

  Regular $118
  Buy one, get 2nd half-off!

  Treads-R-Us

  Regular $118
  Buy 3, Get one free!

  You need 4 new winter tires. Which company has the better deal? Discuss the promotional techniques used by each company.

  Note: Use real examples of sales and promotions that are found in your community in local flyers.

- A particular brand of house paint is available as 4-L cans of paint for $42.95 and 250 mL for $7.50. The first option (4L) requires the purchase of primer, while the second option (250 mL) includes the primer. Which option would you have chosen and what conditions would affect your choice? (**Note:** This is an open question, and each option has its merits. Students should be given the opportunity to explore both options and the merits of each, depending on the situation.)

- You have purchased an iPod Touch for $225. The original price was $300. What percentage discount did you receive?

- Chelsea bought stock in a company for $25. Two weeks later she sold it for $60. What was the percent increase in value?

- Jordan bought an adapter for his computer that cost $29.99. Two weeks later he noticed that the same adapter was priced at $19.99. What was the percent decrease in price?

- Frank decided to buy an iPod for $297. He scratched a discount card at the checkout and got $50 off. What is the percent decrease in cost?

- How are percent increase and percent decrease alike and how are they different? Include examples in your explanation.

- The original price of a car was $19 295. The sale price of the same car was $17 995. Khalid calculated the percent decrease to be 93%. Did Khalid make an error? Explain why or why not.

- Define the terms **selling rate** and **purchasing rate**, and then explain the difference between the two using an appropriate example.

- Juan is planning a trip to Florida. He uses C$300 to buy US dollars at the bank at the current daily rate of 0.9084. Later that day his trip gets cancelled so he changes his money back to Canadian dollars at the rate of 1.0361. Ask students to determine how much money he lost and explain why he would not get exactly $300 back.

- For a school exchange trip to Europe, your parents have given you C$500 as spending money.
  (a) At the bank, you exchanged this for Euros, at a rate of 1 EUR = 1.35 CAD. How many Euros will you receive? At this exchange rate, what is the value of C$1 in Euros?
  (b) On return you have €50 left, and you exchange these Euros back into Canadian dollars at a rate of 1 EUR = 1.27 CAD. How much will you have lost (paid to the bank) for the exchange of this €50 back and forth?

  Note: Actual currency rates can be found online and used in place of rates given.

- Prior to your trip to Mexico, you exchanged some Canadian dollars for pesos at an exchange rate of 1 CAD = 12.35 MXN. You buy a burrito for 30 pesos. Approximately how much has this cost you in Canadian dollars?

- Use currency exchange rates from the Internet or the newspaper to answer the following:
  (a) Stephanie is travelling to the Philippines on vacation.
    (i) What is the name of the currency used in the Philippines?
    (ii) What is the exchange rate of that currency in Canadian dollars?
    (iii) If Stephanie goes into a bank and purchases, in cash, 4500 units of Philippine currency, how many Canadian dollars would this cost her?
    (iv) If Stephanie's trip were cancelled and she returned to the bank to return the Philippine currency, how many Canadian dollars would she get back?
    (v) How much did this exchange process cost Stephanie? How much would it have cost her if she had obtained traveller's cheques rather than cash?

  Note: Groups of students could also be assigned different countries.

- Chantelle gets a job in Malaysia where the currency is the Ringitt (RM). She has the option to get paid C$60 000 per year, or RM 210 000. Ask students to look up today's exchange rates on the Internet to determine the best option for Chantelle.

- Explore the daily exchange rate over the last 30 days and determine how the rates may have been influenced by current events or other factors.

- Answer the following: (You will need to look up exchange rates).
  (a) While in the United States, you wish to purchase a laptop computer for US$385. If the selling rate of the US dollar compared to the Canadian dollar is 1.0375, estimate the cost of the laptop in Canadian funds.
  (b) Alivia is in Mexico bargaining with a local seller. The cloth she wants to buy costs 85 pesos. If the exact value of 1 Canadian dollar is 12.3 pesos, what is a good estimate (in Canadian dollars) for 85 pesos? Why is estimating quickly useful in a situation like this?

(c) Compare shopping prices for a similar product on Canadian and American store websites. Consider which site is more economical for making online purchases based on currency exchange rates.

Extension: Include shipping, duties, brokerage fees, and tax rates.

- A company manufactures and sells a product for $15 (before taxes) here in Canada. If the cost of shipping and exporting it to Europe is $1 per item, determine the equivalent price in Euros for this item when it is sold in Europe. Use the rate of 1 EUR = 1.35 CAD.

- A company is buying desk calendars for their employees. They can buy them in packages of 10 for $32, packages of 15 for $45, or packages of 25 for $70. The company is buying desk calendars for 70 employees. Which combination of packages should be purchased to minimize the cost to the company?

- Liam has decided to order new kitchen doorknobs for his apartment buildings. He has found one supplier who sells them in groups of 20 doorknobs for $66. Another supplier sells them in groups of 50 for $155. He has found a third supplier who is willing to sell individual doorknobs for $4.50 each. If Liam plans to purchase 185 doorknobs, determine what combination of purchases would result in the best buy.

FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should inform instruction.

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SUGGESTED LEARNING TASKS

Effective instruction should consist of various strategies.

Guiding Question

- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

Consider the following sample instructional strategies when planning lessons.

- Activate prior knowledge of proportions and percentage seen in previous years. Determine if students are able to think multiplicatively to solve problems.

- Estimation should be emphasized before a student calculates the answer, to help the student predict whether or not the answer is reasonable.

- Encourage solving problems mentally where possible.

- Develop proportional reasoning through a progression of questions. Start with simple numbers (whole number examples that are easy to double, triple, etc.) to establish the understanding before moving to other numbers such as fractions and decimals as examples.

- "How much for one of something → two of something → four of something" using examples from real life.

- Use flyers, catalogues, and websites to provide real-life examples. Bring in products—such as yogurt, cereal, granola bars, and vitamins—so students can visually make comparisons.

- Check currency rates as a class and then talk about fluctuations that occur on a daily or longer term basis.

- In a class discussion, ask students to explain how fluctuations in the exchange rates of different countries could affect the import and export business.

- Discuss various sales promotion techniques that stores use to help sell items. Stores often sell different quantities of the same product at different prices (e.g., soft drinks sold at 4 for $5 as opposed to 1 for $1.49). Promotions, such as "buy one get one free"/discounted, also encourage consumers to shop in a particular store. Students must realize, however, that to effectively compare the prices of two or more items they must use the same units. For example, deli meat sold at $2 per 100 g may seem less expensive than $20 per kilogram. Using the conversion factor of 1000 g = 1 kg, students should realize that deli meat at $2 per 100 g is equivalent to $20 per kilogram. Although students have explored the relationship between metric units of measurement, it may be necessary to revisit the following conversions:

  1000 g = 1 kg 100 cm = 1 m 1000 mL = 1 L

- After comparing unit prices, discuss other factors that may influence the choice for a "best buy." Students should be reminded that more is not always better. Engage students in a discussion about this. Buying large quantities of items that have a cheaper unit price is not helpful if the consumer ends up wasting some of the product because it was not fully used or has expired. Other factors, such as the travelling distance from stores and the quality of one product over another, must also be considered. Students should realize that decisions to buy an item should not be based on price alone. Present students with a choice such as the following:

- Mustard is sold in a 2-bottle package for $2.49 and a 12-bottle package for $12.99. Which package has the lower unit price? How much would you save by buying a 12-bottle package rather than six 2-bottle packages? When deciding which package size is the better buy for you, what should you consider in addition to unit price?

- Ask students to collect flyers to compare various products. Ask them to create their own problem related to comparing unit price and finding the best buy. After doing the comparison, students should determine other factors that could influence their decision to purchase that item.

- Students could work in centres, with each centre containing similar items in different sizes with prices given, such as soup, cans of juice, dog food, and shampoo. In their journals, students could list the item they would buy and why it is the better deal or the better purchase.

- Gather empty containers of dish detergent, including various different sizes. Ask students to discuss factors that contribute to deciding which size to purchase. They should be encouraged to consider environmental concerns, such as packaging, use of water, and concentration of chemicals.

- Students could compare a jumbo-size and a regular-size liquid laundry detergent. Ask them to determine what constitutes one use for each size. To visualize how much more the jumbo size contains, they could measure out each serving (empty containers with water could be used). They should then determine the cost per usage.

- Ask small groups of students to research the cost of lumber at the local hardware store(s) to determine:
  (a) the store that offers the best buy on one piece of 2"× 4" × 8'
  (b) the best rate per foot of a piece of
  (i) 2" × 4" × 8' (ii) 2" × 4" × 10' (iii) 2" × 4" × 12'

- Students will explore currencies in countries around the world and should recognize the importance of understanding currency rates, especially when travelling and buying and selling goods in different countries. Discuss with students some of the different systems of currencies in a variety of countries. Some samples are provided below.

Currency by Country

Canada	dollar	C$
United States	dollar	US$
Germany	Euro	€
England	pound	£
Japan	yen	¥
Denmark	Krone	kr
Thai	Baht	฿
South Korean	Won	₩
Poland	Zloty	zł

- Engage students in a discussion about businesses that import or export materials and how the fluctuation in the Canadian dollar can affect these businesses.

- Problems involving currency exchange provide opportunities for discussion. Often, for example, both Canadian and American prices are listed on magazines and books. Discuss whether or not

customers would benefit from choosing which price to pay. Travellers to countries that use a different currency from their home country's currency can exchange their money to make purchases while they are travelling. The exchange rate may determine how much Canadian travellers will buy in a foreign country. Before items are bought in a foreign country, students need to be aware of what the item actually costs in their own currency to ensure they are not paying more than they would at home. Estimation can help students compare foreign prices to Canadian prices. Consider the following example:

- When Kalie was vacationing in France, she wanted to purchase a print of the Eiffel Tower costing 190 Euros. What would be the cost in Canadian dollars if the exchange rate were 1.644814?

Exact Value	**Estimated Value**
1 Euro = C$1.644814	1 Euro = C$1.6
190 Euros = C$312.51	200 Euros = C$320

- An engaging activity for students involves setting up an international store with food items from different countries (e.g., a can of olives from Greece, lettuce from the United States, bananas from Chile). The items should be labelled with the cost in the currency of the source country. Each group chooses from provided recipes and selects the required ingredients. The students calculate the cost, in Canadian dollars, of the completed dish. As an extension, they could determine the cost per serving.

Suggested Models and Manipulatives

- coins from various countries
- consumer items
- newspapers
- sales flyers
- various-sized containers

Mathematical Vocabulary

Students need to be comfortable using the following vocabulary.

- buying rate
- currency
- exchange rate
- percentage decrease
- percentage increase
- purchasing rate
- selling rate
- unit price

Resources/Notes

Internet

- Bank of Canada (Bank of Canada 2013)
  www.bank-banque-canada.ca/en/rates/converter.html
  For the current Canadian exchange rates, go to the daily currency converter at the Bank of Canada's website.
- Canadian Bankers Association, "Banks and Financial Literacy" (Canadian Bankers Association 2013)
  www.cba.ca/en/component/content/category/79-banks-and-financial-literacy
  Financial Literacy Information (Banks)

- Financial Consumer Agency of Canada. *The City: A Financial Life Skills Resource* (Government of Canada 2013)
  www.fcac-acfc.gc.ca/eng/education/index-eng.asp
- *Mathematics Learning Commons 7–9, Nova Scotia Virtual School* (Nova Scotia Department of Education and Early Childhood Development 2013):
  - Proportional Reasoning PowerPoint
    http://lrt.ednet.ns.ca/PD/math8support_11/prop_reasoning/01D_proportional_reasoning_ratios.ppt
  - Proportional Reasoning Problems
    http://lrt.ednet.ns.ca/PD/math8support_11/prop_reasoning/01K_question_bank.doc
  - Proportional Reasoning articles:
    http://lrt.ednet.ns.ca/PD/math8support_11/prop_reasoning/01L_problems_encourage_prop_sense.pdf
    http://lrt.ednet.ns.ca/PD/math8support_11/prop_reasoning/01M_multiple_ways_to_solve_proportions.pdf
- Practical Money Skills Canada: Financial Literacy for Everyone, "For Educators"
  Lesson Plans: Choices & Decisions (Visa 2013)
  http://practicalmoneyskills.ca/foreducators/lesson plans
- RBC Royal Bank (Royal Bank of Canada 2013)
  www.rbcroyalbank.com
  Lists both the buy and sell rates.
- RBC Royal Bank, "Foreign Exchange Currency Converter" (Royal Bank of Canada 2013)
  (www.rbcroyalbank.com/cgi-bin/travel/currency-converter.pl.)
- XE, Currency Encyclopedia (XE 2013)
  http://xe.com/currency
  Provides rates and information for every currency.

Print

- *Financial Mathematics* (Etienne and Kalwarowsky, McGraw-Hill 2013)
  - Sections FM.1 and FM.2, pp. 8–31
  - Teacher's Resource Blackline Masters
    - BLM FM–2 Financial Mathematics Warm-Up
    - BLM FM–3 Financial Mathematics Mini Projects
    - BLM FM–4 FM.1 Extra Practice
    - BLM FM–5 FM.2 Extra Practice

Notes

SCO FM02 Students will be expected to demonstrate an understanding of income to calculate gross pay and net pay, including wages, salary, contracts, commissions, and piecework. [C, CN, R, T]			
[**C**] Communication [**T**] Technology	[**PS**] Problem Solving [**V**] Visualization	[**CN**] Connections [**R**] Reasoning	[**ME**] Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have met the corresponding specific curriculum outcome.

FM02.01 Describe, using examples, various methods of earning income.

FM02.02 Identify and list jobs that commonly use different methods of earning income (e.g., hourly wage, wage and tips, salary, commission, contract, bonus, shift premiums).

FM02.03 Determine in decimal form, from a time schedule, the total time worked in hours and minutes, including time and a half and/or double time.

FM02.04 Determine gross pay from given or calculated hours worked when given
- the base hourly wage, with and without tips
- the base hourly wage, plus overtime (time and a half, double time)

FM02.05 Determine gross pay for earnings acquired by
- base wage, plus commission
- single commission rate

FM02.06 Explain why gross pay and net pay are not the same.

FM02.07 Determine the Canadian Pension Plan (CPP), Employment Insurance (EI), and income tax deductions for a given gross pay.

FM02.08 Determine net pay when given deductions (e.g., health plans, uniforms, union dues, charitable donations, payroll tax).

FM02.09 Investigate, with technology, “what if …” questions related to changes in income (e.g., What if there is a change in the rate of pay?)

Scope and Sequence

Mathematics 9	**Mathematics 10**	**Mathematics 12**
N03 Students will be expected to demonstrate an understanding of rational numbers by comparing and ordering rational numbers and solving problems that involve arithmetic operations on rational numbers. **N04** Students will be expected to Students will be expected to explain and apply the order of operations, including exponents, with and without technology.	**FM02** Students will be expected to demonstrate an understanding of income to calculate gross pay and net pay, including wages, salary, contracts, commissions, and piecework.	**FM01** Students will be expected to solve problems that involve compound interest in financial decision making. **FM02** Students will be expected to analyze costs and benefits of renting, leasing, and buying. **FM03** Students will be expected to analyze an investment portfolio in terms of interest rate, rate of return, and total return.

Background

Increasingly, students are working in part-time jobs. In addition to providing an income, work experiences enhance resumés, college applications, and future job applications. In this unit, students will be introduced to the various methods of income payment, deductions, and calculations involving gross and net pay. They will also be presented with flawed solutions involving gross or net pay that require them to identify and correct mistakes.

Students will gain an understanding of income, how it can be earned, and what the advantages and disadvantages of various ways of earning an income might be.

Income is the money received within a specified time frame, usually in return for work completed. This can be in the form of an hourly wage in which a worker is paid at a set rate per hour, a wage as piecework in which a worker is paid a fixed "piece rate" for each unit produced or job completed (such as planting trees, completing a translation job), or a salary which is paid regularly by an employer to an employee, and may be specified in an employment contract.

Working on commission involves an employee performing a service or making a sale for a business and being paid a percentage of the money received by their employer for each service performed or sale made. Workers can work entirely on commission or work for a base salary and receive a commission over and above their salary. Additional pay can also be received if an employee works overtime or on holidays, or extra pay can come in the form of tips, bonuses, or shift premiums.

Applying SCO A01, students will use formulae to calculate income, as well as gross and net pay. They will determine which deductions are required and which are optional depending on circumstances. They will understand that gross pay is what you make before any deductions. Net pay is the actual "take-home" pay after taxes, health benefits, Canada Pension Plan (CPP), Employment Insurance (EI), and other deductions are taken into account.

Additional Details

- There are many methods of earning income.

- Various combinations of these methods of earning income, such as hourly wage and tips, are also common. Students usually begin work with jobs that earn an hourly wage, wage and tips, or a salary.

- Employees sometimes have to work extra hours in addition to their regular hours. Overtime usually begins when they work beyond 40 hours in a workweek. Overtime pay must be received for those additional hours. Overtime pay is typically 1.5 times the employee's regular rate of pay. Students may be familiar with this as time and a half. For example, if regular pay is $12 an hour, then the overtime rate is $18 an hour (12 × 1.5) for every hour worked beyond 40 hours in each week. Other employees, in professions such as nursing, receive a shift premium. In this case, they receive an extra amount of money per hour because they work non-standard hours.

- Gross pay is the total amount earned before any deductions are taken out. There are three types of fixed gross earnings:
  - wages (rate × hours worked)
  - salaries (set amount)
  - bonuses (discretionary)

Students should

- be able to calculate weekly (52 pay periods), bi-weekly (26 pay periods), or monthly (12 pay periods) gross wages given the hourly rate of pay or the annual income (The gross pay calculation is straightforward when the number of hours worked in the pay period and the hourly pay rate are known. Gross pay calculations for salaried employees may require more focus. The total annual pay is divided by the number of pay periods per year.)

- recognize the difference between hourly gross pay and salaried gross pay through exploration of situations such as the following:

Hourly Gross Pay	**Salaried Gross Pay**
An employee works 20 hours a week at $12/h. If he or she is paid weekly, the gross pay for each of the 52 pay periods is $240.	If an employee's annual salary is $30 000 and he or she is paid biweekly, the gross pay for each of the 26 pay periods is $1153.85.

Gross pay is the total of all earnings including regular pay and overtime pay. Net pay is the gross pay minus any deductions. Students may be familiar with this as "take-home" pay. Mandatory government deductions are CPP, EI, and income tax. The employee's share will be deducted from his or her paycheque and the employer's share will be a cost to the company. Students will calculate these deductions given the appropriate percentage rates found on the Canada Revenue Agency website.

Note: Teachers should visit the CRA website yearly for the updated rates for Employment Insurance, Canada Pension Plan, and tax tables.

Employment Insurance offers financial assistance for some people who lose their jobs through no fault of their own. The number of hours or weeks an employee needs to qualify for EI is based on where he or she lives and the unemployment rate in his or her economic region at the time he or she files the claim. In Nova Scotia, most people will need between 420 and 665 insurable hours of work in the last 52 weeks in order to qualify for EI.

EI is a fund into which employees and employers pay. Employers pay 1.4 times the employee's rate. The greater a worker's earnings, the greater the deductions and EI payments will be, if collected. EI contributions on all eligible earnings will continue throughout the year until the maximum contribution levels are reached. How fast an employee reaches that figure, or if he or she reaches it at all, depends on how much the worker earns.

Canada Pension Plan protects families against income loss due to retirement, disability, or death. Both employees and employers contribute a portion to the Canada Pension Plan. The employer matches the contributions made by the employee. For CPP, there are yearly maximum contribution amounts (for example, $2163.15 for 2010) and once these are reached during the calendar year the contributions will cease. In 2012, the CPP contribution rate was 4.95% of any gross earnings above $3500 and the maximum rate for pensionable earnings was $47 200.

CPP = (Earnings – $3500) × 0.0495

Students should be exposed to situations where

- income is below the minimum contribution level
- income is between the minimum and maximum contribution rates
- income is higher than the maximum contribution level

For example,

- Kyle earned $3280 through a summer job, $220 less than the minimum contribution level. As a result, he did not have to contribute to the CPP in 2012. If Kyle's employer deducted any CPP contributions, they would have been refunded to him when he filed his tax return.

- Quentin is employed as a photographer. His annual salary is $45 000. This figure is between the minimum contribution level of $3500 and the 2010 maximum rate of $47 200. His maximum contribution for 2012 was ($45 000 – $3500) × 0.0495 = $2054.25.

- Hana is employed as a dental hygienist. Her annual salary is $68 700. This figure is higher than the maximum pensionable earnings of $47 200 in 2012. She will make her contributions of $130.79 biweekly; when her total deductions for the year reaches the maximum, however, she will see an increase in her net pay as there will no longer be CPP deductions. Beginning with the new year, CPP contributions will recommence until she reaches the maximum level again.

Income tax is a type of deduction used to help pay for everything from maintaining law and order to funding our health service. Most students will be interested in actual deductions for different ranges of income, as well as different methods of payment. Federal and provincial/territorial tax rates vary depending on the employee's taxable income. Taxable income is the gross income less a variety of deductions. Canada, for example, is recognized for its effective health care system. Employees still opt, however, to buy extended coverage from their place of work to cover unforeseen expenses such as vision care, dental care, prescription drugs, and accidental death. Once all the deductions have been totalled, the net pay will then be calculated. This can be demonstrated using a sample pay stub.

Students will next investigate commission earnings and earnings for piecework and contract work. They should explore various methods of calculating regular pay, including commission only, salary plus commission, and wages plus tips. It might be worth noting that, although it is illegal in Canada to have someone working for tips only, in many places in the United States servers do not receive a minimum wage. Instead, they take home their tips only.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Student tasks such as the following could be completed to assist in determining students' prior knowledge.

- Ask students to add brackets where required to make this a true statement.
  13.5 + 4 ÷ 0.75 + (8.1) = 1.9

- Some people are saying that the answer to the skill-testing question (3 × 50) + 20 ÷ 5 is 154 and some say the answer is 34. Which answer is correct and why?

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

- Calculate and compare wage situations involving minimum wage rates, regular pay, overtime pay, gratuities, piecework, straight commission, salary and commission, salary plus quota, and graduated commission.

- Two restaurants have offered Jamir a job. Mario's pays $8/h, and tips average $24 daily. Teppan's pays $5.50/h, and tips average $35 daily. If Jamir works 30 hours weekly, spread over four days, how much would she earn at each restaurant?

- Identify and calculate various payroll deductions, including income tax, CPP, EI, medical benefits, union and professional dues, and life insurance premiums.

- Estimate, calculate, and compare gross and net pay for various wage or salary earners in your community.

- You have a summer job at a local restaurant as a server. The owner presents three choices for your income:
  (a) $14 per hour (no tips)
  (b) $10 per hour (plus tips)
  (c) salary of $320 per week

  Which option would you choose and why? What aspects of the job should you consider before choosing an option?

- Crystal is working at a fish plant for the summer. She makes $12 per hour, and she earns a shift premium of $2 per hour for hours worked between 12 a.m. and 8 a.m. She gets 1.5 times the regular pay for overtime hours worked above 40 hours per week. Her weekly time schedule is shown below:

Monday	12 p.m.–6 p.m.
Tuesday	8 a.m.–4:30 p.m.
Wednesday	6 a.m.–2 p.m.
Thursday	12 a.m.–10 a.m.
Friday	10 p.m.–4 a.m.
Saturday	10 p.m.–4 a.m.
Sunday	4 p.m.–11 p.m.

  (a) calculate Crystal's regular hours
  (b) calculate Crystal's premium hours
  (c) calculate Crystal's overtime hours
  (d) calculate Crystal's gross pay

- Design your own time schedule and create a problem for other students to answer.

- Choose a job that suits your skills and interests. Research the rate of pay in Nova Scotia on the jobbank.ca website (http://jobbank.ca) and post the information on the job wall that your teacher has set up for that purpose.

- Oliver is paid $12.50 per hour for 40 hours per week. If he works more than 40 hours per week, he makes time and a half. If Oliver worked 52 hours this week, what would his gross earnings be?

- Xena is a hairstylist who works for a base hourly wage of $12. She works 35 hours per week and receives $160.50 in tips for the week. Calculate her gross pay.

- Joshua is a waiter at the local restaurant and is paid an hourly rate of $10 plus tips. Joshua earns 6% of the tips received in a shift. During his shift on Tuesday, $1500 in total was received in tips. Calculate Joshua's gross pay for Tuesday.

- Saleem works 48 hours per week at an hourly rate of $16 per hour. After 40 hours he receives time and a half. Saleem calculated his income using the following method.

  Step 1: Regular pay = 40 hr. × $16 = $640
  Step 2: Overtime hours = 48 – 40 = 8 hr.
  Step 3: Overtime pay = 8 hr. × $16 = $128
  Step 4: Gross pay = regular pay + overtime pay
  Step 5: Gross pay = $640 + $128 = $768

  Is Saleem's gross pay correct? If not, identify the step in which the error occurred and determine the correct gross pay.

- Paxton's biweekly gross salary is $2800. He has to pay the following deductions:

  EI: 1.73%
  CPP: 4.95%
  Income Tax: 25%

  (a) calculate each deduction
  (b) determine Jeff's net pay

- Lesley earns $11.50 per hour. She works 35 hours a week. Her weekly deductions are as follows:

  EI: $9.06
  CPP: $14.41
  Income Tax: $49.10
  Company Pension Plan: $10.77
  Health Plan: $4.85

  Determine her
  (a) gross pay
  (b) total deductions
  (c) net pay

- Describe, in your own words, how a higher gross income affects deductions.

- Kadeem wants to move into an apartment and is wondering how much he can afford to pay for rent. Offer him advice on whether he should consider his gross income or his net income. Explain.

- In the role of a business owner, use the Payroll Deductions Online Calculator (see Resources/Notes, p. 243) to determine the CPP, EI, and tax deductions for an employee.

- Working with the job that your teacher has assigned you (and your partner):
  (a) Calculate the gross annual salary.
  (b) Determine which federal tax bracket it fits into and calculate the federal income tax deduction.
  (c) Repeat (b) for Nova Scotia tax.
  (d) Calculate EI and CPP deductions.
  (e) Complete a blank T4 form.
  (f) Draw five cards from a deck of cards containing other considerations, such as childcare, charitable donations, rent, RRSP, tuition amounts, tips, student loan payments, moving expenses, transportation, and dependents. Complete the income tax including your five drawn considerations.

- Daija's biweekly gross salary is $2400. She has to pay the following deductions:

  EI: 1.73%
  CPP: 4.95%
  Income Tax: 25%

Daija used the following steps to calculate her net income.

Step 1: EI = $2400 × 0.0173 = $41.52
Step 2: CPP = $2400 × 0.0495 = $118.80
Step 3: Tax = $2400 × 0.25 = $600
Step 4: Total deductions = $760.32
Step 5: Net income = $2400.00 – $760.32 = $1639.68

Is Daija's net pay correct? If not, identify the step in which the error occurred and calculate Daija's correct net pay.

- Research the business and classified ads section of a newspaper or the government's job bank and find a job that pays by
  (a) a salary
  (b) an hourly wage
  (c) a straight commission
  (d) a salary plus commission
  (e) piecework

- Share job descriptions with other students in your class and discuss which jobs are of interest to you. From the figures you have acquired on your ads, calculate the yearly, monthly and weekly gross wages available for each job. Describe the advantages and disadvantages of one of the methods of earning income you have researched.

- Adair works at Sears selling appliances. His base salary is $300/week and he makes 5% on his sales. During the month of August, he sold appliances worth $120 000. What is his gross pay for that particular month?

- Lan fishes with his grandfather in the summertime. He is paid 6% commission on the amount of catch landed. If $7500 worth of fish is landed, what is his gross pay?

- Monica's monthly gross income is $1916.67
  (a) Calculate her Employment Insurance (EI) payment using the formula given.
  (b) Calculate her monthly Canada Pension Plan (CPP) payment using the formula given.

  Note: Check rates for EI and CPP, as they change yearly, and provide students with the appropriate formula.

- Brogan is a sales clerk in a bicycle shop. He is paid $11.25/hour for a 37.5 hour week, plus a commission of 6% of his sales for the week. In one week Brogan's sales were $2319.75.
  (a) Calculate Brogan's gross pay for the week.
  (b) What was his average hourly wage for that week?
  (c) What would Brogan's sales for the week have to be for him to earn a total of $700 in one week?

- Complete the following time schedule for each employee:

Employee Hours:						Regular Hours: 8					Overtime Rate: 1.5	
Name	Start	Lunch Out	Lunch In	End	Total Hours	Worked Hours	Regular Hours	Over-time Hours	$/hr.	Reg $	Over $	Total
Ryan	9:00	12:00	13:00	18:00	9:00	8:00	8:00	0:00	$11.00	$88.00	$0.00	$88.00
Sheila	8:30	11:30	12:30	18:30					$11.00			
Katelyn	8:45	12:30	13:15	16:30					$12.25			
Simon	22:00	0:30	1:00	9:00					$11.50			

- Claudette has been offered two jobs, one of which offers an annual salary of $37 500, and the other that requires working a 40-hour week for 50 weeks of the year at $18.25/hr.
  (a) Calculate the weekly pay for each option.
  (b) Which option gives Claudette the greatest gross income?

- As a waitress, Carla earns $7.25/hour for a 40-hour week and shares 25% of her tips with other employees. In one week, her tips were $318. What was Carla's gross pay for the week?

- Aadesh is paid $8.50/hr. for a 37.5-hour week and earns double-time for overtime.
  (a) Calculate Aadesh's gross pay if his total over-time was 4.5 hours.
  (b) Determine CPP, EI, and Income Tax deduction amounts
  (c) Calculate net pay if the other deductions total $14.73.

- Identify and correct the error made when solving the following problem:
  A real estate agent earns 2.4% on the sale of a house. The last house she sold earned her a commission of $4128. What was the selling price of the house?
  C = PR
  $4128 = P × 2.4%
  P = $4128 × 2.4%
  P = $9907.20
  The selling price of the house is $9907.20.

FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should inform instruction.

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?

- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SUGGESTED LEARNING TASKS

Effective instruction should consist of various strategies.

Guiding Question

- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

Consider the following sample instructional strategies when planning lessons.

- Students should be provided with a time schedule and asked to compute regular, overtime and shift premium hours. They should be exposed to situations in which an employee works a fraction of an hour (e.g., 15 minutes, 30 minutes, or 45 minutes). In such cases, students should convert minutes to hours in decimal form. They should recognize, for example, that 15 minutes is 0.25 of an hour. When considering an example such as the following, students must be careful not to include overtime hours twice.
  - Alyson works at a local donut shop. Her regular pay is $10 per hour and she earns a shift premium of $1 per hour for hours worked between 12 a.m. and 8 a.m. Alyson gets 1.5 times the regular rate of pay for overtime hours worked above 40 hours per week. Alyson's time schedule is shown below:

Monday	8 a.m.–4 p.m.
Tuesday	8 a.m.–8:00 p.m.
Wednesday	12 a.m.–8 a.m.
Thursday	12 a.m.–5:30 a.m.
Friday	6 a.m.–12:15 p.m.
Saturday	Holiday
Sunday	8 a.m.–4 p.m.

Ask students to

(a) Calculate Alyson's regular hours.
(b) Calculate Alyson's premium hours.
(c) Calculate Alyson's overtime hours.
(d) Calculate Alyson's gross pay for the week.

Note: An extension of this activity would be to ask students to calculate the gross pay ($531.75). It may be a good idea to encourage students to compute overtime hours first. Otherwise, because overtime hours can occur during the regular work day, the tendency could be to include the overtime hours with the regular hours as well.

- A discussion of terminology related to income calculations is necessary before doing the calculations. A pre-assessment of what students know about gross pay, pay periods, deductions, and the types of deductions may be beneficial here. Students could be given an admit card with a job and three options of how to be paid. They choose the best method and justify their choice.

- Technology (e.g., calculator, online payroll calculator, tax programs, spreadsheets) should be used to compare various income rates. Students should examine how changes in rate of pay, number of hours worked, increases in income, or decreases in deductions impact net income.

- Engaging students in error analysis heightens awareness of common errors. Along with providing the correct solutions, students should be able to identify incorrect solutions, including why errors might have occurred and how they can be corrected. Questions requiring error analysis can be effective tools to assess students' understanding of gross and net pay calculations because it requires a deeper understanding than simply "doing the problem." Analyzing errors is a good way to focus discussion on "How did you get that?" rather than being limited to "Is my answer right?" This reinforces the idea that the process of determining the solution is as important as the solution itself.

- Ask students if anyone has had the experience of being surprised when he or she received his or her first paycheque and realized how much money had been deducted. Students need to be aware of the deductions their employers take from their cheques and by how much this will reduce their gross income. Some typical deductions are Employment Insurance (EI), Canada Pension Plan (CPP), pension, union dues, benefits, and income tax.

- In 2012, the EI premium rate was 1.73% of gross earnings and the maximum annual employee premium was $747.36. Illustrate with a pay stub an example of EI being deducted, and one where it is paid up.

- Students should examine federal and provincial tax deduction tables and discuss why provincial and federal amounts are different.

- Students should also discuss optional deductions, such as company health and pension plans, union fees.

- Students should brainstorm, research, and discuss possible advantages and disadvantages of various income earning methods. Some suggestions follow. However, this list is not intended to be exhaustive.

Method	**Advantage**	**Disadvantage**
Hourly Wage	guaranteed income for hours worked	reduced hours during slow periods
Tips	additional income beyond regular salary	job may not pay well
Piecework	more money if you work faster	may ignore safety standards to work faster
Salary	income continues during slow sales periods	work overtime without extra income
Commission	increased income during good sales periods	decreased income during slow sales periods
Contract Work	guaranteed contract income	decreased yearly income if job takes longer than expected or expenses are greater than expected

At the end of this section

- have students generate a list of jobs that interest them and then have them work in teams to categorize the jobs by payment method (salary, hourly wage, commission)
- (as a class) discuss other payment methods and the advantages and disadvantages of each method
- provide students with a scenario and have them complete a pay stub, filling in all the important information (EI, CPP, gross pay, net pay, etc.)

SUGGESTED MODELS AND MANIPULATIVES

- calculator

MATHEMATICAL VOCABULARY

Students need to be comfortable using the following vocabulary.

- contract
- CPP
- deductions
- EI
- gross pay
- hourly wage
- income
- net pay
- piecework
- salary
- wages

Resources/Notes

Internet

- Canada Revenue Agency (Government of Canada 2013)
  www.cra-arc.gc.ca/menu-eng.html
- Canada Revenue Agency, "Payroll Deductions On-line Calculator" (Government of Canada 2013)
  www.cra-arc.gc.ca/esrvc-srvce/tx/bsnss/pdoc-eng.html
  PDOC calculates federal and provincial payroll deductions for provinces and territories.
- Jobbank.ca Website
  http://jobbank.ca
- PayScale, "Get the Right Salary Data for You." (PayScale Inc. 2013)
  www.payscale.com
  Students can visit this site to determine hourly wages and gross annual income for jobs in various Canadian cities.
- XE, Currency Encyclopedia (XE 2013)
  http://xe.com/currency
  Provides rates and information for every currency

Print

- *Financial Mathematics* (Etienne and Kalwarowsky, McGraw-Hill 2013)
  - Sections FM.3, FM.4, and FM.5, pp. 32–67
  - Teacher's Resource Blackline Masters
    - BLM FM–2 Financial Mathematics Warm-Up
    - BLM FM–3 Financial Mathematics Mini Projects
    - BLM FM–6 Calendar Templates
    - BLM FM–7 FM.3 Extra Practice
    - BLM FM–8 Blank Pay Stubs
    - BLM FM–9 Blank T4
    - BLM FM–10 FM.4 Extra Practice
    - BLM FM–11 FM.5 Extra Practice

Notes

SCO FM03 Students will be expected to investigate personal budgets. [C, PS, R, T]			
[**C**] Communication	[**PS**] Problem Solving	[**CN**] Connections	[**ME**] Mental Mathematics and Estimation
[**T**] Technology	[**V**] Visualization	[**R**] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have met the corresponding specific curriculum outcome.

FM03.01 Identify income and expenses that should be included in a personal budget.
FM03.02 Explain considerations that must be made when developing a budget (e.g., prioritizing, and recurring and unexpected expenses).
FM03.03 Create a personal budget based on given income and expense data.
FM03.04 Collect income and expense data, and create a budget.
FM03.05 Modify a budget to achieve a set of personal goals.
FM03.06 Investigate and analyze, with or without technology, "what if …" questions related to personal budgets.

Scope and Sequence

Mathematics 9	**Mathematics 10**	**Mathematics 12**
—	**FM03** Students will be expected to investigate personal budgets.	**FM01** Students will be expected to solve problems that involve compound interest in financial decision making. **FM02** Students will be expected to analyze costs and benefits of renting, leasing, and buying. **FM03** Students will be expected to analyze an investment portfolio in terms of interest rate, rate of return, and total return.

Background

In SCO FM02, students were expected to demonstrate an understanding of income (e.g., net and gross pay) and deductions (e.g., CPP, EI, income tax, health plans, and union dues) including wages, salary, contracts, commissions, and piecework.

Income, deduction, and expense data will now be used by students to prepare budgets. Budgets are useful for

- determining how money is being made and spent
- preventing the accumulation of large amounts of debt
- for planning for future purchases or emergencies

Students are expected to create a budget based on data that are provided, as well as data they collect themselves. At the start of the unit, students should be asked to record their personal expenses and income for one week in preparation for creating their own budget later in the unit. Once completed, students should modify their budget to achieve immediate personal goals (such as purchasing a car, and making car and insurance payments) and analyze changes to their budget necessary to achieve future goals.

To create a personal budget, students should identify

- net pay
- income from other sources (e.g., investments, tax credits, and rental properties)
- fixed expenses (e.g., rent, car payments, and telephone/internet bills)
- variable expenses (e.g., heat, electricity, dining out, and repairs)

It may be useful for students to first identify general categories of expenses (e.g., housing, transportation, insurance, education, food, pets, personal, entertainment, loans, taxes, savings and investments, recreation, miscellaneous) and then give specific expenses under each category.

When developing personal budgets, students need to prioritize expenses to first ensure that the necessities of life are met. Ideally, some money will remain for investing and unexpected expenses (e.g., auto repair, health care, gifts, appliance replacement). Some expenses, such as property tax, are billed annually but can be paid monthly. Students need to be cognizant of this additional expense when making a monthly budget. Students must realize that, for some expenses, the amount budgeted and the amount actually spent may not match. It is suggested that students be provided with the data needed to complete a monthly budget, based on stated assumptions about income levels and expenses. Additional assumptions that should be stated in assignments include owning or renting housing, transportation options such as owning or leasing a car, or using public transit and taxis. Assignments should reflect diversity of lifestyles, values, and economic realities for students and their families.

Once their personal budgets are completed, students should identify some personal goals, the monetary cost of each goal, and a timeline for achieving each goal. Students should then determine how to modify their budget to attain these goals. For example, students may select an educational goal (such as attending a trade program at a local college), the purchase of a new vehicle, or living on their own. Students would then determine what budget changes are necessary to fund their goals. These changes may involve a combination of increasing income and decreasing or eliminating expenses. Students should be helped to realize that a realistic and achievable goal can sometimes be derailed by unexpected and uncontrollable events. This can be investigated by posing “what if” scenarios and determine if previously identified goals are still attainable. Spreadsheets are an excellent tool for investigating the solutions to these scenarios. For example, students could be faced with loss of income, major health expenses, vehicle or property damage, or unexpected family costs.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Student tasks such as the following could be completed to assist in determining students' prior knowledge.

- Determine the tax on a purchase of $120. (Tax rate is 15%).
- Estimate the sum of $43.52, $24.31, and $57.48. Explain your strategy.
- Mentally determine 3 × $8.50 and explain your strategy.
- Mentally determine 25% of $440 and explain your strategy.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

- Make a list of variable and fixed expenses that should be included in your personal budget. Based on a weekly income of $250, prepare a personal budget where you will save at least $25 per week.

- Explore options for when you finish high school (such as post-secondary studies or employment). List the sources of income (e.g., part-time jobs, scholarships, savings, loans) and possible expenses that you would incur (e.g., tuition, transportation, rent). Then, using the collected data, develop a personal budget.

- Using the completed budget and a goal provided by your teacher, determine if the goal is attainable/realistic.

- Michelle is a high school student living at home. She has a part-time job. Michelle's budget for a month is in the following table.

Monthly Net Income	**Budget**
Job	$475
Monthly expenses	Fixed expenses
Savings	$45
Internet access	$20
Cell phone	$25
Transportation (bus fare)	$40
Variable expenses	
Clothing	$250
Entertainment	$50
Personal items	$55
School expenses	$35
Other (gifts)	$50

(a) Calculate her total expenses.
(b) How much more are her planned expenses than her income?
(c) Suggest ways she can alter her spending to keep her planned expenses less than or equal to her income.

- It has been suggested that families have a reserve equal to 6 times the family's monthly income for unexpected events such as job loss. List the advantages and disadvantages of this suggestion. This could be extended to ask how realistic this is for some families (single parent; only one parent working; low-income earners; people on social assistance). For situations in which this is not feasible, what other strategies could be employed to prepare for unexpected events?

- Use a chart similar to the following as a guide to list all the expenses you think you might incur living on your own or with one or more roommates (this can be used as a group activity).

Expense	**Amount ($)**
Getting started costs One-time costs, such as hook-up fees for phone, cable /internet, purchase of furniture, dishes, appliances.	
Rent or Mortgage	
Utilities Electricity, heat	
Food Staples, such as flour, spices, condiments, beans; regular groceries. Home-cooked meals are cheaper and usually healthier than restaurant food.	
Transportation Public transit, bicycle, or car. If you have a car, you will need to budget for insurance, gas, maintenance, and parking.	
Medical/Dental Medical plan payments and/or costs such as glasses, contacts, prescriptions, and dental care not covered by Nova Scotia Health Insurance or by a medical plan.	
Clothing Consider clothes required for work and seasonal clothes, such as boots and a winter coat.	
Miscellaneous This may include laundry, entertainment, toiletries, and cleaning supplies. Also consider purchasing gifts for birthdays and holidays.	
Other This includes anything else that is not included in other categories, such as loan payments, vacations, memberships, or workshop fees.	
TOTAL OF ALL ESTIMATED COSTS	

FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should inform instruction.

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?

- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SUGGESTED LEARNING TASKS

Effective instruction should consist of various strategies.

Guiding Question

- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

Consider the following sample instructional strategies when planning lessons.

- Spreadsheets, such as Excel, could be effectively used in situations such as calculating and revising budgets as well as to keep track of expenses and income.

- Provide students with a budget and a goal that is currently unattainable. Ask them to decide which expenses to reduce or eliminate in order to make the goal attainable.

- After completing a budget from given data, students may be provided with income data and some basic expenses and asked to add at least three expenses of their own. Using these data, students could create a second budget. Eventually, students will be expected to collect their own income and expense data and create a budget. This data may be collected from sources such as telephone and electricity bills, credit card statements, catalogues, and advertising flyers. Using the data collected at the beginning of the unit, and data from other sources, students can create their personal budget.

Sample Monthly Family Budget (2 incomes)

Income ($)	Expected Expenses ($)	Actual Expenses ($)	Difference ($)
Net Pay (1): 2300	Mortgage: 700	700	0
Net Pay (2): 1500	Property Tax: 120	120	0
Rental Income: 500	Groceries: 600	650	-50
	Child Care: 800	800	0
	Dining Out: 250	200	50
	Heat and Light: 200	230	-30
	Telephone: 30	70	-40
	Mobile Phone: 50	30	20
	Cable/Internet: 120	120	0
	Car Payment: 400	400	0
	Gas: 300	350	-50
	Insurance (vehicle): 200	200	0
	Insurance (house): 50	50	0
	Clothing: 150	120	30
	Personal Care: 50	70	-20
	Investments: 120	120	0

Sample Monthly Family Budget (1 income)

Income ($)	Expected Expenses ($)	Actual Expenses ($)	Difference ($)
Net Pay (1): 1650	Rent: 600	600	0
	Property Tax: 0	NA	NA
	Groceries: 300	325	-25
	Child Care: 400	400	0
	Dining Out: 20	25	-5
	Heat and Light: 150	180	-30
	Telephone: 35	70	-40
	Mobile Phone: 0	NA	NA
	Cable/Internet: 0	NA	NA
	Car Payment: 0	NA	NA
	Public Transit: 75	75	0
	Gas: 0	NA	NA
	Insurance (vehicle): 0	NA	NA
	Insurance (house): 0	NA	NA
	Clothing: 40	50	-10
	Personal Care: 30	10	20
	Investments: 0	NA	NA

- Present students with various scenarios—living on their own and working, living as a single parent with an infant or school-aged child, going to school and working part-time, working full-time, living as a two-income family with two children, among other possible scenarios. Have them develop their own budgets that include all of their expenses or complete a budget such as the one found in Chapter 9 of Money and Youth (Canadian Foundation for Economic Education 2013) at http://moneyandyouth.cfee.org/en/resources/pdf/moneyyouth_chap9.pdf.

Note: You may have students who are living in the scenarios listed above. This could also be an opportunity for students to interview someone living in various circumstances listed above to get a true picture of expenses, especially hidden ones.

Suggested Models and Manipulatives

-
-

Mathematical Vocabulary

Students need to be comfortable using the following vocabulary.

- fixed expenses
- variable expenses

Resources/Notes

Internet

Use some of the free online resources available on financial literacy such as

- Investor Education Fund. (Get Smarter About Money.ca 2013)
  www.getsmarteraboutmoney.ca
- Literacy at Every Level, "Skills for Life, Unit 6—Managing My Money." (Quebec Literacy Working Group 2013)
  www.qlwg.ca/index.php/skills-for-life
  A free download, Managing my Money.
- "Create a Budget Calculator—Manage your Debt Effectively." (Royal Bank of Canada 2013)
  www.rbcroyalbank.com/products/personalloans/budget/budget-calculator.html
  A resource provided by RBC to explore the financial implications of "What if?" situations such as job loss, loss of roommate, illness etc.
- Starting Point: Teaching Entry Level Geoscience. "How to use Excel" (Carleton College 2013)
  http://serc.carleton.edu/introgeo/mathstatmodels/xlhowto.html
  Part of Carleton College in Minnesota, it has links to several useful tutorials for using the Excel spreadsheet program.

- Financial Consumer Agency of Canada. *The City: A Financial Life Skills Resource* (Government of Canada 2013)
  www.fcac-acfc.gc.ca/eng/education/index-eng.asp

Print

- *Financial Mathematics* (Etienne and Kalwarowsky, McGraw-Hill 2013)
  - Sections FM.6, pp. 68–81
  - Teacher's Resource Blackline Masters
    - BLM FM–2 Financial Mathematics Warm-Up
    - BLM FM–3 Financial Mathematics Mini Projects
    - BLM FM–12 Budget Tracking
    - BLM FM–13 Setting Up an Apartment
    - BLM FM–14 FM.6 Extra Practice
    - BLM FM–4 FM.1 Extra Practice
    - BLM FM–5 FM.2 Extra Practice
    - BLM FM–7 FM.3 Extra Practice
    - BLM FM–9 Blank T4
    - BLM FM–10 FM.4 Extra Practice
    - BLM FM–11 FM.5 Extra Practice
    - BLM FM–14 FM.6 Extra Practice

Notes

SCO FM04 Students will be expected to explore and give a presentation on an area of interest that involves financial mathematics. [C, CN, ME, PS, R. T, V]			
[**C**] Communication [**T**] Technology	[**PS**] Problem Solving [**V**] Visualization	[**CN**] Connections [**R**] Reasoning	[**ME**] Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have met the corresponding specific curriculum outcome.

FM04.01 Collect primary or secondary data (statistical or informational) related to the topic.
FM04.02 Organize and present a project.
FM04.03 Create and solve a contextual problem that is related to the project.
FM04.04 Make informed decisions and plans related to the project.
FM04.05 Compare advantages and disadvantages as part of the project.

Scope and Sequence

Mathematics 9	**Mathematics 10**	**Mathematics 11**
SP03 Students will be expected to develop and implement a project plan for the collection, display, and analysis of data by ▪ formulating a question for investigation ▪ choosing a data collection method that includes social considerations ▪ selecting a population or a sample ▪ collecting the data ▪ displaying the collected data in an appropriate manner ▪ drawing conclusions to answer the question	**FM04** Students will be expected to explore and give a presentation on an area of interest that involves financial mathematics.	**MR01** Students will be expected to research and give a presentation on a historical event or an area of interest that involves mathematics.

Background

In Mathematics 9, students have developed a project plan, drawn conclusions, communicated their findings to an audience, and created a rubric to assess that project.

The purpose of this financial mathematics exploration and project is to
- prepare students for learning independently
- provide students with the opportunity to explore in more depth,
  - financial mathematics content that they have been exposed to but would like to know more about
  - new financial mathematics content areas not yet explored
  - financial mathematics topics of interest

If the teacher has a sufficient number of students enrolled, it is recommended that students chose a research project from the following topics. If class size is large, additional topics may be selected based on student interest.

Possible topics
- Credit cards and interest
- Cellphone plans
- Buying versus leasing a car
- Student loans (government versus bank)
- Banking charges
- Operating a car
- Transportation options
- Insurance (house, tenant's insurance, personal, medical)
- Viability of small business options
- Saving money (TFSA, RRSP, savings account, stock market)

One of the goals of this project is to give students an opportunity to communicate their learning experience with others and to be creative in their delivery of the presentation.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

Guiding Questions
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Assessing Prior Knowledge

Student tasks such as the following could be completed to assist in determining students' prior knowledge.

- Your friend is unclear what the term **bias** means. Develop an example to help explain the term.

- A friend tells you that she has heard that there are lots of jobs for pharmacists. How could you find out if this is true? How could you determine if this is a job that might be right for you to consider?

Whole-Class/Group/Individual Assessment Tasks

Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

- Create an organizer, such as the flow chart below, to help organize the research project and carry out the plan.

  Step 1: Develop the project plan.
  - Write a statement describing what questions you wish to answer.
  - Describe how you will collect data.

  Step 2: Create a rubric to assess your project.

  Step 3: Continue to develop the project plan.
  - Describe how you will display the data.
  - Describe how you will analyze the data.
  - Describe how you will present your findings.

  Step 4: Complete the project according to your plan.
  - Display the data.
  - Analyze the data.
  - Draw a conclusion or make a prediction.
  - Evaluate the research results.

  Step 5: Present your findings.

  Step 6: Self-assess your project.

- Demonstrate what you have learned from presentations of others by completing a questionnaire that focuses on the highlights of a presentation.

- Collect summaries from students who have presented their research projects. Construct a question from each presentation that could be asked on a test.

- Keep a journal entry describing what you have learned from each of the projects that have been presented in class.

FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should inform instruction.

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SUGGESTED LEARNING TASKS

Effective instruction should consist of various strategies.

Guiding Question

- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

Consider the following sample instructional strategies when planning lessons.

- Teachers might wish to spread the project over one term, with some class periods being designated to complete the project introduction while other periods are checkpoints, each with a particular expectation. Additionally, class time could be allowed for finalization, preparing, and presenting.

- Approximately 10–15 hours of class time should be devoted to this research project. Teachers should allow time for
  - students to present the results of their research to other students and for the student audience to respond to each presentation (presentation and response time of an average of 10–15 minutes per student should be planned. If students are working collaboratively on this project, it is expected that each would be responsible for gathering certain information and, thus, could be held responsible for the oral presentation that deals with that part of the project.)
  - initial discussion of various topics to be explored and an assignment of topics to groups
  - discussing the expectations and developing, with student input, assessment rubrics for the students' presentations and how students will be assessed during the unit
  - brainstorming, topic-webbing, developing action plans and timelines, conferencing
  - peer evaluation

- It is critical to give students class time to complete some of the necessary steps, such as brainstorming, writing an outline, analyzing data, making decisions, and using technology. The time will be most productive if, at the end of each class work session, tasks are assigned to be completed prior to the next meeting.

- Students should brainstorm possible questions, ideas and/or issues relating to their topic.

- Encourage students to create and submit an action plan or graphic organizer outlining their tasks and time frames.

- Ask students questions periodically to ensure that their work is progressing and to give them support. Observe their attitudes, contributions, time on task, and teamwork. Some possible questions are
  - Where are you in the research process?
  - What are the different roles of the individuals in your group?
  - How are you collecting data or using data already collected?
  - What are you doing next?
  - Are you experiencing any problems with the research information?
  - Are you having any technological difficulties?
  - What resources are you using?
  - How are you presenting your result(s)?
  - What questions do you have?

- Ask students about the presentation of the project. To make the presentation interesting, encourage students to choose a format that suits the student's style or the group's style. Will students use a multimedia presentation, such as a video or a slideshow? Will it be in the form of a podcast? Will students create their own website or blog to present their project? Some students may prefer to present without the use of technology and use, for example, a poster board, a pamphlet, a debate, an advertising campaign, or a demonstration.

  Students must distribute summaries of the topic they are presenting to the class. This means that students should summarize what they learned so that other students can read over the summary, see a couple of examples, and have a good foundation in the new topic.

  It is important that some strategy is employed so that students value the information being presented by their peers. One such strategy would be to have the students develop questions, and answers, for the material that their group presented as well as a smaller number of questions for the material presented by other groups. The questions could be placed on file cards (answers on the reverse). [The teacher could assign part of a group's project mark to these cards.] Once all cards have been created by the various student groups, one class can be spent playing a game using these cards. The game could take various forms. For example, it might be similar to Jeopardy. Students could be asked to assign point values to the questions they created based on difficulty level. So for example, in the Category "Buying or Leasing a Car," there could be four 3-point questions, three 5-point questions, two 10-point questions, and one 15-point question.

  Peer evaluation should occur during presentations and from each group member on the group effort. This could include a reflective journal entry on each student's experience within his or her own group, as well as his or her experience in seeing and hearing from other groups.

  A rubric should be used to assess the project, and students should be aware of the criteria before they start their project. Encourage them to participate in the development of the rubric and to work out the appropriate categories and criteria for specific tasks. This involvement will improve student motivation, interest, and performance in the project. Content, organization, sources, and layout are critical components used to evaluate projects. Illustrations, images, and graphics are also important features that should be included in assessment. Remind students that there are many different ways to deliver a project. Therefore, the rubric may have to be modified to fit the format of the presentation.

Ask students to discuss categories and criteria that may be included in a rubric. Suggestions are shown below.

	Excellent	**Good**	**Competent**	**Needs Improvement**
Reasoning	Justifies all mathematical statements in an efficient and accurate manner, and draws valid conclusions.	Justifies most mathematical statements accurately, and draws valid conclusions.	Justifies some of the mathematical statements accurately, and draws valid conclusions.	Does not justify mathematical statements accurately, and does not draw valid conclusions.
Connections	Discusses, in depth, how mathematical concepts interconnect and build on each other.	Discusses how mathematics concepts interconnect and build on each other.	Discusses superficially how mathematics concepts interconnect and build on each other.	Does not discuss the inter-connection of mathematical concepts.

Top Level	▪ Contains a complete report with clear, coherent, unambiguous, and elegant explanations. ▪ Includes clear and simple diagrams, charts, graphs, etc. ▪ Communicates effectively to an identified audience. ▪ Shows understanding of the financial mathematics topic, processes and thinking that indicate careful and thorough consideration. ▪ Identifies all the important elements of the topic. ▪ Includes examples and counterexamples where appropriate. ▪ Gives strong supporting arguments.
Second Level	▪ Contains good solid report with some of the characteristics above. ▪ Explains less elegantly and less completely than desired. ▪ Does not go beyond the requirements of the project (or topic).
Third Level	▪ Contains a complete report, but the explanation is vague in places. ▪ Presents arguments, but they are incomplete at times. ▪ Includes diagrams, but some are inappropriate, unclear, or misplaced. ▪ Indicates some understanding of mathematical ideas, but not expressed clearly enough.
Fourth Level	▪ Omits significant parts. ▪ Has major errors. ▪ Uses inappropriate strategies.

See the books in Resources/Notes for more examples of rubrics for evaluating projects and open-ended activities.

SUGGESTED MODELS AND MANIPULATIVES

- graphic organizer

MATHEMATICAL VOCABULARY

Students need to be comfortable using the following vocabulary.

- credit
- lease
- loan
- insurance

Resources/Notes

Print

- *Financial Mathematics* (Etienne and Kalwarowsky, McGraw-Hill 2013)
  - p. 88
  - Teacher's Resource Blackline Masters
    - BLM FM–16 Project Plan
    - BLM FM–17 Project Rubric
    - BLM FM–18 Presentation Planner
    - BLM FM–19 Presentation Peer Assessment
    - BLM FM–20 Presentation Self-Assessment
    - BLM FM–21 BLM Answers

SUPPLEMENTARY PRINT

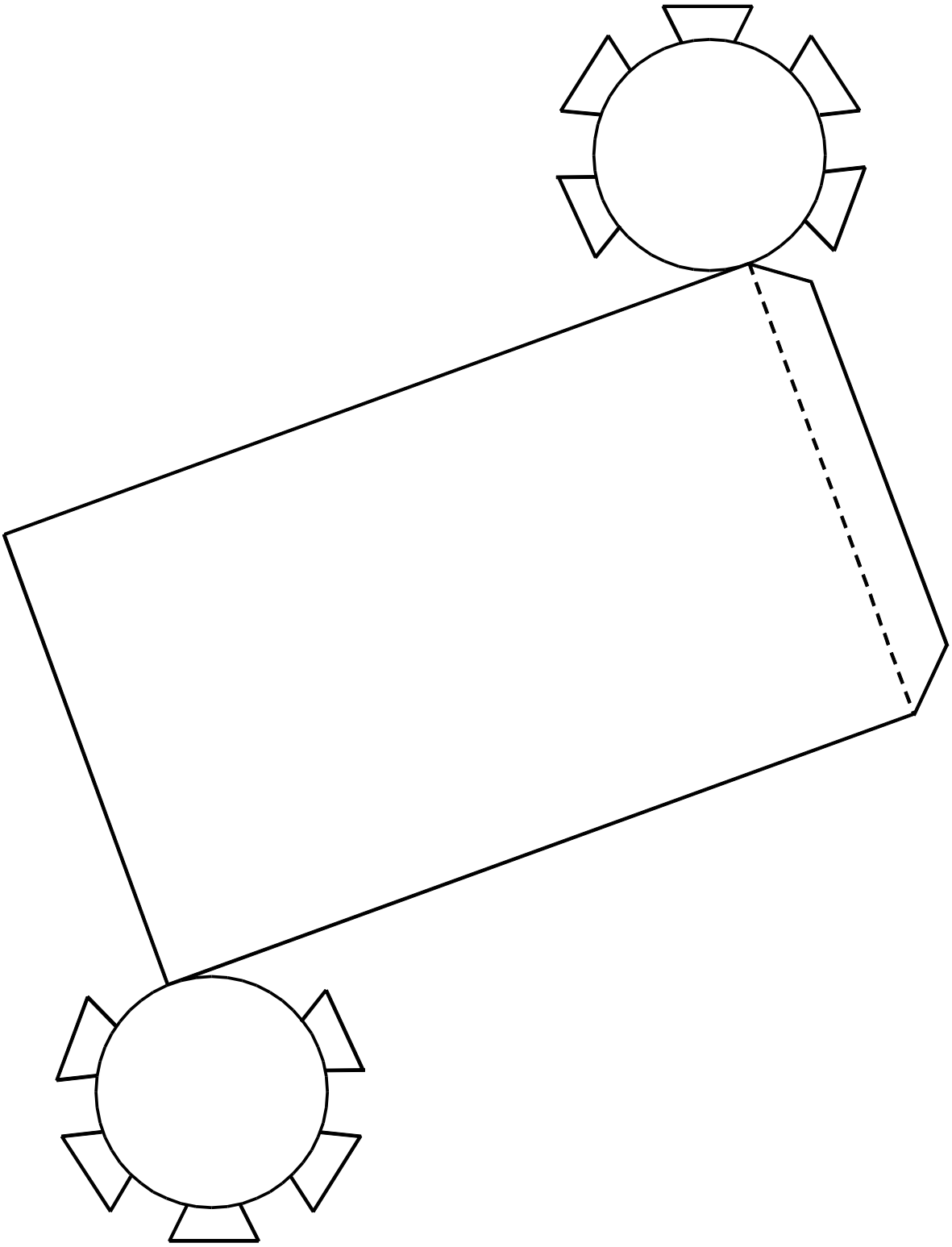
- *Assessment Alternatives in Mathematics* (Stenmark 1989)
- *Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions* (Stenmark 1991)
- *How to Evaluate Progress in Problem Solving* (Randall, Randall, Lester, and O'Daffer 1987)

Notes

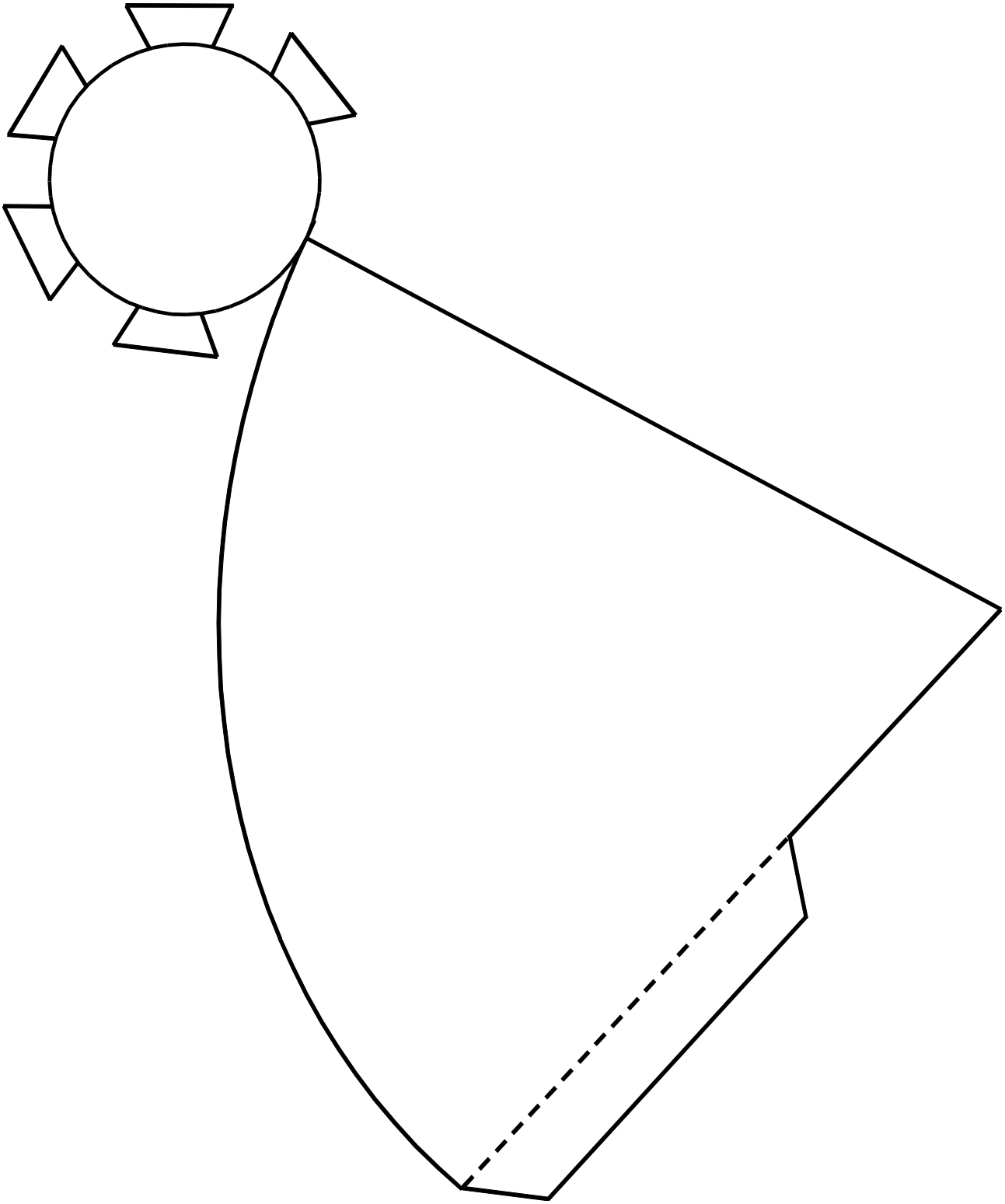
Appendices

Appendix A: Copyable Pages

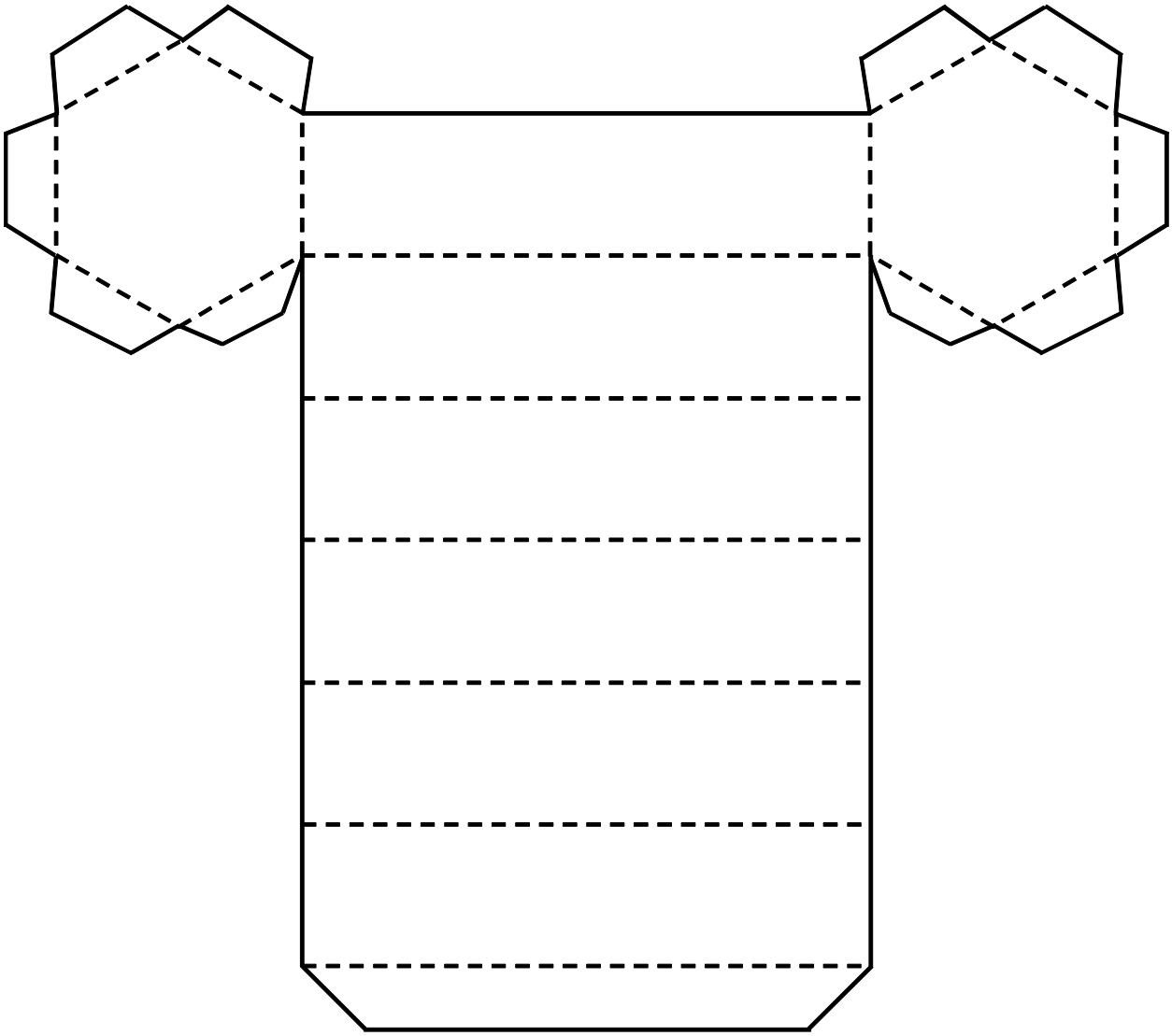
A.1 Net (closed cylinder)



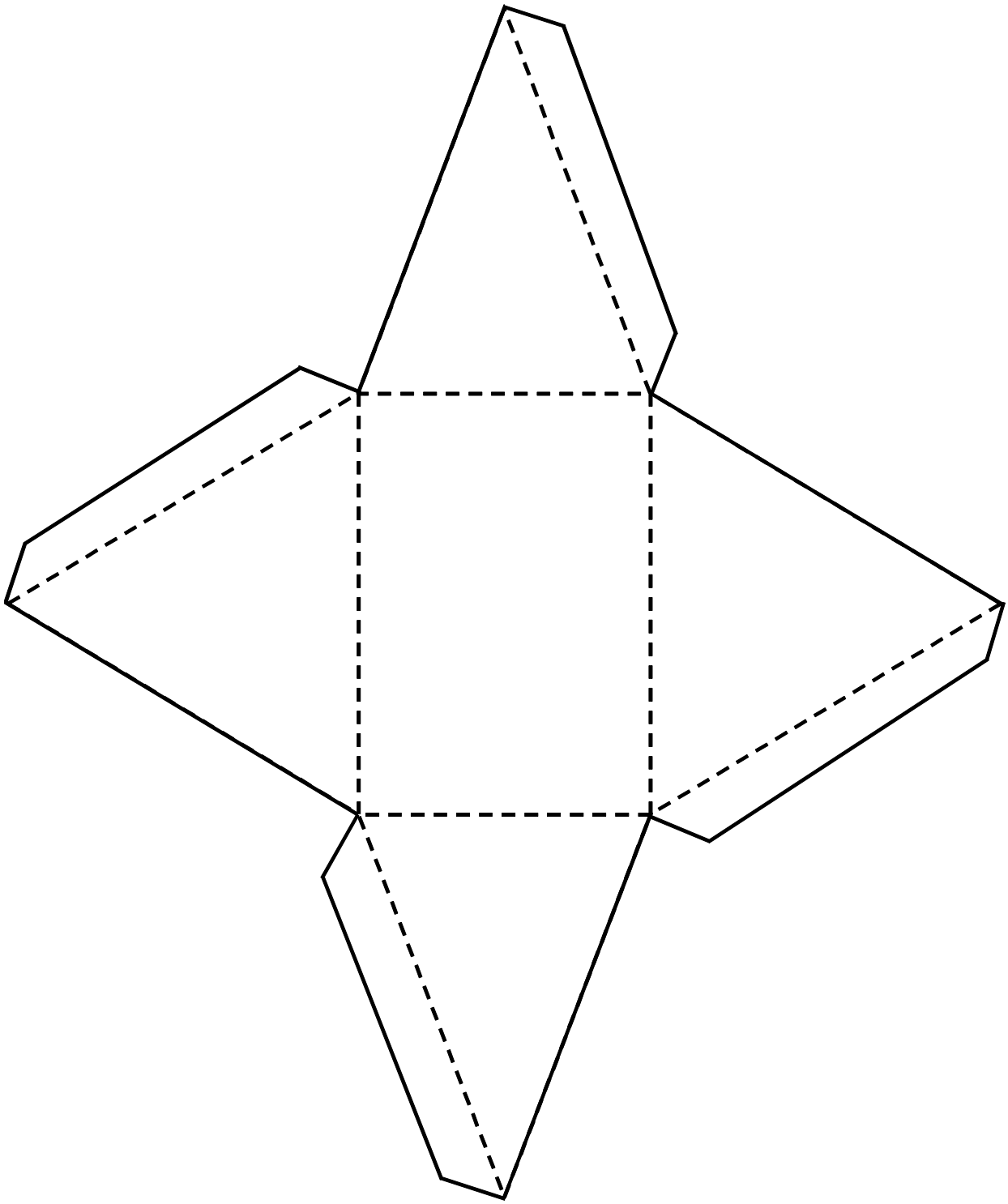
A.2 Net (cone)



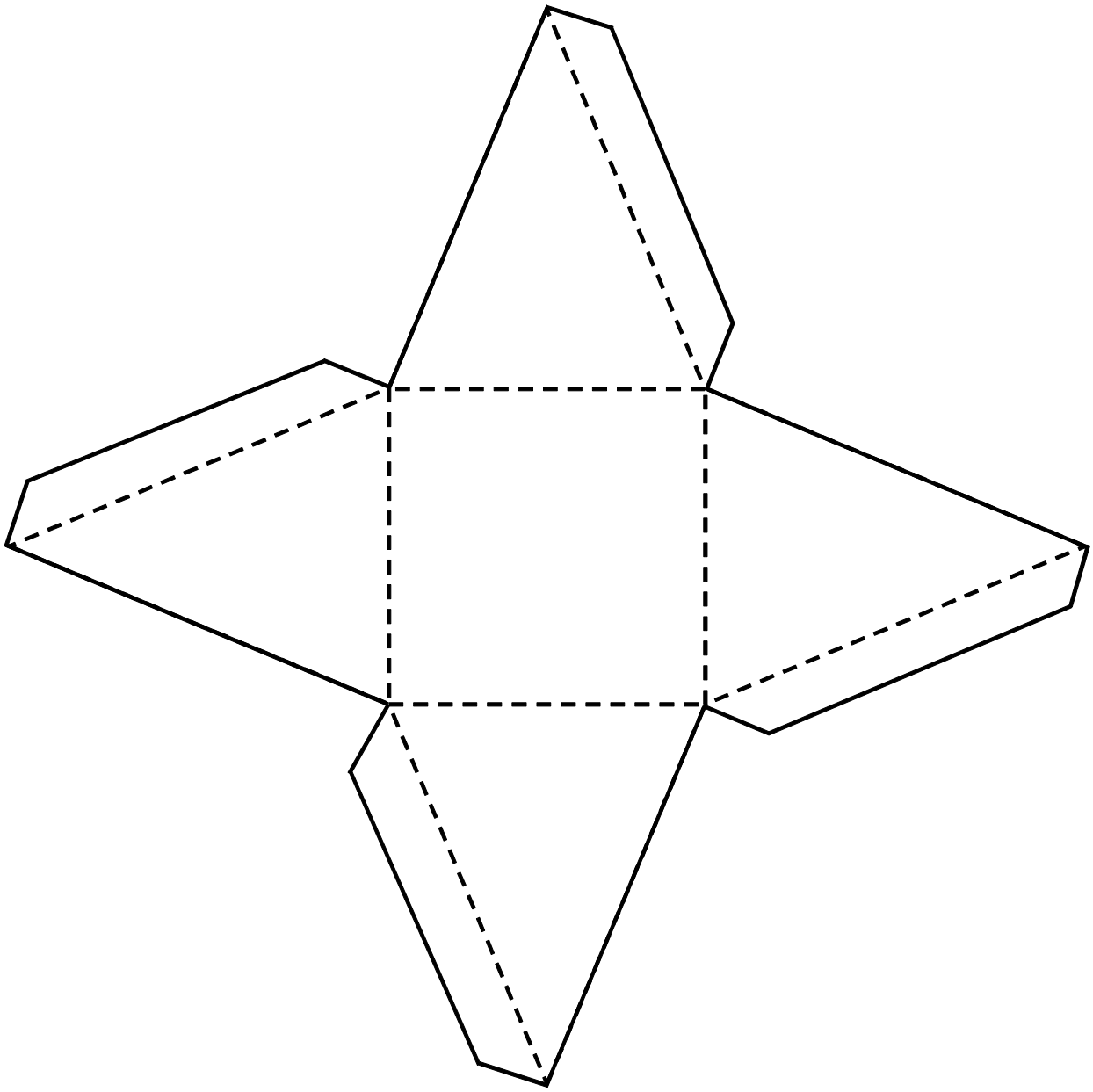
A.3 Net (hexagonal prism)



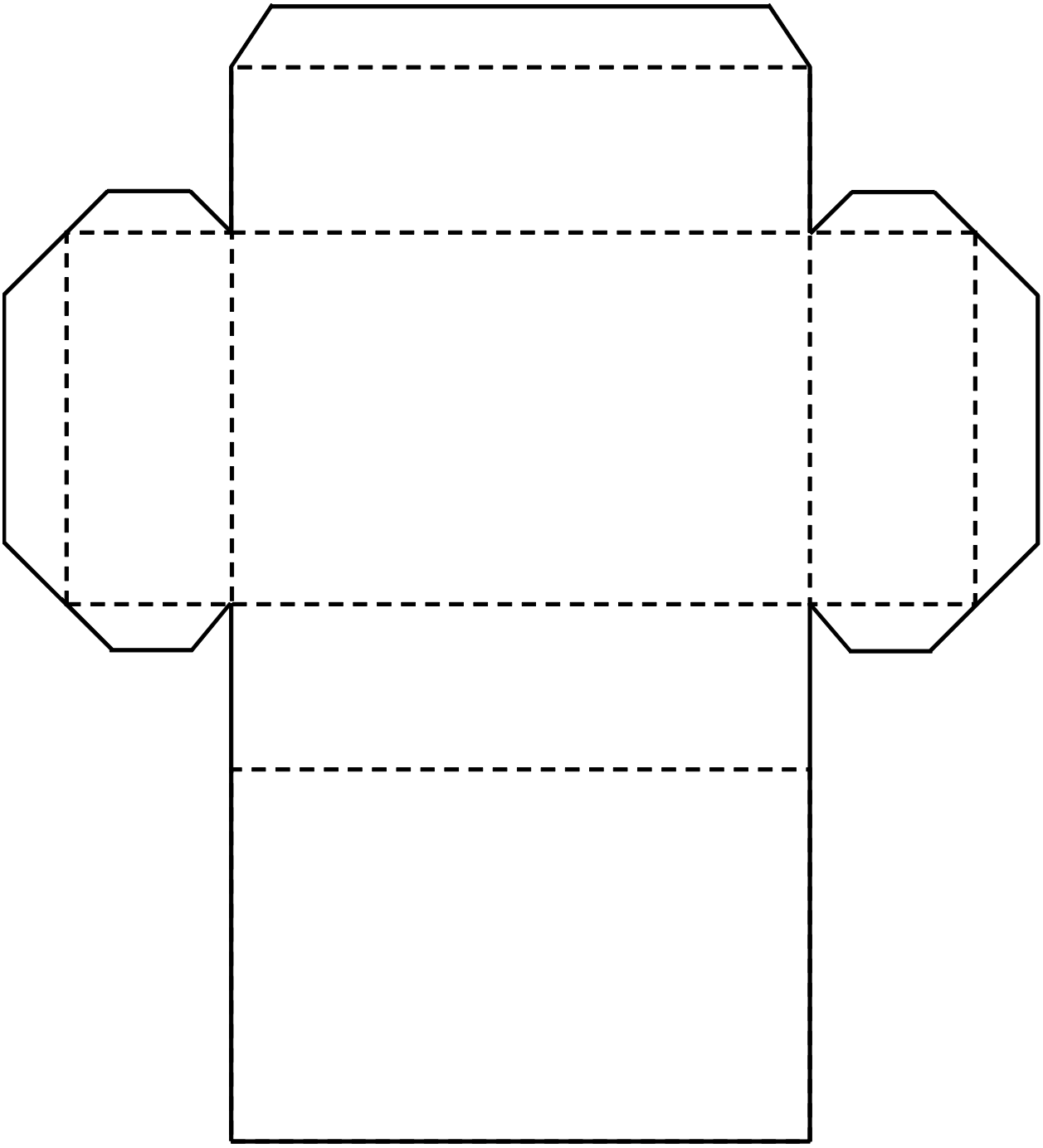
A.4 Rectangular Pyramid



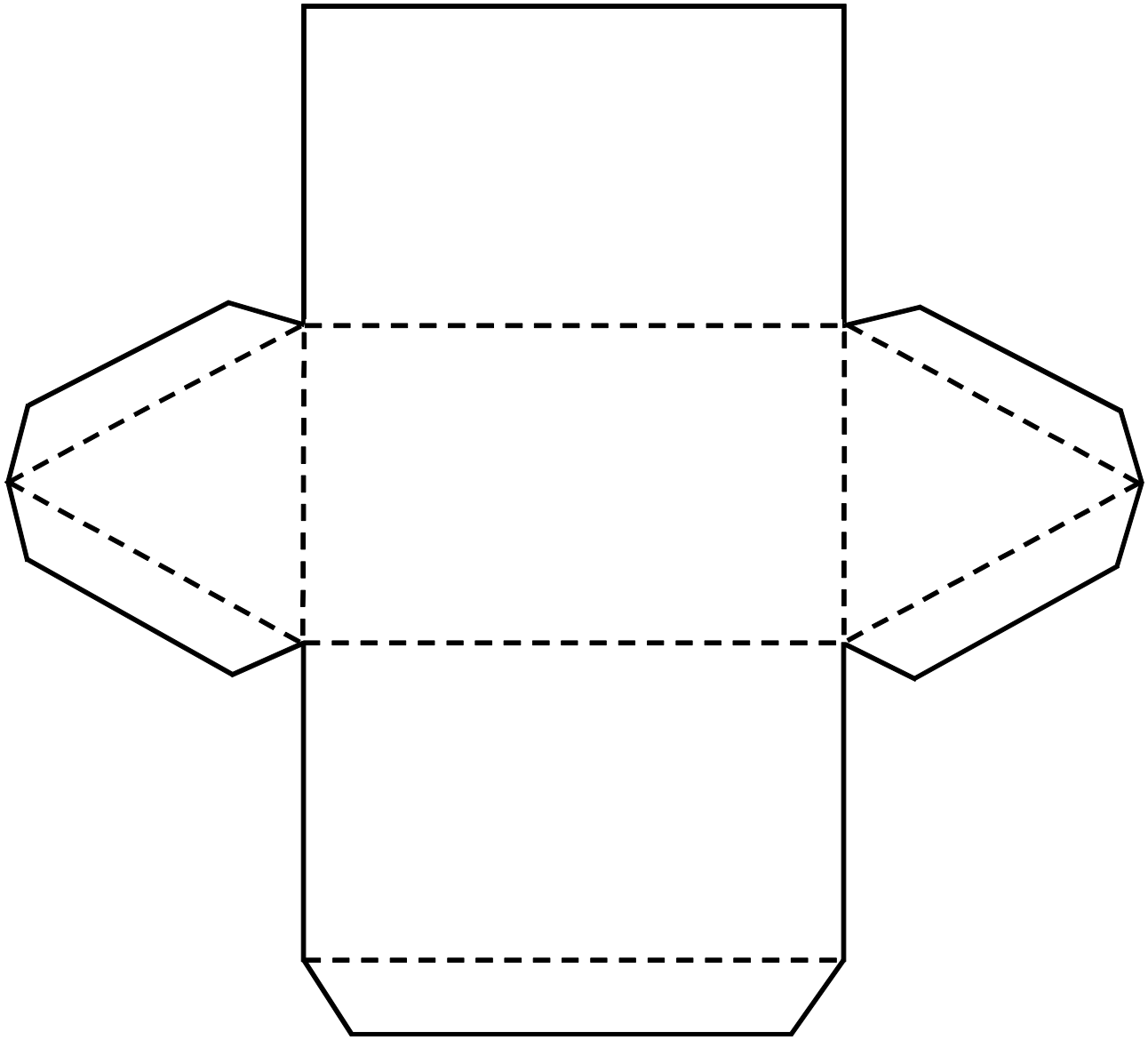
A.5 Net (square-based pyramid)



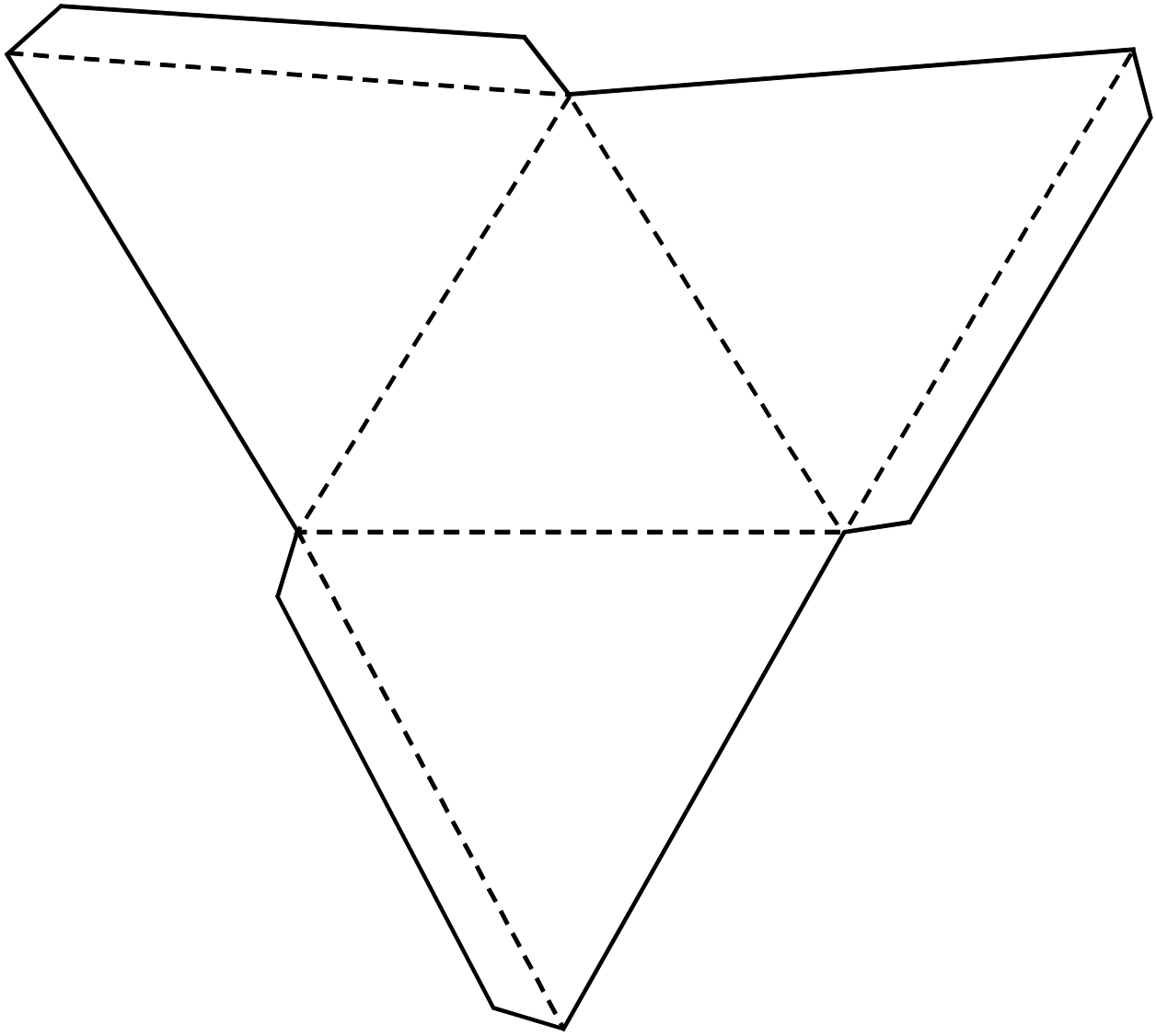
A.6 Net (rectangular prism)



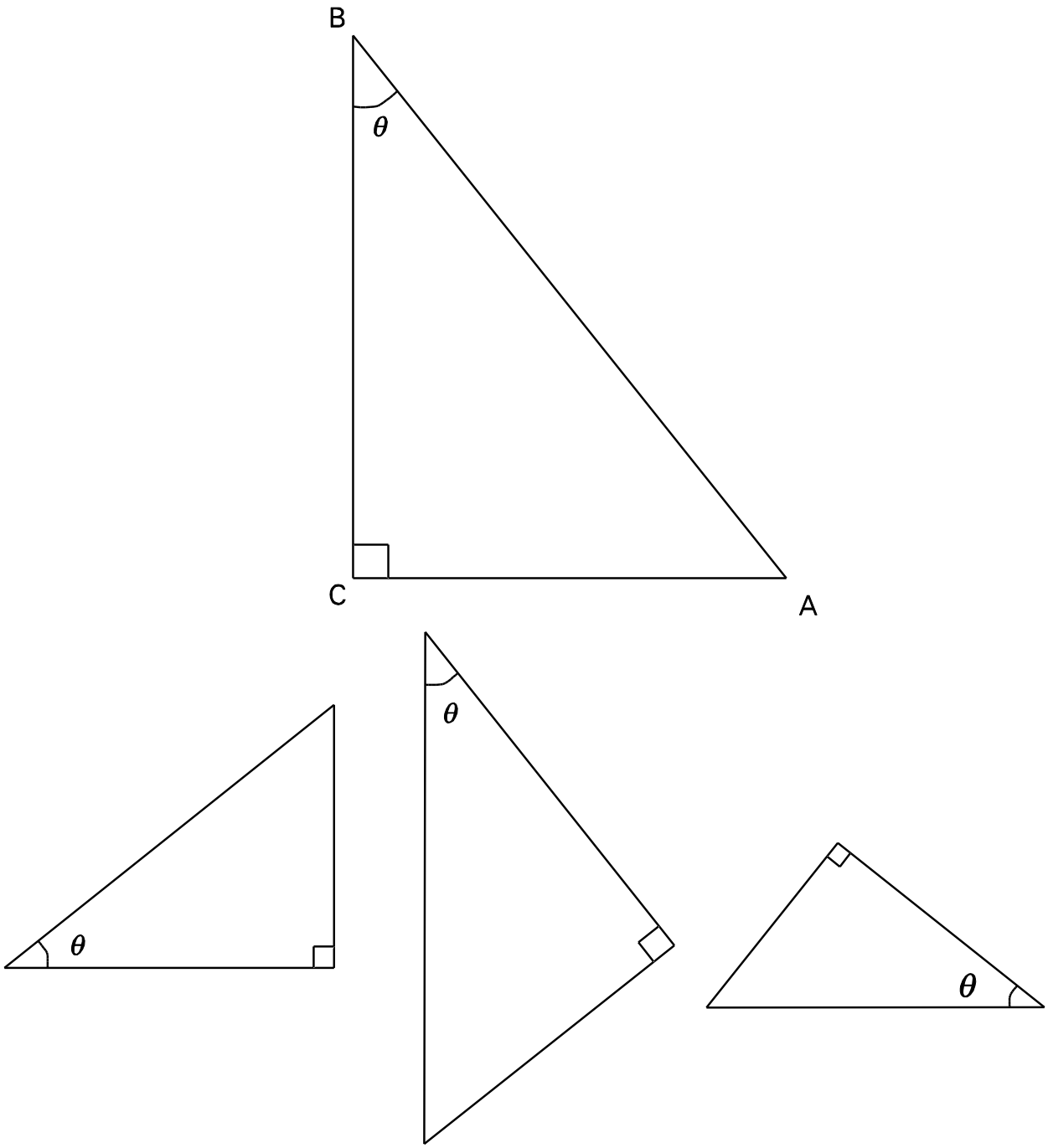
A.7 Net (triangular prism)



A.8 Net (triangular pyramid)



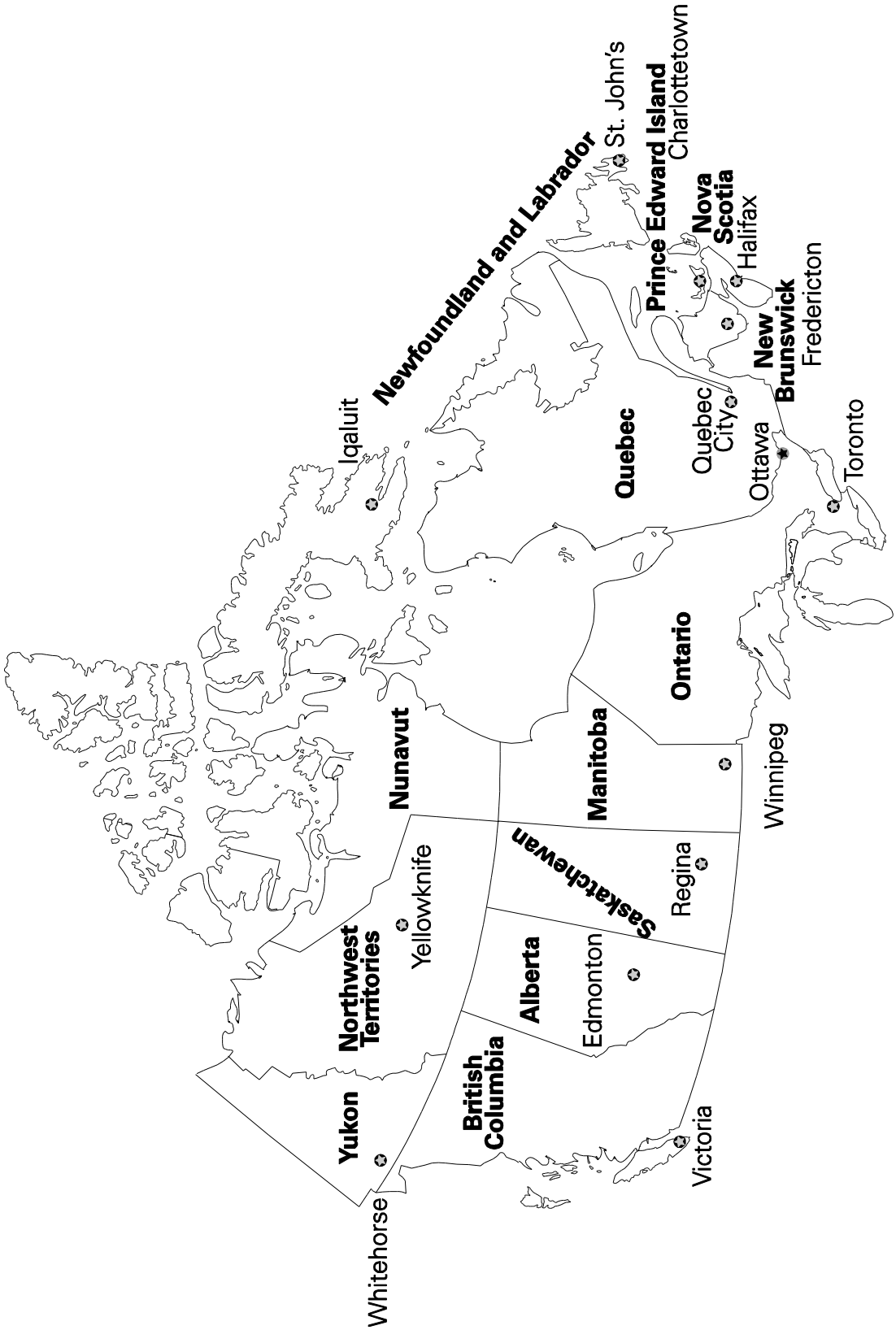
A.9 Triangles



A.10 Hundred Chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

A.11 Map of Canada



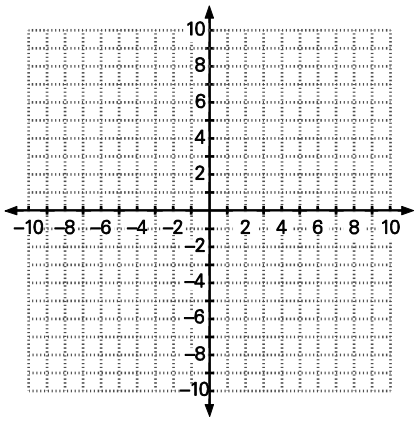
Appendix B: Graphic Organizer

Ordered Pairs

Table of Values

Equations

Graph



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