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Mathematics 4

CURRICULUM GUIDE

NOVA SCOTIA

Mathematics 4

Implementation Draft
July 2014

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Mathematics 4, Implementation Draft

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Contents

Introduction

Background and Rationale

Mathematics curriculum is shaped by a vision that fosters the development of mathematically literate students who can extend and apply their learning and who are effective participants in society. It is essential that the mathematics curriculum reflect current research in mathematics instruction. To achieve this goal, Western and Northern Canadian Protocol's (WNCP) *The Common Curriculum Framework for K–9 Mathematics* (2006) has been adopted as the basis for the new mathematics curriculum in Nova Scotia.

The Common Curriculum Framework was developed by the seven ministries of education (Alberta, British Columbia, Manitoba, Northwest Territories, Nunavut, Saskatchewan, and Yukon Territory) in collaboration with teachers, administrators, parents, business representatives, post-secondary educators, and others. The framework identifies beliefs about mathematics, general and specific student outcomes, and performance indicators agreed upon by the seven jurisdictions. The outcomes and performance indicators have been adapted for Nova Scotia. This document is based on both national and international research by the WNCP and the National Council of Teachers of Mathematics (NCTM).

There is an emphasis in the Nova Scotia curriculum on particular key concepts at each grade that will result in greater depth of understanding and, ultimately, stronger student achievement. There is also a greater emphasis on number sense and operations concepts in the early grades to ensure students develop a solid foundation in numeracy.

Purpose

This document provides sets of outcomes and performance indicators to be used as a mandated common base for defining mathematics curriculum expectations. This common base should result in consistent student outcomes in mathematics within the province of Nova Scotia. It should also enable easier transfer for students moving within the province or from any jurisdiction that has adopted the WNCP framework. This document is intended to clearly communicate to all education partners across the province the high expectations for students' mathematical learning.

Program Design and Components

Assessment

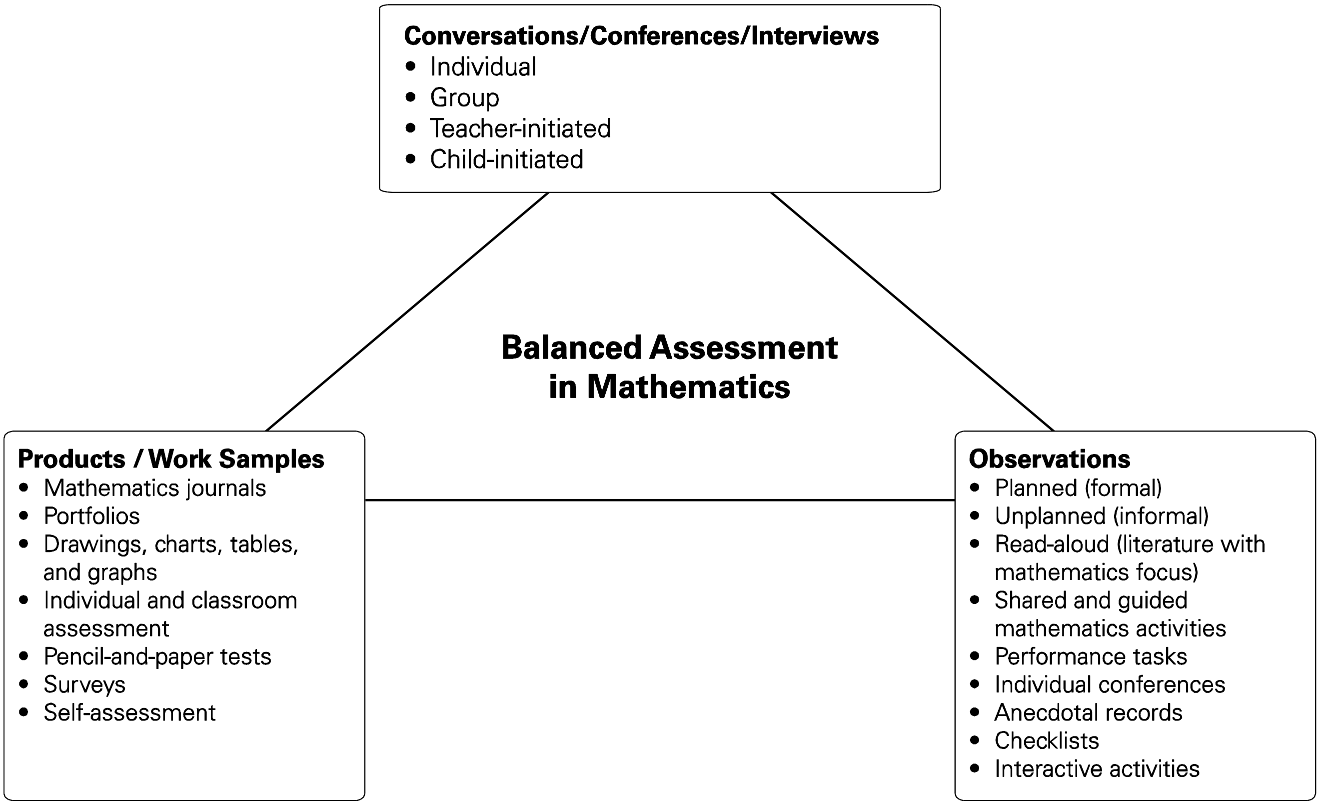
Ongoing assessment for learning is essential to effective teaching and learning. Research has shown that assessment for learning (formative assessment) practices produce significant and often substantial learning gains, close achievement gaps, and build students' ability to learn new skills (Black & Wiliam 1998; OECD 2006). Student involvement in assessment promotes learning. Timely and effective teacher feedback and student self-assessment allow students to reflect on and articulate their understanding of mathematical concepts and ideas.

Assessment in the classroom includes
- providing clear goals, targets, and learning outcomes
- using exemplars, rubrics, and models to help clarify outcomes and identify important features of the work
- monitoring progress towards outcomes and providing feedback as necessary
- encouraging self-assessment
- fostering a classroom environment where conversations about learning take place, where students can check their thinking and performance and develop a deeper understanding of their learning

(Davies 2000)

Assessment for learning practices act as the scaffolding for learning, which only then can be measured through assessment of learning (summative assessment). Assessment of learning tracks student progress, informs instructional programming, and aids in decision making. Both forms of assessment are necessary to guide teaching, stimulate learning, and produce achievement gains.

Assessment of student learning should
- align with curriculum outcomes
- clearly define criteria for success
- make explicit the expectations for students' performance
- use a wide variety of assessment strategies and tools
- yield useful information to inform instruction



Conversations/Conferences/Interviews
- Individual
- Group
- Teacher-initiated
- Child-initiated

Balanced Assessment in Mathematics

Products / Work Samples
- Mathematics journals
- Portfolios
- Drawings, charts, tables, and graphs
- Individual and classroom assessment
- Pencil-and-paper tests
- Surveys
- Self-assessment

Observations
- Planned (formal)
- Unplanned (informal)
- Read-aloud (literature with mathematics focus)
- Shared and guided mathematics activities
- Performance tasks
- Individual conferences
- Anecdotal records
- Checklists
- Interactive activities

Time to Learn for Mathematics

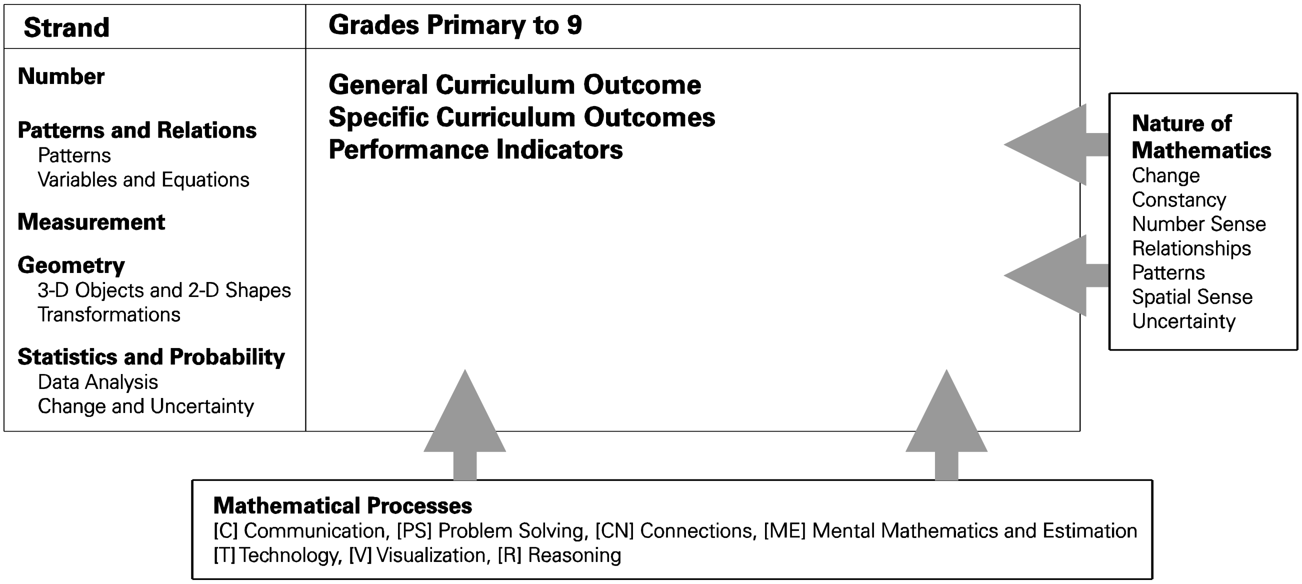
The Time to Learn Strategy *Guidelines for Instructional Time: Grades Primary*–6 (Nova Scotia Department of Education 2002) includes time for mathematics instruction in the "Required Each Day" section. In order to support a constructivist approach to teaching through problem solving, it is highly recommended that the 45 minutes required daily in grades primary–2 and the 60 minutes required daily for grades 3–6 mathematics instruction be provided in an uninterrupted block of time.

Time to Learn guidelines can be found at
www.ednet.ns.ca/files/ps-policies/semestering.pdf
www.ednet.ns.ca/files/ps-policies/instructional_time_guidelines_p-6.pdf

Outcomes

Conceptual Framework for Mathematics Primary–9

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



(Adapted with permission from Western and Northern Canadian Protocol, *The Common Curriculum Framework for K–9 Mathematics*, p. 5. All rights reserved.)

Structure of the Mathematics Curriculum

Strands

The learning outcomes in the Nova Scotia Framework are organized into five strands across grades primary to 9.

- Number (N)
- Patterns and Relations (PR)
- Measurement (M)
- Geometry (G)
- Statistics and Probability (SP)

General Curriculum Outcomes (GCO)

Some strands are further subdivided into sub-strands. There is one general outcome (GCO) per sub-strand. GCOs are overarching statements about what students are expected to learn in each strand/sub-strand. The general curriculum outcome for each strand/sub-strand is the same throughout the grades.

NUMBER (N)

GCO: Students will be expected to demonstrate number sense.

PATTERNS AND RELATIONS (PR)

Patterns
GCO: Students will be expected to use patterns to describe the world and solve problems.

Variables and Equations
GCO: Students will be expected to represent algebraic expressions in multiple ways.

MEASUREMENT (M)

GCO: Students will be expected to use direct and indirect measure to solve problems.

GEOMETRY (G)

3-D Objects and 2-D Shapes
GCO: Students will be expected to describe the characteristics of 3-D objects and 2-D shapes and analyze the relationships among them.

Transformations
GCO: Students will be expected to describe and analyze position and motion of objects and shapes.

STATISTICS AND PROBABILITY (SP)

Data Analysis
GCO: Students will be expected to collect, display, and analyze data to solve problems.

Chance and Uncertainty
GCO: Students will be expected to use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

Specific Curriculum Outcomes (SCOs) and Performance Indicators

Specific curriculum outcomes (SCOs) are statements that identify the specific conceptual understanding, related skills, and knowledge students are expected to attain by the end of a given grade.

Performance indicators are statements that identify specific expectations of the depth, breadth, and expectations for the outcome. Teachers use these statements to determine whether students have achieved the corresponding specific curriculum outcome.

Process Standards Key

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

NUMBER (N)

N01 Students will be expected to represent and partition whole numbers to 10 000. [C, CN, V]

Performance Indicators

N01.01 Read a given four-digit numeral without using the word “and.”
N01.02 Record numerals for numbers expressed orally, concretely, pictorially, and/or symbolically as expressions, using proper spacing without commas.
N01.03 Write a given numeral, 0 to 10 000, in words.
N01.04 Represent a given numeral using a place-value chart or diagrams.
N01.05 Express a given numeral in expanded notation (e.g., 4321 = 4000 + 300 + 20 + 1).
N01.06 Write the numeral represented by a given expanded notation.
N01.07 Explain the meaning of each digit in a given four-digit numeral.
N01.08 Represent a given number in a variety of ways and explain how they are equivalent.
N01.09 Read a given number word, 0 to 10 000.
N01.10 Represent a given number using expressions.

N02 Students will be expected to compare and order numbers to 10 000. [C, CN, V]

Performance Indicators

N02.01 Order a given set of numbers in ascending or descending order, and explain the order by making references to place value.
N02.02 Create and order three different four-digit numerals.
N02.03 Identify the missing numbers in an ordered sequence and on a number line.
N02.04 Identify incorrectly placed numbers in an ordered sequence and on a number line.
N02.05 Place numbers in relative order on an open number line.
N02.06 Place numbers on a number line containing benchmark numbers for the purpose of comparison.
N02.07 Compare numbers based on a variety of methods.

N03 Students will be expected to demonstrate an understanding of addition and subtraction of numbers with answers to 10 000 (limited to three- and four-digit numerals) by

- using personal strategies for adding and subtracting
- estimating sums and differences
- solving problems involving addition and subtraction

[C, CN, ME, PS, R]

Performance Indicators

N03.01 Represent concretely, pictorially, and symbolically the addition and subtraction of whole numbers, limited to three- and four-digit numerals.
N03.02 Determine the sum of two given numbers, limited to three- and four-digit numerals, using a personal strategy, and record the process symbolically.
N03.03 Determine the difference of two given numbers, limited to three- and four-digit numerals, using a personal strategy, and record the process symbolically.
N03.04 Describe a situation in which an estimate rather than an exact answer is sufficient.
N03.05 Estimate sums and differences using different strategies.
N03.06 Create and solve problems that involve addition and subtraction of two or more numbers, limited to three- and four-digit numerals.
N03.07 Explain mental mathematics strategies that could be used to determine a sum or difference.
N03.08 Determine a sum or difference of one-, two-, and three-digit numerals efficiently, using mental mathematics strategies.

N04 Students will be expected to apply and explain the properties of 0 and 1 for multiplication and the property of 1 for division. [C, CN, R]

Performance Indicators

N04.01 Determine the answer to a given question involving the multiplication of a number by 1, and explain the answer using the property of 1 in multiplication.
N04.02 Determine the answer to a given question involving the multiplication of a number by 0, and explain the answer using the property of 0 in multiplication.
N04.03 Determine the answer to a given question involving the division of a number by 1, and explain the answer using the property of 1 in division.

N05 Students will be expected to describe and apply mental mathematics strategies, to recall basic multiplication facts to 9 × 9, and to determine related division facts. [C, CN, ME, R]

Performance Indicators

N05.01 Describe the mental mathematics strategy used to determine basic multiplication or division facts.
N05.02 Use and describe a personal strategy for determining the multiplication facts.
N05.03 Use and describe a personal strategy for determining the division facts.
N05.04 Quickly recall basic multiplication facts up to 9 × 9.

N06 Students will be expected to demonstrate an understanding of multiplication (one-, two-, or three-digit by one-digit numerals) to solve problems by

- using personal strategies for multiplication, with and without concrete materials
- using arrays to represent multiplication
- connecting concrete representations to symbolic representations
- estimating products
- applying the distributive property

[C, CN, ME, PS, R, V]

Performance Indicators

N06.01 Model a given multiplication problem, using the distributive property (e.g., 8 × 365 = (8 × 300) + (8 × 60) + (8 × 5)).

N06.02 Model the multiplication of two given numbers, limited to one-, two-, or three-digit by one-digit numerals, using concrete or visual representations, and record the process symbolically.

N06.03 Create and solve multiplication story problems, limited to one-, two-, or three-digit by one-digit numerals, and record the process symbolically.

N06.04 Estimate a product using a personal strategy (e.g., 2 × 243 is close to or a little more than 2 × 200, or close to or a little less than 2 × 250).

N06.05 Model and solve a given multiplication problem using an array, and record the process.

N06.06 Determine the product of two given numbers using a personal strategy, and record the process symbolically.

N07 Students will be expected to demonstrate an understanding of division (one-digit divisor and up to two-digit dividend) to solve problems by

- using personal strategies for dividing, with and without concrete materials
- estimating quotients
- relating division to multiplication

[C, CN, ME, PS, R, V]

Performance Indicators

N07.01 Model the division of two given numbers without a remainder, limited to a one-digit divisor and up to a two-digit dividend, using concrete or visual representations, and record the process pictorially and symbolically.

N07.02 Model the division of two given numbers with a remainder, limited to a one-digit divisor and up to a two-digit dividend, using concrete or visual representations, and record the process pictorially and symbolically. (It is not intended that remainders be expressed as decimals or fractions.)

N07.03 Solve a given division problem, using a personal strategy, and record the process symbolically.

N07.04 Create and solve division word problems involving a one- or two-digit dividend, and record the process pictorially and symbolically.

N07.05 Estimate a quotient using a personal strategy (e.g., 86 ÷ 4 is close to 80 ÷ 4 or close to 80 ÷ 5).

N07.06 Solve a given division problem by relating division to multiplication (e.g., for 80 ÷ 4, we know that 4 × 20 = 80, so 80 ÷ 4 = 20).

N08 Students will be expected to demonstrate an understanding of fractions less than or equal to 1 by using concrete, pictorial, and symbolic representations to

- name and record fractions for the parts of one whole or a set
- compare and order fractions
- model and explain that for different wholes, two identical fractions may not represent the same quantity
- provide examples of where fractions are used

[C, CN, PS, R, V]

Performance Indicators

N08.01 Represent a given fraction of one whole object, region, or a set using concrete materials.
N08.02 Identify a fraction from its given concrete representation.
N08.03 Name and record the shaded and non-shaded parts of a given whole object, region, or set.
N08.04 Represent a given fraction pictorially by shading parts of a given whole object, region, or set.
N08.05 Explain how denominators can be used to compare two given unit fractions with a numerator of 1.
N08.06 Order a given set of fractions that have the same numerator, and explain the ordering.
N08.07 Order a given set of fractions that have the same denominator, and explain the ordering.
N08.08 Identify which of the benchmarks 0, $\frac{1}{2}$, or 1 is closer to a given fraction.
N08.09 Name fractions between two given benchmarks on a number line.
N08.10 Order a given set of fractions by placing them on a number line with given benchmarks.
N08.11 Provide examples of instances when two identical fractions may not represent the same quantity.
N08.12 Provide, from everyday contexts, an example of a fraction that represents part of a set and an example of a fraction that represents part of one whole.

N09 Students will be expected to describe and represent decimals (tenths and hundredths) concretely, pictorially, and symbolically. [C, CN, R, V]

Performance Indicators

N09.01 Write the decimal for a given concrete or pictorial representation of part of a set, part of a region, or part of a unit of measure.
N09.02 Represent a given decimal using concrete materials or a pictorial representation.
N09.03 Explain the meaning of each digit in a given decimal.
N09.04 Represent a given decimal using money values (dimes and pennies).
N09.05 Record a given money value using decimals.
N09.06 Provide examples of everyday contexts in which tenths and hundredths are used.
N09.07 Model, using manipulatives or pictures, that a given tenth can be expressed as a hundredth (e.g., 0.9 is equivalent to 0.90, or 9 dimes is equivalent to 90 pennies).
N09.08 Read decimal numbers correctly.

N10 Students will be expected to relate decimals to fractions and fractions to decimals (to hundredths). [C, CN, R, V]

Performance Indicators

N10.01 Express, orally and symbolically, a given fraction with a denominator of 10 or 100 as a decimal.
N10.02 Read decimals as fractions (e.g., 0.5 is zero and five tenths).
N10.03 Express, orally and symbolically, a given decimal in fraction form.
N10.04 Express a given pictorial or concrete representation as a fraction or decimal (e.g., 15 shaded squares on a hundredth grid can be expressed as 0.15 or $\frac{15}{100}$).
N10.05 Express, orally and symbolically, the decimal equivalent for a given fraction (e.g., $\frac{50}{100}$ can be expressed as 0.50).

N11 Students will be expected to demonstrate an understanding of addition and subtraction of decimals (limited to hundredths) by

- estimating sums and differences
- using mental mathematics strategies to solve problems
- using personal strategies to determine sums and differences

[C, ME, PS, R, V]

Performance Indicators

N11.01 Predict sums and differences of decimals, using estimation strategies.
N11.02 Solve problems, including money problems, that involve addition and subtraction of decimals (limited to hundredths), using personal strategies.
N11.03 Ask students to determine which problems do not require an exact solution.
N11.04 Determine the approximate solution of a given problem not requiring an exact answer.
N11.05 Count back change for a given purchase.
N11.06 Determine an exact solution using mental computation strategies.

PATTERNS AND RELATIONS (PR)

PR01 Students will be expected to identify and describe patterns found in tables and charts, including a multiplication chart. [C, CN, PS, V]

Performance Indicators

PR01.01 Identify and describe a variety of patterns in a multiplication chart.
PR01.02 Determine the missing element(s) in a given table or chart.
PR01.03 Identify the error(s) in a given table or chart.
PR01.04 Describe the pattern found in a given table or chart.

PR02 Students will be expected to translate among different representations of a pattern (a table, a chart, or concrete materials). [C, CN, V]

Performance Indicators

PR02.01 Create a table or chart from a given concrete representation of a pattern.
PR02.02 Create a concrete representation of a given pattern displayed in a table or chart.
PR02.03 Translate between pictorial, contextual, and concrete representations of a pattern.
PR02.04 Explain why the same relationship exists between the pattern in a table and its concrete representation.

PR03 Students will be expected to represent, describe, and extend patterns and relationships, using charts and tables, to solve problems. [C, CN, PS, R, V]

Performance Indicators

PR03.01 Translate the information in a given problem into a table or chart.
PR03.02 Identify, describe, and extend the patterns in a table or chart to solve a given problem.

PR04 Students will be expected to identify and explain mathematical relationships, using charts and diagrams, to solve problems. [CN, PS, R, V]

Performance Indicators

PR04.01 Complete a given Carroll diagram to solve a problem.
PR04.02 Determine where new elements belong is a given Carroll diagram.
PR04.03 Solve a given problem using a Carroll diagram.
PR04.04 Identify a sorting rule for a given Venn diagram.
PR04.05 Describe the relationship shown in a given Venn diagram when the circles overlap, when one circle is contained in the other, and when the circles are separate.
PR04.06 Determine where new elements belong in a given Venn diagram.
PR04.07 Solve a given problem by using a chart or diagram to identify mathematical relationships.

PR 05 Students will be expected to express a given problem as an equation in which a symbol is used to represent an unknown number. [CN, PS, R]

Performance Indicators

PR05.01 Explain the purpose of the symbol in a given addition, subtraction, multiplication, or division equation with one unknown (e.g., 36 ÷ ⍰ = 6).
PR05.02 Express a given pictorial or concrete representation of an equation in symbolic form.
PR05.03 Identify the unknown in a problem; represent the problem with an equation; and solve the problem concretely, pictorially, and/or symbolically.
PR05.04 Create a problem in context for a given equation with one unknown.

PR06 Students will be expected to solve one-step equations involving a symbol to represent an unknown number. [C, CN, PS, R, V]

Performance Indicators

PR06.01 Represent and solve a given one-step equation concretely, pictorially, or symbolically.
PR06.02 Solve a given one-step equation using guess and test.
PR06.03 Describe, orally, the meaning of a given one-step equation with one unknown.
PR06.04 Solve a given equation when the unknown is on the left or right side of the equation.
PR06.05 Represent and solve a given addition or subtraction problem involving a "part-part-whole" or comparison context using a symbol to represent the unknown.
PR06.06 Represent and solve a given multiplication or division problem involving equal grouping or partitioning (equal sharing) using symbols to represent the unknown.
PR06.07 Solve equations using a symbol to represent the unknown.

MEASUREMENT (M)

M01 Students will be expected to read and record time using digital and analog clocks, including 24-hour clocks. [C, CN, V]

Performance Indicators

M01.01 State the number of hours in a day.
M01.02 Express the time orally and numerically from a 12-hour analog clock.
M01.03 Express the time orally and numerically from a 24-hour analog clock.
M01.04 Express the time orally and numerically from a 12-hour digital clock.
M01.05 Express time orally and numerically from a 24-hour digital clock.
M01.06 Describe time orally as "minutes to" or "minutes after" the hour.
M01.07 Explain the meaning of a.m. and p.m., and provide an example of an activity that occurs during the a.m., and another that occurs during the p.m.

M02 Students will be expected to read and record calendar dates in a variety of formats. [C, V]

Performance Indicators

M02.01 Write dates in a variety of formats (e.g., yyyy/mm/dd, dd/mm/yyyy, March 21, 2014, dd/mm/yy).
M02.02 Relate dates written in the format yyyy/mm/dd to dates on a calendar.
M02.03 Identify possible interpretations of a given date (e.g., 06/03/04).

M03 Students will be expected to demonstrate an understanding of area of regular and irregular 2-D shapes by

- recognizing that area is measured in square units
- selecting and justifying referents for the units square centimetre (cm^2) or square metre (m^2)
- estimating area using referents for cm^2 or m^2
- determining and recording area (cm^2 or m^2)
- constructing different rectangles for a given area (cm^2 or m^2) in order to demonstrate that many different rectangles may have the same area

[C, CN, ME, PS, R, V]

Performance Indicators

M03.01 Describe area as the measure of surface recorded in square units.
M03.02 Identify and explain why the square is the most efficient unit for measuring area.
M03.03 Provide a referent for a square centimetre, and explain the choice.
M03.04 Provide a referent for a square metre, and explain the choice.
M03.05 Determine which standard square unit is represented by a given referent.
M03.06 Estimate the area of a given 2-D shape using personal referents.
M03.07 Determine the area of a regular 2-D shape, and explain the strategy.
M03.08 Determine the area of an irregular 2-D shape, and explain the strategy.
M03.09 Construct a rectangle for a given area.
M03.10 Demonstrate that many rectangles are possible for a given area by drawing at least two different rectangles for the same given area.

GEOMETRY (G)

G01 Students will be expected to describe and construct rectangular and triangular prisms. [C, CN, R, V]

Performance Indicators

G01.01 Identify and name common attributes of rectangular prisms from given sets of rectangular prisms.
G01.02 Identify and name common attributes of triangular prisms from given sets of triangular prisms.
G01.03 Sort a given set of right rectangular and triangular prisms, using the shape of the base.
G01.04 Construct and describe a model of a rectangular and a triangular prism, using materials such as pattern blocks or modelling clay.
G01.05 Construct rectangular prisms from their nets.
G01.06 Construct triangular prisms from their nets.
G01.07 Identify examples of rectangular and triangular prisms found in the environment.

G02 Students will be expected to demonstrate an understanding of congruency, concretely and pictorially. [CN, R, V]

Performance Indicators

G02.01 Determine if two given 2-D shapes are congruent, and explain the strategy used.
G02.02 Create a shape that is congruent to a given 2-D shape, and explain why the two shapes are congruent.
G02.03 Identify congruent 2-D shapes from a given set of shapes shown in different positions in space.

G03 Students will be expected to demonstrate an understanding of line symmetry by

- identifying symmetrical 2-D shapes
- creating symmetrical 2-D shapes
- drawing one or more lines of symmetry in a 2-D shape

[C, CN, V]

Performance Indicators

G03.01 Identify the characteristics of given symmetrical and non-symmetrical 2-D shapes.
G03.02 Sort a given set of 2-D shapes as symmetrical and non-symmetrical.
G03.03 Complete a symmetrical 2-D shape, given one-half the shape and its line of symmetry, and explain the process.
G03.04 Identify lines of symmetry of a given set of 2-D shapes, and explain why each shape is symmetrical.
G03.05 Determine whether or not a given 2-D shape is symmetrical by using an image reflector or by folding and superimposing.
G03.06 Create a symmetrical shape with and without manipulatives and explain the process.
G03.07 Provide examples of symmetrical shapes found in the environment, and identify the line(s) of symmetry.
G03.08 Sort a given set of 2-D shapes as those that have no lines of symmetry, one line of symmetry, or more than one line of symmetry.
G03.09 Explain connections between congruence and symmetry using 2-D shapes.

STATISTICS AND PROBABILITY (SP)

SP01 Students will be expected to demonstrate an understanding of many-to-one correspondence. [C, R, T, V]

Performance Indicators

SP01.01 Compare graphs in which the same data has been displayed using one-to-one and many-to-one correspondences, and explain how they are the same and different.
SP01.02 Explain why many-to-one correspondence is sometimes used rather than one-to-one correspondence.
SP01.03 Find examples of graphs in print and electronic media, such as newspapers, magazines, and the Internet, in which many-to-one correspondence is used; and describe the correspondence used.

SP02 Students will be expected to construct and interpret pictographs and bar graphs involving many-to-one correspondence to draw conclusions. [C, PS, R, V]

Performance Indicators

SP02.01 Identify an interval and correspondence for displaying a given set of data in a graph, and justify the choice.
SP02.02 Create and label (with categories, title, and legend) a pictograph to display a given set of data, using many-to-one correspondence, and justify the choice of correspondence used.
SP02.03 Create and label (with axes and title) a bar graph to display a given set of data, using many-to-one correspondence, and justify the choice of interval used.
SP02.04 Answer a given question, using a given graph in which data is displayed using many-to-one correspondence.

Mathematical Processes

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics.

Students are expected to

- communicate in order to learn and express their understanding of mathematics (Communication [C])
- develop and apply new mathematical knowledge through problem solving (Problem Solving [PS])
- connect mathematical ideas to other concepts in mathematics, to everyday experiences, and to other disciplines (Connections [CN])
- demonstrate fluency with mental mathematics and estimation (Mental Mathematics and Estimation [ME])
- select and use technologies as tools for learning and solving problems (Technology [T])
- develop visualization skills to assist in processing information, making connections, and solving problems (Visualization [V])
- develop mathematical reasoning (Reasoning [R])

The Nova Scotia curriculum incorporates these seven interrelated mathematical processes that are intended to permeate teaching and learning. The key to these process standards is presented in a box, as shown below, with each specific curriculum outcome within the strands.

Process Standards Key

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics. Communication is important in clarifying, reinforcing, and modifying ideas, knowledge, attitudes, and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics.

Students also need to communicate their learning using mathematical terminology. Communication can help students make connections between and among the different representational modes—contextual, concrete, pictorial, linguistic/verbal, and symbolic—of mathematical ideas. Students must communicate *daily* about their mathematics learning. This enables them to reflect, to validate, and to clarify their thinking and provides teachers with insight into students' interpretations of mathematical meanings and ideas.

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, How would you ...? or How could you ...? the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement, perseverance, and collaboration.

Problem solving is also a powerful teaching tool that fosters multiple, creative, and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive, mathematical risk takers.

When students are exposed to a wide variety of problems in all areas of mathematics, they explore various methods for solving and verifying problems. In addition, they are challenged to find multiple solutions for problems and to create their own problem.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to one another or to real-world phenomena, students can begin to view mathematics as useful, relevant, and integrated. Learning mathematics within contexts and making connections relevant to learners can validate past experiences and increase student willingness to participate and be actively engaged. The brain is constantly looking for and making connections.

“Because the learner is constantly searching for connections on many levels, educators need to *orchestrate the experiences* from which learners extract understanding. … *Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching*.” (Caine and Caine 1991, 5).

Mathematics should be viewed as an integrated whole rather than as the study of separate strands or units. Connections must also be made between and among the different representational modes—contextual, concrete, pictorial, linguistic/verbal, and symbolic. The process of making connections, in turn, facilitates learning. Concepts and skills should also be connected to everyday situations and other curricular areas. For example, when developing literacy skills students learn to make text-to-world, text-to-text, and text-to-self connections. Students can also make connections to make mathematics come alive through math-to-world, math-to-math, and math-to-self connections.

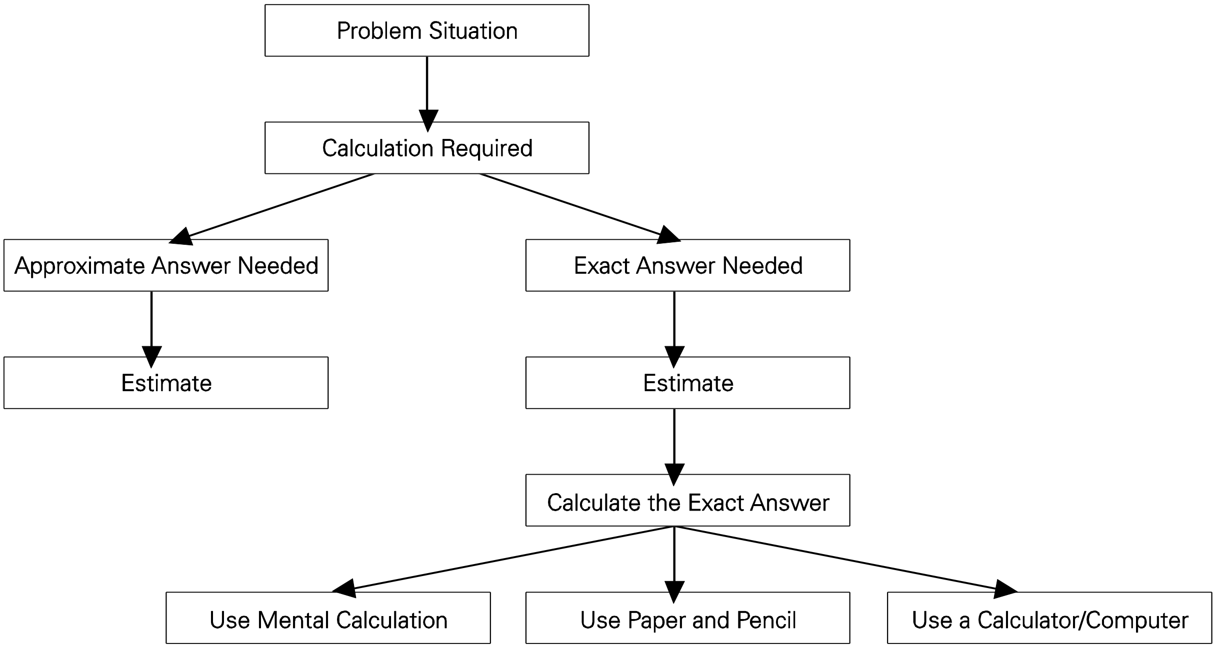
Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally without the use of external aids. Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy, and flexibility. “Even more important than performing computational procedures or using calculators is the greater facility that students need—more than ever before—with estimation and mental math.” (National Council of Teachers of Mathematics, May 2005).

Students proficient with mental mathematics “become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers, and are more able to use multiple approaches to problem solving.” (Rubenstein 2001) Mental mathematics “provides a cornerstone for all estimation processes, offering a variety of alternative algorithms and nonstandard techniques for finding answers.” (Hope 1988, v)

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when, and what strategy to use when estimating. Estimation is used to make mathematical judgments and develop useful, efficient strategies for dealing with situations in daily life.

Students need to develop both mental mathematics and estimation skills through context and not in isolation so they are able to apply them to solve problems. Whenever a problem requires a calculation, students should follow the decision-making process as illustrated below.



The skill of estimation requires a sound knowledge of mental mathematics. Both are necessary to many everyday experiences, and students should be provided with frequent opportunities to practise these skills.

Technology [T]

Technology can be effectively used to contribute to and support the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures, and solve problems. Information and communication technology best improves learning when it is accessible, flexible, responsive, participatory, and integrated thoroughly into all public school programs.

Technology can be used to
- explore and demonstrate mathematical relationships and patterns
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- decrease the time spent on computations when other mathematical learning is the focus

- reinforce the learning of basic facts and test properties
- develop personal procedures for mathematical operations
- create geometric displays
- simulate situations
- develop number sense

The use of calculators is recommended to enhance problem solving, to encourage discovery of number patterns, and to reinforce conceptual development and numerical relationships. Calculators do not, however, replace the development of number concepts and skills. Carefully chosen computer software can provide interesting problem-solving situations and applications.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. While technology can be used in grade 4 to enrich learning, it is expected that students will achieve all outcomes without the use of technology.
The *Integration of Information and Communication Technology within the Classroom, 2012* (P–6) can be found online in several locations, including http://lrt.ednet.ns.ca.

Visualization [V]

Visualization "involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world." (Armstrong 1999). The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number, spatial, and measurement sense. Number visualization occurs when students create mental representations of numbers. These mental images are needed to develop concepts and understand procedures. Images and explanations help students clarify their understanding of mathematical ideas in all strands.

Being able to create, interpret, and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure, when to estimate and to know several estimation strategies. (Shaw and Cliatt 1989)

Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics. Mathematical experiences in and out of the classroom provide opportunities for inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyze observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Mathematics reasoning involves informal thinking, conjecturing, and validating—these help students understand that mathematics makes sense. Students are encouraged to justify, in a variety of ways, their solutions, thinking processes, and hypotheses. In fact, good reasoning is as important as finding correct answers.

Nature of Mathematics

Mathematics is one way of trying to understand, interpret, and describe our world. There are a number of components that define the nature of mathematics, and these are woven throughout this document. These components include change, constancy, number sense, relationships, patterns, spatial sense, and uncertainty.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics. Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, … can be described as

- skip counting by 2s, starting from 4
- an arithmetic sequence, with first term 4 and a common difference of 2
- a linear function with a discrete domain

(Steen 1990, 184).

Constancy

Different aspects of constancy are described by the terms **stability**, **conservation**, **equilibrium**, **steady state**, and **symmetry** (AAAS–Benchmarks 1993, 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems. Examples of constancy include the following:

- The area of a rectangular region is the same regardless of the methods used to determine the solution.
- The sum of the interior angles of any triangle is 180°.
- The theoretical probability of flipping a coin and getting heads is 0.5.

Number Sense

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (British Columbia Ministry of Education 2000, 146). A true sense of number goes well beyond the skills of simply counting, memorizing facts, and the situational rote use of algorithms. Number sense develops when students connect numbers to real-life experiences and use benchmarks and referents. This results in students who are computationally fluent, flexible with numbers, and have intuition about numbers. The evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections.

Relationships

Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects, and concepts. The search for possible relationships involves the collection and analysis of data, and describing relationships visually, symbolically, orally, or in written form.

Patterns

Mathematics is about recognizing, describing, and working with numerical and non-numerical patterns. Patterns exist in all strands, and it is important that connections are made among strands. Working with patterns enables students to make connections within and beyond mathematics. These skills contribute to students' interaction with an understanding of their environment. Patterns may be represented in concrete, visual, or symbolic form. Students should develop fluency in moving from one representation to another. Students must learn to recognize, extend, create, and use mathematical patterns. Patterns allow students to make predictions and justify their reasoning when solving problems. Learning to work with patterns in the early grades helps develop students' algebraic thinking, which is foundational for working with more abstract mathematics in higher grades.

Spatial Sense

Spatial sense involves visualization, mental imagery, and spatial reasoning. These skills are central to the understanding of mathematics. Spatial sense enables students to reason and interpret among and between 3-D and 2-D representations and identify relationships to mathematical strands. Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 3-D objects and 2-D shapes. Spatial sense offers a way to interpret and reflect on the physical environment and its 3-D or 2-D representations. Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to make predictions about the results of changing these dimensions. For example,

- knowing the dimensions of an object enables students to communicate about the object and create representations
- the volume of a rectangular solid can be calculated from given dimensions
- doubling the length of the side of a square increases the area by a factor of four

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty. Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty. The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation. Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

Curriculum Document Format

This guide presents the mathematics curriculum so that a teacher may readily view the scope of the outcomes that students are expected to achieve during that year. Teachers are encouraged, however, to examine what comes before and what follows after, to better understand how students' learning at a particular grade level is part of a bigger picture of concept and skill development.

The order of presentation in no way assumes or prescribes a preferred order of presentation in the classroom, but simply lays out the specific curriculum outcomes in relation to the overarching general curriculum outcomes (GCOs).

The footer of the document shows the name of the course, and the strand name is presented in the header. When a specific curriculum outcome (SCO) is introduced, it is followed by the mathematical processes and performance indicators for that outcome. A scope and sequence is then provided, which relates the SCO to previous and next grade SCOs. Also for each SCO, there is background information, assessment strategies, suggested instructional strategies, suggested models and manipulatives, mathematical language, and a section for resources and notes. For each section, the guiding questions should be used to help with unit and lesson preparation.

SCO

Mathematical Processes
[C] Communication **[PS]** Problem Solving **[CN]** Connections
[ME] Mental Mathematics and Estimation
[T] Technology **[V]** Visualization **[R]** Reasoning

Performance Indicators

Describes observable indicators of whether students have achieved the specific outcome.

Scope and Sequence

Previous grade or course SCOs	Current grade SCO	Following grade or course SCOs

Background

Describes the "big ideas" to be learned and how they relate to work in previous grade and work in subsequent courses.

Additional Information

A reference to Appendix A, which contains further elaborations for the performance indicators.

Assessment, Teaching, and Learning

Assessment Strategies

Guiding Questions
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Sample tasks that can be used to determine students' prior knowledge.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Some suggestions for specific activities and questions that can be used for both instruction and assessment

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Correlations to related resources.

Planning for Instruction

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcome and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Suggested strategies for planning daily lessons.

SUGGESTED LEARNING TASKS

Suggestions for general approaches and strategies suggested for teaching this outcome.

Guiding Questions
- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

SUGGESTED MODELS AND MANIPULATIVES

MATHEMATICAL LANGUAGE

Teacher and student mathematical language associated with the respective outcome.

Resources/Notes

Contexts for Learning and Teaching

Beliefs about Students and Mathematics Learning

“Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.” (National Council of Teachers of Mathematics 2000, 20).

The Nova Scotia mathematics curriculum is based upon several key assumptions or beliefs about mathematics learning that have grown out of research and practice. These beliefs include the following:

- Mathematics learning is an active and constructive process.
- Learning is most effective when standards of expectation are made clear with ongoing assessment and feedback.
- Learners are individuals who bring a wide range of prior knowledge and experiences and who learn via various styles and at different rates.
- Learning is most likely to occur when placed in meaningful contexts and in an environment that supports exploration, risk taking, and critical thinking and that nurtures positive attitudes and sustained effort.

Students are curious, active learners with individual interests, abilities, and needs. They come to classrooms with varying knowledge, life experiences, and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Students develop a variety of mathematical ideas before they enter school. Children make sense of their environment through observations and interactions at home and in the community. Mathematics learning is embedded in everyday activities, such as playing, reading, storytelling, and helping around the home. Such activities can contribute to the development of number and spatial sense in children. Curiosity about mathematics is fostered when children are engaged in activities such as comparing quantities, searching for patterns, sorting objects, ordering objects, creating designs, building with blocks, and talking about these activities. Positive early experiences in mathematics are as critical to child development as are early literacy experiences.

Students learn by attaching meaning to what they do and need to construct their own meaning of mathematics. This meaning is best constructed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of models and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with and translating through a variety of materials, tools, and contexts when constructing meaning about new mathematical ideas. Meaningful discussions can provide essential links among concrete, pictorial, contextual, and symbolic representations of mathematics.

The learning environment should value and respect all students’ experiences and ways of thinking, so that learners are comfortable taking intellectual risks, asking questions, and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must realize that it is acceptable to solve problems in different ways and that solutions may vary.

Goals for Mathematics Education

The main goals of mathematics education are to prepare students to

- use mathematics confidently to solve problems
- communicate and reason mathematically
- appreciate and value mathematics
- make connections between mathematics and its applications
- become mathematically literate adults, using mathematics to contribute to society

Students who have met these goals will

- gain understanding and appreciation of the contributions of mathematics as a science, a philosophy, and an art
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical tasks and projects
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity about mathematics and situations involving mathematics

Opportunities for Success

A positive attitude has a profound effect on learning. Environments that create a sense of belonging, encourage risk taking, and provide opportunities for student success help develop and maintain positive attitudes and self-confidence. Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, participate willingly in classroom activities, persist in challenging situations, and engage in reflective practices.

To experience success, students must be taught to set achievable goals or assess their progress as they work toward these goals. Striving toward success and becoming autonomous and responsible learners are ongoing, reflective processes that involve revisiting the setting and assessing of personal goals.

Engaging All Learners

“No matter how engagement is defined or which dimension is considered, research confirms this truism of education: *The more engaged you are, the more you will learn.*” (Hume 2011, 6)

Student engagement is at the core of learning. Engagement in learning occurs when students are provided with opportunities to become more invested in their learning. This is critical for teachers to take into account when planning and implementing instruction. Effective instruction engages, embraces, and supports all learners through a range of learning experiences that are both age and developmentally appropriate.

This curriculum is designed to provide learning opportunities that are equitable, accessible, and inclusive of the many facets of diversity represented in today’s classrooms. When teachers know their students as individual learners and as individual people, their students are more likely to be motivated to learn, persist in challenging situations, and apply reflective practices.

SUPPORTIVE LEARNING ENVIRONMENTS

A supportive and positive learning environment has a profound effect on students' learning. Students need to feel physically, socially, emotionally, and culturally safe in order to take risks with their learning. In classrooms where students feel a sense of belonging, see their teachers' passion for learning and teaching, are encouraged to actively participate, and are challenged appropriately, they are more likely to be successful.

Teachers recognize that not all students progress at the same pace nor are they equally positioned in terms of their prior knowledge of particular concepts, skills, and learning outcomes. Teachers are able to create more equitable access to learning when

- instruction and assessment are flexible and offer multiple means of representation
- students have options to engage in learning through multiple ways
- students can express their knowledge, skills, and understanding in multiple ways

(Hall, Meyer, and Rose 2012)

In a supportive learning environment, teachers plan learning experiences that support *each* student's ability to achieve curriculum outcomes. Teachers use a variety of effective instructional approaches that help students to succeed, such as

- providing a range of learning opportunities that build on individual strengths and prior knowledge
- providing all students with equitable access to appropriate learning strategies, resources, and technology
- involving students in the creation of criteria for assessment and evaluation
- engaging and challenging students through inquiry-based practices
- verbalizing their own thinking to model comprehension strategies and new learning
- balancing individual, small-group, and whole-class learning experiences
- scaffolding instruction and assignments as needed and giving frequent and meaningful descriptive feedback throughout the learning process
- integrating "blended learning" opportunities by including an online environment that extends learning beyond the physical classroom
- encouraging students to take time and to persevere, when appropriate, in order to achieve a particular learning outcome

MULTIPLE WAYS OF LEARNING

"Advances in neuroscience and education research over the past 40 years have reshaped our understanding of the learning brain. One of the clearest and most important revelations stemming from brain research is that there is no such thing as a 'regular student.'" (Hall, Meyer, and Rose 2012, 2) Teachers who know their students well are aware of students' individual learning differences and use this understanding to inform instruction and assessment decisions.

The ways in which students make sense of and demonstrate learning vary widely. Individual students tend to have a natural inclination toward one or a few learning styles. Teachers are often able to detect learning strengths and styles through observation and through conversation with students. Teachers can also get a sense of learning styles through an awareness of students' personal interests and talents. Instruction and assessment practices that are designed to account for multiple learning styles create greater opportunities for all students to succeed.

While multiple learning styles are addressed in the classroom, the three most commonly identified are:
- auditory (such as listening to teacher-modelled think-aloud strategies or participating in peer discussion)
- kinesthetic (such as examining artifacts or problem-solving using tools or manipulatives)
- visual (such as reading print and visual texts or viewing video clips)

For additional information, refer to *Frames of Mind: The Theory of Multiple Intelligences* (Gardner 2007) and *How to Differentiate Instruction in Mixed-Ability Classrooms* (Tomlinson 2001).

A GENDER-INCLUSIVE CURRICULUM AND CLASSROOM

It is important that the curriculum and classroom climate respect the experiences and values of all students and that learning resources and instructional practices are not gender-biased. Teachers promote gender equity and inclusion in their classrooms when they
- articulate equally high expectations for all students
- provide equal opportunity for input and response from all students
- model gender-fair language, inclusive practices, and respectful listening in their interactions with students
- identify and openly address societal biases with respect to gender and sexual identity

VALUING DIVERSITY: TEACHING WITH CULTURAL PROFICIENCY

"Instruction that is embedded in socially meaningful contexts, and tasks that are meaningful and relevant to the lives of students, will engage students in high-level problem-solving and reasoning and enhance students' engagement (Frankenstein 1995; Gutstein 2003; Ladson-Billings 1997; Tate 1995)." (Herzig 2005)

Teachers appreciate that students have diverse life and cultural experiences and that individual students bring different prior knowledge to their learning. Teachers can build upon their knowledge of their students as individuals, value their prior experiences, and respond by using a variety of culturally-proficient instruction and assessment practices in order to make learning more engaging, relevant, and accessible for all students. For additional information, refer to *Racial Equity Policy* (Nova Scotia Department of Education 2002) and *Racial Equity / Cultural Proficiency Framework* (Nova Scotia Department of Education 2011).

STUDENTS WITH LANGUAGE, COMMUNICATION, AND LEARNING CHALLENGES

Today's classrooms include students who have diverse language backgrounds, abilities, levels of development, and learning challenges. By observing and interacting with students and by conversing with students and/or their families, teachers gain deeper insights into the student as a learner. Teachers can use this awareness to identify and respond to areas where students may need additional support to achieve their learning goals. For students who are experiencing difficulties, it is important that teachers distinguish between those students for whom curriculum content is challenging and those for whom language-based factors are at the root of apparent academic difficulties. Students who are learning English as an additional language may require individual support, particularly in language-based subject areas, while they become more proficient in their English language skills. Teachers understand that many students who appear to be disengaged may be experiencing difficult life or family circumstances, mental health challenges, or low self-esteem, resulting in a loss of confidence that affects their engagement in learning. A caring, supportive teacher demonstrates belief in the students' abilities to learn and uses the students' strengths to create small successes that help nurture engagement in learning and provide a sense of hope.

STUDENTS WHO DEMONSTRATE EXCEPTIONAL TALENTS AND GIFTEDNESS

Modern conceptions of giftedness recognize diversity, multiple forms of giftedness, and inclusivity. Some talents are easily observable in the classroom because they are already well developed and students have opportunities to express them in the curricular and extracurricular activities commonly offered in schools. Other talents only develop if students are exposed to many and various domains and hands-on experiences. Twenty-first century learning supports the thinking that most students are more engaged when learning activities are problem-centred, inquiry-based, and open-ended. Talented and gifted students usually thrive when such learning activities are present. Learning experiences may be enriched by offering a range of activities and resources that require increased cognitive demand and higher-level thinking with different degrees of complexity and abstraction. Teachers can provide further challenges and enhance learning by adjusting the pace of instruction and the breadth and depth of concepts being explored. For additional information, refer to *Gifted Education and Talent Development* (Nova Scotia Department of Education 2010).

Connections across the Curriculum

The teacher should take advantage of the various opportunities available to integrate mathematics and other subjects. This integration not only serves to show students how mathematics is used in daily life, but it helps strengthen the students' understanding of mathematical concepts and provides them with opportunities to practise mathematical skills. There are many possibilities for integrating mathematics in health education, literacy, music, physical education, science, social studies, and visual arts.

Number (N)

GCO: Students will be expected to demonstrate number sense.

SCO N01 Students will be expected to represent and partition whole numbers to 10 000. [C, CN, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N01.01 Read a given four-digit numeral without using the word "and."
N01.02 Record numerals for numbers expressed orally, concretely, pictorially, and/or symbolically as expressions, using proper spacing without commas.
N01.03 Write a given numeral, 0 to 10 000, in words.
N01.04 Represent a given numeral using a place-value chart or diagrams.
N01.05 Express a given numeral in expanded notation (e.g., 4321 = 4000 + 300 + 20 + 1).
N01.06 Write the numeral represented by a given expanded notation.
N01.07 Explain the meaning of each digit in a given four-digit numeral.
N01.08 Represent a given number in a variety of ways and explain how they are equivalent.
N01.09 Read a given number word, 0 to 10 000.
N01.10 Represent a given number using expressions.

Scope and Sequence

Mathematics 3	Mathematics 4	Mathematics 5
N02 Students will be expected to represent and partition numbers to 1000.	**N01** Students will be expected to represent and partition whole numbers to 10 000.	**N01** Students will be expected to represent, partition, and compare whole numbers to 1 000 000.

Background

Students must construct their own understanding of number. This is best accomplished through the use of a variety of materials and through the use of those materials as a representation of student thinking. Students should be provided with many opportunities to model numbers concretely and pictorially and to explore the value of the digits in a number using proportional materials such as straws, paper clips, or base-ten blocks, and non-proportional materials such as money or multi-link cubes on place-value charts. Proper introduction and use of these materials will move student thinking from counting strategies to a deeper understanding of number.

Include situations in which students use a variety of models, such as

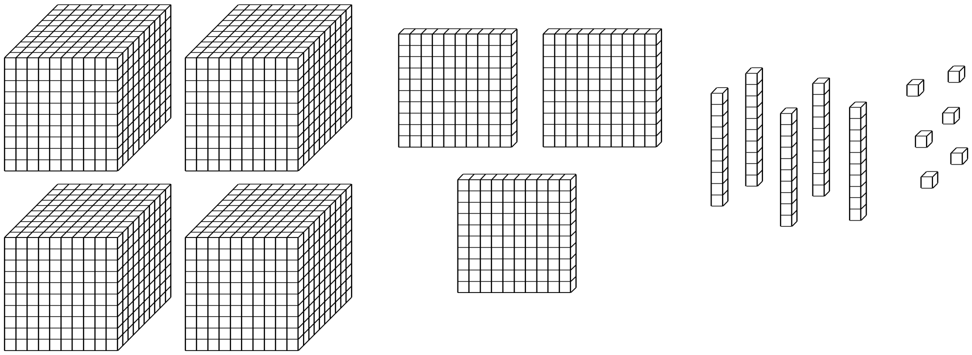
- proportional, groupable items such as straws or paper clips (e.g., place an elastic around a set of 10 straws to create a group of 10, bundle 10 sets of ten straws to create a group of 100, bundle 10 sets of 100 straws to create a group of 1000, and bundle 10 sets of 1000 straws to create a group of 10 000)
- proportional, pregrouped items such as base-ten blocks (e.g., to model 10 000 invite the class to make a long rod with 10 large base-ten cubes. The long rod created will be a ten thousand rod. Students should recognize that this also models 10 000 unit cubes, 1000 rods, and 100 flats.)
- hundreds charts

- money (e.g., How many $100 bills are there in $9347?)
- place-value charts

Through these experiences, students will develop flexibility in identifying, modelling, and representing numbers up to 10 000. It is also important for students to gain an understanding of the relative size (magnitude) of numbers through real-life contexts that are personally meaningful. Use numbers from student's experiences, such as capacity for local arenas or the population of the school or community. Students could search the Internet or newspapers to find contexts that use numbers between any two given numbers less than 10 000, such as between 2500 and 5000. Students can use these personal referents to think of other large numbers. Benchmarks that students may find helpful are multiples of 100 and 1000, as well as 250, 500, 750, 2500, 5000, and 7500.

Students should record numerals for numbers expressed in a variety of ways. For example, students should be able to write the numeral for numbers expressed

- as models in conventional and non-conventional displays



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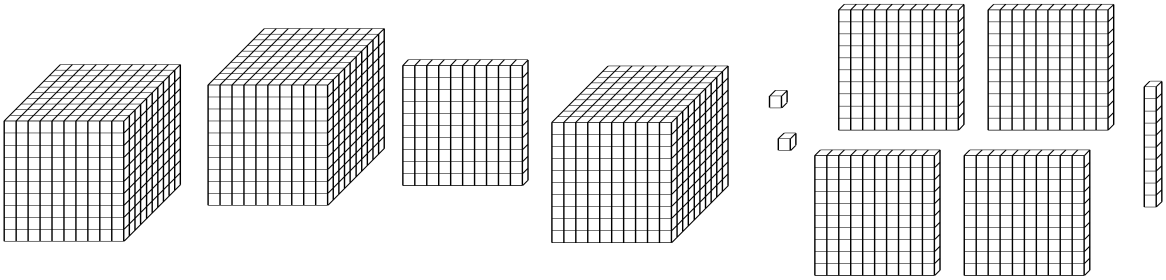

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- in words (e.g., students would record 2860 when they hear the words twenty-eight hundred sixty or they would write 8920 if asked to record the numeral that is eighty less than ninety thousand)
- as expressions (e.g., students would record 2047 when presented with the expression 500 + 500 + 500 + 500 + 40 + 7)
- in expanded notation (e.g., students would record 7453 when presented with 7000 + 400 + 50 + 3)
- in place value charts (e.g., students would record 7453 in a place-value chart as shown below)

Thousands			**Ones**		
H	T	O	H	T	O
		7	4	5	3

The focus of instruction should be on ensuring students develop a strong, flexible sense of number.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to choose a number less than 1000 and represent in at least three ways using base-ten blocks.
- Ask students, Which of the expressions below represents 360? Ask them to explain their thinking.
  - 200 – 160
  - 380 – 30
  - 400 – 40
  - 300 + 60
  - 100 + 100 + 100 + 50 + 10
  - 260 + 75 + 25
  - 357 + 3
  - 260 + 10

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to use base-ten blocks to model 9806 in three different ways and explain their models.
- Provide students with pictures of base-ten blocks displayed in conventional and non-conventional displays. Ask them to record the number represented by each base-ten picture.
- Have students record a series of numbers as numerals that have been read to them (such as eight thousand eighty-two or sixteen hundred five). Include examples such as, Record the greatest four-digit number or record the number that is one hundred less than the greatest four-digit number.
- Ask students to explain how 903 and 9003 are different and how they are similar?

- Tell students that a boat costs $6135. Ask students to explain how many $100 bills they would need to pay for the boat. Extend this by asking students to explain how many $10 bills they would need.
- Ask students to write a number that has 980 tens.
- Have students read a given numeral, such as 7106, 4083, 2456, or 8050 in more than one way.
- Ask students to record numerals for numbers expressed in expanded notation or as expressions.
- Ask students to select a four-digit number and represent that number in as many ways as they can.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 4 (Bauman 2011)
- Checkpoint 1, Tasks 1 and 2, pp. 19–21
- Checkpoint 2, Task 1, pp. 22–24

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Invite students to investigate the length of a line comprising 10 000 dimes. Encourage students to share the various strategies they used to investigate this problem. It is also important to invite them to share strategies that they considered but rejected, and to explain their reasoning.
- Use base-ten blocks or ask students to draw pictorial representations of the blocks. Invite students to use them to explore what numbers might be represented using exactly 10 base-ten blocks. (**Note:** It is important to use the correct vocabulary when referring to the blocks—“flat” not “hundred flat” and “rod” not “ten rod,” etc.—so students are flexible in their thinking about the models when working with decimals.)

SUGGESTED LEARNING TASKS

- Provide a stack of four sets of cards numbered 0 through 9. From the stack of 40 cards, ask students to select four cards and arrange them to make the greatest possible four-digit number. Ask them to record and read that number. (**Note:** Students should record the numeral with correct spacing and without commas and should read the number without using the word “and.”) Then, ask them to rearrange their four cards to make the least possible four-digit number and record that number under the larger number. As an extension related to outcome N02, ask students to estimate the difference between the two numbers. This activity is an ideal opportunity for students to practise front-end subtraction (left-to-right calculations).
- Invite students to work in pairs or in small groups. Provide each group with a sheet of chart paper. Ask each group to select a four-digit number and to represent that number in as many different ways as they can using base-ten blocks. As each base-ten model is created, groups should record it on their chart paper using pictorial representations. Groups should then record the expressions that correspond to each pictorial display. After each group has completed as many different representations for their chosen number as they can, post the chart paper from each group. Have the class examine each chart paper to determine the number represented by the pictorial representations and the expressions. Ask them to explain why all of the pictures and number expressions on a sheet of chart paper are equal.
- Have students rename a number, less than 10 000, as the sum of other numbers.
- Ask students to place benchmark numbers on a number line labelled 0 and 10 000 (e.g., 2500, 5000, 7500).
- Have students create and solve number riddles such as, I have written a secret number between 6000 and 8000. It is an odd number. The sum of the digits is 10. What might it be?
- Ask students, as a class, to create a “ten thousand” chart. Provide each small group of students with hundred grids (or other pictorial representations, such as arrays of dots) and invite them to create a model to represent 1000. Combine these models to create a class representation of 10 000.
- Tell students that Ravi wrote, “five thousand six hundred and seventy-two” for the numeral 5672. Ask students to explain whether Ravi was correct or not and to explain their thinking.
- Have students create a four-digit number using 9, 2, 7, and 5. Tell them the digit in the hundreds place must be two more than the digit in the ones place and have them list all of the possible numbers.
- Ask students to read the number 8503. (**Note:** Students should read the number without using the word “and.”)
- Ask questions about the reasonableness of numbers such as, Would it be reasonable for an elementary school to have 9600 students? Would it be reasonable for an elevator to hold 1000 people? Would someone be able to drive twenty-six hundred kilometres in a day? Would it be reasonable to pay $5000 for a house/book/computer? Investigate and discuss possible answers.
- Invite students to create their own “Is it reasonable?” questions about a variety of topics.
- Using a place-value chart, ask students to represent 6015.

Thousands			**Ones**		
H	T	O	H	T	O
		6	0	1	5

- Ask students to find large numbers from newspapers and magazines and to share and discuss the numbers within their group. Invite students to read, write, and model the numbers in different ways.
- Have students use base-ten blocks to model a number, such as 1594, in three different ways. As each model is created, ask students to record their work using a pictorial representation. Invite them to explain the models and explain why all the number expressions and pictures are equivalent.
- Ask students to explain the meaning of each digit in a given number such as 9618, 3333, or 6030.
- Have students write the numeral represented by the expanded notation of 8060.

SUGGESTED MODELS AND MANIPULATIVES

- base-ten blocks
- hundred chart
- money
- number cards
- number lines
- place-value charts

MATHEMATICAL LANGUAGE

Teacher	Student
▪ expanded notation ▪ number expression ▪ number lines, hundreds charts ▪ number words, symbols, digits ▪ ones, tens, hundreds, thousands ▪ represent, partition numbers	▪ expanded notation ▪ number expression ▪ number lines, hundreds charts ▪ number words, symbols, digits ▪ ones, tens, hundreds, thousands ▪ represent, partition numbers

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students K–8* (Small 2008), pp. 137–143, 146
- *Making Mathematics Meaningful to Canadian Students K–8*, Second Edition (Small 2013), pp. 193–200, 201–202
- *Teaching Student-Centered Mathematics, Grades 3–5*, Volume Two (Van de Walle and Lovin 2006), pp. 19, 43–55

Notes

SCO N02 Students will be expected to compare and order numbers to 10 000. [C, CN, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicator s to determine whether students have achieved the corresponding specific curriculum outcome.

N02.01 Order a given set of numbers in ascending or descending order, and explain the order by making references to place value.
N02.02 Create and order three different four-digit numerals.
N02.03 Identify the missing numbers in an ordered sequence and on a number line.
N02.04 Identify incorrectly placed numbers in an ordered sequence and on a number line.
N02.05 Place numbers in relative order on an open number line.
N02.06 Place numbers on a number line containing benchmark numbers for the purpose of comparison.
N02.07 Compare numbers based on a variety of methods.

Scope and Sequence

Mathematics 3	Mathematics 4	Mathematics 5
N03 Students will be expected to compare and order numbers to 1000.	**N02** Students will be expected to compare and order numbers to 10 000.	**N01** Students will be expected to represent, partition, and compare whole numbers to 1 000 000.

Background

Students should encounter a variety of numbers in context. These contexts help them develop an understanding of number size. In Mathematics 3, students compared and ordered numbers to 1000 using benchmarks, number lines, hundred charts, and place-value materials. In Mathematics 4, students will compare numbers to 10 000 and will order a set of numbers in ascending and descending order using a variety of methods including number charts, number lines, and place value. Visual models encourage reasoning, as students consider how to compare and order numbers. As with all concepts, begin with concrete models before moving to pictorial and symbolic representations.

Comparing and ordering numbers is fundamental to understanding numbers. Students should investigate meaningful contexts to compare and order two or more numbers, both with and without models. It is expected that students will be able to explain why one number is greater or less than another. Some possible examples of contexts for large numbers that might be meaningful for students include scores on video games, apples in a shipping crate in an orchard, number of people in attendance at a special event such as a Stanley Cup Parade in a player’s home town, the population of the community, attendance at a community event, or the price of items in a newspaper flyer.

Students should be provided with opportunities to examine large numbers concretely and pictorially and to use place-value arguments to explain which number is larger. Students must learn that when comparing two numbers with the same number of digits, the digit with the greatest place value needs to be addressed first. For example, when asked to explain why one number is greater or less than another, students might say that 2542 < 3653 because 2542 is less than 3 thousands while 3653 is more than 3 thousands. When comparing 6456 and 6546, students should begin comparing the thousands and then compare each place value to the right. Students must also recognize that when comparing the size of a number, the digit 4 in 4289 has a greater value than the digit 4 is 489, and they should be able to provide an explanation for this difference.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to find a number between 312 and 387 that can be represented using eight base-ten blocks.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Provide students with some numeral cards and ask them to order the numbers from greatest to least. Ask them to explain how they know the order is correct.
- Ask students to explain how they might advise a younger student to determine which of two numbers is greater. This could be an interview question or a journal activity.
- Have students identify two numbers that meet the following requirements: the first number has 3 in the thousands place, but is less than the second number which has 3 in the hundreds place. Ask them to explain their reasoning.
- Ask students to write a number that would fall about halfway between 9490 and 10 000.
- Tell students that you are thinking of a four-digit number that has 2 thousands, a greater number of tens, and an even greater number of ones. Ask them to give three possibilities for your number.

- Invite students to create all of the possible numbers using the digits 8, 9, 7, 6. Have students place their created numbers on a number line and explain their thinking.
- Tell students that Jodi's number had 9 hundreds, but Fran's number had only 6 hundreds. Fran's number was greater. Ask students to explain how this is possible.
- Ask students to explain which number below is greater?

  4☐☐2 or 9☐☐3
- Ask students to identify how many whole numbers are greater than 8000, but less than 8750.
- Present students with a sequence of numbers containing an error or missing number. Ask students to correct the error and to explain their thinking.
- Provide students with a set of four numeral cards, each showing a four-digit number. Ask them to place the cards in order from least to greatest and to explain their thinking.
- Provide students with a set of four numeral cards, each showing a four-digit number. Ask them to place the number cards on an open number line and to explain their thinking.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 4 (Bauman 2011)
- Checkpoint 1, Tasks 3 and 4, pp. 19–21
- Checkpoint 2, Task 2, pp. 22–24

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

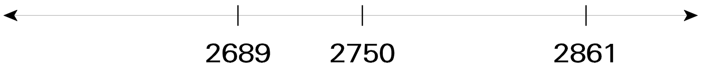
Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Provide students with opportunities to practise comparing numbers, such as 9098 and 9089, and ask them to explain their reasoning.
- Ask students to identify and explain the most appropriate benchmarks for various number lines labelled with different start and end points, such as 0 to 50, 90 to 150, 200 to 1000, 243 to 2448, or 4000 to 6000.
- Provide situations in which students
  - name numbers that are greater than or less than a given number (**Note:** In some cases the amount greater or less could be specified, such as 29 more or 3000 less, 1000 less than 8567, 100 more than 4987, 10 more than 3999, etc.)
  - name numbers which are between given numbers
- Use a variety of number lines, including open number lines, on which students may place numbers and/or correct numbers that are already placed.



SUGGESTED LEARNING TASKS

- Display a four-digit numeral. Invite students to enter a number on their calculators that differs from the displayed number by one digit. Invite students to read their numbers and ask others to determine if they are greater than or less than the number displayed. Collect five or more of these numbers and ask students to order them on a number line. Invite students to explain how they determined the order of the numbers.
- Assign pairs of students the task of making two-, three-, and four-digit numeral cards for their classmates to place in order.
- Provide a list of populations of local communities ranging from a few hundred to about ten thousand. Ask students to order the populations from least to greatest.
- Provide the following riddle for students to solve: I am thinking of a number. It is between 8000 and 10 000. All the digits are even and the sum of the digits is 16. What are some possibilities? After students have identified possible solutions, use an open number line to display their numbers. Invite students to write their own riddles.
- Place a four-digit numeral card on each student’s back. Ask students to order the numbers from least to greatest, without seeing their own numbers and without talking to each other.
- Ask students to find large numbers, up to 10 000, from newspapers, magazines, and the Internet and to create a collage that would illustrate the order of the numbers from least to greatest.
- Prepare cards for students to order from least to greatest (e.g., 6183, 9104, 9080, 7102, 6604, 1999, 6540).
- Ask students to decide which is worth more: 4356 quarters, 8462 dimes, or 9999 pennies. Ask students to predict first, and then use calculators to help solve the problem.
- Use a number line with benchmarks marked on it. Invite students to place a variety of numbers on the line using the benchmark numbers as a guide.

- Ask students to use reference books or the Internet to find the populations of two communities. Then ask them to find another population that is greater than one of the communities, but less than the other.

SUGGESTED MODELS AND MANIPULATIVES

- base-ten blocks
- number lines
- place-value charts

MATHEMATICAL LANGUAGE

Teacher	Student
▪ benchmark numbers	▪ benchmark numbers
▪ compare, order, relative order, ascending, descending order	▪ compare, order, relative order
▪ hundreds chart, number line	▪ hundreds chart, number line
▪ least, greatest	▪ least, greatest
▪ less than, more than, closer to, greater than,	▪ less than, more than, closer to, greater than,
▪ missing numbers, errors	▪ missing numbers, errors

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students K–8* (Small 2008), pp. 87–89
- *Making Mathematics Meaningful to Canadian Students K–8*, Second Edition (Small 2013), pp. 144–148
- *Teaching Student-Centered Mathematics, Grades 3–5*, Volume Two (Van de Walle and Lovin 2006), pp. 45–46

Notes

SCO N03 Students will be expected to demonstrate an understanding of addition and subtraction of numbers with answers to 10 000 (limited to three- and four-digit numerals) by

- using personal strategies for adding and subtracting
- estimating sums and differences
- solving problems involving addition and subtraction

[C, CN, ME, PS, R]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N03.01 Represent concretely, pictorially, and symbolically the addition and subtraction of whole numbers, limited to three- and four-digit numerals.
N03.02 Determine the sum of two given numbers, limited to three- and four-digit numerals, using a personal strategy, and record the process symbolically.
N03.03 Determine the difference of two given numbers, limited to three- and four-digit numerals, using a personal strategy, and record the process symbolically.
N03.04 Describe a situation in which an estimate rather than an exact answer is sufficient.
N03.05 Estimate sums and differences using different strategies.
N03.06 Create and solve problems that involve addition and subtraction of two or more numbers, limited to three- and four-digit numerals.
N03.07 Explain mental mathematics strategies that could be used to determine a sum or difference.
N03.08 Determine a sum or difference of one-, two-, and three-digit numerals efficiently, using mental mathematics strategies.

Scope and Sequence

Mathematics 3	Mathematics 4	Mathematics 5
N08 Students will be expected to apply estimation strategies to predict sums and differences of one-, two-, or three-digit numerals in a problem-solving context. **N09** Students will be expected to demonstrate an understanding of addition and subtraction of numbers (limited to one-, two-, and three-digit numerals) with answers to 1000 by ▪ using personal strategies for adding and subtracting with and without the support of manipulatives ▪ creating and solving problems in context that involve addition and subtraction of numbers concretely, pictorially, and symbolically	**N03** Students will be expected to demonstrate an understanding of addition and subtraction of numbers with answers to 10 000 (limited to three- and four-digit numerals) by ▪ using personal strategies for adding and subtracting ▪ estimating sums and differences ▪ solving problems involving addition and subtraction	**N02** Students will be expected to use estimation strategies, including front-end, front-end adjusted, rounding, compatible numbers, and compensation in problem-solving contexts.

Background

Computational fluency is a balance between conceptual understanding (thinking about the structure of numbers and the relationship among numbers and the operations) and computational proficiency (includes both efficiency and accuracy) (NCTM 2000, p. 35). This outcome involves the development of two critical abilities—the ability to solve the full range of addition and subtraction story problems efficiently, and the ability to add and subtract up to four-digit numbers efficiently. For the most part, these two abilities should be taught simultaneously; however, there will be times when some lessons should focus on one or the other.

Students should have many opportunities to solve and create word problems for the purpose of answering real-life questions, preferably choosing topics of interest to them. These opportunities provide students with a chance to practise their computational skills and to clarify their mathematical thinking. Students should be presented with addition and subtraction story problems of all structures.

- Join (result, change, and start unknown)
- Separate (result, change, and start unknown)
- Part-part-whole (part and whole unknown)
- Compare (difference, smaller, and larger unknown).

Join story problems all have an action that causes an increase, while separate story problems have an action that causes a decrease. Part-part-whole story problems, on the other hand, do not involve any actions, and compare story problems involve relationships between quantities rather than actions.

When a problem requires an exact answer, students should first determine if they are able to calculate it mentally. (This should become an automatic response.) Students in grade 4 should be able to mentally add and subtract one-, two-, and three-digit numbers. Provide opportunities to practise a variety of mental mathematics strategies. It is suggested that a strategy be introduced by giving students a question for which that strategy would be efficient to use, asking students to calculate it mentally, and asking them to share the strategies they used. Very often, the strategy being targeted is already being used by some students. These students can be invited to explain their thinking with their classmates.

It is expected that students will be able to symbolically add and subtract two four-digit numbers using reliable, accurate, and efficient strategies. While some of these strategies may have emerged directly from students work with base-ten blocks, other strategies should be modelled by students using the base-ten blocks to help understand the logic behind them. Students should be able to explain the strategy used and whether the solution is reasonable based on their prior estimate. Through the sharing of strategies, students will be exposed to a variety of possible addition and subtraction strategies, and each student will adopt ones that he or she understands well and has made his or her own. That is why these strategies are often referred to as “personal strategies.” The most appropriate strategy used may vary depending on the student and the numbers involved in the problem.

Personal strategies make sense to students and are as valid as the traditional algorithm. Therefore, emphasis should be on students’ algorithms rather than on the traditional algorithm. The paper-and-pencil recording of students’ personal strategies should reflect their thinking and must be reliable, accurate, and efficient. Most important is that student can justify how and why an algorithm works. Students should be encouraged to refine their strategies to increase their efficiency, and teachers should monitor each student’s symbolic recording of the strategy to ensure that the recording is accurate, mathematically correct, organized, and efficient.

Examples of personal strategies and their symbolic recordings are shown below. Additional examples are provided in Appendix A.

- If students are asked to add 4537 and 2178, they could use a personal strategy to determine the sum.

  A student's personal strategy could have been as follows:
  - Start by writing the two addends in expanded notation; 4537 as 4000 + 500 + 30 + 7 and 2178 as 2000 + 100 + 70 + 8.
  - Add 4000 and 2000 to get a sum of 6000.
  - Add 500 and 100 to get a sum of 600.
  - Add 30 and 70 to get a sum of 100.
  - Add 7 and 8 to get a sum of 15.
  - Add 6000, 600, 100, and 15 to get a sum of 6715.

  This personal strategy may be recorded on paper as

  4537 + 2178 = 4000 + 500 + 30 + 7 + 2000 + 100 + 70 + 8
  4000 + 2000 = 6000
  500 + 100 = 600
  30 + 70 = 100
  6000 + 600 + 100 + 15 = 6715

  or

$$\begin{array}{r} 4537 \\ \underline{+\ 2178} \\ 6000 \\ 600 \\ 100 \\ \underline{+\quad 15} \\ 6715 \end{array}$$

- If we introduce subtraction using word problems, students can begin modelling their solutions. Consider the following problem: On my vacation, I went to visit my aunt in Toronto. On the first day, I drove 739 km. If the distance to my aunt's house is 1826 km, how much further do I have to drive?

  A student's personal strategy could have been
  - I knew I had to subtract 739 from 1826.
  - So, I started by showing 1826 with 1 large cube, 8 flats, 2 rods, and 6 small cubes.
  - I removed 7 flats (which is 700) and had 1126 left.
  - I removed 3 rods (which is 30) and had 1096 left.
  - I removed 9 small cubes and had 1087 left.

  This personal strategy could be recorded on paper as

  1826 – 700 = 1126
  1126 – 30 = 1096
  1096 – 9 = 1087
  I have to travel 1087 km more.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to explain two different ways to estimate the difference for 54 – 26.
- Ask students to add 125 and 78 and describe the process using an open number line.
- Ask students to solve 675 – 234 in at least two different ways and to explain their thinking.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Have students model the addition of 1273 and 2485 using concrete and/or visual representations and to record the process symbolically. Invite students to explain their method.
- Have students model the subtraction of 248 from 5073 using concrete and/or visual representations and to record the process symbolically. Invite students to explain their method.
- Ask students to create an addition or subtraction story problem for the number sentence 5330 – 185 = ☐ or 185 + ☐ = 2330.
- Ask students to determine the sum/difference of 3185 and 628 using a personal strategy and to explain how their strategy works.
- Present students with the following problem:
  - You drink 250 mL of milk on the first day, 375 mL of milk the second day, and 450 mL of milk on the third day. About how many millilitres of milk did you drink during these three days? Stimulate the students' thinking by asking whether 900 mL would be a good estimate for the answer.
- Tell students that Jari said, "To estimate 583 – 165, I think, 575 subtract 175." Ask them if the estimate will be high or low, and to explain why Jari might have chosen to estimate in this way.
- Tell students that to mentally add 498 and 767, Kyesha said, "5 hundreds plus 7 hundreds is 12 hundreds; 12 hundreds plus 67 is 1267; 1267 less 2 is 1265." Ask students to explain whether Kyesha's sum is correct.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 4 (Bauman 2011)

- Checkpoint 1, Tasks 3 and 4, pp. 19–21
- Checkpoint 2, Task 2, pp. 22–24
- Checkpoint 4, Task 1, pp. 28–30
- Checkpoint 6, Tasks 1 and 2, pp. 34–37
- Checkpoint 7, Task 1, pp. 38–40

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Encourage students to estimate prior to doing calculations.
- Encourage students to consider whether an exact answer can be calculated using mental mathematics strategies or whether paper-and-pencil calculations are required.
- Use a variety of models, such as base-ten blocks and number lines, to assist in estimation and calculation.
- Provide students with a variety of mental mathematics and estimation strategies.
- Use problem-solving strategies, such as skip counting on a number line using place-value knowledge.
- Explore personal strategies such as Add Tens, Add Ones, Then Combine or Take Extra Tens, Then Add Back (Van de Walle and Lovin 2006b, pp. 109–111).
- Reinforce proper mathematics vocabulary. **Regrouping** or **trading** are preferred to using terms such as **borrowing** or **carrying** to describe the addition and subtraction process.

- Ask students to paraphrase various story problems to enhance understanding and to recognize which numbers in a problem refer to a part or to a whole. Share solutions.

SUGGESTED LEARNING TASKS

- Provide students with an addition number sentence, such as 328 + 462 = 330 + 460. Ask students to decide whether the number sentence is true or false and to explain how they know. Remind students to think of the equal sign as meaning “the same as” so that they are deciding whether the two sides of the equation balance each other.
- Ask students to find two numbers with a difference of about 150 and a sum of about 500.
- Provide students with a set of addition and subtraction questions. Ask them to determine how best to calculate each of the questions. If they decide to use mental mathematics strategies, ask them to calculate the answer and to share their strategies.
- Present the students with problems and have them decide which problems can be answered with an estimate only and which problems require calculation as well as an estimate. For example, Will a container that holds 2000 mL be large enough to hold 1350 mL of water from another container as well as 1015 mL of water from a different container?
- Ask students to use two different strategies to prove that 3457 – 1898 = 1559.
- Ask students to find and correct the error Sam made while recording his calculation of 6789 + 2345. Sam recorded the following:

  6789 + 2345 = 6000 + 2000 = 8000 = 700 + 300 = 1000 = 80 + 40 = 120 = 9 + 5 = 14 = 8000 + 1000 + 120 + 14 = 9134

SUGGESTED MODELS AND MANIPULATIVES

- balance scale
- base-ten blocks
- calculators
- number lines (including open number lines)
- place-value charts

MATHEMATICAL LANGUAGE

Teacher	Student
▪ addition, subtraction ▪ benchmarks ▪ compensation ▪ estimate ▪ front-end addition ▪ making a friendly number ▪ number sentence ▪ one-digit, two-digit, three-digit, four-digit ▪ rounding ▪ sum, difference	▪ addition, subtraction ▪ benchmarks ▪ compensation ▪ estimate ▪ front-end addition ▪ making a friendly number ▪ number sentence ▪ one-digit, two-digit, three-digit, four-digit ▪ rounding ▪ sum, difference

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students K–8* (Small 2008), pp. 107–113, 144–145
- *Making Mathematics Meaningful to Canadian Students K–8*, Second Edition (Small 2013), pp. 160–168, 200–201
- *Teaching Student-Centered Mathematics, Grades 3–5*, Volume Two (Van de Walle and Lovin 2006), pp. 53–55, 108–112

Videos

- *Using a Hands-on Approach to Develop Mental Strategies for Addition* (11:04 min.) (ORIGO Education 2010)
- *Using a Hands-on Approach to Develop Mental Strategies for Subtraction* (6:45 min.) (ORIGO Education 2010)

Notes

SCO N04 Students will be expected to apply and explain the properties of 0 and 1 for multiplication and the property of 1 for division.
[C, CN, R]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N04.01 Determine the answer to a given question involving the multiplication of a number by 1, and explain the answer using the property of 1 in multiplication.
N04.02 Determine the answer to a given question involving the multiplication of a number by 0, and explain the answer using the property of 0 in multiplication.
N04.03 Determine the answer to a given question involving the division of a number by 1, and explain the answer using the property of 1 in division.

Scope and Sequence

Mathematics 3	Mathematics 4	Mathematics 5
N11 Students will be expected to demonstrate an understanding of multiplication to 5 × 5 by ▪ representing and explaining multiplication using equal grouping and arrays ▪ creating and solving problems in context that involve multiplication ▪ modelling multiplication using concrete and visual representations and recording the process symbolically ▪ relating multiplication to repeated addition ▪ relating multiplication to division	**N04** Students will be expected to apply and explain the properties of 0 and 1 for multiplication and the property for division.	**N03** Students will be expected to describe and apply mental mathematics strategies and number properties to recall, with fluency, answers for basic multiplication facts to 81 and related division facts.

Background

It is important to address the property of zero in multiplication, and the property of one in multiplication and division. Students need to develop story problems that demonstrate multiplication by 0 and 1, and division by 1 that employ linear, set, and area models. For example, to explore that the product is 0 when multiplying by 0, 3 × 0 can be shown by making 3 hops of 0 or making 0 hops of 3 (Van de Walle and Lovin 2006a, p.85) on a number line. The property of multiplying and dividing by 1 can be similarly explored on the number line by making 3 hops of 1 or making 1 hop of 3. It may also be explored with an area model showing 1 row of 3 or 3 rows of 1, or a set model showing 1 group of 3 or 3 groups of 1.

Provide opportunities for students to not only solve multiplication and division problems, but to create their own problems requiring the use of these operations. These opportunities will support students in developing understanding of how multiplication by 0 and 1 are different, and how multiplication and division by 1 are similar.

It is important to avoid teaching arbitrary rules such as “any number multiplied by one is that number” or “any number multiplied by zero is zero.” Students will come to these generalizations on their own given opportunities to develop understanding using models.

This outcome should be connected to outcome N05.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students’ prior knowledge.

- Ask students to use tiles to create a realistic story problem to go with a given number sentence (e.g., 4 × 5) or ask them to describe a situation for which they might have to find the answer to 5 × 3.
- Show students an array (4 × 2) and ask them to identify the related multiplication and division sentences represented by the array.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample tasks (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to create a story problem in which they are dividing a number by one.
- Ask students to create a story problem in which they are multiplying a number by zero.
- Ask students to describe a general statement they could make about multiplying any number by zero. Ask them to prove their statement using pictures or models.

- Ask students to describe a general statement they could make about multiplying any number by one. Ask them to prove their statement using pictures or models.
- Ask students to describe a general statement they could make about dividing any number by one. Ask them to prove their statement using pictures or models.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 4 (Bauman 2011)
- Checkpoint 5, Task 1, pp. 31–33

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Use various concrete materials and pictorial representations to demonstrate the multiplication and division of zero. For example, use paper plates for the concept of multiplying by zero. Show six plates with zero counters on each. Ask, How many plates are there? (six) How many counters are there on each plate? (zero) Six groups of zero are how many? ($6 \times 0 = 0$).
- Address the misconception that multiplication always makes the product greater. For example, any number multiplied or divided by one remains unchanged.

SUGGESTED LEARNING TASKS

- Tell students that Jeremiah said that he can multiply any number by zero and his answer will always be zero. Ask students to tell whether Jeremiah is correct or not and to explain how they know.
- Ask students to choose any three numbers and to model with blocks or a number line the multiplication of each of those numbers by one. Have students explain what they notice about their answers.
- Ask students to choose any three numbers and to model with blocks or a number line the multiplication of each of those numbers by zero. Have students explain what they notice about their answers.
- Ask students to choose any three numbers and to model with blocks or a number line the division of each of those numbers by one. Have students explain what they notice about their answers.
- Ask students to build one set of five objects. Then, ask them to build five sets of one object. Have them explain how the two displays are the same and how they are different.
- Ask students to use seven counters to show seven divided into one group. Then, ask them to use another set of seven counters to show seven divided into groups of one. Have them explain how the two displays are the same and how they are different.

SUGGESTED MODELS AND MANIPULATIVES

- area models
- arrays
- counters
- number line

MATHEMATICAL LANGUAGE

Teacher	**Student**
dividing one	dividing one
multiplying, dividing zero	multiplying, dividing zero
no change nature	
properties	properties
zero, one	zero, one

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students K–8* (Small 2008), pp. 128–129
- *Making Mathematics Meaningful to Canadian Students K–8*, Second Edition (Small 2013), pp. 183–184
- *Teaching Student-Centered Mathematics, Grades 3–5*, Volume Two (Van de Walle and Lovin 2006), pp. 66–67

Notes

SCO N05 Students will be expected to describe and apply mental mathematics strategies, to recall basic multiplication facts to 9 × 9, and to determine related division facts.
[C, CN, ME, R]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N05.01 Describe the mental mathematics strategies used to determine basic multiplication or division facts.
N05.02 Use and describe a personal strategy for determining the multiplication facts.
N05.03 Use and describe a personal strategy for determining the division facts.
N05.04 Quickly recall basic multiplication facts up to 9 × 9.

Scope and Sequence

Mathematics 3	Mathematics 4	Mathematics 5
N10 Students will be expected to apply mental mathematics strategies and number properties to develop quick recall of basic addition facts to 18 and related basic subtraction facts. **N11** Students will be expected to demonstrate an understanding of multiplication to 5 × 5 by ▪ representing and explaining multiplication using equal grouping and arrays ▪ creating and solving problems in context that involve multiplication ▪ modelling multiplication using concrete and visual representations and recording the process symbolically ▪ relating multiplication to repeated addition ▪ relating multiplication to division	**N05** Students will be expected to describe and apply mental mathematics strategies, to recall basic multiplication facts to 9 × 9, and to determine related division facts.	**N03** Students will be able to describe and apply mental mathematics strategies and number properties to recall, with fluency, answers for basic multiplication facts to 81 and related division facts.

Background

Developing basic multiplication facts to 9 × 9 and related division facts requires that the students have a strong foundation in patterns, number relationships, and place value, as well as the meaning, relationships, and properties of operations as described below.

- Patterns are used in developing mental strategies, such as skip-counting from a known fact and using the constant sum of the digits in products with the 9s facts.
- Number relationships are evident when using the properties of operations or other strategies, such as repeated doubling (e.g., $4 \times 6 = (2 \times 6) \times 2 = 24$).
- Number sense is used extensively in various strategies, such as doubling and adding or subtracting one more group (e.g., $3 \times 7 = (2 \times 7) + 7 = 14 + 7 = 21$; $9 \times 9 = (10 \times 9) - 9 = 81$).
- The meaning of multiplication and division and the connection between the operations is crucial as the students develop understanding of multiplication and division facts.

By the end of Mathematics 4, students are expected to be proficient with their multiplication facts. Proficiency with their division facts is not expected until the end of grade 5. Proficiency is defined as the ability to recall the multiplication facts quickly and accurately when needed. This could be achieved through learning a series of strategies, each of which addresses a cluster of facts. Each strategy is introduced, reinforced, and assessed before being integrated with previously learned strategies. It is important that students understand the logic and reasoning of each strategy, so the introduction of each strategy is very important. As students master each cluster of facts for a strategy, it is recommended that they record these learned facts on a multiplication chart. By doing this, they visually see their progress and are aware of which facts they should be practising. A description of these strategies and a suggested sequence for them is provided in Appendix A.

The focus for mastering the multiplication facts must be based on relating the facts to previous knowledge. It is important that this work be spread over several months focusing daily on the acquisition of the facts, and the facts should be revisited often throughout the remainder of the year. While the expectation of this outcome is to have students become quick and accurate with their recall of the multiplication facts (three- to five-second recall for facts), it is important to remember that some students may require more time to process questions. Students should be encouraged to set their pace by acknowledging when they feel they have the facts mastered. Guessing the answer should be avoided, and timed tests should not be used.

Although this outcome focuses on developing recall of basic multiplication facts and using strategies to determine basic division facts, students are also expected to be able to model multiplication and division of one-digit numbers contextually, concretely, pictorially, and symbolically. Please refer to outcomes N06 and N07 for a description of these expectations.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to use models to solve a contextual problem such as, Jacques has 3 bags of apples. Each bag has 4 apples. How many apples does he have?

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample tasks (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to explain how they would determine the answer to a division fact, for example 30 ÷ 5 =?, by relating it to multiplication.
- Have students illustrate two different ways to think about 6 × 7.
- Ask students to explain how knowing 4 × 5 helps someone find the product of 8 × 5.
- Ask students to explain how knowing 8 × 10 helps someone find the product of 8 × 9.
- Have students use counters to show why 6 × 8 is the same as (4 × 8) + (2 × 8). (**Note:** Students are not required to use parentheses.)

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 4 (Bauman 2011)
- Checkpoint 5, Tasks 1 and 2, pp. 31–33

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

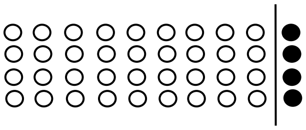
Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Introduce a strategy with the use of materials, practise the strategy, and continue to introduce and practise new strategies. When students have two or more strategies, it is important to focus on strategy selection; choosing the strategy that will be most efficient to determine a particular fact.
- Use the properties of multiplication in developing mental strategies.
  - the associative property (e.g., $(2 \times 2) \times 6 = 2 \times (2 \times 6)$)
  - the commutative property: 3×4 is read 3 sets or groups of 4; the product however, is the same if the factors are reversed (4×3)
  - distributive property: $4 \times 8 = (4 \times 5) + (4 \times 3) = 20 + 12 = 32$
- Encourage students to visualize the process for the strategy they are using (e.g., 4×9; think 4×10 is 40, and then subtract a set 4. So 4×9 is to equal 36).



- Ask students to begin with what they know. For example, to figure out 6×8, one student might think, I know $5 \times 8 = 40$ and one more set of 8 is 48. Another might think, I know 3×8 is 24 and twice 24 is 48.

SUGGESTED LEARNING TASKS

- Place students in pairs to practise a particular strategy for facts. Invite students to take turns asking facts and providing answers by repeated doubling.

- Invite students to play the Target Game. Provide students with multiplication questions that show a known number multiplied by an unknown number and a “target” they are trying to reach. The goal for students is to determine the unknown factor that will result in a product closest to the target number without going over.

  5 × ☐ → 43 (Target) ☐ are left over
- Ask students to practise the five facts using an analog clock.
- Tell the students that the “6” button on the calculator is not working. Ask students to suggest ways to solve 6 × 9 without using this button.
- Ask students to examine the nine facts and describe a strategy that they could use to determine those facts.
- Invite students to model with counters the “double plus one more set” (three facts).
- Invite student to work in groups to develop strategies for the six facts (e.g. double three or fives and one more set), the seven facts (e.g., fives and a double), and the eight facts (e.g., double, double, and double again).

SUGGESTED MODELS AND MANIPULATIVES

- area models
- arrays
- clocks
- counters
- geo-boards (10 × 10)
- ten-frames
- number line (skip counting)

MATHEMATICAL LANGUAGE

Teacher	Student
▪ factors, product, quotient, divisor, dividend ▪ groups of, rows of, jumps of ▪ mental mathematics strategy ▪ multiplication, division facts ▪ relating division to multiplication ▪ repeated doubling, using halving, skip counting, ten facts, five facts (clock facts) ▪ repeated addition, equal groups, number of groups	▪ factors, product, quotient, divisor, dividend ▪ groups of, rows of, jumps of ▪ mental mathematics strategy ▪ multiplication, division facts ▪ relating division to multiplication ▪ repeated doubling, using halving, skip counting, ten facts, five facts (clock facts) ▪ repeated addition, equal groups, number of groups

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students K–8* (Small 2008), pp. 129–131
- *Making Mathematics Meaningful to Canadian Students K–8*, Second Edition (Small 2013), pp. 184–186
- *Teaching Student-Centered Mathematics, Grades 3–5*, Volume Two (Van de Walle and Lovin 2006), pp. 12, 74, 88–93

Videos

- *An Introduction to Teaching Multiplication Number Facts* (15:18 min.) (ORIGO Education 2010)
- *Teaching the Use-Ten Strategy for Multiplication Number Facts* (10:21 min.) (ORIGO Education 2010)
- *Teaching the Double Strategy for Multiplication Number Facts* (11:06 min.) (ORIGO Education 2010)
- *Teaching the Build-Up/Build-Down Strategy for Multiplication Number Facts* (16:01 min.) (ORIGO Education 2010)

Notes

SCO N06 Students will be expected to demonstrate an understanding of multiplication (one-, two- or three-digit by one-digit numerals) to solve problems by

- using personal strategies for multiplication, with and without concrete materials
- using arrays to represent multiplication
- connecting concrete representations to symbolic representations
- estimating products
- applying the distributive property

[C, CN, ME, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N06.01 Model a given multiplication problem, using the distributive property (e.g., 8 × 365 = (8 × 300) + (8 × 60) + (8 × 5)).

N06.02 Model the multiplication of two given numbers, limited to one-, two-, or three-digit by one-digit numerals, using concrete or visual representations, and record the process symbolically.

N06.03 Create and solve multiplication story problems, limited to one-, two-, or three-digit by one-digit numerals, and record the process symbolically.

N06.04 Estimate a product using a personal strategy (e.g., 2 × 243 is close to or a little more than 2 × 200, or close to or a little less than 2 × 250).

N06.05 Model and solve a given multiplication problem using an array, and record the process.

N06.06 Determine the product of two given numbers using a personal strategy, and record the process symbolically.

Scope and Sequence

Mathematics 3	**Mathematics 4**	**Mathematics 5**
N11 Students will be expected to demonstrate an understanding of multiplication to 5 × 5 by ▪ representing and explaining multiplication using equal grouping and arrays ▪ creating and solving problems in context that involve multiplication ▪ modelling multiplication using concrete and visual representations and recording the process symbolically ▪ relating multiplication to repeated addition ▪ relating multiplication to division	**N06** Students will be expected to demonstrate an understanding of multiplication (one-, two-, or three-digit by one-digit numerals) to solve problems by ▪ using personal strategies for multiplication, with and without concrete materials ▪ using arrays to represent multiplication ▪ connecting concrete representations to symbolic representations ▪ estimating products ▪ applying the distributive property	**N02** Students will be expected to use estimation strategies, including front-end, front end adjusted, rounding, compatible numbers, and compensation in problem-solving contexts. **N05** Students will be expected to demonstrate, with and without concrete materials, an understanding of multiplication (two-digit by two-digit) to solve problems.

Background

Students should use a variety of concrete and pictorial models to investigate multiplication to help them develop an understanding of the connection between the models and the symbols. Base-ten blocks serve as a tool for understanding the multiplication operation, and it is important that the students use correct mathematical language as they manipulate the materials and pictorially record their work with base-ten blocks. It is important to start with a word problem and then have students use materials to model the problem and to determine the product. For example, A marching band has 24 rows with 5 players in each row. How many people are in the band?

Students should have many opportunities to solve and create word problems for the purpose of answering real-life questions, preferably choosing topics of interest to them. These opportunities provide students with a chance to practise their computational skills and clarify their mathematical thinking. To understand multiplication, students must have meaningful experiences with the many situations in which this operation is used. For addition and subtraction, students were exposed to 11 different structures of story problems. For multiplication (and division) there are three categories of story problem structures: equal groups, comparing, and combining. The structures of story problems for multiplication and division have been given special attention here because these situational contexts help students to see the circumstances under which they might use multiplication and division. Students should be encouraged to both solve and create problems related to these structures. Students should solve problems by building and sketching models and explaining their discoveries both symbolically and verbally (either written or oral). Students should be exposed to multiplication and division situations that enable them to understand the various ways in which we use multiplication and division. Multiplication and division situations should involve sets, arrays, and linear models.

It is expected that, by the end of the year, students will be able to symbolically multiply one-, two- and three-digit numbers by a one-digit number using reliable, accurate, and efficient strategies. Students should be able to model their strategies with base-ten blocks to help them understand the logic behind them. Students should be able to explain the strategy used and whether the solution is reasonable based on the prior estimate. Through the sharing of strategies, students will be exposed to a variety of possible multiplication strategies, and each student will adopt ones that he or she understands well and has made his or her own. That is why these strategies are often referred to as “personal strategies.” The most appropriate strategy used may vary depending on the student and the numbers involved in the problem.

It is not expected that students would be explicitly taught all possible multiplication algorithms. Instead, teachers should provide opportunities for students to develop their personal algorithms for multiplication. Personal strategies make sense to students and are as valid as the traditional algorithm. Therefore, emphasis should be on students’ algorithms rather than on the traditional algorithm. The paper-and-pencil recording of students’ personal strategies should reflect their thinking and must be reliable, accurate, and efficient. Most important is that students can justify how and why an algorithm works. Students should be encouraged to refine their strategies to increase their efficiency, and teachers should monitor each student’s symbolic recording of the strategy to ensure that the recording is accurate, mathematically correct, organized, and efficient. Students can begin by using concrete materials to solve the problem while the teacher assists by recording the student’s thinking symbolically and facilitating the documentation. Through these opportunities to model and record solutions to story problems, students discover most efficient algorithms for the numbers included in a given problem.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to represent a given repeated addition as multiplication and vice versa.
- Ask students to represent, concretely or pictorially, equal groups for a given number sentence.
- Ask students to model as many arrays as possible with 16 counters. Have them write the related multiplication and division facts for each array.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample tasks (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Tell students that they have $60. Ask, Do you have enough money to buy three CDs if each costs $17? Explain how you know.
- Ask students to determine whether they can reach a cottage that is 1200 km away if they travel 375 km each day for 3 days. Invite them to explain their thinking.
- Ask students to write all the possible number sentences that are represented in the following array and explain how each number sentence relates to the array.

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********** ********** ********** * *

********** ********** ********** * *

********** ********** ********** * *
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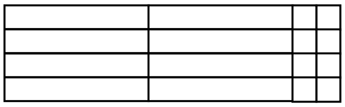
- Ask students to model 24 × 6 and explain their model.
- Tell students that for a school assembly, 9 rows of 38 chairs have been placed in the gym. Ask them to explain whether there are enough chairs for 370 students and invite them to explain their thinking.

- Ask students to create and solve a realistic problem that includes the factors 6 and 329.
- Ask students to solve problems such as, You save six times as much money this year as you saved last year. If you saved $125 last year, how much money did you save this year?
- Ask students to solve 243 × 5 using a personal strategy and record their work symbolically.
- Ask students to choose a two-digit number and a one-digit number and create a multiplication story problem using the two numbers they selected. Have them solve their problem and record their work symbolically.
- Show students the following multiplication array.



Ask them to create a story problem that would be represented with the array. Have them solve their problem and record their work symbolically.

- Tell students that Mya solved 4 × 123 as shown below:

$$\begin{array}{r} 123 \\ \underline{\times\ 4} \\ 12 \\ 8 \\ \underline{+\ 4} \\ 24 \end{array}$$

Ask students to explain why Mya’s solution is not correct and to use base-ten blocks to explain the correct solution.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 4 (Bauman 2011)

- Checkpoint 2, Tasks 1 and 2, pp. 22–24
- Checkpoint 5, Tasks 1 and 2, pp. 31–33
- Checkpoint 7, Task 1, pp. 38–40

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Provide regular practice in estimation, accompanied by the sharing of strategies. When assessing estimation, the amount of time provided must be controlled in order to determine whether students are proficient in this skill. The goal is for students to routinely estimate in problem-solving situations, not only when instructed to do so.
- Ask students to estimate the product to the problem before calculating so that they are better able to determine the reasonableness of their answers.
- Provide a variety of problems representing the different multiplication situations with varying degrees of difficulty to differentiate instruction.
- Provide time for students to create their personal strategies to solve the problem and share these strategies with members of their group or with the entire class.
- Challenge students to solve a problem another way, do a similar problem without models, or clarify the explanation of their personal strategies.

SUGGESTED LEARNING TASKS

- Ask students how they would use the front-end mental multiplication strategy for questions such as 3 × 125 = 375 ((3 × 100) + (3 × 20) + (3 × 5)) and encourage strategies such as (3 × 100) + (3 × 25).
- Ask students to fill in the blanks with the digits 3, 4, and 5 in three different ways and find all the possible products. □ □ × □
- Provide students with problems to solve such as,
  - If you travel 412 km each day for three days, will you reach a cabin that is 1200 km away by the end of the third day?
  - You set up 6 rows of chairs with 28 chairs in each row in the gym. Are there enough chairs to seat 180 people? How many chairs did you set up?
  - A kangaroo jumps 135 cm on the first jump and twice as far on the second jump. About how far does the kangaroo jump in all?
  - You jog for 175 minutes each week. How many minutes do you jog in 28 days?
- Have students use an array to model 4 × 32. Ask them to record their work symbolically in two different ways.
- Tell students that Avril estimated 47 × 7 as 500. Ask them what strategy they think Avril was using and if they would estimate it differently.
- Tell students that Ruby said it was just as easy to mentally solve 2 × 525 as it is to give an estimate. Ask how she might have found the answer mentally.
- Invite students to use a supermarket flyer and select 6 of one item, 4 of another item, and 10 of a third item, and give an estimate for the total cost.

- Ask students to use a number line to represent 5 × 25 and to explain their thinking.
- Have students sketch an array on grid paper to represent 6 × 24. Ask them to identify the partial products and the final product.

SUGGESTED MODELS AND MANIPULATIVES

- area models
- arrays
- base-ten blocks
- grid paper
- number lines
- sets
- tables

MATHEMATICAL LANGUAGE

Teacher	Student
▪ distributive property ▪ factors, product ▪ groups of, rows of, jumps of ▪ multiplication ▪ number line ▪ number sentence, number expression ▪ repeated addition, equal groups, number of groups ▪ sets, arrays	▪ distributive property ▪ factors, product ▪ groups of, rows of, jumps of ▪ multiplication ▪ number line ▪ number sentence, number expression ▪ repeated addition, equal groups, number of groups ▪ sets, arrays

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students K–8* (Small 2008), pp. 118–119
- *Making Mathematics Meaningful to Canadian Students K–8*, Second Edition (Small 2013), pp. 172–173
- *Teaching Student-Centered Mathematics, Grades 3–5*, Volume Two (Van de Walle and Lovin 2006), pp. 13, 18–19, 98–99, 107, 113,

Videos

- *Using Mental Strategies to Multiply* (26:16 min.) (ORIGO Education 2010)
- *Using Language Stages to Develop Multiplication Concepts* (16:97 min.) (ORIGO Education 2010)

Notes

SCO N07 Students will be expected to demonstrate an understanding of division (one-digit divisor and up to two-digit dividend) to solve problems by

- using personal strategies for dividing, with and without concrete materials
- estimating quotients
- relating division to multiplication

[C, CN, ME, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N07.01 Model the division of two given numbers without a remainder, limited to a one-digit divisor and up to a two-digit dividend, using concrete or visual representations, and record the process pictorially and symbolically.

N07.02 Model the division of two given numbers with a remainder, limited to a one-digit divisor and up to a two-digit dividend, using concrete or visual representations, and record the process pictorially and symbolically. (It is not intended that remainders be expressed as decimals or fractions.)

N07.03 Solve a given division problem, using a personal strategy, and record the process symbolically.

N07.04 Create and solve division word problems involving a one- or two-digit dividend, and record the process pictorially and symbolically.

N07.05 Estimate a quotient using a personal strategy (e.g., 86 ÷ 4 is close to 80 ÷ 4 or close to 80 ÷ 5).

N07.06 Solve a given division problem by relating division to multiplication (e.g., for 80 ÷ 4, we know that 4 × 20 = 80, so 80 ÷ 4 = 20).

Scope and Sequence

Mathematics 3	**Mathematics 4**	**Mathematics 5**
N12 Students will be expected to demonstrate an understanding of division by ▪ representing and explaining division using equal sharing and equal grouping ▪ creating and solving problems in context that involves equal sharing and equal grouping ▪ modelling equal sharing and equal grouping using concrete and visual representations, and recording the process symbolically ▪ relating division to repeated subtraction ▪ relating division to multiplication (Limited to division related to multiplication facts up to 5 × 5.)	**N07** Students will be expected to demonstrate an understanding of division (one-digit divisor and up to two-digit dividend) to solve problems by ▪ using personal strategies for dividing, with and without concrete materials ▪ estimating quotients ▪ relating division to multiplication	**N06** Students will be expected to demonstrate, with and without concrete materials, an understanding of division (three-digit by one-digit), and interpret remainders to solve problems.

Background

The concept of division needs to be taught in conjunction with multiplication. The teacher should ensure that students recognize that multiplication and division are two ways of looking at the same situation—this is very clear when they examine models or pictures. Some students might think, What do I multiply 3 by to get 18? when asked to find 18 ÷ 3. Other students might imagine the area model and think, How many will be in each row if I organize 18 objects into 3 rows? Allowing students multiple opportunities to make connections between multiplication and division, and the concrete and pictorial representations of these operations, will help students to develop understanding of the operations.

Students should use a variety of models to investigate division to help them develop an understanding of the connection between the models and the symbols. Base-ten blocks serve as a tool for understanding the division operation, and it is important that the students use correct mathematical language as they manipulate the materials and record their work with base-ten blocks pictorially. It is important to start with a word problem and then have students use materials to determine the quotient. For example, There are 24 students in the gym. They want to make teams of four students. How many teams can they make?

Students should have many opportunities to solve and create word problems for the purpose of answering real-life questions, preferably choosing topics of interest to them. These opportunities provide students with a chance to practise their computational skills and clarify their mathematical thinking. To understand division, students must have meaningful experiences with the many situations in which this operation is used. For division (and multiplication) there are three categories of story problem structures: equal groups, comparing, and combining. These structures of story problems should be given special attention because situational contexts help students to see the circumstances under which they might use multiplication and division. Students should be encouraged to both solve and create problems related to these structures. Students should solve problems by building and sketching models and explaining their discoveries both symbolically and verbally (either written or oral). Students should not be expected to do symbolic manipulation in isolation. Students should be exposed to multiplication and division situations that enable them to understand the various ways in which we use multiplication and division. Multiplication and division situations should involve sets, arrays, and linear models.

It is not expected that students would be explicitly taught all possible division algorithms. Instead, teachers should provide opportunities for students to develop their personal algorithms for division. Students can begin by using concrete materials to solve the problem while the teacher assists by recording the thinking symbolically and facilitating the documentation. Through these opportunities to model and record solutions to story problems, students discover the most efficient algorithms for the numbers included in a given problem.

Students should estimate quotients prior to exploring their own methods or procedures for finding the quotient.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Show students the multiplication sentence 5 × 3 = 15. Ask them to write related division sentences.
- Show students an array of up to 25 counters. Ask students to identify the multiplication and division facts shown by the array.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Have students use/draw models to show 83 ÷ 3 and explain their thinking.
- Ask students to explain how many digits there would be in the quotient of $4\overline{)57}$? Ask students to explain how they know.
- Present students with the following problem: You have 72 marbles to share equally among four friends. How many marbles will each friend receive? Explain how you know.
- Ask students to explain the connection between multiplication and division by using counters or base-ten blocks. If necessary, suggest to the student to make an array.
- Have the student estimate 93 ÷ 5 and tell whether the estimate is probably high or low and why. Ask students to suggest another division question for which the same estimate would be appropriate.
- Provide the students with a set of base-ten blocks. Ask students to model three different division questions of their choice and to write the division sentence for each.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 4 (Bauman 2011)
- Checkpoint 2, Tasks 1 and 2, pp. 22–24
- Checkpoint 7, Task 1, pp. 38–40

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Ensure students explore the relationship between multiplication and division.
- Provide regular practice in estimation, accompanied by the sharing of strategies.
- Present division questions in context to identify various meanings of division.
- Provide a variety of problem structures that include the various meanings of division used in a real-life context.
- Invite students to create and share problems that include the various meanings of division. It is helpful for students to explore a variety of models for solving division questions.

SUGGESTED LEARNING TASKS

- Ask students to use a model to explain to a classmate how to share 86 marbles among five people. Discuss the different strategies used.
- Ask students to make up division problems about situations in the classroom and post them. Encourage them to give examples of both the partition and the repeated subtraction meanings of division. Invite others to try to guess what the division situations are. For example, 25 ÷ 6 (Classmates divided into groups of 6. How many groups?).
- Provide a list of division questions to pairs of students and ask them to estimate a quotient and explain their strategy to their partner and tell whether the estimate is too high or too low and why.

- Present the students with a problem and have them choose which of the number sentences provided could be used to solve the problem and why they chose it. For example, Jamal earned $96 this month by doing odd jobs for the neighbours. Last month, he earned $8. How many times more did he earn this month than last month?

96 × 8 = □	□ = 8 × 96	[✓ 8 × □ = 96]
96 × □ = 8	[✓ 96 ÷ 8 = □]	8 ÷ 96 = □
□ ÷ 8 = 96	[✓ 96 ÷ □ = 8]	8 ÷ □ = 96

- Present students with a variety of problems to solve such as,
  - There are 77 baseball cards to be shared between 2 students. Ask them how they know that there will be a remainder. What about sharing them among 5 students? 7 students?
  - Tyra rode her bicycle every day for 8 days. She cycled 68 km in total. About how far did she ride each day?
- Use base-ten blocks to solve the following problem: If the area of a rectangular field is 182 m^2 and the length is 7 m, how wide is the field? (This question is related to SCO M03.)
- Ask students to estimate 73 ÷ 4 and indicate whether their estimates are too high or too low and how they know.
- Ask students to solve 86 ÷ 4 using base-ten blocks and to record their work pictorially and symbolically.

SUGGESTED MODELS AND MANIPULATIVES

- arrays
- base-ten blocks
- counters
- grid paper
- money

MATHEMATICAL LANGUAGE

Teacher	Student
▪ arrays, repeated subtraction ▪ divided into ▪ estimating ▪ number in each group, number of groups ▪ number sentence, number expression ▪ quotient, divisor, dividend ▪ relating division to multiplication ▪ remainder	▪ repeated subtraction ▪ divided into ▪ estimating ▪ number in each group, number of groups ▪ number sentence, number expression ▪ quotient, divisor, dividend ▪ relating division to multiplication ▪ remainder

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students K–8* (Small 2008), pp. 120–121, 123
- *Making Mathematics Meaningful to Canadian Students K–8*, Second Edition (Small 2013), pp. 174–175, 178
- *Teaching Student-Centered Mathematics, Grades 3–5*, Volume Two (Van de Walle and Lovin 2006), pp. 64–65, 93, 121–124

Notes

SCO N08 Students will be expected to demonstrate an understanding of fractions less than or equal to one by using concrete, pictorial, and symbolic representations to

- name and record fractions for the parts of one whole or a set
- compare and order fractions
- model and explain that for different wholes, two identical fractions may not represent the same quantity
- provide examples of where fractions are used

[C, CN, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N08.01 Represent a given fraction of one whole object, region, or a set using concrete materials.
N08.02 Identify a fraction from its given concrete representation.
N08.03 Name and record the shaded and non-shaded parts of a given whole object, region, or set.
N08.04 Represent a given fraction pictorially by shading parts of a given whole object, region, or set.
N08.05 Explain how denominators can be used to compare two given unit fractions with a numerator of 1.
N08.06 Order a given set of fractions that have the same numerator, and explain the ordering.
N08.07 Order a given set of fractions that have the same denominator, and explain the ordering.
N08.08 Identify which of the benchmarks 0, $\frac{1}{2}$, or 1 is closer to a given fraction.
N08.09 Name fractions between two given benchmarks on a number line.
N08.10 Order a given set of fractions by placing them on a number line with given benchmarks.
N08.11 Provide examples of instances when two identical fractions may not represent the same quantity.
N08.12 Provide, from everyday contexts, an example of a fraction that represents part of a set and an example of a fraction that represents part of one whole.

Scope and Sequence

Mathematics 3	Mathematics 4	Mathematics 5
N13 Students will be expected to demonstrate an understanding of fractions by ▪ explaining that a fraction represents a part of a whole ▪ describing situations in which fractions are used ▪ comparing fractions of the same whole with like denominators	**N08** Students will be expected to demonstrate an understanding of fractions less than or equal to 1 by using concrete, pictorial, and symbolic representations to ▪ name and record fractions for the parts of one whole or a set ▪ compare and order fractions ▪ model and explain that for different wholes, two identical fractions may not represent the same quantity ▪ provide examples of where fractions are used	**N07** Students will be expected to demonstrate an understanding of fractions by using concrete, pictorial, and symbolic representations to create sets of equivalent fractions ▪ compare and order fractions with like and unlike denominators

Background

Students construct an understanding of fractions when they begin with models. Students need to see fractions modelled through the use of many different concrete materials. The models should include number lines, area models using pattern blocks or fraction circles, and set models using counters, money, or egg cartons. We might ask students to demonstrate different ways they may cut a square in halves, fourths, or eighths.

In order for students to construct a firm foundation for fraction concepts, they need to experience and discuss activities that promote the following understandings.

- “Fractional parts are equal shares or equal-sized portions of a [one] whole or unit.
- A unit can be an object or a collection of things. More abstractly, the unit is counted as 1. On the number line, the distance from 0 to 1 is the unit.
- Fractional parts have special names that tell how many parts of that size are needed to make the whole. For example, *thirds* require three parts to make a [one] whole.
- The more fractional parts used to make a [one] whole, the smaller the parts. For example, eighths are smaller than fifths.

  The denominator of a fraction indicates by what number the whole has been divided in order to produce the type of part under consideration. Thus, the denominator is a divisor. In practical terms, the denominator names the kind of fractional part that is under consideration. The numerator of a fraction counts or tells how many of the fractional parts (or the type indicated by the denominator) are under consideration. Therefore, the numerator is a multiplier—it indicates a multiple of the given fractional part.” (Van de Walle and Lovin 2006a, p. 251).

$$\frac{\text{numerator}}{\text{denominator}} = \frac{\text{parts being considered}}{\text{total number of parts in one whole}}$$

For example, consider the fraction $\frac{3}{8}$. One whole is being divided into eight equal parts. Each part represents one-eight ($\frac{1}{8}$). 8 is the divisor. 3 is the multiplier because we want three one-eighths or three-eighths.

Presenting fractions in context will make them more meaningful to students. They could estimate fractions based on their class; for example, What fraction of the students in the class are left-handed, wear glasses, ride the bus to school, etc. Then the data could be gathered to give precise answers to the questions. It is important that students develop visual images for fractions and be able to tell “about how much” a particular fraction represents and learn common benchmarks, such as one-half.

Students should model fractions using a variety of materials. To strengthen their fraction number sense, it is also recommended that students explore fractions with models where the size of the whole is not the same. In Mathematics 4, the focus is on students developing a firm understanding of fractions less than one. Students may have a strong mindset about numbers that may cause them difficulties with the relative size of fractions. In their experience, larger numbers mean “more.” A common misconception is for students to transfer previously learned whole-number concepts to fractions, thinking seven is more than four, so sevenths should be larger than fourths. The inverse relationship between number of parts and size of parts is better understood by students when they explore and discover this on their own rather than being told.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

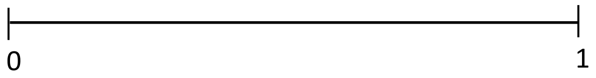
- Ask students, If you are really hungry and want a large piece of vegetarian pizza, would you cut the pizza into thirds, fourths, or tenths? Ask them to explain their thinking.
- Provide students with a square piece of paper and ask them to show fourths by folding the paper. Ask students to compare their fourths. Are they the same shape? Are they all really fourths?
- Provide students with different-sized square pieces of paper. Ask them to show fourths by folding the paper they have been given. Ask students to compare their fourths. Are they the same shape? Are they the same size? Why or why not? Are they all really fourths?

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample tasks (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to place the following fractions on the number line below and verify their positions using models.

  $\frac{1}{2}, \frac{1}{4}, \frac{1}{10}, \frac{3}{4}, \frac{1}{3}, \frac{7}{8}, \frac{4}{6}$

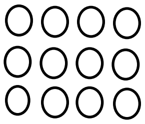


- Present the following problem to students: Kiri ate $\frac{1}{4}$ of her pizza and David ate $\frac{3}{4}$ of his pizza. Kiri said that she ate more pizza than David. Ask students to explain, using diagrams and words, how Kiri could be correct.

- Place the following pairs of fractions before the students, one pair at a time. Tell students to circle the larger fraction and explain in words how they know that the fraction is larger. Then, have students select a manipulative and model the fractions to verify their selection.

  $\frac{1}{5}$ $\frac{3}{5}$ $\frac{3}{8}$ $\frac{3}{5}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{4}{8}$ $\frac{3}{6}$ $\frac{3}{4}$ $\frac{9}{10}$

- Ask students to tell why, whenever there is a representation of $\frac{1}{3}$, there is always a $\frac{2}{3}$ associated with it.
- Ask students to colour $\frac{1}{4}$ of the set of circles.



FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 4 (Bauman 2011)
- Checkpoint 3, Tasks 1, 2, and 3, pp. 25–27

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

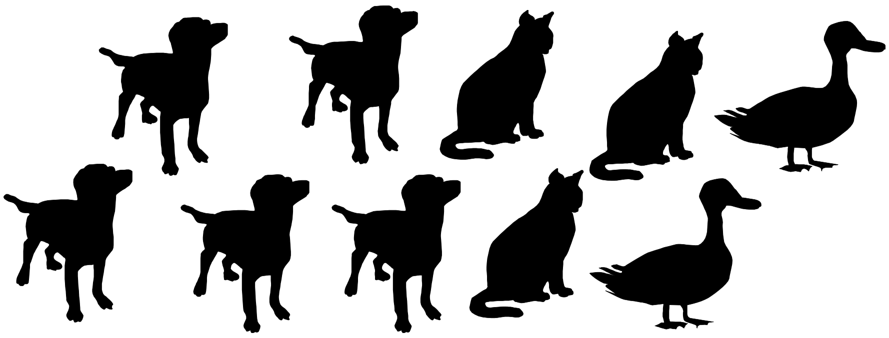
CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

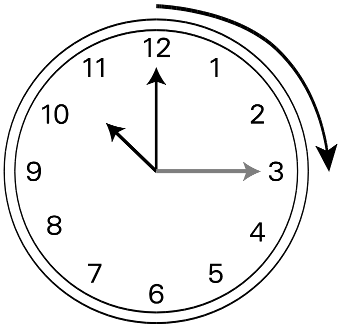
- Present three models for fractions: part of a region; part of a set; part of a length or measures.
- Ensure students develop an understanding that a fraction is not meaningful without knowing the "whole" that it is part of.
- Develop conceptual understanding of comparing fractions through a variety of ways to relate fractions, including the following:
  - more of the same size in which the denominators of the fractions are the same (e.g., five-eighths is greater than three-eighths)
  - same number of parts, but parts of different sizes in which the numerators of the fractions are the same (e.g., three-quarters is greater than three-fifths)
  - more or less than one-half or one whole in which the numerator of the fraction is compared to the denominator in deciding its relation to a given benchmark (e.g., three-eighths is less than one-half because three is less than half of eight (Van de Walle and Lovin 2006a, p. 265).
- Use a horizontal line when writing fractions, instead of a slash (e.g., $\frac{3}{4}$ not 3/4).

SUGGESTED LEARNING TASKS

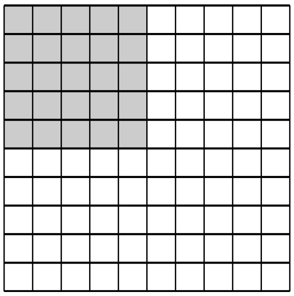
- Ask students to find what fraction of the letters in their names (or other words) are vowels.
- Have students explore fraction relationships among pattern blocks, fraction pieces, Cuisenaire rods, or other materials.
- Show examples and non-examples of specified fractional parts. Ask students to identify the wholes that are correctly divided into requested fractional parts and those that are not. For each response, have students explain their reasoning. This activity should be done with a variety of models.
- Tell students that you have eight coins. Half of them are dimes. More than one-eighth of them are quarters. The others are nickels. Invite students to use coins to represent the situation, and to explain how much money they might have? Ask students to create other coin problems using proper fraction notation.
- Provide students with different sizes and shapes of paper and have them estimate and then tear-off different fractional parts, such as one-fifth. Ask them to explain their thinking. Students can compare their "fifths" as the size of these will vary depending on the size of the whole.
- Ask students to name the fraction of the animals that are cats?



- Ask students to tell the fraction represented when the hand rotates 15 minutes around the clock.



- Ask students to tell what fraction is represented by the shaded area on a given hundred grid.



The grid represents one, then one-fourth of it is shaded.

- Ask students to order a set of fractions. Using sticky notes, place a fraction on the forehead of each student in a small group of students (about four to eight). Ask students to place themselves in order without talking.

SUGGESTED MODELS AND MANIPULATIVES

- colour tiles
- Cuisenaire rods
- egg cartons
- fraction circles
- fraction pieces
- geo-boards (10 × 10)
- grid paper
- hundred grids
- hundredths circles
- metre sticks
- money
- number lines
- paper (plain)
- pattern blocks
- ruler
- tangrams

MATHEMATICAL LANGUAGE

Teacher	Student
▪ benchmark ▪ compare, order ▪ fact families: halves, thirds, fourths, fifths, sixths, eighths, tenths, twelfths ▪ fractions ▪ numerator, denominator ▪ one-half, half, one-fourth ▪ part of a whole, equal parts, fair shares ▪ whole, one whole, one, region	▪ benchmark ▪ compare, order ▪ halves, thirds, fourths, fifths, sixths, eighths, tenths, twelfths ▪ fractions ▪ numerator, denominator ▪ one-half, one-fourth ▪ equal parts ▪ whole, one whole, one, region

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students K–8* (Small 2008), pp. 198–201, 203–206, 207–210, 222
- *Making Mathematics Meaningful to Canadian Students K–8*, Second Edition (Small 2013), pp. 257–260, 262–264, 276
- *Teaching Student-Centered Mathematics, Grades 3–5*, Volume Two (Van de Walle and Lovin 2006), pp. 133–135, 137, 148–149

Notes

SCO N09 Students will be expected to describe and represent decimals (tenths and hundredths) concretely, pictorially, and symbolically.
[C, CN, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N09.01 Write the decimal for a given concrete or pictorial representation of part of a set, part of a region, or part of a unit of measure.
N09.02 Represent a given decimal using concrete materials or a pictorial representation.
N09.03 Explain the meaning of each digit in a given decimal.
N09.04 Represent a given decimal using money values (dimes and pennies).
N09.05 Record a given money value using decimals.
N09.06 Provide examples of everyday contexts in which tenths and hundredths are used.
N09.07 Model, using manipulatives or pictures, that a given tenth can be expressed as a hundredth (e.g., 0.9 is equivalent to 0.90, or 9 dimes is equivalent to 90 pennies).
N09.08 Read decimal numbers correctly.

Scope and Sequence

Mathematics 3	Mathematics 4	Mathematics 5
—	**N09** Students will be expected to describe and represent decimals (tenths and hundredths) concretely, pictorially, and symbolically.	**N08** Students will be expected to describe and represent decimals (tenths, hundredths, thousandths), concretely, pictorially, and symbolically.

Background

In the beginning, students need experience representing decimal numbers with proportional concrete and pictorial models. These may include ten-frames, base-ten blocks, or grids. Students can begin work with decimal tenths using ten-frames with the whole ten-frame representing one and each block of the frame representing one-tenth. Students may then work with base-ten blocks and determine the value of other pieces if the rod represents one or if the flat represents one or if the large cube represents one. Later, they may model decimals using non-proportional models, such as money, and explain relationships, such as two dimes is two-tenths or twenty-hundredths of a dollar.

Number sense with decimals requires that students develop a conceptual understanding of decimals as numbers. To work effectively with decimals, students should demonstrate the ability to represent decimal numbers using words, models, pictures, and symbols and make connections among various representations.

Conceptual understanding of decimals requires that students connect decimals to whole numbers and to fractions. Decimals are shown as an extension of the whole number system by introducing a new place value, the tenth's place, to the right of the one's place. The tenth's place follows the pattern of the base-ten number system by iterating one-tenth ten times to make one whole or a unit (Wheatley and Abshire 2002, p. 152). Similarly, the hundredth's place to the right of the tenth's place iterates one-hundredth ten times to make one-tenth.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to use a hundred grid to shade a capital "T" that takes up more than 0.20 or more of the grid and one that takes less than 0.20 of the grid. Express the shaded and unshaded areas as fractions.
- Ask students where they would find decimal numbers in their daily lives.
- Have students use a model of choice to explain why 0.40 and 0.4 are equivalent.
- Provide the student with a number, such as 3.94 and ask students to
  - give the number that is 0.1 more than
  - 1 less than
  - 0.01 more than
- Explain to students that someone forgot to put the decimal in the number 1427. Ask where it could be if the number is less than 100.
- Ask students to read decimal numbers orally (e.g., 2.5, 26.9, $127.60, 44.09, 0.02).
- Have students select two different models with which to show a given decimal, such as 0.38 or 1.3.
- Ask students to read 1.53 in two different ways.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 4 (Bauman 2011)

- Checkpoint 3, Task 3, pp. 25–27

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Ensure students have an understanding of the meaning of decimal numbers. When discussing the meaning of numbers such as 1.1, attention should be given to the multiple meanings of the number (e.g., one whole and one tenth; 11 tenths, one and 10 hundredths).
- Help students extend the place-value system to decimals by focusing on the basic pattern of ten. Remind students that 10 ones make 1 ten, 10 tens make 1 hundred, etc. Then, extend this pattern to help students understand that it takes 10 equal parts (tenths) to make 1 whole and 100 equal parts (hundredths) to make 1 whole. Students should recognize that the place value of the digits to the right of the one’s place are tenths and hundredths.
- Investigate the relationship between 1.0, 0.1, and 0.01 by making analogies and using real-life objects that are sized proportionally.

SUGGESTED LEARNING TASKS

- “Recall how to make the calculator “count” by pressing + 1 = = ... Now have students press + 0.1 = = ... When the display shows 0.9, stop and discuss what this means and what the display will look like with the next press. Many students predict 0.10 (thinking that 10 comes after 9). This prediction is even more interesting if, with each press, the students have been accumulating base-ten strips as models for tenths. One more press would mean one more strip, or 10 strips. Why should the calculator not show 0.10? When the tenth press produces a display of 1 (calculators never display trailing zeros to the right of the decimal), the discussion should revolve around trading 10 strips for a square. Continue to count to 4 or 5 by tenths. How many presses to get from one whole number to the next? Try counting by 0.01." (Van de Walle and Lovin 2006c, p. 187).

- Ask students to show 2 tenths, if
  - a large base-ten cube represents one whole
  - a flat represents one whole
  - a rod represents one whole

  Extend this to explore hundredths.
- Ask students to record twenty-three cents as a decimal.
- Ask students to use a 10 × 10 geo-board to represent 0.26.
- Ask students to record 0.13 using money values.
- Ask students to model, concretely and/or pictorially, a decimal number of their choice. Ask them to provide examples of situations in which that number might be used.

SUGGESTED MODELS AND MANIPULATIVES

- base-ten blocks
- decimal squares
- decimal strips
- geo-board (10 × 10)
- hundred grid
- hundredths circles
- metre stick
- money
- number lines
- ten-frames

MATHEMATICAL LANGUAGE

Teacher	Student
▪ decimals ▪ measure ▪ money value ▪ one whole ▪ part of a set, part of a region, part of a unit of ▪ tenths, hundredths, equivalent	▪ decimals ▪ measure ▪ money value ▪ one whole ▪ part of a set, part of a region, part of a unit of ▪ tenths, hundredths, equivalent

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students K–8* (Small 2008), pp. 228–231, 234, 242–245
- *Making Mathematics Meaningful to Canadian Students K–8*, Second Edition (Small 2013), pp. 282–285, 288, 296–297
- *Teaching Student-Centered Mathematics, Grades 3–5*, Volume Two (Van de Walle and Lovin 2006), pp. 183, 184–185

Notes

SCO N10 Students will be expected to relate decimals to fractions and fractions to decimals (to hundredths).
[C, CN, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N10.01 Express, orally and symbolically, a given fraction with a denominator of 10 or 100 as a decimal.
N10.02 Read decimals as fractions (e.g., 0.5 is zero and five tenths).
N10.03 Express, orally and symbolically, a given decimal in fraction form.
N10.04 Express a given pictorial or concrete representation as a fraction or decimal (e.g., 15 shaded squares on a hundredth grid can be expressed as 0.15 or $\frac{15}{100}$).
N10.05 Express, orally and symbolically, the decimal equivalent for a given fraction (e.g., $\frac{50}{100}$ can be expressed as 0.50).

Scope and Sequence

Mathematics 3	Mathematics 4	Mathematics 5
—	**N10** Students will be expected to relate decimals to fractions and fractions to decimals (to hundredths).	**N09** Students will be expected to relate decimals to fractions and fractions to decimals (to thousandths). **N10** Students will be expected to compare and order decimals (to thousandths) by using benchmarks, place value, and equivalent decimals.

Background

Decimals are fractional parts, and therefore, it is essential that the relationship between decimals and fractions be regularly addressed. The connection between decimals and fractions is developed conceptually when the students read decimals as fractions and represents them using the same visuals. For example, 0.8 is read as eight-tenths and can be represented using fraction strips or decimal strips (Wheatley and Abshire 2002). Students should use a variety of materials to model and interpret decimal tenths and hundredths.

Foster an understanding of decimals by ensuring that students read them correctly. For example, 3.4 should be read as 3 and 4 tenths, not 3 point 4, or 3 decimal 4. It is also important that students understand the relationship between fractions and decimals. For example, 12.56 is read as 12 and 56 hundredths. Saying decimal numbers correctly will assist students in gaining an understanding of how decimals relate to fractions. By saying 12 and 56 hundredths, 56 is the numerator and 100 is the denominator. Saying the number correctly also reinforces the idea that the digits to the right of the decimal are part of the whole number.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to write the numbers that you say to them (e.g., one and twenty-three hundredths, three and two-tenths, eighty-seven and six-hundredths, fourteen hundredths, five dollars and forty cents, eleven-tenths).
- Invite students to plot common fraction and decimal equivalents on a number line (e.g., $\frac{5}{10}$ and 0.5; $\frac{25}{100}$ and 0.25; $\frac{8}{10}$ and 0.8; $\frac{1}{10}$ and 0.1).
- Ask students to count forward and backward from any number (e.g., count on in tenths from 4.7 or count backwards in hundredths from 4.05).

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 4 (Bauman 2011)
- Checkpoint 3, Tasks 1, 2, and 3, pp. 25–27

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Use a double number line to show equivalence between decimals and fractions.
- Encourage the correct use of language when reading decimal numbers (e.g., five-tenths instead of point five or decimal five).
- Use a variety of materials to model numbers with decimals to the hundredths. Ensure that some models show equivalent fractions and decimals.
- Provide students with ample opportunities to write the decimal and the fraction represented by each model they create.

SUGGESTED LEARNING TASKS

- Invite students to shade in two-tenths of a hundred grid. Ask them to use the model to explain why this same model represents the equivalent fraction and decimal of 20 hundredths.
- Ask students to orally read the following decimals and then write the fraction

  0.45 0.5 0.42 0.2 0.62

- Ask students to show 0.5 on a hundredths circle and then name the equivalent fractions.
- Show students pictorial models of fractions. Ask them to record the fraction in decimal form. Ask them to explain why the fraction and decimal can be represented by the same model.

SUGGESTED MODELS AND MANIPULATIVES

- base-ten blocks
- geo-board (10 × 10)
- hundred grid
- hundredths circles
- metre stick
- money
- number lines (including double number lines

MATHEMATICAL LANGUAGE

Teacher	Student
▪ hundredth grid ▪ equivalent ▪ relate decimals to fractions and fractions to decimals	▪ hundredth grid ▪ equivalent ▪ relate decimals to fractions and fractions to decimals

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students K–8* (Small 2008), pp. 206, 228, 232–233
- *Making Mathematics Meaningful to Canadian Students K–8*, Second Edition (Small 2013), pp. 260, 282, 286–287
- Teaching Student-Centered Mathematics, Grades 3–5, Volume Two (Van de Walle and Lovin 2006), pp. 181–192

Notes

SCO N11 Students will be expected to demonstrate an understanding of addition and subtraction of decimals (limited to hundredths) by

- estimating sums and differences
- using mental mathematics strategies to solve problems
- using personal strategies to determine sums and differences

[C, ME, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N11.01 Predict sums and differences of decimals, using estimation strategies.
N11.02 Solve problems, including money problems, which involve addition and subtraction of decimals (limited to hundredths), using personal strategies.
N11.03 Ask students to determine which problems do not require an exact solution.
N11.04 Determine the approximate solution of a given problem not requiring an exact answer.
N11.05 Count back change for a given purchase.
N11.06 Determine an exact solution using mental computation strategies.

Scope and Sequence

Mathematics 3	Mathematics 4	Mathematics 5
N06 Students will be expected to describe and apply mental mathematics strategies for adding two two-digit numerals. **N07** Students will be expected to describe and apply mental mathematics strategies for subtracting two two-digit numerals.	**N11** Students will be expected to demonstrate an understanding of addition and subtraction of decimals (limited to hundredths) by ▪ estimating sums and differences ▪ using mental mathematics strategies to solve problems ▪ using personal strategies to determine sums and differences	**N11** Students will be expected to demonstrate an understanding of addition and subtraction of decimals (limited to thousandths).

Background

Students should begin to develop strategies for adding and subtracting decimals using concrete materials such as base-ten blocks and number lines. At this grade level, students are just developing an understanding of decimals and should not be expected to complete complex addition and subtraction problems symbolically in isolation. Instead, they should be exposed to opportunities to add and subtract simple decimals in meaningful contexts using concrete materials. They should record pictures of their work with the concrete materials. They should develop and use personal strategies for solving addition and subtraction problems, such as adding by counting on or subtracting by counting back to develop an understanding for regrouping with decimals. It is essential that students understand that all of the properties and strategies established for the addition and subtraction of whole numbers also apply to decimal numbers.

Students have had experience with a variety of story problem structures working with whole numbers in previous grades. When they learn how to operate on decimals, they should also be exposed to the same variety of story problem structures so that they get a full picture of the various contexts in which decimals are used. Please refer to SCO N03 for the addition and subtraction story structures.

Students need to recognize that estimation is a useful skill in their lives. To be efficient when estimating sums and differences mentally, students must be able to access a strategy quickly, and they need a variety from which to choose. Situations must be provided regularly to ensure that students have sufficient practice with mental mathematics strategies and that they use their skills as required. When a problem requires an exact answer, students should first determine if they are able to calculate it mentally; this should be determined every time a calculation is required.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students’ prior knowledge.

- Ask students to explain how knowing 8 + 8 = 16, helps someone solve 58 + 8?
- Ask students to describe a strategy for solving 76 – 11 mentally using models, numbers, words, or pictures.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to count back (or count on) the change from $5, if the bill totalled $3.59.
- Invite students to make up a problem with multi-digit numbers for which the calculation could be done mentally. Ask them solve it and explain their thinking.
- Ask students to explain how they would know that 3.65 + 5.35 < 10 without actually completing the addition. (Observe if the student applied the compatible number strategy).

- Show students the equation, \$4.98 + \$3.98 + \$9.99. Ask them to calculate the sum mentally and explain the strategy they used.
- Ask students to find the difference for 2.3 – 1.8 or other similar computations and explain how they got their answer.
- Tell students that to solve 9.7 – 8.6, Yoshi thought 86 + 11 is 97. Ask students to explain how this might help Yoshi solve 9.7 – 8.6.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 4 (Bauman 2011)
- Checkpoint 4, Task 1, pp. 28–30
- Checkpoint 6, Tasks 1 and 2, pp. 34–37
- Checkpoint 7, Task 1, pp. 38–40

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Encourage students to estimate prior to calculating answers.
- Use a variety of appropriate models, such as base-ten blocks and number lines, to assist students in their initial consideration of estimation.
- Encourage students to use a variety of strategies to estimate prior to calculating sums and differences.
- Encourage students to use mental computation strategies and explain their thinking.

- Ask students to determine how best to calculate various problems without a calculator. If they decide to use mental mathematics strategies, ask them to compute and share their strategies.
- Ask students which questions from a group of computations could be solved mentally. Ask them to explain their thinking and to identify the strategy they used.

SUGGESTED LEARNING TASKS

- Give students word problems that require the addition and/or subtraction of whole numbers and decimals. Particularly appropriate contexts are money and measurement (e.g., 3.45 m + 1.63 m; 12.4 kg – 7.25 kg).
- Ask students to generate addition or subtraction number sentences using only decimal numbers that would result in an answer that is close to 50. Ask them to share their work.
- Have students use a calculator, the digits 7, 5, 1, and 2, and the symbols "+," "=," and "." (decimal point) to produce 7.8 on the display.
- Invite students to model a subtraction problem as subtraction or a missing addend using a variety of models and pictorial representations. Ask students to record their procedure using symbols.
- Have students subtract $0.56 from $6.
- Ask students to add $3.25 + $2.75.
- Tell students that Maxime saved the following amounts over a four-month period:

June	$12.45
July	$6.62
August	$19.95
September	$12.53

  Ask them to give an estimate of the total amount saved. Have them explain how their estimates were determined.
- Ask students to model the solution for a given question, such as
  - 5.43 – 2.33
  - 6 – 4.53
  - 1.43 – 0.09
  - 2.64 – 0.99
  - 3.32 + 2.14
- Ask students to work in pairs. One student will be the shopper; the other will be the store clerk. Provide each pair of students with a newspaper flyer, or a sales flyer you have prepared, and play money. The play money given to the clerk should include both coins and bills. The shopper should be given bills only. The shopper will select an item to purchase from the flyer and will pay the clerk for the item. The store clerk must count back the change from the purchase. Students should switch roles in order to ensure that both students have the opportunity to count back change.

SUGGESTED MODELS AND MANIPULATIVES

- base-ten blocks
- calculators
- hundredths grids
- number lines
- place-value charts
- ten-frames

MATHEMATICAL LANGUAGE

Teacher	Student
▪ change ▪ compatible numbers, front-end addition, front-end subtraction, rounding, compensate, counting on / counting back ▪ estimating sums and differences ▪ exact and approximate ▪ tenths, hundredths ▪ money problems	▪ change ▪ compatible numbers, front-end addition, front-end subtraction, rounding, compensate, counting on / counting back ▪ estimating sums and differences ▪ exact and approximate ▪ tenths, hundredths ▪ money problems

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students K–8* (Small 2008), pp. 234, 236–238
- *Making Mathematics Meaningful to Canadian Students K–8*, Second Edition (Small 2013), pp. 288, 290–292
- *Teaching Student-Centered Mathematics, Grades 3–5*, Volume Two (Van de Walle and Lovin 2006), pp. 197–198

Notes

Patterns and Relations (PR)

GCO: Students will be expected to use patterns to describe the world and solve problems.

GCO: Students will be expected to represent algebraic expressions in multiple ways.

SCO PR01 Students will be expected to identify and describe patterns found in tables and charts, including a multiplication chart. [C, CN, PS, V]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

PR01.01 Identify and describe a variety of patterns in a multiplication chart.
PR01.02 Determine the missing element(s) in a given table or chart.
PR01.03 Identify the error(s) in a given table or chart.
PR01.04 Describe the pattern found in a given table or chart.

Scope and Sequence

Mathematics 3	Mathematics 4	Mathematics 5
PR01 Students will be expected to demonstrate an understanding of increasing patterns by describing, extending, comparing, and creating numerical patterns (numbers to 1000) and non-numerical patterns using manipulatives, diagrams, sounds, and actions. **PR02** Students will be expected to demonstrate an understanding of decreasing patterns by describing, extending, comparing, and creating numerical patterns (numbers to 1000) and non-numerical patterns using manipulatives, diagrams, sounds, and actions.	**PR01** Students will be expected to identify and describe patterns found in tables and charts, including a multiplication chart.	**PR01** Students will be expected to determine the pattern rule to make predictions about subsequent terms (elements).

Background

Mathematics is often referred to as the science of patterns, since patterns are found in every mathematical concept and in everyday contexts. “Patterns are found in physical and geometric situations as well as in numbers. The same pattern can be found in many different forms” (Van de Walle and Lovin 2006b, p. 290).

Students will continue to work with, and expand on, the many patterns found in different tables and charts. The hundreds charts and addition tables (up to 9 + 9) should be familiar to students, as they have worked extensively with them in Mathematics 2 and Mathematics 3. Students should be encouraged to identify and explain patterns that can be found in these familiar tables and charts. Students in Mathematics 3 began representing basic multiplications facts (up to 5 × 5) concretely, contextually, and pictorially, but have not worked extensively with a multiplication table or chart (9 × 9). Therefore, students should be encouraged to identify and explain the patterns in the multiplication table or chart. The patterns found in the addition and multiplication tables can then be used to help students determine an unknown sum, difference, product, or quotient. Students should also be encouraged to find and explain place-value patterns.

Students should use a variety of vocabulary, including vertical, horizontal, diagonal, row, column, starting point, increasing, decreasing, and repeating, to help describe the patterns that they find in charts and tables.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Have students identify the pattern rule of the following increasing patterns and extend the pattern to include 3 more terms.

  4, 7, 10, 13, 16, …

  13, 18, 23, 28, 33, …
- Ask students to identify the errors in the following decreasing patterns and correct them.

  138, 128, 118, 108, 88, 78

  30, 28, 24, 21, 19, 15, 12, 9, 6, 3

  40, 35, 29, 25, 20, 15, 10, 5

  576, 566, 556, 546, 536, 516, 506, 486

- Provide students with a chart such as the one below. Ask students to identify where the pattern has errors. Ask students to explain their thinking in writing.

1	4
2	8
3	12
4	18
5	20
6	22
7	28
8	32

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to explain why some columns/rows on a multiplication grid have both even and odd numbers.
- Provide a chart or grid with missing numbers and ask students to fill in the missing numbers.
- Provide students with a multiplication grid. Ask them to describe some of the patterns they observe.
- Create a chart/grid/table that has not been used in the class as a model and ask students to identify and explain the patterns that can be found on the chart/grid/table.
- Create a chart/grid containing errors and ask students to identify the errors and correct them.
- Invite students to identify and describe at least two different patterns found on the hundred chart.
- Invite students to identify and describe at least two different patterns found on an addition table or chart.
- Invite students to identify and describe at least two different patterns found on a multiplication table or chart.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 4 (Bauman 2011)

- Checkpoint 5, Tasks 1 and 2, pp. 31–33
- Checkpoint 8, Task 1, pp. 41–44

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Encourage students to identify and describe more than one pattern on each table or chart.
- Encourage students to use words and numbers to describe the patterns they discover.
- Ask students to identify similarities and differences in patterns. How are the patterns of counting by twos and counting by fours on a hundred chart the same? How are they different?

SUGGESTED LEARNING TASKS

- Provide students with a hundred chart, an addition chart, or a multiplication chart. Ask students to describe some of the patterns they observe in the chart provided.
- Ask students to find the even and the odd numbers on hundred charts, addition charts, and multiplication charts and ask them to describe the patterns they find.
- Invite students to extend several hundreds charts so they can see the numbers from 1 to 100, 101 to 200, up to 999. On these charts, use coloured counters to cover numbers forming a pattern and explore the place-value representation of the covered numbers (e.g., displaying the pattern 13, 23, 33, 43, ..., depicted as a vertical column of counters). This represents an increase of 10 in the number each time.
- Ask students to explore patterns in different versions of hundred charts by changing the order of the numbers. For example, skip counting by twos looks different when represented on a hundred chart that starts with 0, than it does on a hundred chart that starts with 1. Provide students with a blank hundred chart so they can create their own versions.
- Ask students to show how one could use the multiplication chart to practise skip counting.
- Provide students with a hundred chart. Ask students to find all the multiples of 2 and colour them in. Ask students to describe the pattern. Repeat for the multiples of 3, 4, 5, 6, 7, 8, and 9.
- Ask students to describe in writing the patterns they can find on a given chart.

SUGGESTED MODELS AND MANIPULATIVES

- addition and multiplication charts (tables)
- blank grids
- hundreds chart

MATHEMATICAL LANGUAGE

Teacher	Student
▪ errors ▪ missing elements ▪ multiplication charts ▪ patterns ▪ tables and charts	▪ errors ▪ missing elements ▪ multiplication charts ▪ patterns ▪ tables and charts

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students K–8* (Small 2008), pp. 573–577
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 612–615

Videos

- *Analyzing Patterns (Skip Counting) on a Hundred Board* (27:16 min.) (ORIGO Education 2010)

Notes

SCO PR02 Students will be expected to translate among different representations of a pattern (a table, a chart, or concrete materials). [C, CN, V]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

PR02.01 Create a table or chart from a given concrete representation of a pattern.
PR02.02 Create a concrete representation of a given pattern displayed in a table or chart.
PR02.03 Translate between pictorial, contextual, and concrete representations of a pattern.
PR02.04 Explain why the same relationship exists between the pattern in a table and its concrete representation.

Scope and Sequence

Mathematics 3	Mathematics 4	Mathematics 5
PR01 Students will be expected to demonstrate an understanding of increasing patterns by describing, extending, comparing, and creating numerical patterns (numbers to 1000) and non-numerical patterns using manipulatives, diagrams, sounds, and actions. **PR02** Students will be expected to demonstrate an understanding of decreasing patterns by describing, extending, comparing, and creating numerical patterns (numbers to 1000) and non-numerical patterns using manipulatives, diagrams, sounds, and actions.	**PR02** Students will be expected to translate among different representations of a pattern (a table, a chart, or concrete materials).	**PR01** Students will be expected to determine the pattern rule to make predictions about subsequent terms (elements).

Background

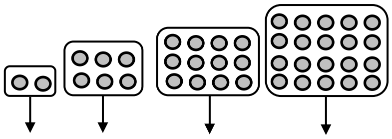
Students have had extensive experience with increasing, decreasing, and repeating patterns and have represented these patterns concretely, pictorially, orally, and symbolically in previous grades. In Mathematics 4, students will be expected to apply this knowledge to translate flexibly among different representations of increasing and decreasing patterns.

Students should begin by representing a pattern with concrete materials and/or pictures. Then, they should represent the same pattern in a table or chart. Once a table or chart is developed, students have two representations of a pattern: the one created with the drawing or materials and the numeric version that is in the table or chart. They can then explain how these patterns are mathematically alike, that is, why the same relationship exists between the pattern in a table and its concrete representation.

Students should also be given opportunities to reproduce a pattern using concrete materials when presented with a pattern displayed in a table or chart. Students should also be asked to describe what is happening as the pattern increases (or decreases) and how the next step is related to the previous one. It is helpful for students to think of a pattern rule and apply it when analyzing tables or charts for errors.

“Growing [increasing] patterns also have a numeric component; the number of objects in each step.” (Van de Walle and Lovin 2006b, p. 294) A table or “T-chart” can be constructed to explore this. Once a table is used for the growing pattern, the materials may become unnecessary. This leads to the next step in pattern exploration that would be to predict what will happen at a particular step.

Input	Output
1	2
2	6
3	12
4	20
5	30
6	?
7	?



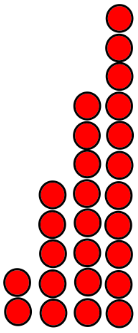
Step	1	2	3	4	5	6	?	…	10
No. of dots	2	6	12	20	30	?	?	…	?

(Source: Van de Walle and Lovin 2006b, p. 295)

“When looking for relationships, some students will focus on the table and others will focus on the physical pattern. It is important for students to see that whatever relationships they discover, they exist in both forms. So if a relationship is found in a table, challenge students to see how that plays out in a physical version.” (Van de Walle and Lovin 2006b, p. 295).

When helping students recognize patterns, it is important to remember that all students may not see the pattern in the same way. Therefore, it is important to ask students to explain their thinking. Expecting students to describe their reasoning can also help them learn that there is often more than one way to look at a pattern.

The elements that make up increasing and decreasing patterns are called **terms**. Each term builds on the previous term. Using a table to model an increasing or decreasing pattern can help students organize their thinking. It can also help them generalize the patterns symbolically (create a rule). A pattern rule must describe how each and every element of the pattern is generated, including the first term. For example, 2, 5, 8, 11, ... can be described as start at two and add three each time.



Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to identify the errors in the following increasing patterns and correct them
  3, 6, 9, 12, 15, 19, 21, 24, 28, 30, …
  40, 45, 50, 60, 65, 75, …
- Have students show different ways these decreasing patterns could be extended.
  80, 40, …
  925, 825, ….
  1000, 500, …

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to create an increasing or decreasing pattern using concrete materials. Ask them to represent the same pattern in a table or chart and explain why the same relationship exists between the pattern in the table or chart and in the concrete representation.
- Provide a table or chart and have students use a model to create a concrete representation of the given pattern displayed in the table or chart. Ask them to explain their thinking.
- Provide several examples of tables and their concrete representations. Ask students to match each table to its concrete representation and to explain their thinking.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 4 (Bauman 2011)

- Checkpoint 8, Task 1, pp. 41–44

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Ask students to translate patterns from one medium to another. For example, red and blue pattern blocks become letters or triangles, and squares translate to coloured tiles. Ask students to explain how these patterns are mathematically alike.

SUGGESTED LEARNING TASKS

- Ask students to start at 3 and skip count by 10s using base-ten blocks and have them represent that skip-counting pattern on a hundreds chart. Ask students to record the pattern in a table and explain why the same relationship exists between the pattern in a table, on the chart, and its concrete form.
- Present students with an increasing pattern constructed with colour tiles. Ask student to make a table of values for the pattern and explain the connection of how the two representations are the same.
- Ask students to create an increasing or decreasing pattern and to represent it concretely and in a table. Ask them to explain how the two representations of the pattern are mathematically alike.

- Present students with a pattern and ask them to extend the pattern for three more terms. Ask them to develop a T-chart that represents the same pattern. Ask students to identify the tenth term in the pattern. For example, present students with the following pattern:

1	2	3	4
□	□□	□□□	□□□□

Students would extend the pattern and then create a T-chart.

Design #	Number of Squares in the Design
1	1
2	2
3	3
4	4

- Provide a table involving one arithmetic operation in the pattern, such as the one below. Ask students to describe what the data could be about, complete the table, and create a concrete representation of the pattern using linking cubes.

Display number	1	2	3	4	5	6	7	8	9	10
Number of cubes	3	6	9	12	?	?	?	24	?	?

- Present students with a problem that involves a pattern. Ask them to represent the pattern in at least two different ways and to explain their reasoning.

SUGGESTED MODELS AND MANIPULATIVES

- base-ten blocks
- charts
- colour tiles
- counters
- grids
- linking cubes
- number lines
- tables
- toothpicks

MATHEMATICAL LANGUAGE

Teacher	Student
▪ decreasing patterns ▪ increasing patterns ▪ tables and charts ▪ term, element ▪ translate	▪ decreasing patterns ▪ increasing patterns ▪ tables and charts ▪ term, element ▪ translate

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students K–8* (Small 2008), p. 573
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), p. 611
- *Teaching Student-Centered Mathematics, Grades 3–5*, Volume Two (Van de Walle and Lovin 2006), p. 295

Videos

- *Analyzing Patterns (Skip Counting) on a Hundred Board* (27:16 min.) (ORIGO Education 2010)

Notes

SCO PR03 Students will be expected to represent, describe, and extend patterns and relationships, using charts and tables, to solve problems.
[C, CN, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

PR03.01 Translate the information in a given problem into a table or chart.
PR03.02 Identify, describe, and extend the patterns in a table or chart to solve a given problem.

Scope and Sequence

Mathematics 3	Mathematics 4	Mathematics 5
PR01 Students will be expected to demonstrate an understanding of increasing patterns by describing, extending, comparing, and creating numerical patterns (numbers to 1000) and non-numerical patterns using manipulatives, diagrams, sounds, and actions. **PR02** Students will be expected to demonstrate an understanding of decreasing patterns by describing, extending, comparing, and creating numerical patterns (numbers to 1000) and non-numerical patterns using manipulatives, diagrams, sounds, and actions.	**PR03** Students will be expected to represent, describe, and extend patterns and relationships, using charts and tables, to solve problems.	**PR01** Students will be expected to determine the pattern rule to make predictions about subsequent terms (elements).

Background

Students should be able to translate flexibly from one representation of a pattern to another. A table or T-chart can be constructed to represent a pattern. Once a table is used for the increasing patterns, the materials may become unnecessary. Students should then learn that they can extend a pattern without building a model each time. This also leads to the next step which would be to predict what will happen at a particular step. (Van de Walle and Lovin, 2006)

When beginning their study of the concept of relations, students need experiences representing patterns with concrete materials, charts, and diagrams used with contexts that are engaging and meaningful to them. Furthermore, they need ample opportunity to connect patterns to numbers.

Once a table or chart is developed, students have two representations of a pattern, the one created concretely or pictorially, and the numeric version that is in a table or chart. When looking for

relationships, some students will focus on the table while others will focus on the physical patterns. It is important for students to see that the relationships discovered exist in a variety of forms.
As students' ability to recognize and create patterns become more refined, they are better prepared to use this knowledge to solve problems. The ability to solve problems is further developed as they systematically investigate a variety of patterns. Students move from a basic recognition of patterns to a more sophisticated use of patterns as a problem-solving strategy.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Have students identify the pattern rule of the following decreasing patterns and extend the pattern three more terms.

  25, 22, 19, 16, ...

  24, 20, 16, 14, 10, 6, ...

  83, 78, 73, 68, 63, ...

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

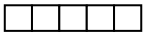
Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to fill in the missing parts of a table or chart. Drawings or materials may be used to help discover the missing parts.

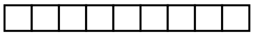
- Ask students to solve the following problem:

  Roberto was making trains using linking cubes.

First Train



Second Train



Third Train

If he continues to build trains this way, how many blocks will he use in the seventh train?

Ask students to look for a pattern and create a table to display the information and solve the problem.

Train	Number of cubes
1	1
2	5
3	9
4	
5	
6	
7	

- Present students with the following problem:

  Luigi was trying out for the swimming team. He had to be able to swim 30 laps in one day by the end of the second week. He was not able to swim on the weekends. On the first day, Luigi swam one lap. On the second day, he swam five laps. On the third day, he swam nine laps. If he continues with this pattern, will he be able to swim enough laps by the end of the second week to make the team?

  Ask students to use a table to represent the pattern, and then extend the pattern to solve the problem.

- Present students with the following problem:

  Emma agrees to walk dogs for three weeks while her friend, Kalila, is away on vacation.

  Kalila asks Emma to choose how she would like to be paid. She can either be paid according to Payment Plan 1 or Payment Plan 2 below.

Payment Plan 1

Day	Payment in Dollars
1	\$2
2	\$4
3	\$6
4	\$8
…	…
21	?

Payment Plan 2

Day	1	2	3	4	…	21
Payment in Cents	2¢	4¢	8¢	16¢	…	?

Ask students to explain which payment option Emma should choose and explain the reasoning for their choice.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 4 (Bauman 2011)

- Checkpoint 8, Task 1, pp. 41–44

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

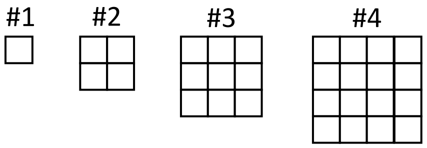
Consider the following strategies when planning daily lessons.

- Engage students in constructing increasing patterns with different materials (toothpicks, linking cubes, etc.). They may draw increasing patterns on grid paper as well. Ask students to describe what is happening as the pattern continues and to explain how each new step related to the previous one.

SUGGESTED LEARNING TASKS

- Give students a diagram showing a square table with 4 chairs (one on each side). Tell students that if 2 tables were put together, the tables would seat 6 people. Ask students to tell how many people could be seated with 6 tables, with 8 tables, or with 10 tables. Ask students what would happen if they started with a table of 6 and then explain their reasoning.

- Present students with a design made of squares and ask them to describe the pattern rule, extend the pattern, and develop a T-chart to represent the same pattern. Ask students to identify a specific term in the pattern. For example, How many squares would be in the 5th step? The 7th, the 9th? For example,



Design #	Number of squares
1	1
2	4
3	9
4	16

- Provide a table or T-chart involving one arithmetic operation in the pattern, such as the one below. Describe what the data could be about and complete the table.

Display number	1	2	3	4	5	6	7	8	9
Number of cubes	5	10	15	20	?	?	?	?	?

- Present students with the following problem:

  Mrs. Settle's grade 2 class was studying addition. One of her students began an investigation about the sum of two numbers. He wrote

  1 + 0 = 1 2 + 0 = 2 3 + 0 = 3
  0 + 1 = 1 0 + 2 = 2 0 + 3 = 3
  1 + 1 = 2 1 + 2 = 3
  2 + 1 = 3

  A grade 4 student who saw the grade 2 student's work recorded the same pattern in a table like this.

Sum	1	2	3	4	5	6	7	…	75
Number of equations with two addends for the identified sum	2	3	4	?	?	?	?	…	?

  Ask students to continue the pattern and extend the table four more terms. Ask them to predict the number of ways to find the sum of 75 using two numbers and explain their thinking.

SUGGESTED MODELS AND MANIPULATIVES

- charts
- colour tiles
- grids
- linking cubes
- number lines
- tables
- toothpicks

MATHEMATICAL LANGUAGE

Teacher	Student
▪ extend ▪ charts and tables ▪ translate	▪ extend ▪ charts and tables ▪ translate

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students K–8* (Small 2008), pp. 45, 571–573
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 104, 609–611
- *Teaching Student-Centered Mathematics, Grades 3–5*, Volume Two (Van de Walle and Lovin 2006), pp. 293–295

Videos

- *Analyzing Patterns (Skip Counting) on a Hundred Board* (27:16 min.) (ORIGO Education 2010)

Notes

SCO PR04 Students will be expected to identify and explain mathematical relationships, using charts and diagrams, to solve problems.
[CN, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

PR04.01 Complete a given Carroll diagram to solve a problem.
PR04.02 Determine where new elements belong in a given Carroll diagram.
PR04.03 Solve a given problem using a Carroll diagram.
PR04.04 Identify a sorting rule for a given Venn diagram.
PR04.05 Describe the relationship shown in a given Venn diagram when the circles overlap, when one circle is contained in the other, and when the circles are separate.
PR04.06 Determine where new elements belong in a given Venn diagram.
PR04.07 Solve a given problem by using a chart or diagram to identify mathematical relationships.

Scope and Sequence

Mathematics 3	Mathematics 4	Mathematics 5
PR01 Students will be expected to demonstrate an understanding of increasing patterns by describing, extending, comparing, and creating numerical patterns (numbers to 1000) and non-numerical patterns using manipulatives, diagrams, sounds, and actions. **PR02** Students will be expected to demonstrate an understanding of decreasing patterns by describing, extending, comparing, and creating numerical patterns (numbers to 1000) and non-numerical patterns using manipulatives, diagrams, sounds, and actions.	**PR04** Students will be expected to identify and explain mathematical relationships, using charts and diagrams, to solve problems.	**PR01** Students will be expected to determine the pattern rule to make predictions about subsequent terms (elements).

Background

In everyday life things are sorted by comparison relationships, for example by colour and size. Such comparison relationships also apply to number, as numbers also have attributes that make them similar or different from other numbers. Students need to explore this particular concept of numbers by being involved in experiences where they are expected to recognize, describe, and identify relationships and number characteristics.

Sorting and classifying objects and numbers will help students with organizing and categorizing data. Sorting is the action of grouping (or organizing) objects (or data). Classification (or categorization) is the naming of the groups of objects (or data). “Before sorting and classifying objects, it is important that students understand that any object has many *attributes*.” (Small 2013, p. 560). “An attribute is a way to compare objects, (e.g., by colour) while a characteristic describes how the attribute is reflected in a particular object (e.g., red, blue, green).” (Small 2013, p. 561)

Sorting Rule-Attribute-Comparison Relationship	**Classification/Characteristics**
Colour	Red, yellow, green, …
Type of polygon	Triangle, rectangle, square, …
Even numbers	22, 2014, …
Prime numbers	2, 3, 5, …

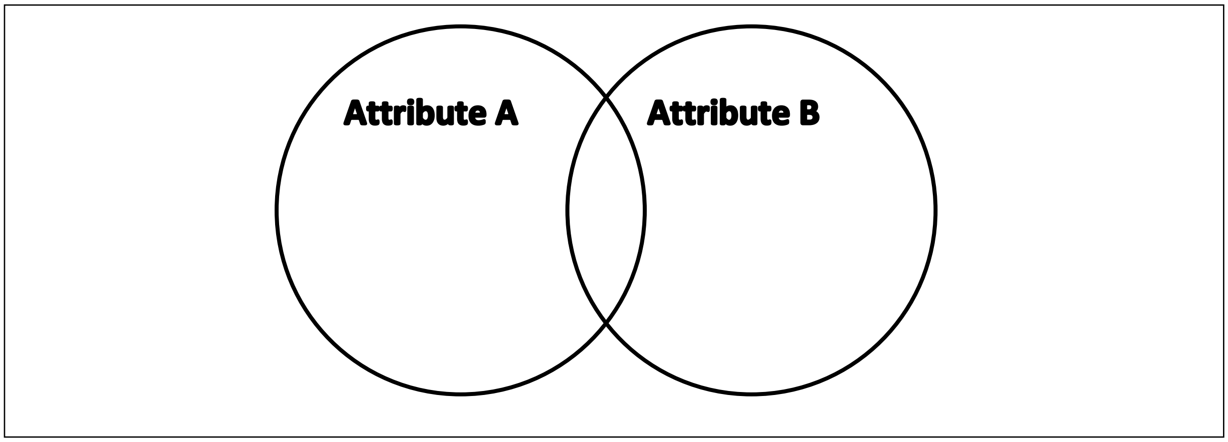
By grade 4, students are expected to use more sophisticated sorting tools such as a Carroll or Venn diagram. These organizational tools are particularly useful as a form of data display when the categories for the sorting situation overlap. These tools should be used within meaningful contexts throughout the year.

The following graphics may help in making the link between Carroll and Venn diagrams.

Carroll Diagram

	Attribute B	Not Attribute B
Attribute A		
Not Attribute A		

Venn Diagram



It is important to draw a rectangle around Venn diagrams to represent the entire group that is being sorted. This will show the items that do not fit the attributes of the circle(s) outside of them, but within the rectangle. Therefore, elements of the set that do belong to Attribute A or Attribute B are shown within the rectangle but not within the circles in the rectangles.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Give students an increasing pattern modelled with tiles and ask them to describe, recreate, and extend the pattern.
- Give students a decreasing pattern modelled with tiles and ask them to describe, recreate, and extend the pattern.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Prepare various presorted 2-D shapes or 3-D objects in Venn diagrams. Hold up different additional objects and ask students where each object should go in the Venn diagram.
- Provide an unlabelled Venn diagram, containing presorted sets of numbers, and ask students to determine the sorting rule and add one more number to each subset.
- Give students various numeral cards containing numbers up to four-digits and ask students to create a labelled Venn or Carroll diagram. Ask them to explain their thinking.
- Provide a completed Carroll diagram and present students with additional numbers that might have one attribute, both attributes, or neither attribute. Ask them to explain where each number should be placed in the diagram and explain their thinking.
- Ask students to compare a completed Venn diagram to a related Carroll diagram and determine if the two displays show the same information. Ask them to explain their thinking.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 4 (Bauman 2011)
- Checkpoint 8, Task 1, pp. 41–44

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

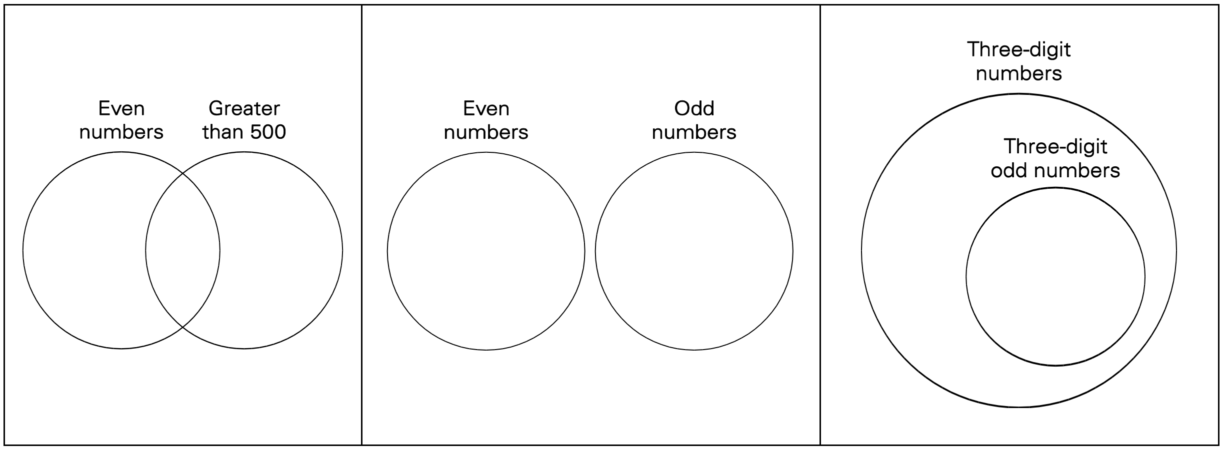
- Reinforce the proper mathematics vocabulary during sorting activities. The word “and” indicates that each item in the group would have all attributes of both categories where “or” makes the distinction between the two categories under consideration.
- Ensure that students include all of the data being considered from their sorting situation in their Venn or Carroll diagram. The rectangle drawn around the Venn diagram is used to show that all of the data, including items that did not match the sorting criteria, has been considered.
- Post a list of possible attributes of numbers, and encourage students to refer to the list as they examine Venn or Carroll diagrams involving numbers.

SUGGESTED LEARNING TASKS

- Give students various 3-D objects or 2-D shapes. One student selects six objects or shapes, chooses two attributes, and then sorts them. The other(s) then attempt to guess the sorting rule.
- Ask students to create a set of ten three-digit or four-digit numbers and sort them using two attributes. Request that they write the sorting rule.

- Provide students with data to organize using both a Venn and a Carroll diagram. Ask them to reflect on which is their preferred sorting organizational tool and to justify their choice.
- Ask students to sort a set of numbers in different ways and explain their sorting rule(s).
- Invite students to work in groups of two. Give each group a set of cards displaying various numerals. One student selects six cards, chooses two mystery attributes, sorts the cards according to the attributes, and places the cards in a Venn diagram. The other student then identifies the sorting rule and labels the Venn diagram.
- Provide students with the set of data below and ask them to complete the problem.

  Jennifer listed the numbers for her raffle tickets in the Spring Fair: 723, 694, 496, 501, 360, 999, and 222. Sort these numbers using the three types of Venn Diagrams shown below.



- Invite students to work with a partner. Provide each pair of students with a set of attribute blocks, labelled cards, and hoops or string to create a Venn diagram. Ask one student to be player A; the other student will be player B. Player A will set up a Venn diagram with one or two circles and will secretly select two cards with sorting rules written on them. The cards are then placed face down in the circles. Player B selects one attribute block and asks player A in which set the block belongs. This continues until all attribute blocks have been placed in the Venn diagram or until Player B identifies the rule. If Player B correctly identifies the rule before all the attribute blocks are placed, they score a point and the two students reverse roles. If Player B is unable to identify the rule before all attribute blocks are placed, Player A scores a point and another round is played but students do not reverse roles.
- Invite students to use Venn diagrams to solve problems, such as the following:

  In a class of 22 students, 10 play hockey and 15 play basketball.

  - Is it possible that there are some students who play neither sport? Explain your thinking using a Venn diagram.
  - What is the greatest possible number of students who do not play either sport? Explain your thinking using a Venn diagram.
  - Is it possible that all 22 students in the class are involved in one sport or the other or both? Explain your thinking using a Venn diagram.

SUGGESTED MODELS AND MANIPULATIVES

- 2-D shapes
- 3-D objects
- attribute blocks
- cards or tiles (commercial or teacher-made)
- collection of various objects to sort
- money

MATHEMATICAL LANGUAGE

Teacher	Student
▪ attributes	▪ attributes
▪ Carroll diagram	▪ Carroll diagram
▪ characteristics	▪ characteristics
▪ charts and diagrams	▪ charts and diagrams
▪ classifying	
▪ mathematical relationships	▪ mathematical relationships
▪ new element	▪ new element
▪ sorting rule	▪ sorting rule
▪ Venn diagram	▪ Venn diagram

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students K–8* (Small 2008), pp. 520–523
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 564–567

Videos

- *Analyzing Patterns (Skip Counting) on a Hundred Board* (27:16 min.) (ORIGO Education 2010)

Notes

SCO PR05 Students will be expected to express a given problem as an equation in which a symbol is used to represent an unknown number.
[CN, PS, R]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

PR05.01 Explain the purpose of the symbol in a given addition, subtraction, multiplication, or division equation with one unknown (e.g., 36 ÷ ⍰ = 6).
PR05.02 Express a given pictorial or concrete representation of an equation in symbolic form.
PR05.03 Identify the unknown in a problem; represent the problem with an equation; and solve the problem concretely, pictorially, and/or symbolically.
PR05.04 Create a problem in context for a given equation with one unknown.

Scope and Sequence

Mathematics 3	Mathematics 4	Mathematics 5
PR03 Students will be expected to solve one-step addition and subtraction equations involving symbols representing an unknown number.	**PR05** Students will be expected to express a given problem as an equation in which a symbol is used to represent an unknown number.	**PR02** Students will be expected to solve problems involving single-variable, one-step equations with whole number coefficients and whole number solutions.

Background

An equation is a mathematical statement that includes an equal sign and is used to express relationships between two quantities. Equations may have been called number sentences in earlier grades.

Prior to Mathematics 4, students explored equality and inequality concretely, pictorially, and symbolically. They solved one-step addition and subtraction equations involving symbols representing an unknown number and they represented equations with the equal sign and/or the unknown symbol in different locations, such as the following:

6 + 3 = ◯
5 + ◊ = 8
Δ + 4 = 24
8 – 5 = ?

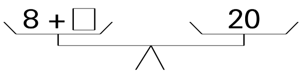
8 – ? = 3
Δ = 12
◊ – 15 = 5
6 = 3 + Δ

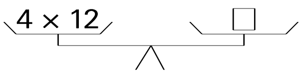
6 = ? + 5
Δ = 16 – 12
4 + ? = 5 + 7

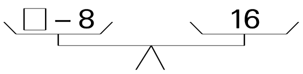
They read and interpreted equations in meaningful ways. In reading 9 + Δ = 16 students may have said, What do I need to add to 9 to get 16? or If 16 is made up of two parts, and one part is 9, how many are in the other part? Students translated addition and subtraction word problems into equations and then solved them. Initially, students solved these addition and subtraction problems using concrete and pictorial representations and later learned to solve them symbolically. This work will be extended in Mathematics 4 to include multiplication and division situations with one unknown.

The various representations of patterns, including unknowns, provide valuable tools to help students make generalizations of mathematical relationships. Equality is used to express relationships. The symbols used on either side of the equal sign represent a quantity. The equal sign is "a symbol of equivalence and balance" (NCTM 2000, p. 39). Students should be given opportunities to explore equivalence using models and pictures before they begin to represent the equations symbolically. Students should be comfortable using various symbols to represent the unknown number in an equation (e.g., a square, circle, triangle, or other shapes).

Display a number of samples of balance scales, such as those shown below. Invite students to write the equation represented by each balance scale and then solve it. For example, for the first model, students would write the equation, $8 + \square = 20$, and the solution would be $\square = 12$. Include examples of scales with the single number on the left side of the scale, so students have opportunities to write equations in different structures (e.g., $36 = \square \times 9$).







Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Present students with two numbers and ask them to create equations where one of the numbers is unknown. For example, for 15 and 8 some possible equations are $15 - 8 = \square$, $8 + \square = 15$, $15 = \square + 8$, and $\square = 15 - 8$. Ask students to explain what a symbol represents in an equation (e.g., it represents an unknown).

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to explain what the box represents in the following equations:
  15 – □ = 8   □ = 383 + 98   □ = 4 × 3   62 = □ × 31   16 ÷ □ = 2   155 × 2 = □
- Ask students to write an equation (with an unknown) to represent a given story problem.
- Provide students with linking cubes of two different colours. Pose the problem below. Invite students to represent the problem with an equation and then model the solution with their linking cubes.
  – Gregory has 13 red marbles and 22 blue marbles. How many more blue marbles than red marbles does Gregory have?
- Ask students to read a given story problem, such as the following:

  Marthe had some money in her piggy bank. Her grandmother gave her $25 for her birthday and Marthe put it in her piggy bank. Then, Marthe had $118 in her piggy bank. How much money was in the piggy bank before her birthday?

  Present students with a set of cards on which equations have been written. Ask students to select an equation that could be used to solve the problem and to explain their thinking. Equations could include the following:

  $25 + $118 = ○
  $118 – $25 = ○
  ○ + $25 = $118
  $25 + ○ = $118
  $118 + $25 = ○
  $25 – $118 = ○
  ○ = $118 – $25

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 4 (Bauman 2011)
- Checkpoint 4, Task 1, pp. 28–30
- Checkpoint 5, Task 1, pp. 31–33

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

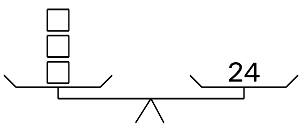
CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Build on the students' knowledge from the previous grade in using equations to write addition, subtraction, multiplication, and division equations. Connect the concrete, pictorial, and symbolic representations as the students develop and demonstrate understanding of equations.
- Use everyday contexts for problems to which the students can relate so that they can translate the meaning of the problem into an appropriate equation using a symbol to represent the unknown.
- Review the relationship between addition and subtraction equations, as well as the relationship between multiplication and division equations.
- Ask students to create problems for a variety of equations using the four operations.
- Reinforce the notion of the equal sign as a balance, rather than as "here comes the answer."
- Connect the concrete, pictorial, and symbolic representations consistently as students develop and demonstrate understanding of equations.

SUGGESTED LEARNING TASKS

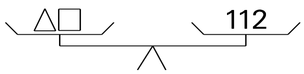
- Show students a balance scale and ask them to work with a partner to find an equation that represents each of the examples below. Possible equations are shown under the balances.



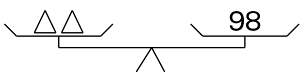
$3 \times \square = 24$

or

$\square + \square + \square = 24$



$\triangle + \square = 112$



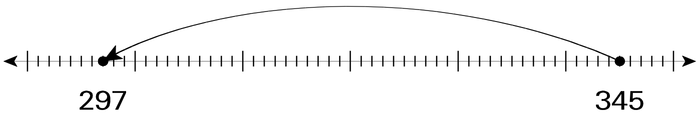
$2 \times \triangle = 98$

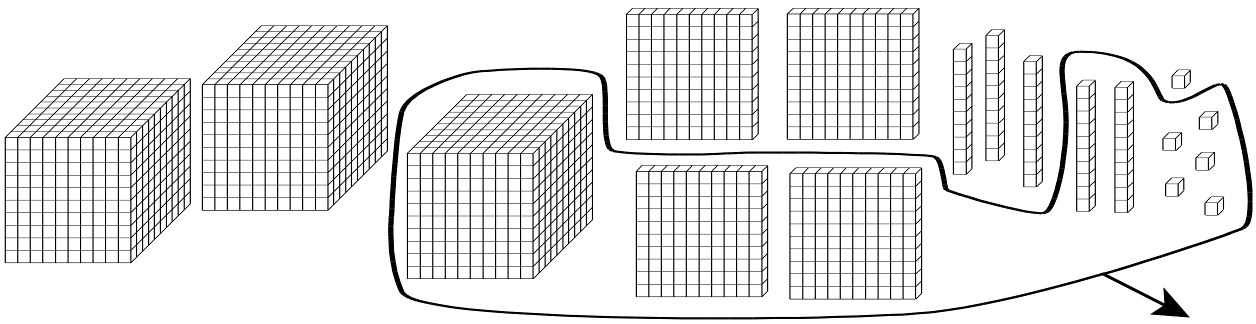
or

$\triangle + \triangle = 98$

- Mathieu has 72 hockey cards and wants to share them equally among his four friends. Ask students to represent the problem as an equation with an unknown.
- Present students with an addition or subtraction story problem, such as Yolanda has 187 cards and Karl has 52 cards. How many more cards does Yolanda have than Karl? Ask students to write two different equations that could be used to represent the problem.
- Ask students to read a given story problem and write an equation to show what is happening in the problem. Then, ask them to use their equation to solve the problem.
- Present students with an equation with an unknown. Ask them to create a story problem that would match the equation.
- Provide students with an equation with an unknown, such as 5 × 4 = Δ. Ask students to use concrete materials to model their equation and determine a solution.
- Tell students that Leila said that the box in the following equation stands for more than one number. 6 + 8 = □ + 4. Ask students to explain whether Leila is correct?
- Ask students to explain how to find the missing number in the equation 2 × Δ = 12.
- Tell students that Beth and Julio were solving the following story problem: Mrs. Oulton placed her 36 books on the 4 shelves of the bookcase. She placed the same number of books on each shelf. How many books did she place on each shelf?

  Beth wrote an equation to represent the problem. She wrote 4 × Δ = 36. Julio wrote an equation to represent the same problem, but he wrote 36 ÷ 4 = □. Ask students whether it is possible for both students to have written correct equations and to explain their thinking.
- Ask students to write equations with unknowns to represent problem situations such as the following:
  - The perimeter of a triangle is 12 cm. One side measures 3 cm and another side measures 4 cm. What is the length of the third side of the triangle?
  - The librarian wanted to know what kind of books to buy for the library. She surveyed 48 students. Twenty-three of the students chose science books and some chose picture books. How many students chose picture books?
  - Gina was making pentagons with toothpicks. She has 30 toothpicks. How many pentagons can she make?
- Provide students with various concrete and pictorial displays, such as base-ten blocks and number lines. Ask students to represent the displays as equations.





SUGGESTED MODELS AND MANIPULATIVES

- balance scales
- linking cubes

MATHEMATICAL LANGUAGE

Teacher	Student
addition, subtraction, multiplication, and division equation solve symbol unknown number	addition, subtraction, multiplication, and division equation solve symbol unknown number

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students K–8* (Small 2008), pp. 585–586
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 624–625
- *Teaching Student-Centered Mathematics, Grades 3–5*, Volume Two (Van de Walle and Lovin 2006), pp. 306–307

Videos

- *Using Structured Patterns to Develop Number Combinations* (18:11 min.) (ORIGO Education 2010)

Notes

SCO PR06 Students will be expected to solve one-step equations involving a symbol to represent an unknown number. [C, CN, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

PR06.01 Represent and solve a given one-step equation concretely, pictorially, or symbolically.
PR06.02 Solve a given one-step equation using guess and test.
PR06.03 Describe, orally, the meaning of a given one-step equation with one unknown.
PR06.04 Solve a given equation when the unknown is on the left or right side of the equation.
PR06.05 Represent and solve a given addition or subtraction problem involving a “part-part-whole” or comparison context using a symbol to represent the unknown.
PR06.06 Represent and solve a given multiplication or division problem involving equal grouping or partitioning (equal sharing) using symbols to represent the unknown.
PR06.07 Solve equations using a symbol to represent the unknown.

Scope and Sequence

Mathematics 3	Mathematics 4	Mathematics 5
PR03 Students will be expected to solve one-step addition and subtraction equations involving symbols representing an unknown number.	**PR06** Students will be expected to solve one-step equations involving a symbol to represent an unknown number.	**PR02** Students will be expected to solve problems involving single-variable, one-step equations with whole number coefficients and whole number solutions.

Background

This outcome is closely related to outcome PR05 with solving the equation as the next step.

To solve an equation means determining the value of the unknown that will balance both sides of the equation. Solving equations using the operations of multiplication and division is new to Mathematics 4. Initially, students need to model the solution to equations concretely using balances. Students should be asked to solve word problems for the four operations and represent the story problems using equations. They should explore the idea that a symbol represents a specific, unknown quantity, as they translate the problem into a written equation. Please refer to SCO N03, N06, and N07 for story-problem structures expected at this grade level.

Model the use of “guess and test” as one strategy that can be used to solve the equation. “For this strategy, a student guesses an answer and then tests it to see if the guess works. If it doesn’t, the student revises the guess based on what was learned and then tries again. This repetitive process continues until the answer is found. Some students are able to think through several guesses at once; others need to go one step at a time. Although we often talk about guessing as bad, this strategy reinforces the value of taking risks and learning from the information that is garnered.” (Small 2008, p. 44)

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to write the corresponding equation for a word problem and solve it. For example, Gabrielle had some stickers and gave her friend 9. Now she has 8 left. How many did she have at the start? ($\square - 9 = 8$)

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Tell students the following story problem: You have 24 marbles. Your friend gives you some more marbles. Now, you have 32 marbles in all. How many marbles did your friend give you? Ask students to write an equation to show what is happening in this problem. Then, ask students to solve the problem and to explain their thinking.
- Ask students to solve the following equation and explain their thinking.
  $34 + 5 = \square + 12$
- Ask students to solve the following equation and explain their thinking.
  $20 = \Delta - 13$
- Tell students that Reika said that the box in the following equation represents more than one number. Is Reika correct? Why or why not?
  $\square + 4 = 6 + 8$
- Ask students to explain how to find the missing number in $4 \times \Delta = 100$.
- Ask students to explain how to find the missing number in $56 \div \Delta = 8$.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 4 (Bauman 2011)

- Checkpoint 4, Task 1, pp. 28–30
- Checkpoint 5, Task 1, pp. 31–33
- Checkpoint 6, Tasks 1 and 2, pp. 34–37

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Encourage the students to write equations in various ways to represent the meaning of a given problem. For example, 14 + Δ = 37 or 37 = Δ + 14; 5 × □ = 30 or 30 = □ × 5 or □ × 5 = 30. Note that the order (commutative) property does not apply to subtraction and division.
- Explain that if the same variable, or unknown number, is used repeatedly in the same equation, then there is only one possible solution for that variable or unknown (e.g., for □ × □ = 25; the unique solution is to place 5 in each of the rectangles. If, however, two different symbols are used, there may be a number of possible solutions (e.g., □ × Δ = 24; some solutions include 3 × 8, 2 × 12, or 6 × 4).
- Explore interactive websites that have equation balancing activities, such as NCTM's Illumination activity site 2013 (http://illuminations.nctm.org/ActivityDetail.aspx?ID=26).

SUGGESTED LEARNING TASKS

- Invite students to create problems to represent the following equations:

  $15 + \square = 24$ $\Delta + 15 = 24$ $24 = 15 + \bigcirc$ $24 = \Diamond + 15$

  $24 - \square = 15$ $24 - 15 = \triangledown$ $15 = 24 - ▭$ $\triangleright = 24 - 15$

- Ask students to create problems to represent the following equations:

  $63 \div \square = 3$ $275 \times \square = 1925$ $52 \times 8 = \square$

  $726 \times 5 = \square$ $96 \div 4 = \square$ $\square \div 7 = 8$

  $\square = 13 \times 5$

SUGGESTED MODELS AND MANIPULATIVES

- linking cubes

MATHEMATICAL LANGUAGE

Teacher	Student
▪ addition, subtraction, multiplication, division equations ▪ guess and test ▪ one-step equations ▪ solve ▪ symbol ▪ unknown number	▪ addition, subtraction, multiplication, division equations ▪ guess and test ▪ one-step equations ▪ solve ▪ symbol ▪ unknown number

Resources/Notes

Internet

- Illumination (National Council of Teachers of Mathematics 2013) http://illuminations.nctm.org/ActivityDetail.aspx?ID=26).

Print

- *Making Mathematics Meaningful to Canadian Students K–8* (Small 2008), pp. 585–586
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 624–625
- *Teaching Student-Centered Mathematics, Grades 3–5*, Volume Two (Van de Walle and Lovin 2006), pp. 306–307

Videos

- *Using Structured Patterns to Develop Number Combinations* (18:11 min.) (ORIGO Education 2010)

Notes

Measurement (M)

GCO: Students will be expected to use direct and indirect measure to solve problems.

SCO M01 Students will be expected to read and record time using digital and analog clocks, including 24-hour clocks. [C, CN, V]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

M01.01 State the number of hours in a day.
M01.02 Express the time orally and numerically from a 12-hour analog clock.
M01.03 Express the time orally and numerically from a 24-hour analog clock.
M01.04 Express the time orally and numerically from a 12-hour digital clock.
M01.05 Express time orally and numerically from a 24-hour digital clock.
M01.06 Describe time orally as “minutes to” or “minutes after” the hour.
M01.07 Explain the meaning of a.m. and p.m., and provide an example of an activity that occurs during the a.m., and another that occurs during the p.m.

Scope and Sequence

Mathematics 3	Mathematics 4	Mathematics 5
M01 Students will be expected to relate the passage of time to common activities using non-standard and standard units (minutes, hours, days, weeks, months, years). **M02** Students will be expected to relate the number of seconds to a minute, the number of minutes to an hour, the number of hours to a day, and the number of days to a month in a problem-solving context.	**M01** Students will be expected to read and record time, using digital and analog clocks, including 24-hour clocks.	—

Background

Although students have not had any explicit teaching related to reading and recording time using clocks, they have had opportunities in previous grades to explore the passage of time and have learned that there are 60 seconds in a minute and 60 minutes in an hour. In Mathematics 4, students learn that there are 24 hours in a day. Prior to Mathematics 4, students will have had many opportunities to learn about the passage of time through their own experiences.

By the end of Mathematics 4, students should be able to read and record time on 12-hour and 24-hour analog and digital clocks. They will benefit from ongoing practice telling time during morning/daily routines through the use of a demonstration clock.

Students should read times on clocks to provide information about relevant situations, focusing on times when special events are going to happen. The units **minutes** and **hours** are usually introduced before the unit **seconds** because students use them more often throughout their daily lives. Students will need to learn these standard units of time and then have many opportunities to explore the relationship among the units. It is important for students to be able to precisely describe time, telling how many "minutes to" and how many "minutes after" the hour.

Students will learn that there are 24 hours in a day; however, time is often described using the 12-hour clock. Although the world has become increasingly digital, there are still many analog clocks in use, and students must learn to tell time on both analog and digital clocks, as well as on a 24-hour analog clock. On a 12-hour analog clock, the hours go from 1:00 in the morning until 12:00 noon, and then the cycles repeat from 1:00 in the afternoon until midnight.

Throughout the school day, students should be provided with numerous opportunities to read and record time using a variety of clocks such as digital, analog, and 24-hour clocks. Students should read times on clocks to provide information about relevant situations such as comparing start and finish times to determine how much time has passed; estimate how long before an event begins (e.g., how long until lunchtime?); planning events; and reading schedules. The focus of teaching should be through problem-solving experiences. Ask students questions such as, If my brother left home at 10 a.m. and was away for seven hours, what time did he return home?

Time can be represented using a linear model such as a timeline because time is actually linear. However, our descriptions of time with words such as **days**, **weeks**, **months**, and **year**, describe cycles. This is a common misconception. Although time is linear because nothing actually repeats, we still can use a time circle when teaching time to show the cyclical nature of the words that are "descriptions" of time.

Students may want to investigate the meaning for some terminology such as, a.m. and p.m. (**Note:** It is also acceptable to write these as AM and PM). The abbreviation, a.m., is the short form for *ante meridiem* meaning being before noon and p.m. is the abbreviation for *post meridiem* meaning being after noon. Students will likely encourage other representations of a.m. and p.m., such as AM and PM or A M and P M (with or without periods/capitals/lower case). Students can use the terms "noon" and "midnight" for those precise times since the use of 12:00 a.m. and 12:00 p.m. may be a source of confusion.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students,
  - What is something you can do in a second? In a minute?
  - What is something you can do about 10 times in a minute? In an hour?
- Tell students that
  - Ashram took 90 seconds to run a race and Logan took 3 minutes. Ask students to explain who was faster?
  - It took Meika 120 minutes to drive to her grandparent's house. Ask students to explain how many hours it took for Meika to make the trip?

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students what time it might be if the minute hand and hour hand are opposite one another.
- Have students move the hands of an analog clock to match the time shown on a digital clock.
- Ask students to express orally and numerically the time that has been created on a 12-hour analog clock, 24-hour analog clock, and 12-hour digital clock.
- Ask students to name an activity they would typically do in the p.m.? a.m.?
- Have students explain how many hours are in one and one-half days.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 4 (Bauman 2011)

- Checkpoint 9, Task 1, pp. 45–47

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Invite students to explore an analog clock and share their findings. Students will be aware that
  - the minute hand is at 6 for the “:30” on a digital clock and at 12 or 24 for the “:00”
  - the hour hand moves during the course of an hour; at the :30, it is halfway between two numbers
  - the hour and minute hands are different lengths
- Use an analog clock to introduce the terms **half past**, **quarter after**, and **quarter to** as well as how many “minutes to” and how many “minutes after.”
- Ask students to read time to the nearest five minutes. It is important that students are comfortable with skip counting by five. This provides the opportunity for students to relate the numbers on a clock to the five times table.
- Use a clock that shows not only the numbers from 1 to 12, but also the minute amounts from 5 to 55 beside the numbers from 1 to 11. Students should be aware that there are 5 minutes between the numbers on the clock. The short hand on the 3 represents 15 minutes, so two one-minute spaces past the 3 is 17 minutes, etc.

SUGGESTED LEARNING TASKS

- Present students with a time shown on an analog clock that just has the hour hand. Ask them to predict what the time might be. For example, if the hour hand is somewhere between the 4 and the 5, the time could be anything from *five past four* to *five to five* depending on the exact placement of the hour hand. Students could also be asked to name an event/activity that often happens at about that time of day.
- Introduce the terms of analog, **a.m.** and **p.m.** Discuss the difference between the two terms and brainstorm activities that would take place during each.
- Ask students to show, on an analog clock, the time (to the nearest half hour) at which they arrive at school, have lunch, go to bed, etc.
- Discuss when a 24-hour clock would be more appropriate to use than a 12-hour clock.

- Ask students to track events throughout a specific day by means of a timeline divided into 15-minute segments. Students should record the time of the activity or event and note it at the appropriate spot on a timeline.
- Invite students to make a list of the times when the minute hand and the hour hand almost line up as well as other patterns, such as all of the times that include a 4 in a 24-hour period.
- Invite students to work in pairs to set up a schedule in which every student will get 10 minutes on the computer, starting at 8:30 a.m. Ask students if everyone in the class can have time on the computer before noon and, if not, how long it will take to finish after lunch. Ask them to tell at what time the last student will finish? (Remind students to leave time for recess.)

SUGGESTED MODELS AND MANIPULATIVES

- digital clock that reads 12-hours and 24-hours
- 12-hour analog clock
- 24-analog clock

MATHEMATICAL LANGUAGE

Teacher	Student
▪ clock ▪ estimate, measure ▪ seconds, minutes, hours, day ▪ time	▪ clock ▪ estimate, measure ▪ seconds, minutes, hours, day ▪ time

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students K–8* (Small 2008), pp. 444–447
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 490–493
- *Teaching Student-Centered Mathematics, Grades 3–5*, Volume Two (Van de Walle and Lovin 2006), p. 270

Notes

SCO M02 Students will be expected to read and record calendar dates in a variety of formats. [C, V]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

M02.01 Write dates in a variety of formats (e.g., yyyy/mm/dd, dd/mm/yyyy, March 21, 2014, dd/mm/yy).
M02.02 Relate dates written in the format yyyy/mm/dd to dates on a calendar.
M02.03 Identify possible interpretations of a given date (e.g., 06/03/04).

Scope and Sequence

Mathematics 3	**Mathematics 4**	**Mathematics 5**
M01 Students will be expected to relate the passage of time to common activities using non-standard and standard units (minutes, hours, days, weeks, months, years). **M02** Students will be expected to relate the number of seconds to a minute, the number of minutes to an hour, the number of hours to a day, and the number of days to a month in a problem-solving context.	**M02** Students will be expected to read and record calendar dates in a variety of formats.	—

Background

By grade four, students should already know the days of the week, the months of the year, and the four seasons. As well, students will have already developed a sense of the arrangement of our year in relation to the months and seasons (e.g., January is the first month of a new year and is early in our winter season).

Using calendars throughout the school year strengthens the students' sense of time. Each month brings a new calendar to explore. Students should be familiar with calendars through their home and school experiences by grade 4. In previous grades, teachers may have explored calendars during explorations of units of time, such as days, weeks, months, and years. Calendars may also have been used to assist in developing number sense and for exploring patterns.

Students need to become aware of the variety of ways dates can be recorded. In Mathematics 4, students are expected to read, record, and interpret calendar dates in a variety of ways, including words and numbers. It is important for students to be familiar with different formats for dates since there are several that are acceptable that will be encountered in their daily lives. The International Organization for Standardization (ISO) has identified a standard notation that many countries, including Canada, have adopted. It always starts with the year, then the month, and the last digits are the day (yyyy-mm-dd). It always uses four digits for the year and all other numbers less than 10 are recorded with a leading zero (e.g., 2014-01-04 is how January 4, 2014, would be recorded).

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to name an activity that takes minutes (hours, weeks, months, or years) to complete.
- Show students a calendar for the year and ask them to
  - identify ways in which months are the same and ways in which they differ
  - point out today's date and to find out what the date will be in six weeks

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Show students a calendar for the year and ask them to point out the day's date. Have them record it using the format month/day/year.
- Ask students to identify two calendar dates that cannot be confused with other dates when they are interpreted regardless of the format.
- Invite students to write their birthdate using three different formats.
- Invite students to identify their favourite day of year and write the date in ISO standard notation (yyyy-mm-dd).

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 4 (Bauman 2011)

- Checkpoint 9, Task 2, pp. 45–47

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Provide groups of students with yearly calendars. Have students explore the calendars and share their findings. Students should focus on the different formats of how dates are written.
- Send students on a scavenger hunt and have them bring in different dates from magazines, posters, items printed from the Internet, cheques, and newspapers. Share, discuss, and display the variety of formats as a class.
- Ask students to predict how many days and/or weeks there are in a year. Verify using calendars.
- Ask them to explore what calendar dates can be confused with other dates when they are interpreted using various formats.
- Investigate a special holiday that has a date that fluctuates, such as Labour Day. Have students record the date(s) of this holiday over the past five years in different formats. Share their findings.

SUGGESTED LEARNING TASKS

- Ask students to write about their favourite format for recording a calendar date and justify their choice.
- Ask students to interpret a particular date such as 06/04/03. Discuss that there is no standard or consistent format and why some dates may be misinterpreted unless the format is known.
- Provide students with a list of dates recorded in the ISO standard notation (yyyy-mm-dd) and ask them to order them from past to present.
- Students could investigate famous events using the Internet and then record them in a variety of formats.
- Students could read expiry dates on food products.

SUGGESTED MODELS AND MANIPULATIVES

- calendars
- electronic devices that display dates

MATHEMATICAL LANGUAGE

Teacher	Student
▪ calendar ▪ days, weeks, months, years ▪ estimate, measure ▪ time	▪ calendar ▪ days, weeks, months, years ▪ estimate, measure ▪ time

Resources/Notes

Notes

SCO M03 Students will be expected to demonstrate an understanding of area of regular and irregular 2-D shapes by

- recognizing that area is measured in square units
- selecting and justifying referents for the units square centimetre (cm^2) or square metre (m^2)
- estimating area using referents for cm^2 or m^2
- determining and recording area (cm^2 or m^2)
- constructing different rectangles for a given area (cm^2 or m^2) in order to demonstrate that many different rectangles may have the same area

[C, CN, ME, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

M03.01 Describe area as the measure of surface recorded in square units.
M03.02 Identify and explain why the square is the most efficient unit for measuring area.
M03.03 Provide a referent for a square centimetre, and explain the choice.
M03.04 Provide a referent for a square metre, and explain the choice.
M03.05 Determine which standard square unit is represented by a given referent.
M03.06 Estimate the area of a given 2-D shape using personal referents.
M03.07 Determine the area of a regular 2-D shape, and explain the strategy.
M03.08 Determine the area of an irregular 2-D shape, and explain the strategy.
M03.09 Construct a rectangle for a given area.
M03.10 Demonstrate that many rectangles are possible for a given area by drawing at least two different rectangles for the same given area.

Scope and Sequence

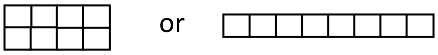
Mathematics 3	**Mathematics 4**	**Mathematics 5**
M05 Students will be expected to demonstrate an understanding of perimeter of regular, irregular, and composite shapes by ▪ estimating perimeter using referents for centimetre or metre (cm, m) ▪ measuring and recording perimeter (cm, m) ▪ create different shapes for a given perimeter (cm, m) to demonstrate that many shapes are possible for a perimeter	**M03** Students will be expected to demonstrate an understanding of area of regular and irregular 2-D shapes by ▪ recognizing that area is measured in square units ▪ selecting and justifying referents for the units square centimetre (cm2) or square metre (m2) ▪ estimating area using referents for cm2 or m2 ▪ determining and recording area (cm2 or m2) ▪ constructing different rectangles for a given area (cm2 or m2) in order to demonstrate that many different rectangles may have the same area	**M01** Students will be expected to design and construct different rectangles, given a perimeter or an area, or both (whole numbers), and make generalizations.

Background

In Mathematics 4, students should participate in explorations that serve to deepen and expand upon previously learned measurement ideas and skills. Through investigations, students should come to understand that the area of a shape can be expressed as the number of units required to cover a certain region. Van de Walle and Lovin define area as “a measure of the space inside a region or how much it takes to cover a region” (Van de Walle and Lovin 2006b, p. 261). The square unit is the most efficient unit to use for measuring area.

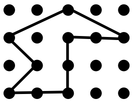
In the beginning, students need to use concrete materials to construct a square centimetre and a square metre. Using these squares, students should estimate the area of various shapes in the classroom such as the top of their desks or the classroom floor and then measure the area using their squares. Students need to make sense of the real-life applications of finding area and should identify everyday contexts in which people need to know the amount of surface covered, such as painting a wall or tiling a floor.

It is helpful for students to use a referent for the single unit of measure and iterate this unit mentally to obtain the estimate (e.g., use the size of the fingernail on your finger or thumb as a referent for 1 cm^2). Once students have developed the meaning of measurement, it is time to move on to connect multiplication in an array format to determine the area of rectangles (Van de Walle and Lovin 2006b, p. 263). Students should relate the area of a rectangle to the product of the numbers describing its length and width. Conversely, any factor of the number representing the area of a rectangle can be one dimension of a rectangle with that area. For example, consider rectangles with an area of 8 square units.



It is important for students to explore not only the areas of rectangles, but areas of other shapes as well. Through these investigations students should recognize that objects of different shapes can have the same area. Encourage students to find shapes using partial squares.

Opportunities should be provided for students to estimate and calculate the area of various surfaces. Laying an acetate centimetre grid over objects is helpful when determining surface area. Students might investigate the area of shapes drawn on centimetre dot paper. Strategies for doing this include adding squares and half squares within the figure or placing a rectangle around the shape, determining its area, and subtracting the area of the “extra” pieces.



Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

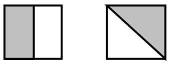
- Provide students with a geo-board. Have them create
  - a rectangle with a perimeter of 12 units
  - a second rectangle with a perimeter of 12 units, but with different dimensions
  - a different shape (not a rectangle or triangle) with a perimeter of 12 units

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

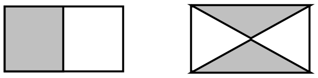
Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to predict how many different arrays can be made to represent 36 cm^2. Have them draw all of the arrays to check their prediction.
- Have students estimate the area for each of the following pairs of congruent shapes. Ask them to decide if the shaded part has the same area in each pair of shapes and to explain their thinking.

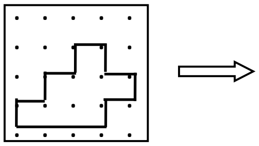
a.

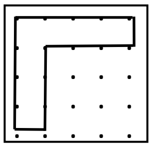


b.

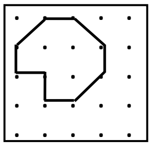


- Ask students to estimate the area of a rectangle and explain what referent was used to determine the estimate.
- Invite students to explain why area is measured in square units.
- Ask students to circle the letters of the shapes that have the same area as the first one on the left.

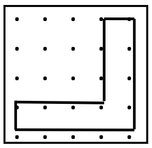




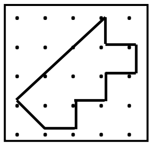
A



B

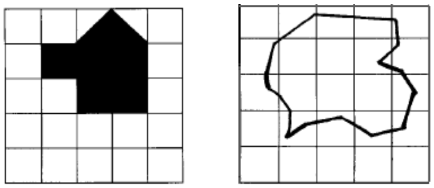


C



D

- Ask students why it is easier to find the area of the shape on the left than the area of the shape on the right?



- Tell students that the area of the entire design below is 12 m^2. Ask them to find the area of the shaded part and to explain their thinking.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 4 (Bauman 2011)

- Checkpoint 12, Task 1, pp. 54–56

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

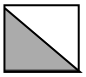
CHOOSING INSTRUCTIONAL STRATEGIES

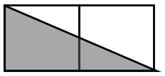
Consider the following strategies when planning daily lessons.

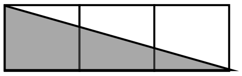
- Use referents for estimating area. Review that referents are familiar objects to which students can refer for estimating (e.g., the width of the “little” finger is about 1 cm). Ask students to suggest a suitable referent for 1 cm^2 and explain why they think it would work. Have them use this referent to estimate the area of a book cover. Ask them to check their estimate by finding the area of the book cover. Discuss possible referents for 1 m^2. Have students use their referents to estimate the area of a large tabletop or a section of the classroom floor and check their estimates.
- Invite students to use colour tiles or grid paper to investigate the numbers from 1 to 30 to see how many different rectangles can be made for each. Students should record their results and look for patterns.
- Use a transparency of a centimetre grid to confirm the estimate of an area of an irregular shape.

SUGGESTED LEARNING TASKS

- Ask students to use centimetre grid paper to explore how a diagonal of a rectangle divides the area of the rectangle in half.



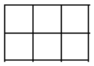




- Provide students with rectangular papers that each measure 10 cm × 13 cm. Have them estimate the area of the paper and explain their thinking.
- Make the design to the right on an overhead geo-board or interactive whiteboard, and ask students to explain various ways to find the area of the shape.

- Provide students with tiles and centimetre grid paper. Give them the following instructions: For each of the areas from 1 cm^2 to 20 cm^2, find all the possible rectangular arrays using whole numbers. For example, the possible arrays for an area of 6 cm^2 would be as follows:

1 row of 6
square units



2 rows of 3
square units

SUGGESTED MODELS AND MANIPULATIVES

- centimetre grid paper
- colour tiles
- geo-boards
- pattern blocks
- transparency grid paper

MATHEMATICAL LANGUAGE

Teacher	Student
- area, measure of surface - measure, estimate - personal referent - standard units: square metre or square centimetre	- area, measure of surface - measure, estimate - standard units: square metre or square centimetre

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students K–8* (Small 2008), pp. 388–397
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 434–442
- *Teaching Student-Centered Mathematics, Grades 3–5*, Volume Two (Van de Walle and Lovin 2006), pp. 262, 288–289

Notes

Geometry (G)

GCO: Students will be expected to describe the characteristics of 3-D objects and 2-D shapes and analyze the relationships among them.

GCO: Students will be expected to describe and analyze position and motion of objects and shapes.

SCO G01 Students will be expected to describe and construct rectangular and triangular prisms. [C, CN, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

G01.01 Identify and name common attributes of rectangular prisms from given sets of rectangular prisms.
G01.02 Identify and name common attributes of triangular prisms from given sets of triangular prisms.
G01.03 Sort a given set of right rectangular and triangular prisms, using the shape of the base.
G01.04 Construct and describe a model of a rectangular and a triangular prism, using materials such as pattern blocks or modelling clay.
G01.05 Construct rectangular prisms from their nets.
G01.06 Construct triangular prisms from their nets.
G01.07 Identify examples of rectangular and triangular prisms found in the environment.

Scope and Sequence

Mathematics 3	**Mathematics 4**	**Mathematics 5**
G01 Students will be expected to describe 3-D objects according to the shape of the faces and the number of edges and vertices.	**G01** Students will be expected to describe and construct rectangular and triangular prisms.	**G01** Students will be expected to describe and provide examples of edges and faces of 3-D objects, and sides of 2-D shapes that are parallel, intersecting, perpendicular, vertical, and horizontal.

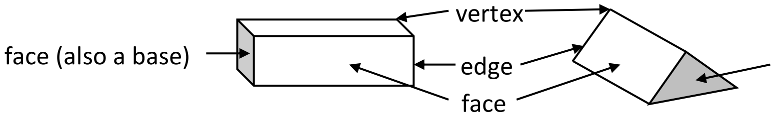
Background

Students should draw upon their previous knowledge of two-dimensional polygons to assist them in their identification and description of prisms. In the earlier grades, students classified geometric shapes by general characteristics and will now develop more detailed ways to describe objects. Students will identify properties of shapes and objects and learn to use proper mathematical vocabulary to describe them.

All prisms have faces, two of which are customarily referred to as bases. These two bases may take the shape of any polygon. For clarification purposes, prisms can be thought of as having a two-part name. The first part refers to the shape of the bases, and the second part, which is “prism” (e.g., triangular prism, rectangular prism). Some students may be keen to identify other prisms such as hexagonal prisms or square prisms (square prisms fall into the category of rectangular prisms because a square is a rectangle). In Mathematics 4, instruction is focused on rectangular prisms and triangular prisms. Sets of 3-D objects usually include a variety of prisms. Students also need to be able to identify examples of rectangular and triangular prisms in their environment.

A good way to explore prisms is to use smaller prisms or tiles to create larger prisms. Pattern blocks are very good for this, but many other materials can be used. Although the pattern-block pieces have been primarily used to represent 2-D shapes, they are prisms. Stacking a number of triangle or square pattern blocks would provide examples of different prisms. This stacking would help students conceptualize the uniform nature of prisms. Students can also make skeletal models for prisms, using rolled newspapers and tape, straws and string, or toothpicks and miniature marshmallows.

Students should be given copies of nets of rectangular and triangular prisms to cut out and fold to construct the prisms. They should be encouraged to unfold them and examine the 2-D shapes that are connected to make each net. Students should recall from Mathematics 3 that these shapes are the faces of the 3-D object and are one of the key attributes students should use to identify rectangular and triangular prisms. Other attributes students should consider when identifying 3-D objects are the number of edges and vertices (from grade three) and congruency. In addition to cutting out and assembling prepared nets, it is also expected that students will trace the various faces of the different prisms to make nets for rectangular and triangular prisms and explore other possible nets for these prisms. Have students visualize the folding and unfolding of the nets and then use materials to explore whether the net will successfully construct the prism.



Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Have students identify the shape of the faces of a given 3-D object.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Invite students to name the prism that best represents various real-life examples of 3-D objects (e.g., a book would be represented by a rectangular prism).
- Invite students to build skeletal models of two different triangular prisms and explain how they are the same and different.
- Have students work together to sort a collection of 3-D objects into two groups: rectangular prisms and triangular prisms. Ask students to explain which attributes of the objects made them alike? Ask them to explain how the objects are different, why some rectangular prisms are cubes, and the kind of prism that would result if it was built from a rectangular base?
- Ask students to construct nets on geo-boards and/or using grid paper and to explain their thinking. Note whether students are discussing the attributes of the object and using the correct vocabulary (i.e., faces, edges, vertices).
- Have students add additional faces to partially completed nets that will successfully construct a rectangular or triangular prism.
- Give small groups of students a set of four or five nets of rectangular or triangular prisms. Each set should consist of one net that can be made into the 3-D object, and three or four others that will not construct the 3-D object. Have students analyze the nets, without manipulating them, to determine which one of the nets in the group could be used to create the 3-D object. Have students justify their selection, and then test their prediction.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 4 (Bauman 2011)
- Checkpoint 13, Task 1, pp. 57–59
- Checkpoint 14, Tasks 1 and 2, pp. 60–62

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?

- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

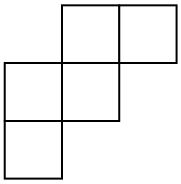
CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

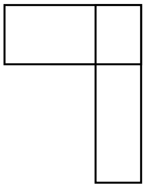
- Provide groups of students with a variety of rectangular and triangular prisms. Have students explore the attributes of these objects and share their findings.
- Encourage students to use the attributes of any prism (number of faces, number of edges, number of vertices, or shapes of the faces) to describe prisms.
- Determine if students recognize that the same prism can be built by piling pattern blocks vertically or horizontally when building objects from the base, (i.e., orientation).
- Work with nets to investigate attributes, including alignment of faces, to determine if the net could successfully construct the 3-D object.

SUGGESTED LEARNING TASKS

- Stack pattern blocks to make rectangular prisms and triangular prisms. Describe how they are alike and how they are different.
- Ask students to build skeletal models of two different rectangular prisms. Have them explain how the two models are the same and how they are different?
- Provide students with various nets of prisms for them to construct. Have students label each face of their model using the words “face” and “base” as well as identify their 3-D object.
- Ask students to trace on paper the various faces of the different prisms to make its net. Have students cut out the net and fold it up around the shape to check if it works. Ask students to record this net on grid paper. Have them cut off one of the faces and investigate the possible places it could be reattached to make a new successful net and record each new net on grid paper.
- Provide students with a square or rectangular prism and an 11-pin × 11-pin geo-board. Ask students to use elastics to construct a net for the prism and discuss how they might move one of the faces to make a new net for the same prism. Have them check by recording the new net on square dot paper and cutting it out.
- Provide students with one of the 12 pentomino pieces (2-D shapes made by joining five squares along full sides). Ask if it could “fold” to make a box with no top. Have students trace this pentomino piece and then add a square for the top of the box. Ask, In how many places can this square be added? (**Note:** This can be cut from grid paper.) To the right is an example of a pentomino that could make a box.



- Tell students that this diagram is part of a net for a rectangular prism. Ask them to complete the net by drawing the three additional faces that are needed.



- Provide students with a variety of rectangular and triangular prisms and ask them to sort using the shape of the base.

SUGGESTED MODELS AND MANIPULATIVES

- dot paper
- geo-boards
- grid paper
- linking cubes
- modelling clay
- pattern blocks
- pentominos
- polydrons
- toothpicks/straws

MATHEMATICAL LANGUAGE

Teacher	Student
▪ 3-D objects ▪ attributes ▪ base ▪ faces, edges/sides, vertices/corners ▪ models ▪ nets ▪ rectangular and triangular prisms	▪ 3-D objects ▪ base ▪ faces, edges/sides, vertices/corners ▪ models ▪ nets ▪ rectangular and triangular prisms

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students K–8* (Small 2008), pp. 287–290, 292, 305–306
- Mak*ing Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 343–345, 348, 360–361
- *Teaching Student-Centered Mathematics, Grades 3–5*, Volume Two (Van de Walle and Lovin 2006), pp. 223–224

Notes

SCO G02 Students will be expected to demonstrate an understanding of congruency, concretely and pictorially. [CN, R, V]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

G02.01 Determine if two given 2-D shapes are congruent, and explain the strategy used.

G02.02 Create a shape that is congruent to a given 2-D shape, and explain why the two shapes are congruent.

G02.03 Identify congruent 2-D shapes from a given set of shapes shown in different positions in space.

Scope and Sequence

Mathematics 3	Mathematics 4	Mathematics 5
—	**G02** Students will be expected to demonstrate an understanding of congruency, concretely and pictorially.	**G02** Students will be expected to name, identify, and sort quadrilaterals, including rectangles, squares, trapezoids, parallelograms, and rhombi, according to their attributes.

Background

Congruency is a geometric property that determines what makes shapes alike and different. Two 2-D shapes are congruent if they are identical in shape and size. Students sometimes do not understand the difference between the mathematics term **congruent** and the everyday term **the same**. It is important to recognize that the term **congruent** applies only to size and shape, not colour or orientation.

Students need to have experience creating a congruent shape by observing a shape or by receiving verbal instructions on how to build a congruent shape. Given two shapes that are congruent, students should be able to make markings that indicate which sides and angles that match.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

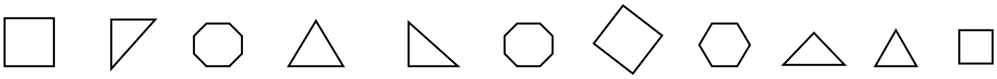
Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

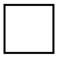
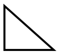
WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Provide the diagrams of 2-D shapes some of which are congruent, such as the following:



Ask students to

- put a check mark on shapes that are congruent to 
- circle the shapes that are congruent to 
- shade in the shape that is congruent to 

Invite students to explain the strategy they used to determine if the shapes were congruent. Suggest that they trace and cut out the three shapes and then superimpose them on the given shapes to prove congruency.

Have students use a Venn diagram to sort a set of shapes based on congruency.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Use everyday contexts to introduce congruence, drawing on the students' prior experiences in the real world and knowledge of 2-D shapes.
- Include many hands-on activities to establish the concept of congruence.

SUGGESTED LEARNING TASKS

- Ask students to create a square using geo-boards or multi-link cubes. Tell students that some of these squares are congruent and some are not. Give clues, such as Greg's square is not congruent to Susan's square, but it is congruent to Jane's. Continue giving clues until students discover what congruence means.
- Ask students to create congruent designs on geo-boards and draw the designs on square dot paper or grid paper. Students could cut out one design from the dot paper and superimpose it on the other design to check for congruency. It is important to test for congruency because shapes in different orientations may not appear to be congruent even when they are.
- Ask students to fold and cut rectangles into triangles and test the resulting triangles for congruency.

SUGGESTED MODELS AND MANIPULATIVES

- 2-D shapes
- geo-boards
- geometric dot paper
- grid paper
- pattern blocks
- power polygons

MATHEMATICAL LANGUAGE

Teacher	Student
▪ 2-D shapes ▪ congruency ▪ identical ▪ position in space ▪ size	▪ 2-D shapes ▪ congruency ▪ identical ▪ position in space ▪ size

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students K–8* (Small 2008), pp. 316–319
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 371–372
- *Teaching Student-Centered Mathematics, Grades 3–5*, Volume Two (Van de Walle and Lovin 2006), pp. 216–217

Notes

SCO G03 Students will be expected to demonstrate an understanding of line symmetry by
- identifying symmetrical 2-D shapes
- creating symmetrical 2-D shapes
- drawing one or more lines of symmetry in a 2-D shape

[C, CN, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

G03.01 Identify the characteristics of given symmetrical and non-symmetrical 2-D shapes.
G03.02 Sort a given set of 2-D shapes as symmetrical and non-symmetrical.
G03.03 Complete a symmetrical 2-D shape, given one-half the shape and its line of symmetry, and explain the process.
G03.04 Identify lines of symmetry of a given set of 2-D shapes, and explain why each shape is symmetrical.
G03.05 Determine whether or not a given 2-D shape is symmetrical by using an image reflector or by folding and superimposing.
G03.06 Create a symmetrical shape with and without manipulatives and explain the process.
G03.07 Provide examples of symmetrical shapes found in the environment, and identify the line(s) of symmetry.
G03.08 Sort a given set of 2-D shapes as those that have no lines of symmetry, one line of symmetry, or more than one line of symmetry.
G03.09 Explain connections between congruence and symmetry using 2-D shapes.

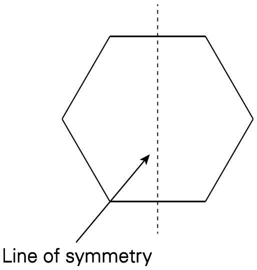
Scope and Sequence

Mathematics 3	Mathematics 4	Mathematics 5
—	**G03** Students will be expected to demonstrate an understanding of line symmetry by ▪ identifying symmetrical 2-D shapes ▪ creating symmetrical 2-D shapes ▪ drawing one or more lines of symmetry in a 2-D shape	**G04** Students will be expected to identify and describe a single transformation, including a translation, rotation, and reflection of 2-D shapes.

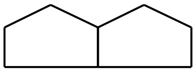
Background

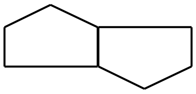
Symmetry is a geometric property. Symmetrical 2-D shapes are geometric figures that can be folded in half so that the two parts match; that is the two parts are congruent. When children are learning about symmetry, they need to spend a lot of time manipulating the shapes rather than simply looking at them. Taking the time to allow students to fold, draw, and work with models to find properties of 2-D shapes is important as it promotes visualization and is helpful in problem solving. Teachers should promote precise vocabulary usage and encourage students to regularly use mathematical language in class. Teachers can model this by using the correct terms, in context, repeatedly.

Line symmetry and congruency are closely connected. Any symmetrical shape can be divided into two congruent parts along the line of symmetry. However, not every composite shape made up of congruent figures is symmetrical. For example, this regular hexagon is symmetrical. The line of symmetry shown in the diagram divides the hexagon into two congruent shapes (pentagons).

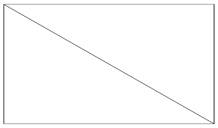


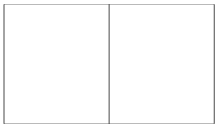
The two composite shapes below are constructed using two congruent pentagons. The first composite shape has line symmetry, but the second one does not have line symmetry.





The first rectangle below is divided into two congruent triangles, but does not have line symmetry. The second triangle below is divided into two congruent rectangles, and has line symmetry.





Students should begin to appreciate that line symmetry is a characteristic of some polygons and not others. These polygons can be described by stating how many lines of symmetry they have.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

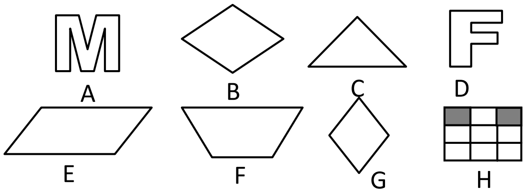
Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Provide students with different 2-D shapes. Ask them to identify how many lines of symmetry each has and to show where the lines of symmetry are.
- Ask students to give three examples of symmetrical shapes in their everyday world.
- Ask students to explain how congruent shapes are part of symmetrical shapes.
- Place the following labelled 2-D shapes before the student.



- Ask students to circle all the symmetrical shapes. Then instruct them to draw all the lines of symmetry on the symmetrical shapes. Finally, have them sort the shapes by the number of lines of symmetry in each shape: no lines of symmetry, one line of symmetry, more than 1 line of symmetry. Ask students to show the lines of symmetry.
- Given one-half of a design, invite students to create the other half and identify/explain the line(s) of symmetry.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 4 (Bauman 2011)

- Checkpoint 15, Tasks 1, 2, and 3, pp. 63–65

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?

- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

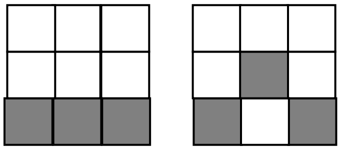
Consider the following strategies when planning daily lessons.

- Use everyday contexts to introduce symmetry, drawing on the students' prior experiences in the real world and knowledge of 2-D shapes.
- Explore that a line of symmetry can be vertical, horizontal, or diagonal.
- Ask students to explore that line symmetry is a characteristic of some polygons, but not all. Polygons can be described by stating the number of lines of symmetry they have. For example, students should discover that a square has four lines of symmetry.
- Provide students with experiences to help them understand that a line of symmetry is where a polygon can be folded onto itself so that each half matches exactly, or where a mirror can be placed so that the reflection on one side matches the shape on the other.
- Ask various students to choose one shape to present to the class and explain how they know this shape is symmetrical or not.
- Create a class "Symme-Tree." Distribute several cut-outs (some of which are symmetrical and some of which are not) in a sandwich bag to each student. Review the characteristics of symmetry and have each student test (by folding or using a Mira) each shape for symmetry. Students then place only the symmetrical shapes on the class "Symme-tree."
- Invite students to use tiles, fraction pieces, or pattern blocks to create a symmetrical design and to explain to their partner how they know their design is symmetrical.

SUGGESTED LEARNING TASKS

- Provide students with a variety of shapes and have them sort the shapes, grouping those with line symmetry and those without line symmetry.
- Ask each student to draw a picture of a shape or create a design that has line symmetry.
- Have students draw examples of triangles with symmetry and triangles without symmetry.
- Invite students to draw on squared dot paper examples of the different quadrilaterals. Cut them out and fold them to find the lines of symmetry. Use pictures of shapes with Miras also. Share and discuss.
- Provide examples of 2-D shapes with one line of symmetry, two lines of symmetry, and no lines of symmetry. Have the students draw the lines of symmetry, sort the shapes, and explain their thinking.

- Provide examples of 3 × 3 squares on grid paper. Shade three small squares so that the figure has one line of symmetry. Challenge students to make as many different patterns as they can with one line of symmetry by shading in three small squares or make figures with more than two lines of symmetry.



SUGGESTED MODELS AND MANIPULATIVES

- 2-D shapes
- geo-boards
- geometric dot paper
- grid paper
- Miras
- pattern blocks

MATHEMATICAL LANGUAGE

Teacher	Student
▪ 2-D shapes ▪ congruent ▪ lines of symmetry, mirror lines ▪ symmetry	▪ 2-D shapes ▪ congruent ▪ lines of symmetry, mirror lines ▪ symmetry

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students K–8* (Small 2008), pp. 298–301, 324, 325
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 354–356, 378, 379
- *Teaching Student-Centered Mathematics, Grades 3–5*, Volume Two (Van de Walle and Lovin 2006), pp. 234–235

Notes

Statistics and Probability (SP)

GCO: Students will be expected to collect, display, and analyze data to solve problems.

GCO: Students will be expected to use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

SCO SP01 Students will be expected to demonstrate an understanding of many-to-one correspondence.
[C, R, T, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

SP01.01 Compare graphs in which the same data has been displayed using one-to-one and many-to-one correspondences, and explain how they are the same and different.

SP01.02 Explain why many-to-one correspondence is sometimes used rather than one-to-one correspondence.

SP01.03 Find examples of graphs in print and electronic media, such as newspapers, magazines, and the Internet, in which many-to-one correspondence is used; and describe the correspondence used.

Scope and Sequence

Mathematics 3	Mathematics 4	Mathematics 5
SP01 Students will be expected to collect first-hand data and organize it using tally marks, line plots, charts, and lists to answer questions. **SP02** Students will be expected to construct, label, and interpret bar graphs to solve problems.	**SP01** Students will be expected to demonstrate an understanding of many-to-one correspondence.	**SP02** Students will be expected to construct and interpret double bar graphs to draw conclusions.

Background

In earlier grades, students have had opportunities to collect and display data in **pictographs** and **bar graphs**. In Mathematics 4, as they investigate a wider range of topics, they may discover that the data they collect is too large to display in a graph using a one-to-one correspondence (i.e., having each symbol or number on the bar graph represent one piece of data). Students need to be introduced to the concept of using a **many-to-one correspondence** or **scale** (i.e., a scale that allows a single symbol to represent a number of items) when they are creating graphs to display large amounts of data. “Once students are introduced to the concept of scale, they need to learn how to choose one that is appropriate for a given situation.” (Small 2008, p. 478). In Mathematics 4, students should begin to make decisions about what symbol to use and what that symbol should represent. These decisions are based on the data being used.

Students need to be given many opportunities to explore and choose what scale is most appropriate for their set of data. For example, if they want to display a graph to show their marble collection and they have 36 blue, 28 red, and 42 black, students may decide to draw symbols that each represent 5 marbles or create a scale in a bar graph that increases by 2. If the numbers are all less than 20, it is usually more appropriate to use a one-to-one correspondence. For larger numbers, however, students may find it better to use intervals (increments) of 2, 5, 10, 25, 100, or 1000 based on the data being graphed. Students should discuss their data displays and be able to explain why they chose their scale. Students would not be expected to use the term **interval** in their explanations, but may justify their choice by telling how they skip counted. It is important for students to ensure that the interval in their data display is consistent. For example, if they are creating a bar graph that has a scale of two, all of the numbers need to increase by two (2, 4, 6, 8, 10, 12, … and not 2, 4, 6, 7, 8, 9, 10, 12, …). Depending on the data and the scale that is selected, it may become necessary to create partial pictograph symbols and bars that fall between numbers on the scale.

Students should examine various graphs from different sources (e.g. web pages, newspapers, and magazines) to see and discuss what decisions must have been made in order to display the data. Emphasis should be placed on the analysis of various displays of data in ways that will cause students to do more than merely read information from them. They should be starting to analyze data displays to draw conclusions, to make decisions, and to stimulate other questions (e.g., If presented with a bar graph for fat and protein in foods, some students might notice that the bars for fat are always higher than the bars for protein). Other students may need to be asked questions such as, What do you notice about the bars for fat? What do you notice about the bars for protein? Is it possible to conclude from this that snack foods are not healthy choices? What questions might you want to ask a nutritionist? If you want a snack high in protein but with the lowest amount of fat intake, which food would you choose?

If many-to-one correspondence or scale is used for a pictograph, the scale must be clearly stated in a scale statement or legend. If a scale is used, the symbol chosen should allow for partial symbols that are easy to interpret. A circle or square is the ideal symbol, as it can easily be divided into fourths and halves and then the symbols are easy to interpret. If a many-to-one correspondence is used for a bar graph, the scale must be clearly shown along a numbered vertical or horizontal axis.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students’ prior knowledge.

- Ask students how they would represent the sports the children in their class play and how many students play each sport?
- Show students a bar graph on a topic of interest to students. Have them answer questions about the graphs and make up questions about the graph.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask why a symbol in a pictograph usually represents more than 1.
- Provide students with two graphs, one that displays one-to-one correspondence and the other displays many-to-one correspondence and ask them to explain the similarities and differences.
- Provide students with a set of data with large numbers and have them create a scale and a graph to display it. Ask students to justify their choice of scale and graph.
- Ask students for an example of when it would be appropriate to use a one-to-one correspondence in a real-life context.
- Ask students for an example of when it would be more appropriate to use a many-to-one correspondence in a real-life context.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 4 (Bauman 2011)
- Checkpoint 16, Task 1, pp. 67–69
- Checkpoint 17, Tasks 1 and 2, pp. 70–72
- Checkpoint 18, Task 1, pp. 73–75

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

SUGGESTED LEARNING TASKS

- Invite students to redraw a pictograph so that each symbol represents four, instead of two. Ask students which graph they prefer and give reasons for the choice. Ask if there is another way to display the data that might be clearer.
- Ask students to determine the scale for a bar graph to display the number of students travelling on each different school bus in the morning. Each step along a bar is to represent more than one student.
- Pose a question such as the following: How much television do grade 4 students watch? Have students estimate how many hours of television (or video games or computer time) they have watched in a week. Have students construct two pictographs for the same data. The intervals in one can be constructed using one-to-one correspondence and the other using many-to-one correspondence (e.g., ⌂ = 5 hours). Have students explain which of the two graphs they prefer and justify their preference.
- Invite students to explore other applications of many-to-one correspondence, such as the use of scale in mapping.
- Investigate the importance of using a consistent scale. Present the following pictograph and ask students if more people watch soccer than basketball. Discuss why the graph is misleading.

People Watching Basketball and Soccer

SUGGESTED MODELS AND MANIPULATIVES

- grid paper
- various collections of objects

MATHEMATICAL LANGUAGE

Teacher	Student
▪ appropriate scale ▪ collect, organize, display, interpret data ▪ consistent intervals (increments) ▪ graphs ▪ many-to-one correspondence ▪ symbols	▪ appropriate scale ▪ collect, organize, display ▪ skip counted ▪ graphs ▪ many-to-one correspondence ▪ symbols

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students K–8* (Small 2008), p. 505
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), p. 547

Notes

SCO SP02 Students will be expected to construct and interpret pictographs and bar graphs involving many-to-one correspondence to draw conclusions.
[C, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

SP02.01 Identify an interval and correspondence for displaying a given set of data in a graph, and justify the choice.

SP02.02 Create and label (with categories, title, and legend) a pictograph to display a given set of data, using many-to-one correspondence, and justify the choice of correspondence used.

SP02.03 Create and label (with axes and title) a bar graph to display a given set of data, using many-to-one correspondence, and justify the choice of interval used.

SP02.04 Answer a given question, using a given graph in which data is displayed using many-to-one correspondence.

Scope and Sequence

Mathematics 3	Mathematics 4	Mathematics 5
SP01 Students will be expected to collect first-hand data and organize it using tally marks, line plots, charts, and lists to answer questions. **SP02** Students will be expected to construct, label, and interpret bar graphs to solve problems.	**SP02** Students will be expected to construct and interpret pictographs and bar graphs involving many-to-one correspondence to draw conclusions.	**SP02** Students will be expected to construct and interpret double bar graphs to draw conclusions.

Background

“The emphasis or goal of this instruction should be to help students see that graphs and charts tell about information, that different types of representations tell different things about the same data. The value of having students actually construct their own graphs is not so much that they learn the techniques but that they are personally invested in the data and that they learn how a graph conveys information. Once a graph is constructed, the most important activity is discussing what it tells the people who see it, especially those who were not involved in making the graph. Discussions about graphs of real data that students have themselves been involved in gathering will help them interpret other graphs and charts that they see in newspapers and on TV.” (Van de Walle and Lovin 2006b, p. 329)

Once students have constructed a graph, it is important for students to have an opportunity to make observations and interpret the data. They should also be given experiences discussing other graphs that they can find, such as in newspapers and magazines, and on television and the Internet.

Questioning should be ongoing whenever students use graphs to encourage students to interpret the data presented and to draw inferences. It is important to ask questions that go beyond simplistic reading of a graph. Both literal questions and inferential questions should be posed. For example,

- How many … ?
- How many more/less than … ?
- Order from least to greatest / greatest to least …
- Based on the information presented in the graph, what other conclusions can be drawn?
- Why do you think ... ?

Have students discuss what kinds of information they can get from reading bar graphs and pictographs that display the use of many-to-one correspondence.

Students extend their understanding of constructing graphs and interpreting data from previous grades by exploring vertical and horizontal displays that require a many-to-one correspondence.

When creating bar graphs, the bars are spaced an equal distance apart making it easier to read and to show that each bar represents a separate or discrete category. Students should also be sure to include the labels and titles for graphs. When creating **pictographs** and **bar graphs**, it is important for students' displays to include a **title**, **labels**, and a **legend** or **key** (when applicable) on their displays. Both axes should be labeled and include headings.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

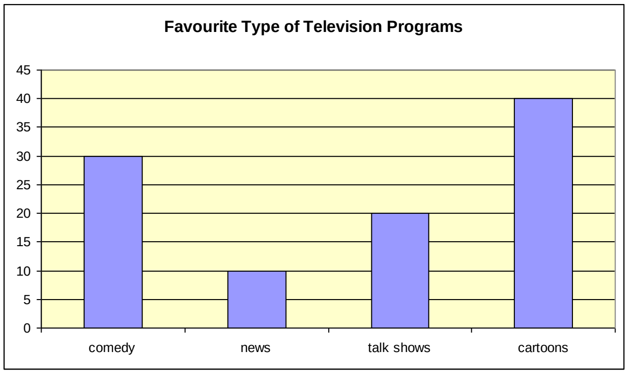
Tasks such as the following could be used to determine students' prior knowledge.

- Show students a set of data presented in chart form. Ask them to represent the data in another way, such as tally marks or a line plot.
- Provide students with data and have them construct a bar graph on grid paper. Ensure that students include a title and labels on both axes.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students what questions might be answered by interpreting this graph?



- Create and label (with categories, title, and legend) a pictograph and bar graph using the data from the table below about “Favourite Movies” using many-to-one correspondence, and justify the choice of scale or correspondence used.

Favourite Movies	
Adventure	29
Comedy	28
Drama	25
Science Fiction	35

- Throughout the year provide opportunities for students to self-assess their graphs. The following are some suggestions for students to complete.
  - I know I constructed a good graph because …
  - Some things that are similar between my graph and my classmate’s graph are …
  - Some things that are different about my graph and my classmate’s graph are …
  - When I make a graph, I choose intervals of 2 (or 5 or 10, etc.) when …
  - When I make a graph, I choose to use an interval of 1 when …

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 4 (Bauman 2011)

- Checkpoint 16, Task 1, pp. 67–69
- Checkpoint 17, Tasks 1 and 2, pp. 70–72
- Checkpoint 18, Task 1, pp. 73–75

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

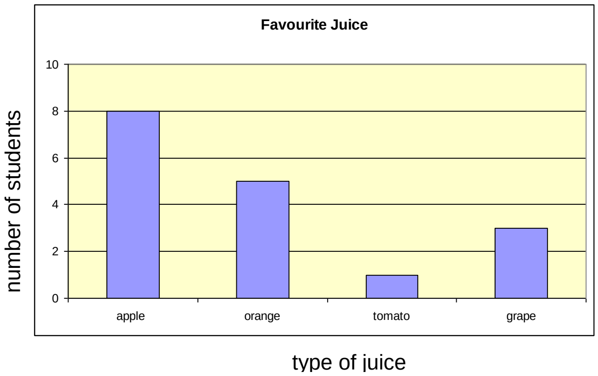
- Use pictographs based on a many-to-one correspondence (i.e., a symbol represents a group of items) and bar graphs that have intervals of more than one (i.e., increments of 2, 5, 10, 25, 100, etc.).
- Provide students with a variety of experiences to ensure that when creating bar graphs and pictographs, students have an understanding of the importance of including a title and labels and using appropriate intervals (scales) and correspondence for their data.
- Ask students to interpret and create various bar graphs and pictographs that run horizontally and vertically.
- Create graphs primarily in the context of other investigations, including other subject areas, rather than as an isolated activity to achieve the curriculum outcome.
- Allow opportunities for students to decide which scales to use for their graphs.
- Help students to investigate that many questions can be answered by looking at graphs.

SUGGESTED LEARNING TASKS

- Provide data for a bar graph, such as Favourite Sports (hockey: 36, baseball: 20, basketball: 15, soccer: 26). Have the student select a scale and create a bar graph.
- Suggest that students create a graph that shows the most popular authors, movies, types of food, etc., of class members. Have some students create a bar graph that shows the results of the data in

a scale of 2 and other groups using a scale of 3, 4, and 5. Have students explain which graph displays the most appropriate use of the data.

- Show a graph like the one below. Explain that the spacing between each horizontal line represents two students and ask questions such as, How many students like apple juice? How many more like apple juice than tomato juice? How many students answered the question about their favourite juice? Can the juices be ordered from most popular to least popular?



- Invite students to discuss what kinds of information they can get from reading existing bar graphs and pictographs that display the use of many-to-one correspondence.

SUGGESTED MODELS AND MANIPULATIVES

- grid paper
- pre-made bar graphs
- pre-made pictographs

MATHEMATICAL LANGUAGE

Teacher	Student
▪ bar graph, bar(s) ▪ collect, organize, display, interpret data ▪ consistent intervals (increments) ▪ data, collect, organize, display, interpret ▪ inferences ▪ title, label, horizontal axis, dots, crosses ▪ title, labels, scale, axis, axes, legend ▪ vertical, horizontal	▪ bar graph, bar(s) ▪ collect, organize, display ▪ skip counting ▪ data, collect, organize, display ▪ conclusions ▪ title, label, horizontal axis, dots, crosses ▪ title, labels, scale, axis, axes, legend ▪ vertical, horizontal

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students K–8* (Small 2008), pp. 503–510
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 547–554
- Teachi*ng Student-Centered Mathematics, Grades 3–5*, Volume Two (Van de Walle and Lovin 2006), pp. 329, 331, 333

Notes

Appendices

Appendix A: Performance Indicator Background

Number (N)

SCO N01 Students will be expected to represent and partition whole numbers to 10 000.
[C, CN, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

N01.01 Read a given four-digit numeral without using the word “and”.
N01.02 Record numerals for numbers expressed orally, concretely, pictorially, and/or symbolically as expressions, using proper spacing without commas.
N01.03 Write a given numeral, 0 to 10 000, in words.
N01.04 Represent a given numeral using a place-value chart or diagrams.
N01.05 Express a given numeral in expanded notation (e.g., 4321 = 4000 + 300 + 20 + 1).
N01.06 Write the numeral represented by a given expanded notation.
N01.07 Explain the meaning of each digit in a given four-digit numeral.
N01.08 Represent a given number in a variety of ways and explain how they are equivalent.
N01.09 Read a given number word, 0 to 10 000.
N01.10 Represent a given number using expressions.

Performance Indicator Background

N01.01 Students must be able to record numbers heard and read numbers written symbolically. Students should read a given four-digit numeral without using the word “and.” For example, 5321 is read as five thousand three hundred twenty-one, not five thousand three hundred and twenty-one. When reading numbers, the word **and** is reserved for the decimal, which will be discussed in outcome N09. Students should also have experience reading numbers in several ways. For example, 9347 may be read as nine thousand three hundred forty-seven, but might also be read as 93 hundreds, 4 tens, 7 ones; 9 thousands, 34 tens, 7 ones; or 9 thousands, 33 tens, 17 ones.

N01.02 Students should be given many opportunities to record numbers in symbolic form. Students should write a given numeral using proper spacing without commas. We do not use the comma because in many countries using the metric system, the comma is used as the decimal point. The accepted convention for four-digit numbers is to not leave a space (e.g., 4567). For numbers with five or more digits, leave a small space between each group of three digits starting from the right (e.g., 10 000). If too large a space is used, the number may be misinterpreted as two separate numbers. When provided with a number represented as a model, expression, expanded notation, place-value chart, or words, students need to be able to record the number symbolically in more than one way. For example, if presented with a model or picture of 5 large cubes, 2 flats, 3 rods, and 4 small cubes, the number can be recorded in many ways including, 5234; 5000, 200, 30, 4; or 5 thousands, 2 hundreds, 3 tens, 4 ones.

N01.03 and **N01.09** Students will also need to be able to write the number words for the numbers they encounter and read numbers written in words. The accepted convention for writing number words is as follows:

- fifty-six
- three hundred fifty-six
- four thousand three hundred fifty-six
- twenty-six thousand nine hundred fifty-six
- one hundred forty-six thousand nine hundred fifty-six
- one million one hundred forty-six thousand nine hundred fifty-six

N01.04 Students will learn that the position of a digit determines its value. Use of the place-value chart can support the development of this understanding. Students should represent given numbers in place-value charts. For example, students would record the number 7453 in a place-value chart as follows:

Thousands			**Ones**		
H	T	O	H	T	O
		7	4	5	3

When students examine large numbers, they develop a greater sense of the patterning in the place-value system. This exploration will help students to recognize the regularity of the patterns that are inherent in the place-value system. Students should be able to explain that the digits 0–9 are used cyclically to indicate the number of units in any given place. They should also be able to explain the relationship between each place-value position and its neighbour positions, namely a group of ten in one position makes a group of one in the position to the left and a group of one in any position makes a group of ten in the position to the right. Students have used this principle to regroup and trade in previous grades and are now able to state that this pattern continues to work regardless of the size of the number. One area of place value that may cause some confusion for students is that one number, such as 8921, can be represented in a variety of ways such as 8000 + 900 + 20 + 1; 8900 + 21; or 8920 + 1.

N01.02, **N01.05**, **N01.06**, and **N01.10** Once students have ample opportunities with concrete, pictorial, and verbal representations of base-ten models, they can record the base-ten partitions as an expression, such as 3159 is 3100 + 59. Expressions may also be recorded in the additive expanded form (expanded notation), such as 4256 is 4000 + 200 + 50 + 6. It is important to model the correct use of the term **expression** to students. An expression names a number. Sometimes an expression is a number such as 1500. Sometimes an expression shows an arithmetic operation, such as 1250 + 250 or 2000 – 500. The number 1500 may also be represented by its partitions, such as 800 + 700; 1000 + 500; and 500 + 500 + 500. Numbers can also be represented by a difference expression, such as 2000 – 500 or 1750 – 250. Students should also be provided with opportunities to write the numeral represented by a given expression.

Expanded form can be demonstrated in either of the following ways:

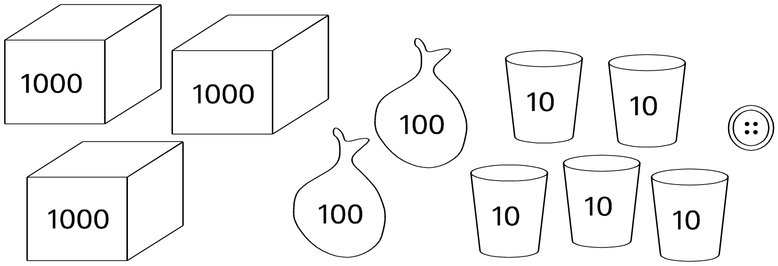
- 4123 = 4000 + 100 + 20 + 3
- 4123 = (4 × 1000) + (1 × 100) + (2 × 10) + (3 × 1)

N01.07 Students should recognize and work with the idea that the value of a digit varies depending on its position or place in a numeral. Students should recognize the value represented by each digit in a number, as well as what the number means as a whole. The digit “2” in 2300 represents 2 thousands whereas the digit “2” in 3200 represents 2 hundreds. Students should be able to explain the meaning of the digits, including numerals with all digits the same (e.g., for the numeral 2222, the first digit represents 2 thousands; the second digit, 2 hundreds; the third digit, 2 tens; and the fourth digit, 2 ones).

It is important to spend time developing a good understanding of the meaning and use of zero in numbers. Students need many experiences using base-ten materials to model numbers with zeros as digits. Teachers should ask students to write the numerals for numbers such as seven thousand five hundred forty or nine thousand two hundred eight. When a number, such as seven thousand five hundred forty, is written in its symbolic form using digits, the digit 0 is called a place holder. If the digit 0 was not used, the number would be recorded as 754, and you would mistakenly think that the 5 represented 50 instead of 500. Students need many experiences using base-ten materials to make connections with the symbols for numbers with zeros as digits.

N01.08 Students who have a deep understanding of numbers up to 10 000 will be able to represent numbers in a variety of ways. For example, 9842 is the same as 98 hundreds and 42 ones; 9 thousands, 84 tens and 2 ones; 9 thousands, 8 hundreds, 4 tens, and 2 ones; or 8 thousands, 18 hundreds, 3 tens, and 12 ones. Provide opportunities for students to represent each digit in a four-digit number using concrete materials, explaining the value of each digit.

It is important that students see the numbers up to 10 000 in different ways in order to understand that a number can cover a big area or a small area, depending on the size of the items being used. Provide opportunities for students to use hundreds charts and collections of materials such as straws, buttons, commercial counters, kidney beans, and paper clips to represent given numbers. Students will decide on various ways to organize the objects, perhaps grouping them in tens and/or hundreds and/or thousands, then presenting their numbers in pictures.



Students should recognize that 10 000 is just another expression for 10 thousands, 100 hundreds, or 1000 tens.

SCO N02 Students will be expected to compare and order numbers to 10 000.
[C, CN, V]

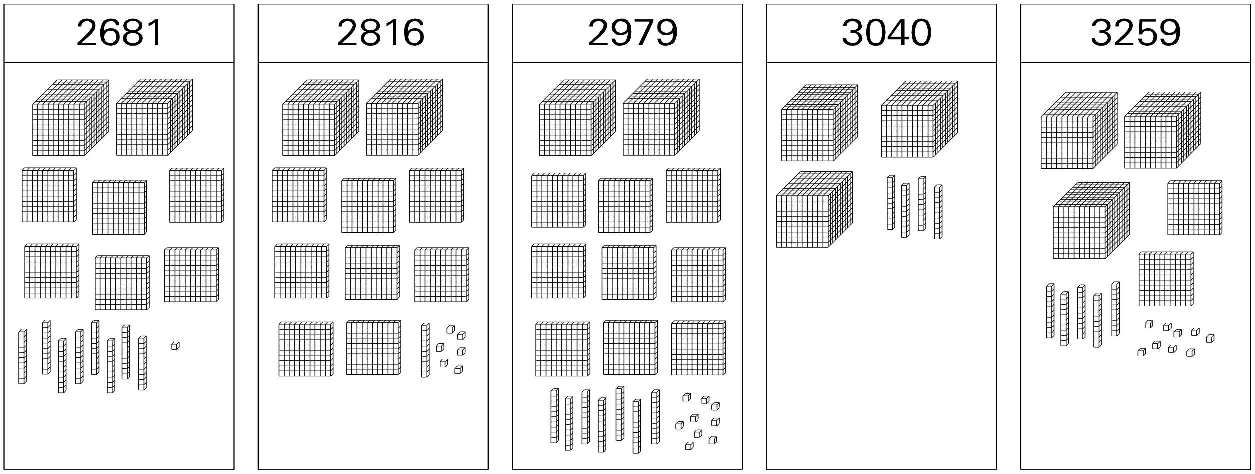
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

N02.01 Order a given set of numbers in ascending or descending order, and explain the order by making references to place value.
N02.02 Create and order three different four-digit numerals.
N02.03 Identify the missing numbers in an ordered sequence and on a number line.
N02.04 Identify incorrectly placed numbers in an ordered sequence and on a number line.
N02.05 Place numbers in relative order on an open number line.
N02.06 Place numbers on a number line containing benchmark numbers for the purpose of comparison.
N02.07 Compare numbers based on a variety of methods.

Performance Indicator Background

N02.01 Students should be given opportunities to place a given set of numbers in ascending or descending order. For example, a student may be given six or eight base-ten cards with different amounts on them and be asked to sort them from least to greatest, or vice versa. It is important for teachers to mix up the amounts so that the sets do not always represent consecutive numbers. Students could also be given cards with numerals on them and be asked to model and sort the numbers. This type of task reinforces modelling while providing an opportunity to put numbers in order. Students could verify the order by looking for the numbers on hundreds charts, by drawing a number line, or by referencing place value.



Students should not only recognize numbers that are greater or less than a number, but also be able to place numbers between two given numbers. For example, students should be able to identify a school that has a population larger than the population of their school but smaller than the population of the local high school.

N02.02 Given any four digits, students should make as many four-digit numbers as they can, then put them in order from least to greatest. For example, if given cards containing the digits 6, 3, 5, and 2, students could make any of the following numbers: 6235, 6253,6325, 6352, 6523, 6532, 5623, 5632, 5326, 5362, 5236, 5263, 3625, 3652, 3526, 3562, 3256, 3265, 2635, 2653, 2536, 2563, 2356, or 2365. Students should be able to explain how they determined all possible numbers and how they ordered them. Students should also be able to arrange the numbers from greatest to least or should be able to place the numbers on an open number line.

N02.03 and **N02.04** Students should be able to identify when a given sequence of numbers is not in the correct order and be able to correct it by rearranging them. They should be encouraged to talk about how they made their corrections.

Students should have enough familiarity with number lines that they are able to identify errors or the values of missing numbers. A student could be given a number line with numbers missing and be asked to fill in the missing values and to explain how they decided what number went in each empty position.

N02.05 Students have had experiences working with number lines that began with 0 and ended in a specific number. Now, students should have opportunities for working with number lines that begin with numbers other than 0 and have a variety of end numbers, with and without hatch marks. They should also work with open number lines. It is important that students share their thinking strategies for placing the numbers where they do. Students should use logical reasoning when identifying the approximate location of numbers on a number line, for example, when locating 1500, they reason that the number will be between 1000 and 2000 and that it will be halfway between these two numbers. Students should be encouraged to focus on reasoning and justification for their approximations, rather than focusing on determining the exact location of a number.

N02.06 Students should be able to place large numbers in approximate positions on a number line when given benchmarks. Benchmarks that students may find helpful are multiples of 100 and 1000, as well as 250, 500, 750, 2500, 5000, and 7500. Teachers should use number lines often and provide opportunities for students to construct various numbers lines.

N02.07 When comparing two numbers, students should be encouraged to make use of benchmarks. Students should say that 4850 is less than 6850 since both numbers are to the left of 10 000 on a number line but only 4850 is to the left of 5000. Similarly, 3716 is greater than 2716 since 3716 is to the right of 3000 and 2716 is to the left of 3000 on a number line. This reasoning process is part of having number sense.

Students will often refer to the number of thousands in a number in order to compare it to another; for example, 4752 is greater than 2198 since 4752 is more than 4 thousand, but 2198 is only a bit more than 2 thousand. This type of language is preferable to 4 is more than 2 so 4752 is greater, particularly since students should focus on the fact that the 4 in 4752 represents 4000, not 4, and the 2 in 2198 represents 2000, not 2. This work should be connected to the use of base-ten materials, number lines, and place-value charts.

An understanding of place value is essential for students to compare and order numbers. For example, to compare 6067 and 6607, students should notice that both numbers have 6 thousands, but that the 6607 is greater than 6067 because it has more hundreds in the hundreds place. The numbers could also be compared by considering their relative position on a number line—6067 comes before 6607, so 6667 is greater than 6067. Students should be able to compare two or more numbers, each less than 10 000, to determine relative sizes. Include situations in which numbers are located on hundreds charts and number lines. When numbers are represented in their standard or symbolic form, students can use the number of digits to get a sense of their size in order to compare them. Four-digit whole numbers are less than 10 000 but are greater than any two- or three-digit whole number.

Students should be able to compare and order a set of numbers in using a variety of methods. Students should continue to model numbers with base-ten materials, both concretely and pictorially, and should be encouraged to visualize the base-ten block representations of numbers. They should also use contextual situations, number lines, hundreds charts, and place value to help them in comparing and ordering numbers.

SCO N03 Students will be expected to demonstrate an understanding of addition and subtraction of numbers with answers to 10 000 (limited to three- and four-digit numerals) by

- using personal strategies for adding and subtracting
- estimating sums and differences
- solving problems involving addition and subtraction

[C, CN, ME, PS, R]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

N03.01 Represent concretely, pictorially, and symbolically the addition and subtraction of whole numbers, limited to three- and four-digit numerals.
N03.02 Determine the sum of two given numbers, limited to three- and four-digit numerals, using a personal strategy, and record the process symbolically.
N03.03 Determine the difference of two given numbers, limited to three- and four-digit numerals, using a personal strategy, and record the process symbolically.
N03.04 Describe a situation in which an estimate rather than an exact answer is sufficient.
N03.05 Estimate sums and differences using different strategies.
N03.06 Create and solve problems that involve addition and subtraction of two or more numbers, limited to three- and four-digit numerals.
N03.07 Explain mental mathematics strategies that could be used to determine a sum or difference.
N03.08 Determine a sum or difference of one-, two-, and three-digit numerals efficiently, using mental mathematics strategies.

Performance Indicator Background

N03.01 When introducing addition and subtraction with three- and four-digit numbers, it is important to use base-ten blocks to model the operations concretely, before representing the calculation pictorially or symbolically. Students should be able to model the sum and difference of two given numbers up to four-digits using base-ten blocks, and use symbols to record the processes that reflect their actions with those blocks. For example, to subtract 437 from 1265, students may represent 437 with 4 flats, 3 rods, and 7 small cubes in one set; in a second set they may place 8 more flats saying 1237, 2 rods saying 1257, and 8 small cubes saying 1265; determine that 828 is the total in the second set; and state that the difference between 437 and 1265 is 828. The students would record 437 + 800 + 20 + 8 = 1265 and 800 + 20 + 8 = 828 to reflect the counting-up strategy they did with the blocks.

Pictorial representations may include student-generated pictures, those described on page 67 of *Teaching Student-Centered Mathematics, Grades K–3* by John Van de Walle and LouAnn Lovin (2006), or strip diagrams as described below. It is important that the pictures students draw represent **their** thinking and should mirror their work with models.

After students have modelled and solved a number of addition and subtraction situations, they may be introduced to strip diagrams as another way to represent the situations. For example, Bobby was given 363 green stamps. He already had 2127 stamps. How many stamps does he have now? The strip diagram for this problem is as follows:

<table>
<tr><td>2127</td><td>363</td></tr>
<tr><td colspan="2">?</td></tr>
</table>

As another example, Bobby had 987 stamps. He was given more by his friend. Then he had 1537 stamps. How many stamps did his friend give him? The strip diagram for this problem is as follows:

987	?
1537	

The principal use of strip diagrams is as a strategy to help students interpret story problems. Because students have to decide where to place in the diagram the two given numbers in the story problem, they have to carefully read the problem to determine whether each given quantity is a part or the whole. If the quantity is a part, it would be placed in one section of the top rectangle; if the quantity is the whole it would be placed in the bottom rectangle. They should put a question mark in the bottom rectangle or one of the sections in the top rectangle, depending upon what is missing (what they are asked to find).

N03.02 and **N03.03** It is expected that students will be able to symbolically add and subtract two four-digit numbers using reliable, accurate, and efficient strategies. Students should be able to explain their strategy and whether their solution is reasonable based on their prior estimate. Through the sharing of strategies, students will be exposed to a variety of possible addition and subtraction strategies, and each student will adopt ones that he or she understands well and has made his or her own. That is why these strategies are often referred to as “personal strategies.” The most appropriate strategy used may vary depending on the student and the numbers involved in the problem.

Personal strategies make sense to students and are as valid as the traditional algorithm. Therefore, emphasis should be on students’ algorithms rather than on the traditional algorithm. Most important is that students can justify how and why an algorithm works. Students should be encouraged to refine their strategies to increase their efficiency, and teachers should monitor each student’s symbolic recording of the strategy to ensure that the recording is accurate, mathematically correct, organized, and efficient.

Examples of personal strategies and symbolic recordings are shown below.

ADDITION EXAMPLE 1

4237 + 3478

If students are asked to add 4237 and 3478, students could determine the sum by doing the following:

- Start by writing 4237 as 4000 + 200 + 30 + 7 and 3478 as 3000 + 400 + 70 + 8.
- Add 4000 and 3000 to get a sum of 7000.
- Add 200 and 400 to get a sum of 600.
- Add 30 and 70 to get a sum of 100.
- Add 7 and 8 to get a sum of 15.
- Add 7000, 600, 100, and 15 to get a sum of 7715.

This may be recorded on paper as

4237 + 3478 = 4000 + 200 + 30 + 7 + 3000 + 400 + 70 + 8
4000 + 3000 = 7000
200 + 400 = 600
30 + 70 = 100
7 + 8 = 15
7000 + 600 + 100 + 15 = 7715

or

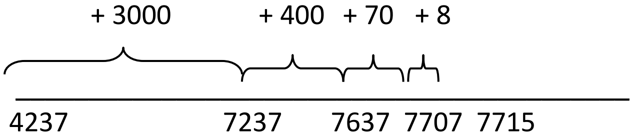
$$\begin{array}{r} 4237 \\ \underline{+\ 3478} \\ 7000 \\ 600 \\ 100 \\ \underline{+\quad 15} \\ 7715 \end{array}$$

ADDITION EXAMPLE 2

4237 + 3478

- Start with the larger number 4237.
- Add 3000 to get a sum of 7237.
- Add 400 to 7237 get a sum of 7637.
- Add 70 to 7637 to get a sum of 7707.
- Add 8 to 7707 to get a sum of 7715.

This can be represented as jumps on a number line.



This may be recorded on paper as follows:

4237 + 3478
4237 + 3000 = 7237
7237 + 400 = 7637
7637 + 70 = 7707
7707 + 8 = 7715

ADDITION EXAMPLE 3

4237 + 3478

- Start by placing one addend below the other.
- Add 7 and 8 and place the sum of 15 below the two addends in line 1.
- Add 30 and 70 and place the sum of 100 in line 2.
- Add 200 and 400 and place the sum of 600 in line 3.
- Add 4000 and 3000 and place the sum of 7000 in line 4.
- Add the four lines to get a sum of 7715.

This may be recorded on paper as follows:

$$\begin{array}{rl} 4237 & \\ \underline{+\ 3478} & \\ 15 & \text{(line 1)} \\ 100 & \text{(line 2)} \\ 600 & \text{(line 3)} \\ \underline{+\ 7000} & \text{(line 4)} \\ 7715 & \end{array}$$

This same strategy might also be recorded as follows:

4237 + 3478 = 7 + 8 + 30 + 70 + 200 + 400 + 4000 + 3000
7 + 8 = 15
30 + 70 = 100
200 + 400 = 600
4000 + 3000 = 7000
15 + 100 + 600 + 7000 = 7715

ADDITION EXAMPLE 4

4237 + 3478

- Start by decomposing 3478 into 63 and 3415.
- Combine the 63 with 4237 to get a sum of 4300.
- Add 3415 to 4300 to get a sum of 7715.

This may be recorded on paper as follows:

4237 + 3478 = 4237 + 3415 + 63
4237 + 63 = 4300
4300 + 3415 = 7715

ADDITION EXAMPLE 5

4237 + 3478

- Start by adding 3500 to 4237 to get a sum of 7737.
- Subtract 22 from 7737 to get a difference of 7715.

This may be recorded on paper as follows:

4237 + 3478
4237 + 3500 = 7737
7737 – 22 = 7715
4237 + 3478 = 7715

ADDITION EXAMPLE 6

4237 + 3478

- Start by placing one addend below the other.
- Add 7 ones and 8 ones to get a sum of 15 ones.
- Regroup the 15 ones into 1 ten and 5 ones.
- Record a 1 in the tens' place above the addends.
- Record a 5 in the ones' place below the line.
- Add 3 tens, 7 tens, and 1 ten (from regrouping the ones) to get a sum of 11 tens.
- Regroup the 11 tens into 1 hundred (10 of the tens) and 1 ten.
- Record a 1 in the hundreds' place above the addends.
- Record 1 in the tens' place below the line.
- Add 2 hundreds, 4 hundreds, and 1 hundred (from the regrouping of the tens) to get a sum of 7 hundreds.
- Record a 7 in the hundreds' place below the line.
- Add 4000 thousand and 3000. Record a 7 in the thousands' place below the line.

This may be recorded on paper as follows:

$$\begin{array}{rcccc} & & {\scriptstyle 1} & {\scriptstyle 1} & \\ & 4 & 2 & 3 & 7 \\ \underline{+} & \underline{3} & \underline{4} & \underline{7} & \underline{8} \\ & 7 & 7 & 1 & 5 \end{array}$$

SUBTRACTION EXAMPLE 1

1526 – 239
We knew we had to subtract 239 from 1526. So we started with 1 large cube, 5 flats, 2 rods, and 6 small cubes to show 1526. We remove 2 flats. We needed to remove 3 rods, so we traded one flat cube for 10 rods (giving us 12 rods in all) and removed 3 rods. Finally we removed 9 small cubes, after we traded 1 rod for 10 small cubes.

This could be recorded on paper as follows:

1526 – 239 = ?
1526 – 200 = 1326
1326 – 30 = 1296
1296 – 9 = 1287

SUBTRACTION EXAMPLE 2

1526 – 239
We used an empty number line to find the difference between 239 and 1526. We put 239 and 1526 on the line. We made a jump of 1 from 239 to 240. Next, we made a jump of 60 from 240 to 300. Then we made a jump of 200 from 300 to 500. Then we made a jump of 1000 from 500 to 1500. Then we made a jump of 26, from 1500 to 1526. So, we combined all of our jumps, 1 + 60 + 200 + 1000 + 26, to get the difference of 1287.

This could be recorded on paper as follows:

239 + ? = 1526
239 + 1 = 240
240 + 60 = 300
300 + 200 = 500
500 + 1000 = 1500
1500 + 26 = 1526

1 + 60 + 200 + 1000 + 26 = 1287

SUBTRACTION EXAMPLE 3

1526 – 239
To solve 239 + ? = 1526, we started with 2 flats, 3 rods, and 9 small cubes to show 239. We added 1 large cube and had 1239. We added 2 flats to get to 1439. We added 1 small cube to get to 1440. Then, we added on 6 rods to get to 1500. Then we added on 2 rods and 6 small cubes to get to 1526. So, we

looked at everything we had added on (1 large cube, 2 flats, 1 small cube, 6 rods, 2 rods, and 6 small cubes) and knew that we had added on 1287.

This could be recorded on paper as follows:

239 + ? = 5526
239 + 1000 = 1239
1239 + 200 = 1439
1439 + 1 = 1440
1440 + 60 = 1500
1500 + 26 = 1526
1000 + 200 + 1 + 60 + 26 = 1287

SUBTRACTION EXAMPLE 4

1526 – 239
We knew we had to subtract 239 from 1526. We decided to subtract 240 instead because it was easier to work with. So, we started at 1526, jumped back 200 to 1326. Then we jumped back 20 to 1306, and then jumped back another 20 to 1286. But we knew we had jumped back 1 too many and so we moved to 1287.

This could be recorded on paper as follows:

1526 – 200 = 1326
1326 – 20 = 1306
1306 – 20 = 1286
1286 + 1 = 1287

SUBTRACTION EXAMPLE 5

1526 – 239
We wanted to subtract a friendly number. It would be nice to subtract 300. So, we changed 239 to 300 by adding on 61. Since we added 61 to 239, we had to add 61 to 1526 to keep our constant difference. Then, we had a nice question to solve mentally, 1587 – 300 = 1287.

This could be recorded on paper as follows:

239 + 61 = 300
1526 + 61 = 1587
1587 – 300 = 1287
1526 – 239 = 1587 – 300 = 287

SUBTRACTION EXAMPLE 6

1526 – 239

- Start by placing the subtrahend below the minuend.
- You cannot subtract 9 ones from 6 ones. You must regroup 1 ten for 10 ones. Cross out the 2 and record a 1 above it in the tens' place.
- Combine the 10 ones with the 6 ones, and record a 1 to the left of the 6 in the ones' place. You have 16 ones.
- 16 ones subtract 9 ones is 7 ones. Record a 7 below the line in the ones place.
- You cannot subtract 3 tens from 1 ten. You must regroup 1 hundred for 10 tens. Cross out the 5 and record a 4 above it in the hundreds' place.
- Combine the 10 tens with the 1 ten and record a 1 to the left of the 1 in the tens' place.
- 11 tens subtract 3 tens is 8 tens. Record an 8 below the line in the tens' place.
- Subtract 2 hundreds from 4 hundreds. Record a 2 below the line in the hundreds' place.
- There are no thousands to subtract, so record a 1 below the line in the thousands' place.

This may be recorded on paper as follows:

$$\begin{array}{r} \scriptstyle 1\,1 \\ 15\cancel{2}6 \\ \underline{-\ 239} \\ 7 \end{array} \qquad \begin{array}{r} \scriptstyle 4\ 11\ 1 \\ 1\cancel{5}\cancel{2}6 \\ \underline{-\ 239} \\ 87 \end{array} \qquad \begin{array}{r} \scriptstyle 4\ 11\ 1 \\ 1\cancel{5}\cancel{2}6 \\ \underline{-\ 239} \\ 287 \end{array} \qquad \begin{array}{r} \scriptstyle 4\ 11\ 1 \\ 1\cancel{5}\cancel{2}6 \\ \underline{-\ 239} \\ 1287 \end{array}$$

Subtraction Example 7

1526 – 239

Think addition to solve subtraction. To solve 239 + ? = 1526, we started with 2 flats, 3 rods, and 9 small cubes to show 239. We added 1 large cube. Then, we added 3 flats, but we knew that was too much because we had 1539. So, we removed 9 small cubes, and we had 1530. We still needed to remove 4 more small cubes to get to 1526. So, we traded 1 rod for 10 small cubes, and removed the 4 small cubes.

This could be recorded on paper as follows:

1526 – 239 = ?
239 + ? = 1526
239 + 1000 = 1239
1239 + 300 = 1539
1539 – 9 = 1530
1530 – 4 = 1526
1300 – 13 = 1287

CORRECTING STUDENT RECORDING ERRORS

Regardless of the personal strategy used, the teacher must monitor each student's paper-and-pencil recording of the strategy to ensure that it is mathematically correct, organized, and efficient. Particular attention must be paid to students' recordings to ensure they are not using the equal sign incorrectly. For example, to solve 237 + 478, a student could accurately record his or her thinking as follows:

Method A

237 + 478 = 200 + 30 + 7 + 400 + 70 + 8
200 + 400 = 600
30 + 70 = 100
7 + 8 = 15
600 + 100 + 15 = 715

Method B

$$\begin{array}{r} 237 \\ \underline{+\ 478} \\ 600 \\ 100 \\ \underline{+\ \ \ 15} \\ 715 \end{array}$$

Method C

237 + 478 = 200 + 400 + 30 + 70 + 7 + 8 = 600 + 100 + 15 = 715

However, if a student recorded his or her thinking as
237 + 478 = 200 + 30 + 7 + 400 + 70 + 8 = 200 + 400 = 600 + 30 + 70 = 700 + 7 + 8 = 715,
it would be necessary to work with the student to correct the recording error. Correction would be necessary, as this is an example of the incorrect use of the equal sign. It may result from a student's misunderstanding of the meaning of the equal sign. One way to address this is to have students verify the accuracy of the recording by asking them to read the equal sign as "is the same as." In the example above, it is correct to say

237 + 478 is the same as 200 + 30 + 7 + 400 + 70 + 8
237 + 478 is the same as 600 + 100 + 15
237 + 478 is the same as 715
200 + 30 + 7 + 400 + 70 + 8 is the same as 600 + 100 + 15
200 + 30 + 7 + 400 + 70 + 8 is the same as 715
600 + 100 + 15 is the same as 715

However, it is incorrect to say

237 + 478 is the same as 200 + 400
237 + 478 is the same as 600 + 30 + 70
237 + 478 is the same as 200 + 400
200 + 30 + 7 + 400 + 70 + 8 is the same as 200 + 400, etc.

N03.04 and **N03.05** The ability to estimate computations is a major goal of any modern computational program. For most people in their everyday lives, an estimate is all that is needed to make decisions, and to be alert to the reasonableness of numerical claims and answers generated by others and with technology. The ability to estimate rests on a strong and flexible command of facts and mental calculation strategies.

Before attempting pencil-and-paper or calculator computations, students must determine estimates, so they are alert to the reasonableness of those pencil-and-paper or calculator answers. Teachers should also model this process before personally doing any calculations in front of the class, and should constantly remind students to estimate before calculating. While teaching estimation strategies, it is important to use the language of estimation. Some of the common words and phrases are *about*, *just about*, *between*, *a little more than*, *a little less than*, *close*, *close to*, and *near*.

Students need to learn that estimation is a very useful skill in their lives. Often, an estimate, rather than an exact answer is sufficient in everyday life. To be efficient when estimating sums and differences mentally, students must be able to access a strategy quickly, and they need a variety of strategies from which to choose. Some strategies to consider are using benchmarks, rounding, front-end addition, making a friendly number, compensation, subtraction (left-to-right calculations), and clustering of compatible numbers.

FRONT-END ESTIMATION

This strategy involves adding or subtracting only the values in the highest place-value positions to get an estimate, only if they have the same number of digits. Such estimates are adequate in many circumstances, including getting an estimate before computations with technology in order to be alert to the reasonableness of the answers. For addition, because only the highest place values are added, the estimated front-end sum will always be less than the actual sum. For subtraction, this is not always the case.

ADJUSTED FRONT-END ESTIMATION

This strategy begins by getting a front-end estimate and then adjusting that estimate to get a better, or closer, estimate by either (a) considering the second highest place values or (b) by clustering all the values in the other place values to determine whether there would be enough together to account for an adjustment. These two adjustment strategies will not always result in the same adjustment being made.

ROUNDING IN ADDITION

This strategy involves rounding each number to the highest, or the highest two, place values and adding the rounded numbers. Rounding to the highest place value would enable most students to keep track of the rounded numbers and do the calculation in their heads; however, rounding to two highest place values would probably require most students to record the rounded numbers before performing the calculation mentally.

ROUNDING IN SUBTRACTION

For rounding in subtraction situations, the process is similar to that for addition, except for the situations in which both numbers involve 5, 50, or 500, and in situations in which both numbers are close to 5, 50, or 500. For rounding in these situations, both numbers should be rounded up because you are looking for the difference between the two numbers; therefore, you don’t want to increase this difference by rounding one up and one down. This will require careful introduction for students to be convinced. So often students only associate subtraction with *take-away* and need to reminded that subtraction also finds the difference between two numbers. (Help them make the connection to the balancing-for-a-constant-difference strategy in mental mathematics.)

CLUSTERING OF NEAR COMPATIBLES

This strategy is useful when you need to estimate the sums and differences in a list of numbers. You examine the list to search for pairs of numbers that are near known compatibles that make 100s or 1000s. These pairs provide estimates for 100 or 1000, and are combined with other such estimates, as well as estimates of any leftovers, to get a total estimate for the list.

N03.06 Students should create and solve addition and subtraction story problems of all structures.

- Join (result, change, and start unknown)
- Separate (result, change, and start unknown)

- Part-part-whole (part and whole unknown)
- Compare (difference, smaller, and larger unknown)

Join story problems all have an action that causes an increase, while separate story problems have an action that causes a decrease. Part-part-whole story problems, on the other hand, do not involve any actions, and compare story problems involve relationships between quantities rather than actions.

Examples of these various types of problems appear in the table provided below.

Join			**Part-Part-Whole**	**Compare**
Result Unknown	**Change Unknown**	**Start Unknown**	**Whole Unknown**	**Difference Unknown**
Zaire earned $728 last year selling newspapers. This year he earned $815. How much money did he earn in all? 728 + 815 = ?	Last week 2115 kg of blueberries were picked in Oxford. Some more blueberries were picked this week, giving a total of 4236 kg of blueberries picked. How many kilograms of blueberries were picked this week? 2115 + ? = 4236 or 4236 – 2115 = ?	The grade 4 class is fundraising for a community centre. A donor just gave them $563 and now they have $4,998. How much money did they have before the donation? ? + 563 = 4998 or 4998 – 563 = ?	There are 317 boys and 248 girls in a school. How many students are in the school? 317+ 248= ?	Mary sold 1278 greeting cards for the school fundraiser. Chantella sold 195. How many more greeting cards did Mary sell than Chantella sold? 195 + ? = 1278 or 1278 – 195 = ?
Separate			**Part-Part-Whole**	**Compare**
Result Unknown	**Change Unknown**	**Start Unknown**	**Part Unknown**	**Smaller or Larger Unknown**
Gavin collected 239 seashells in his bucket. He gave his brother 103 of those seashells. How many seashells does he have left? 239 – 103 = ?	Kayla had 156 g of sugar. She used some to make cookies and has 83 g left. How much sugar did she use? 156 – ? = 83 or 156 – 83 = ?	A company had some books to donate to schools. They gave the first school 2356 of them. They still have 3517 books to give away. How many books did they have to begin with? ? – 2356 = 3517 or 2356 + 3517= ?	There were 4735 people at a concert. If 1352 of them were children, how many were adults? 1352 + ? = 4735 or 4735 – 1352 = ?	Our school collected 4387 bottles for the recycling project. Another school collected 2185 more bottles than our school. How many bottles did the other school collect? 4387 + 2185 = ?

Students should be able to solve story problems of different types by writing the most efficient open number sentences and computing the sums or differences to find the solutions. They should be able to do this either directly upon reading the problem, or by drawing or visualizing pictures that represent the problem. In the table, there are number sentences that students may generate depending upon how they think about the problem. For example, consider the Join (change unknown) problem: Last week 2115 kg of blueberries were picked in Oxford. Some more blueberries were picked this week, giving a total of 4236 kg of blueberries. How many kilograms of blueberries were picked week? If students solved this by starting with 2115, adding on until they reached 4236, and determining what they added on, they would represent the problem in symbols as 2115 + ? = 4236. On the other hand, if students solved it by starting with 4236, removing 2115, and determining what was left, they would represent the problem as 4236 – 2115 = ?

Students should be encouraged to model the story problems with base-ten blocks, and write number sentences that reflect their thinking. All these story problems can also be modelled using a variety of pictorial representations including student-generated pictures, those described on page 67 of *Teaching Student-Centered Mathematics, Grades K–3* (2006) by John Van de Walle and LouAnn Lovin, or strip diagrams as described above. It is important that the pictures students draw represent their thinking and should mirror their work with models.

Students should be able to create story problems given an addition or subtraction number sentence. In order for their story problems to go beyond simple result-unknown types, they will need to have very specific experiences in which they create story problems similar to ones that are modelled. For example, students should be presented with four or five join (change unknown) story problems and, after they solve those problems, they should be asked to create a story problem similar to the join problems they were presented, but in a different context.

N03.07 and **N03.08** Mental mathematics for addition and subtraction is an expectation for this grade. In general, a mental mathematics computational strategy should be introduced in isolation from other strategies, a variety of different reinforcement activities should be provided until it is mastered, the strategy should be assessed in a variety of ways, and then it should be combined with other previously learned strategies.

INTRODUCING A STRATEGY

The approach to highlighting a computational strategy is to give the students an example of a computation for which the strategy would be useful to see if any of the students already can apply the strategy. If so, the student(s) can explain the strategy to the class. If not, the teacher could share the strategy themselves. The explanation of a strategy should include anything that will help students see the pattern and logic of the strategy, be that concrete materials, visuals, and/or contexts. The introduction should also include explicit modelling of the mental processes used to carry out the strategy, and explicit discussion of the situations for which the strategy is most appropriate and efficient. Discussion should also include situations for which the strategy would not be the most appropriate and efficient one. Most important is that the logic of the strategy be well understood before it is reinforced, otherwise its long-term retention will be very limited.

REINFORCING A STRATEGY

Each strategy for building mental computational skills should be practised in isolation until students can give correct solutions in a reasonable time frame. Students must understand the logic of the strategy, recognize when it is appropriate, and explain the strategy. The amount of time spent on each strategy should be determined by the students' abilities and previous experiences.

The reinforcement activities for a strategy should be varied in type and should focus as much on the discussion of how students obtained their answers as on the answers themselves. The reinforcement activities should be structured to ensure maximum participation. Time frames should be generous at first, then narrowed as students internalize the strategy. Student participation should be monitored and their progress assessed in a variety of ways to help determine how long should be spent on a strategy.

After most of the students have internalized the strategy, help them integrate it with other strategies they have developed. Do this by providing activities that include a mix of number expressions for which this strategy and others would apply. Students should complete the activities and discuss the strategy/strategies that could be used or students could match the number expressions included in the activity to a list of strategies and discuss the attributes of the number expressions that prompted them to make the matches.

Students should hear and see the teacher use a variety of language associated with each operation so they do not develop a single word-operation association. Through rich language usage students are able to quickly determine which operation and strategy they should employ. For example, when students hear the teacher say, “Sixty plus fifty,” “Sixty and fifty more,” “The total of sixty and fifty,” “The sum of sixty and fifty,” or “Fifty more than sixty,” they should be able to quickly determine that they must add 60 and 50.

Present students with a variety of contexts for each operation in some of the reinforcement activities, so they are able to transfer the use of operations and strategies to situations found in their daily lives. By using contexts, the numbers and operations become more meaningful to the students. Contexts also provide opportunities for students to recall and apply other common knowledge that should be well known. For example, when a student hears you say, “How many days in eight weeks?” they should be able to recall that there are 7 days in a week and that 8 groups of 7 days would be 56 days.

The recognition and extension of number patterns can reinforce strategy development. For example, when a student is asked to extend the pattern “30, 60, 120, …,” one possible extension is to double the previous term to get 240, 480, 960. Another possible extension, found by adding multiples of 30, would be 210, 330, 480. Both possibilities require students to mentally calculate numbers using a variety of strategies.

ASSESSING A STRATEGY

Assessment of computational strategies should take a variety of forms. In addition to the traditional quizzes that involve students recording answers to questions given one at a time in a certain time frame, teachers should also record any observations made during the reinforcement activities. Students should also be asked for oral responses and written explanations of strategies. Individual interviews can provide many insights into a students’ thinking, especially in situations where pencil-and-paper responses are weak.

Assessments, regardless of their form, should shed light on students’ abilities to compute efficiently and accurately, to select appropriate strategies, and to explain their thinking.

Response time is an effective way to see if students can use the computational strategies efficiently and to determine if students have automaticity of their facts.

For the facts, the goal is to get a response in three seconds or less. Students should be given more time than this in the initial strategy introduction and during reinforcement activities. The response time can be reduced as the students become more proficient applying the strategy until the three-second goal is reached. In Mathematics 4, when the addition and subtraction facts are extended to 10s, 100s, and 1000s, a three-second response should also be expected by the end of the year. The three-second response goal is a guideline for teachers and does not need to be shared with students if it will cause undue anxiety.

With mental-calculation strategies and computational estimation, allow 5 to 10 seconds, depending upon the complexity of the mental activity required. Again, in the initial application of these strategies, allow as much time as needed to ensure success, and gradually decrease the wait time until students attain solutions in a reasonable time frame.

After students have achieved competency using one strategy, provide opportunities for them to integrate it with other strategies they have learned. The ultimate goal is for students to have a network of mental strategies that they can flexibly and efficiently apply whenever a computational situation arises. This integration can be aided in a variety of ways, some of which are described below.

Provide students with a variety of questions, some of which could be done just as efficiently by two or more different strategies and some of which are most efficiently done by one specific strategy. It is important to have a follow-up discussion of the strategies and the reasons for the selection of specific strategies. Take every opportunity that arises in regular mathematics class time to reinforce the strategies learned in mental mathematics time. Include written questions in regular mathematics time. This could be as a journal entry, a quiz/test question, part of a portfolio, or other assessment for which students will get individual feedback. Students should be asked to explain how they could mentally compute a given question in one or more ways, to comment on a student response that has an error in thinking, or to generate sample questions that would be efficiently done by a specified strategy.

MENTAL MATHEMATICS STRATEGIES: ADDITION

Facts Applied to Multiples of 10, 100, and 1000

At the beginning of Mathematics 4, it is important that students review the addition facts to 18 and the fact-learning strategies. Recalling the addition facts with proficiency was a Mathematics 2 expectation. These facts are then applied to 10s and 100s in Mathematics 3. In Mathematics 4, they should be extended to 1000s. It would be beneficial to connect these sums to the addition of two groups of base-ten blocks. For example, for 5 small cubes and 6 small cubes or 5 rods and 6 rods or 5 flats and 6 flats or 5 large cubes and 6 large cubes, the results will all be 11 blocks, be they 11 ones, 11 tens, 11 hundreds, or 11 thousands. The sums of tens are a little more difficult than the sums of hundreds and thousands because when the answer is more than 10 tens, students have to translate the number. For example, for 70 + 80, 7 tens and 8 tens are 15 tens, or one hundred fifty, while 700 + 800 is 7 hundreds and 8 hundreds which is 15 hundreds, or fifteen hundred.

Front-End Addition

This strategy is applied to questions that involve two combinations of non-zero digits having the same number of digits, one combination of which may require regrouping. The strategy involves first adding the digits in the highest place-value position, then adding the non-zero digits in any other or the next place-value position, and doing any needed regrouping. This was applied to all two two-digit numbers in Mathematics 3 and should be extended in Mathematics 4 to those two three-digit and four-digit whole number calculations that will require only two combinations. For example, 375 + 542, add the 3 hundreds and the 5 hundreds to make 8 hundreds, then add the 7 tens and 4 tens to make 11 tens, trade 10 tens for 1 hundred to make 9 hundreds with 1 ten left, then add 5 ones and 2 ones to make 7 ones to get 917.

Sample Tasks:

- For 37 + 26, think, 30 and 20 is 50, 7 and 6 is 13, and 50 plus 13 is 63.
- For 450 + 380, think, 400 and 300 is 700, 50 and 80 is 130, and 700 plus 130 is 830.
- 190 + 430 =

- I read 340 pages in one novel and 280 pages in another novel. How many pages did I read in total?
- Teesha likes to travel. She drove 290 km the first day and 120 km the following day. What is the sum of the kilometres Teesha travelled?
- Dan walked halfway around the rectangular ball field. He walked 470 m down one side and 360 m down another side. How far did Dan walk?
- $\begin{array}{r} \$607 \\ \underline{+\ \$309} \end{array}$

Quick Addition
This strategy is actually the front-end strategy, applied to questions that involve more than two combinations and with no regrouping needed. The questions are always presented visually, and students quickly record their answers on paper. While this is a pencil-and-paper strategy because answers will always be recorded on paper before answers are read, it is included here as a mental mathematics strategy because most students will do all the combinations in their heads starting at the front end.

This strategy requires students to holistically examine each question to confirm there will be no regrouping. This habit of holistically examining each question as a first step in determining the most efficient strategy needs to pervade all mental mathematics lessons. (One suggestion for an activity during the discussion of this strategy is to present students with a list of 20 questions, some of which do require regrouping, and direct students to apply quick addition to the appropriate questions and leave out the other ones.) It is important to present examples of these addition questions in both horizontal and vertical formats. Most likely, students will add the digits in corresponding place values of the two addends without consciously thinking about the names of the place values. Therefore, in the discussion of the questions, encourage students to read the numbers correctly and to use place-value names. This will reinforce place-value concepts at the same time as addition.

Examples:

- For 543 + 256, think and record each resultant digit: 5 and 2 is 7, 4 and 5 is 9, and 3 and 6 is 9, so the sum is 799 (seven hundred ninety-nine); or think, 500 and 200 is 700, 40 and 50 is 90, 3 and 6 is 9, for a sum of 799.
- For 2341 increased by 3415, think and record each resultant digit: 2 and 3 is 5, 3 and 4 is 7, 4 and 1 is 5, and 1 and 5 is 6, so the sum is 5756 (five thousand seven hundred fifty-six); or think, 2000 and 3000 is 5000, 300 and 400 is 700, 40 and 10 is 50, 1 and 5 is 6, for a sum of 5756.

Sample Tasks:

- $715 + $123 =
- $\begin{array}{r} 314 \\ \underline{+\ 263} \end{array}$
- 770 increased by 129
- One bulletin board has a length of 870 cm. Another bulletin board has a length of 109 cm. What is the combined length of the two bulletin boards?
- The total of 6621 km and 2100 km
- 1452 increased by 8200
- $4678 + $3211
- $\begin{array}{r} 6334 \\ \underline{+\ 2200} \end{array}$

- 3700 more than 5200
- The sum of 6245 and 1712 is

Finding Compatibles

This strategy for addition involves looking for pairs of numbers that combine easily to make a sum that will be easy to work with. In Mathematics 4, this should involve searching for pairs of numbers that have a sum of 10, 100, or 1000. Some examples of common compatible numbers are 1 and 9, 40 and 60, 300 and 700, and 75 and 25. (In some resources, these compatible numbers are referred to as *friendly* numbers or nice numbers.) Students should be certain that the numbers in an addition expression can be combined in any order (the associative property of addition).

Examples:

- For 3 + 8 + 7 + 6 + 2, think, 3 and 7 is 10, 8 and 2 is 10, so 10 and 10 and 6 is 26.
- For 25 + 47 + 75, think, 25 and 75 is 100, so 100 plus 47 is 147.
- For 400 + 720 + 600, think, 400 and 600 is 1000, and 1000 plus 720 is 1720.

Sample Tasks:

- 6 + 9 + 4 + 5 + 1 =
- Students measured the capacity of 5 different containers in mL. Find the total capacity.
  The containers were 7 mL, 1 mL, 3 mL, 5 mL, and 9 mL.
- 60 + 30 + 40 =
- How much money would Elijah need to buy three items that cost 75, 95, and 25 cents?
- 300 plus 437 plus 700
- What is the total mass of three bunches of bananas that weigh 310 g, 600 g, and 400 g?
- What is the sum of $750 + $250 + $330?
- Susan walked 700 metres on Monday, half a 500 metres on Tuesday, and 300 metres on Wednesday. How many metres did she walk altogether?
- $$\begin{array}{r} 200 \\ 225 \\ \underline{+\ 800} \end{array}$$

Break Up and Bridge

This strategy involves starting with the first number in its entirety and adding the second number, one place value at a time, starting with the largest place value. In Mathematics 4, this involves extending the questions to numbers involving hundreds and thousands. Remember that the problems should only include sums that involve two combinations and one regrouping. For example, 424 + 705 = 424 + 700 + 5 = 1129. In the introduction of this strategy, teachers should model both numbers with base-ten blocks and model their addition by combining the blocks, starting with the largest blocks of the second number, in the same way symbols are combined for break up and bridge. Similarly, modelling on a number line, start with the first number in its entirety.

Sample Tasks:

- 563 + 355 = 563 + 300 + 50 + 5
- 727 + 462 = 727 + 400 + 60 + 2
- 422 + 378 = 422 + 300 + 70 + 8
- 3450 + 2349 = 3450 + 2000 + 40 + 9

- 1424 + 2385 = 1424 + 2000 + 300 + 80 + 5

Compensation

This strategy involves changing one number in the addition question to a nearby multiple of ten or hundred, carrying out the addition using that multiple of ten or hundred, and adjusting the answer to compensate for the original change. Students should understand that the number is changed to make it more compatible, and that they have to hold in their memories the amount of the change. In the last step, it is helpful if they remind themselves that they added too much so they will have to take away that amount. Some students may have used this strategy when learning their facts involving 9s in Mathematics 2 (e.g., for 9 + 7, they may have found 10 + 7 and then subtracted 1).

Examples:

- For 52 + 39, think, 40 is easier to work with than 39. Then 52 plus 40 is 92, but I added 1 too many; so, to compensate, I subtract 1 from my answer, 92, to get 91.
- To find the sum of 345 and 198, think, 200 is easier to work with than 198. The sum of 345 + 200 is 545, but I added 2 too many, so, I subtract 2 from 545 to get 543.

Sample Tasks:

- $43 + $9 =
- 8 more than 56 =
- The sum of 65 + 29 =
- 44 cents plus 28 cents =
- 255 + 49 =
- The total of the number of days in one year and 18 days.
- 526 increased by 799
- I bought 355 mL of grape juice and 298 mL of orange juice. How much juice did I buy?
- 999 + 154

Make Multiples of 10, 100

This strategy involves changing both addends in an addition question by distributing part of one addend to the other addend in order to make that addend a multiple of ten or hundred. Students should understand that this strategy centres on creating a more compatible addend. A common error occurs when students forget that both addends have changed; therefore, some students may need to record one addend as an interim step. This strategy should be compared to the compensation strategy to see how it is alike and how it is different. For example, 475 + 125, move 25 from the second addend to the first addend to make 500, then add 500 to 100 to make 600.

MENTAL MATHEMATICS STRATEGIES: SUBTRACTION

Facts Applied to Multiples of 10, 100, and 1000

This strategy applies to calculations involving the subtraction of two numbers that are both multiples of 10, 100, or 1000. A simple strategy for these questions, only if they have the same number of digits, is to combine the front-end digits as if they were subtraction facts, and then attach the appropriate place-value name and symbols. This strategy should be modelled with base-ten blocks so students understand that 7 blocks subtract 3 blocks will be 4 blocks whether those blocks are small cubes, rods, flats, or large cubes. Since this strategy rests on students' knowledge of subtraction facts, the facts should be reviewed and consolidated. This strategy was introduced in Mathematics 3 to the tens and hundreds that are related to the subtraction facts with minuends of 10 or less.

Quick Subtraction

This strategy is actually the front-end strategy applied to subtraction questions that involve no regrouping. If questions only require two subtractions to get an answer, students should be able to do them mentally. However, questions involving three, or more, subtractions should be presented visually with students quickly recording their answers on paper. While this is a pencil-and-paper strategy for these questions because answers will always be recorded on paper before answers are read, it is included here as a mental mathematics strategy because most students will do all the subtractions in their heads starting at the front end. This strategy requires students to holistically examine the demands of each question as a first step in choosing a strategy. This habit of thinking needs to pervade all mental mathematics lessons. The practice items are presented visually instead of orally. It is important to present these subtraction questions both horizontally and vertically. In Mathematics 3, students would have applied this to two two-digit numbers in mental mathematics; this is extended to four-digit numbers in Mathematics 4. Most likely, students will subtract the digits in corresponding place values of the minuend and subtrahend without consciously thinking about the names of the place values. Therefore, in the discussion of the questions, encourage students to read the numbers correctly and to use place-value names. This will reinforce place-value concepts at the same time as subtraction is reinforced.

Examples:

- For 87 – 23, think, I see there is no regrouping needed, so I simply subtract 20 from 80 and 3 from 7, recording as I do each subtraction, to get 64.
- For 568 – 135, think, I see there is no regrouping needed, so I simply subtract 100 from 500, 30 from 60, and 5 from 8, recording as I do each subtraction, to get 433.
- For 4568 – 1135, think, I see there is no regrouping needed, so I simply subtract 1000 from 4000, 100 from 500, 30 from 60, and 5 from 8, recording as I do each subtraction, to get 3433.

Sample Tasks:

- 38 – 25 =
- $$\begin{array}{r} 85 \\ \underline{-\ 31} \end{array}$$
- How many hours less is a day than 76 hours?
- The teacher had 27 m of yarn for a craft. The students used 15 m of yarn. How many metres of yarn are remaining?
- $$\begin{array}{r} 537 \\ \underline{-101} \end{array}$$
- 304 fewer people than 8605 people
- $475 less than $699
- It is 745 m from Jan’s house to the school. She walks 23 m to meet Stephan at his house and then they continue on to school. How far is it from Stephan’s house to the school?
- 7898 – 5237

Back through Multiples of 10 and 100

This strategy extends the back-through-10 strategy students learned in Mathematics 3 for fact learning. This strategy involves subtracting a part of the subtrahend to get to the nearest multiple of ten or hundred, and then subtracting the rest of the subtrahend. **Note:** This strategy is most effective when the subtrahend is much less than the minuend.

Up through Multiples of 10 and 100

This strategy is an extension of the up-through-10 strategy that students learned in Mathematics 3 to help learn the subtraction facts. This strategy involves finding the difference between the two numbers in two steps from the smaller: first, find the difference between the subtrahend and the next multiple of ten or hundred, then find the difference between that multiple of ten or hundred and the minuend, and finally add these two differences to get the total difference. **Note:** This strategy is most effective when the two numbers involved are quite close together.

Break Up and Bridge

This strategy involves starting with the first number in its entirety and subtracting the numbers in the second number, one at a time, starting with the largest place value. Two-digit examples are easily modelled on a hundreds chart and/or a metre stick. If subtraction is modelled on a number line, it is natural to model it in the same manner as this strategy.

Compensation

This strategy for subtraction involves changing the subtrahend to the nearest multiple of ten or hundred, carrying out the subtraction, and adjusting the answer to compensate for the original change. Students should understand that the number is changed to make it more compatible, and that they have to hold in their memories the amount of that change. In the last step, it is helpful if they remind themselves that they subtracted too much or too little, so they will have to add or subtract that amount back on.

Balancing for a Constant Difference

In subtraction situations that require regrouping, this strategy can be used most effectively. By adding the same amount to both numbers in order to get the subtrahend to a ten or a hundred, regrouping is eliminated. This strategy needs to be carefully introduced because students need to be convinced it actually works! They need to understand that by adding the same amount to both numbers, the two new numbers have the same difference as the original numbers. Examining possible numbers on a metre stick that are a fixed distance apart can help students with the logic of this strategy. (For example, place a highlighter that is more than 10 cm long against a metre stick so that its bottom end is at the 18-cm mark, note where its top end is located, and write the subtraction sentence that gives the length of the highlighter. Repeat by placing the bottom end of the highlighter at the 20-cm mark. Ask, Is the length of the highlighter the same in both number sentences? Which subtraction would be easier to do?) **Note:** Because both numbers change in carrying out this strategy, many students may need to record the changed minuend to keep track, especially for numbers greater than two-digits.

SCO N04 Students will be expected to apply and explain the properties of 0 and 1 for multiplication and the property of 1 for division.
[C, CN, R]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

N04.01 Determine the answer to a given question involving the multiplication of a number by 1, and explain the answer using the property of 1 in multiplication.

N04.02 Determine the answer to a given question involving the multiplication of a number by 0, and explain the answer using the property of 0 in multiplication.

N04.03 Determine the answer to a given question involving the division of a number by 1, and explain the answer using the property of 1 in division.

Performance Indicator Background

N04.01 Multiplying by 1 is unique, 1 × __ simply means one group of __. On a number line students can see that 1 hop of 3 moves them to 3. When building sets, 1 set of 5 is 5.

N04.02 Multiplying by 0 is unique, __ × 0 = 0 since many zeros still equal zero. To help them develop good understandings, use contexts and pictures (e.g., On a number line students can see that 3 hops of 0 or 0 hops of 3 leaves them still on 0.

N04.03 Dividing by 1 is unique, __ ÷ 1 simply means how many ones in __? or How much is in the group if I put it all in one group?

SCO N05 Students will be expected to describe and apply mental mathematics strategies, to recall basic multiplication facts to 9 × 9, and to determine related division facts.
[C, CN, ME, R]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

N05.01 Describe the mental mathematics strategies used to determine basic multiplication or division facts.
N05.02 Use and describe a personal strategy for determining the multiplication facts.
N05.03 Use and describe a personal strategy for determining the division facts.
N05.04 Quickly recall basic multiplication facts up to 9 × 9.

Performance Indicator Background

N05.01 In general, a computational strategy should be introduced in isolation from other strategies, a variety of different reinforcement activities should be provided until proficiency is achieved, the strategy should be assessed in a variety of ways, and then it should be combined with other previously learned strategies.

INTRODUCING A STRATEGY

The approach to highlighting a computational strategy is to give the students an example of a computation for which the strategy would be useful to see if any of the students already can apply the strategy. If so, the student(s) can explain the strategy to the class. If not, the teacher could share the strategy. The explanation of a strategy should include anything that will help students see the pattern and logic of the strategy, be that concrete materials, visuals, and/or contexts. The introduction should also include explicit modelling of the mental processes used to carry out the strategy, and explicit discussion of the situations for which the strategy is most appropriate and efficient. Discussion should also include situations for which the strategy would not be the most appropriate and efficient one. Most important is that the logic of the strategy be well understood before it is reinforced; otherwise, its long-term retention will be very limited.

REINFORCING A STRATEGY

Each strategy for building mental computational skills should be practised in isolation until students can give correct solutions in a reasonable time frame. Students must understand the logic of the strategy, recognize when it is appropriate, and explain the strategy. The amount of time spent on each strategy should be determined by the students’ abilities and previous experiences.

The reinforcement activities for a strategy should be varied in type and should focus as much on the discussion of how students obtained their answers as on the answers themselves. The reinforcement activities should be structured to ensure maximum participation. At first, time frames should be generous and then narrowed as students internalize the strategy. Student participation should be monitored and their progress assessed in a variety of ways to help determine how long should be spent on a strategy.

After most of the students have internalized the strategy, help them integrate it with other strategies they have developed. Do this by providing activities that include a mix of number expressions for which this strategy and others would apply. Students should complete the activities and discuss the strategy/strategies that could be used or they could match the number expressions included in the activity to a list of strategies, and discuss the attributes of the number expressions that prompted them to make the matches.

As strategies are reinforced, students should hear and see the teacher use a variety of language associated with each operation so they do not develop a single word-operation association. Through rich language usage, students are able to quickly determine which operation and strategy they should employ. For example, when a student hears, “two groups of five,” “two rows of five,” “The product of two and five,” they should be able to quickly determine that they must multiply 2 and 5, and that an appropriate strategy to do this is the doubling strategy.

Present students with a variety of contexts for each operation in some of the reinforcement activities, so they are able to transfer the use of operations and strategies to situations found in their daily lives. By using contexts, the numbers become more real to the students. Contexts also provide opportunities for students to recall and apply other common knowledge that should be well known. For example, when a student hears the question, How many days in two weeks? they should be able to recall that there are seven days in a week and that double seven is 14 days.

ASSESSING STRATEGIES

Assessments of computational strategies should take a variety of forms. In addition to the traditional quizzes, teachers should also record any observations made during the reinforcement activities. Students should be asked for oral responses and written explanations of the strategies used. Individual interviews can provide many insights into a students’ thinking, especially in situations where pencil-and-paper responses are weak.

Assessments, regardless of their form, should shed light on students’ abilities to compute efficiently and accurately, to select appropriate strategies, and to explain their thinking.

After students have achieved competency using one strategy, they should be provided with opportunities for them to integrate it with other strategies they have learned. The ultimate goal is for students to have a network of mental mathematics strategies that they can flexibly and efficiently apply whenever a computational situation arises. This integration can be aided in a variety of ways, some of which are described below.

Take every opportunity that arises in regular mathematics class time to reinforce the strategies learned in mental mathematics time. Include written questions in regular mathematics time. This could be as a journal entry, a quiz/test question, part of a portfolio, or other assessment for which students will get individual feedback. Students should be asked to explain how they could mentally compute a given question in one or more ways, to comment on a student response that has an error in thinking, or to generate sample questions that would be efficiently done by a specified strategy.

N05.02 and **N05.04** In Mathematics 4, students are expected to recall the multiplication facts quickly and accurately by the end of the year. This could be done through learning a series of strategies, each of which addresses a cluster of facts. Each strategy is introduced, reinforced, and assessed before being integrated with previously learned strategies. It is important that students understand the logic and reasoning of each strategy, so the introductions of the strategies are very important. As students master each cluster of facts for a strategy, it is recommended that they record these learned facts on a multiplication chart. By doing this, they visually see their progress and are aware of which facts they should be practising. What follows is a suggested sequence for these strategies.

THE TWOS FACTS (DOUBLES)

This strategy involves connecting the addition doubles to the related “two-times” multiplication facts. It is important to make sure students are aware of the equivalence of commutative pairs (2 × ? and ? × 2) (e.g., 2 × 7 is the double of 7 and that 7 × 2, while it means 7 groups of 2, has the same answer as 2 × 7. When students see 2 × 7 or 7 × 2, they should think, 7 and 7 are 14. Flash cards displaying the facts involving 2 and the times 2 function on the calculator are effective reinforcement tools to use when learning the multiplication doubles. It is suggested that 2 × 0 and 0 × 2 be left until later when all the zeros facts will be done.

Examples:

- For 2 × 9, think, This is 9 plus 9, so the answer is 18.
- For 6 × 2, think, This is 6 plus 6, so the answer is 12.

If students are proficient with doubling, the fours facts (repeated doubling), rather than the 9s, might be the next strategy explored.

THE NIFTY NINE FACTS

The introduction of the facts involving nines should concentrate on having students discover two patterns in the answers; namely, the tens’ digit of the answer is one under the number of nines involved, and the sum of the ones’ digit and tens’ digit of the answer is 9. For example, for 6 × 9 = 54, the tens’ digit in the product is one less than the factor 6 (the number of nines) and the sum of the two digits in the product is 5 + 4 or 9. Because multiplication is commutative, the same thinking would be applied to 9 × 6. Therefore, when asked for 3 × 9, think, the answer is in the 20s (the decade of the answer) and 2 and 7 add to 9, so the answer is 27. Help students master this strategy by scaffolding the thinking involved; that is, practise presenting the multiplication expressions and just asking for the decade of the answer; practise presenting students with a digit from 1 to 8 and asking them the other digit that they would add to the digit to get 9; and conclude by presenting the multiplication expressions and asking for the answers and discussing the steps in the strategy.

Another strategy that some students may discover and/or use is a compensation strategy, where the computation is done using 10 instead of 9 and then adjusting the answer to compensate for using 10, rather than 9. For example, for 6 × 9, think, 6 groups of 10 is 60 but 6 is too many (1 extra in each group), so 60 subtract 6 is 54. This strategy can be modelled nicely using ten-frames. Students can build six sets of nine on ten-frames and see that they have almost six full ten-frames (60) but each ten-frame has one counter missing (6 less than 60) so there are 54 counters in all. This model can help them to visualize multiples of nine and make sense of this compensation strategy.

While 2 × 9 and 9 × 2 could be done by this strategy, these two nines facts were already handled by the twos facts. This nifty-nine strategy is probably most effective for factors 3 to 9. Zeroes and ones will be addressed with the properties of 0 and 1.

Examples:

- For 5 × 9, think, The answer is in the 40s, and 4 and 5 add to 9, so 45 is the answer.
- For 9 × 9, think, The answer is in the 80s, and 8 and 1 add to 9, so 81 is the answer.

THE FIVES FACTS

Many students probably have been using a skip-counting-by-five strategy when 5 has been a factor; however, this strategy is not always the quickest for all combinations and often results in students using their fingers to keep track. Therefore, students need to adopt a more efficient strategy.

If the students know how to read the various positions of the minute hand on an analog clock, it is easy to make the connection to the multiplication facts involving fives. For example, if the minute hand is on the 6 and students know that means 30 minutes after the hour, then the connection to 6 × 5 = 30 is easily made. This is why the five facts may be referred to as the “clock facts.” This would be the best strategy for students who can proficiently tell time on an analog clock.

Another possible strategy involves the patterns in the products. While most students have observed that the five facts have a 0 or a 5 as a ones’ digit, some have also noticed other patterns. One pattern is that the ones’ digit is a 0 if the number of fives involved is even or the ones’ digit is 5 if the number of fives involved is odd.

Another pattern is that the tens’ digit of the answer is half the numbers of fives involved, or half the number of fives rounded down. For example, the product of 8 and 5 ends in 0 because there are 8 fives and the tens’ digit is 4 because 4 is half of 8; therefore, 8 × 5 is 40. The product of 7 and 5 ends in 5 because 7 is odd and the tens’ digit is 3 because half of 7 rounded down is 3; therefore, 7 × 5 is 35.

While these strategies apply to 2 × 5, 5 × 2, 5 × 9, and 9 × 5, these facts were also part of the twos facts, and nines facts. The fives facts involving zeros are probably best left for the zeros facts since the minute-hand approach has little meaning for zero.

Examples:

- For 5 × 8, think, When the minute hand is on 8, it is 40 minutes after the hour, so the answer is 40.
- For 3 × 5, think, When the minute hand is on 3, it is 15 minutes after the hour, so the answer is 15.

THE ONES FACTS

While the ones facts are the “no change” facts, it is important that students understand why there is no change. Many students get these facts confused with the addition facts involving 1. To understand the ones facts, knowing what is happening when we multiply by one is important. For example 6 × 1 means six groups of 1 or 1 + 1 + 1 + 1 + 1 + 1 and 1 × 6 means one group of 6. It is important to avoid teaching arbitrary rules such as “any number multiplied by one is that number.” Students will come to this rule on their own given opportunities to develop understanding. Be sure to present questions visually and orally; for example, “4 groups of 1” and 4 × 1; and “1 group of 4” and 1 × 4. While this strategy applies to 2 × 1, 1 × 2, 1 × 5, and 5 × 1, these facts have also been handled previously with the other strategies.

Examples:

- For 8 × 1, think, Eight 1s make 8.
- For 1 × 7, think, One 7 is 7.

THE ZEROS FACTS

As with the ones facts, students need to understand why these facts all result in zero because they are easily confused with the addition facts involving zero; thus, the zeros facts are often "tricky." To understand the zeros facts, students need to be reminded what is happening by making the connection to the meaning of the number sentence. For example, 6 × 0 means "six 0's or "six sets of nothing." This could be shown by drawing six boxes with nothing in each box. 0 × 6 means "zero sets of 6." This is much more difficult to conceptualize; however, if students are asked to draw two sets of 6, then one set of 6, and finally zero sets of 6, where they do not draw anything, they will understand why zero is the product. Similar to the previous strategy for teaching the ones facts, it is important not to teach a rule such as "any number multiplied by zero is zero." Students will come to this rule on their own, given opportunities to develop understanding.

Examples:

- For 7 × 0, think, Having seven zeros means having a total of zero.
- For 0 × 8, think, Having no eights means having zero.

THE THREES FACTS (DOUBLE PLUS ONE MORE SET)

The way to teach the threes facts is to develop a "double plus one more set" strategy. Invite students to examine arrays with three rows. If they cover the third row, they easily see that they have a "double" in view, so adding "one more set" to the double should make sense to them. For example, for 3 × 7, think, 2 sets of 7 (double) plus one set of 7 or (7 × 2) + 7 = 14 + 7 = 21. This strategy uses the doubles facts that should be well-known before this strategy is introduced; however, there will need to be a discussion and practise of quick addition strategies to add on the third set. While this strategy can be applied to all facts involving three, the emphasis should be on 3 × 3, 3 × 4, 4 × 3, 3 × 6, 6 × 3, 3 × 7, 7 × 3, 3 × 8, and 8 × 3, all of which have not been addressed by earlier strategies.

Examples:

- For 3 × 6, think, Two 6s make 12, plus one more 6 is 18.
- For 4 × 3, think, Two 4s make 8, plus one more 4 is 12.

THE FOURS FACTS (REPEATED DOUBLING)

The way to teach the fours facts is to develop a "double-double" strategy. Invite students to examine arrays with four rows. If they cover the bottom two rows, they easily see they have a "double" in view and another "double" covered; so, doubling twice should make sense.

For example, for 4 × 7, think, 2 × 7 (double) is 14 and 2 × 14 is 28. Discussion and practise of quick mental strategies for the doubles of 12, 14, 16, and 18 will be required for students to master their fours facts. (One efficient strategy is front-end whereby the ten is doubled, the ones are doubled, and the two results are added together. For example, for 2 × 16, think, 2 times 10 is 20, 2 times 6 is 12, so 20 and 12 is 32.)

While this strategy can be applied for all facts involving 4, the emphasis should be on 4 × 4, 4 × 6, 6 × 4, 4 × 7, 7 × 4, 4 × 8, and 8 × 4, all of which have not been addressed by earlier strategies.

Examples:

- For 4 × 6, think, Double 6 is 12, and double 12 is 24.
- For 8 × 4, think, Double 8 is 16, and double 16 is 32.

THE LAST NINE FACTS

After students have worked on the above seven strategies for learning the multiplication facts, there are only nine facts left to be learned. These include, 6 × 6, 6 × 7, 6 × 8, 7 × 7, 7 × 8, 8 × 8, 7 × 6, 8 × 7, and 8 × 6. At this point, the students themselves can probably suggest strategies that will help with quick recall of these facts. Each fact may be presented to students and then they can be asked for their suggestions. Among the strategies suggested might be one that involves decomposition and the use of helping facts.

Examples:

- For 6 × 6, think, 5 sets of 6 is 30 plus one more set of 6 is 36.
- For 6 × 7 or 7 × 6, think, 5 sets of 6 is 30 plus two more sets of 6 is 12, so 30 plus 12 is 42.
- For 6 × 8 or 8 × 6, think, 5 sets of 8 is 40 plus one more set of 8 is 48. Another strategy is to think, 3 sets of 8 is 24 and double 24 is 48.
- For 7 × 7, think, 5 sets of 7 is 35, 2 sets of 7 is 14, so 35 and 14 is 49. (This is more difficult to do mentally than most of the others; however, many students seem to commit this one to memory quite quickly, perhaps because of the uniqueness of 49 as a product.)
- For 7 × 8, think, 5 sets of 8 is 40, 2 sets of 8 is 16, so 40 plus 16 is 56. (Some students may notice that 56 uses the two digits 5 and 6 that are the two counting numbers before 7 and 8.)
- For 8 × 8, think, 4 sets of 8 is 32, and 32 doubled is 64. (Some students may know this as the number of squares on a chess or checker board.

The distributive property relates to the fact that sets can be broken down into subsets, for example, 5 sets of 3 can be

- 4 sets of 3 + 1 set of 3
- 3 sets of 3 + 2 sets of 3
- 5 sets of 2 + 5 sets of 1

Understanding this principle will help students when they have to master the multiplication and division facts (e.g., 6 × 8 can be thought of as (5 × 8) + (1 × 8); or 36 ÷ 6 as (30 ÷ 6) + (6 ÷ 6)). Students learn division facts by thinking about corresponding multiplication facts. They can reduce the number of separate multiplication facts to be learned by drawing on a relationship previously explored (e.g., any multiple of 4 is twice the same multiple of 2). To help students learn to determine one fact based on what they know about another, include, on a regular basis, questions such as, How does knowing 5 × 4 = 20 help you to know 6 × 4? or What other division fact could help you solve 48 ÷ 6?

The distributive property is illustrated by

XXXXX	XXX	$4 \times 8 = (4 \times 5) + (4 \times 3)$
XXXXX	XXX	$= 20 + 12$
XXXXX	XXX	$= 32$
XXXXX	XXX	

N05.03 It is important for students to see that multiplication and division are related. When students learn that 2 × 3 = 6, they also see that 3 × 2 = 6, that 6 ÷ 2 = 3, and that 6 ÷ 3 = 2. These fact families should be discussed as a whole and not as four separate concepts. When faced with a division question, many people scan their memories for the related multiplication fact. For example, if asked to determine 36 ÷ 4, they ask themselves, four times what number is 36? This think multiplication strategy can be used for determining the division facts. Some students however, may think of the whole and imagine the ways in which it can be partitioned to help determine division facts; for example, they may visualize 36 squares as forming 4 rows of 9, 2 rows of 18, 3 rows of 12, and so on. Students should be encouraged to talk about the strategies they are using to determine their division facts.

To help students determine the division facts, it might be helpful to break down the facts into clusters. These clusters can relate to the corresponding multiplication fact clusters. For example, the 17 non-zero twos facts in multiplication have 17 corresponding division facts: 18 ÷ 2, 18 ÷ 9, 16 ÷ 2, 16 ÷ 8, 14 ÷ 2, 14 ÷ 7, 12 ÷ 2, 12 ÷ 6, 10 ÷ 2, 10 ÷ 5, 8 ÷ 2, 8 ÷ 4, 6 ÷ 2, 6 ÷ 3, 4 ÷ 2, 2 ÷ 2, 2 ÷ 1.

Possible clusters of division facts are as follows:

- From the twos facts in multiplication (17): 18 ÷ 2, 18 ÷ 9, 16 ÷ 2, 16 ÷ 8, 14 ÷ 2, 14 ÷ 7, 12 ÷ 2, 12 ÷ 6, 10 ÷ 2, 10 ÷ 5, 8 ÷ 2, 8 ÷ 4, 6 ÷ 2, 6 ÷ 3, 4 ÷ 2, 2 ÷ 2, 2 ÷ 1
- From the nifty nines facts in multiplication (15): 81 ÷ 9, 72 ÷ 9, 72 ÷ 8, 63 ÷ 9, 63 ÷ 7, 54 ÷ 9, 54 ÷ 6, 45 ÷ 9, 45 ÷ 5, 36 ÷ 9, 36 ÷ 4, 27 ÷ 9, 27 ÷ 3, 9 ÷ 9, 9 ÷ 1
- From the fives facts in multiplication (13): 40 ÷ 5, 40 ÷ 8, 35 ÷ 5, 35 ÷ 7, 30 ÷ 5, 30 ÷ 6, 25 ÷ 5, 20 ÷ 5, 20 ÷ 4, 15 ÷ 5, 15 ÷ 3, 5 ÷ 5, 5 ÷ 1
- From the ones facts in multiplication (11): 8 ÷ 1, 8 ÷ 8, 7 ÷ 1, 7 ÷ 7, 6 ÷ 1, 6 ÷ 6, 4 ÷ 1, 4 ÷ 4, 3 ÷ 1, 3 ÷ 3, 1 ÷ 1
- From the threes facts in multiplication (9): 24 ÷ 3, 24 ÷ 8, 21 ÷ 3, 21 ÷ 7, 18 ÷ 3, 18 ÷ 6, 12 ÷ 3, 12 ÷ 4, 9 ÷ 3
- From the fours facts in multiplication (7): 32 ÷ 4, 32 ÷ 8, 28 ÷ 4, 28 ÷ 7, 24 ÷ 4, 24 ÷ 6, 16 ÷ 4
- The last facts (9): 64 ÷ 8, 56 ÷ 8, 56 ÷ 7, 49 ÷ 7, 48 ÷ 8, 48 ÷ 6, 42 ÷ 7, 42 ÷ 6, 36 ÷ 6

N05.04 Response time is an effective way to see if students have automaticity of their facts. For the multiplication facts, the goal is for students to respond in three seconds or less by the end of the year. Students should be given more time than this in the initial strategy reinforcement activities. The time can be reduced as the students become more proficient applying the strategy until the three-second goal is reached. The three-second response goal is a guideline for the teacher and does not need to be shared with students if it will cause undue anxiety.

Note: Quick recall of basic division facts is not expected until the end of Mathematics 5.

SCO N06 Students will be expected to demonstrate an understanding of multiplication (one-, two- or three-digit by one-digit) to solve problems by

- using personal strategies for multiplication, with and without concrete materials
- using arrays to represent multiplication
- connecting concrete representations to symbolic representations
- estimating products
- applying the distributive property

[C, CN, ME, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

N06.01 Model a given multiplication problem, using the distributive property (e.g., 8 × 365 = (8 × 300) + (8 × 60) + (8 × 5)).

N06.02 Model the multiplication of two given numbers, limited to one-, two-, or three-digits by one-digit numerals, using concrete or visual representations, and record the process symbolically.

N06.03 Create and solve multiplication story problems, limited to one-, two-, or three-digits by one-digit numerals, and record the process symbolically.

N06.04 Estimate a product using a personal strategy (e.g., 2 × 243 is close to or a little more than 2 × 200, or close to or a little less than 2 × 250).

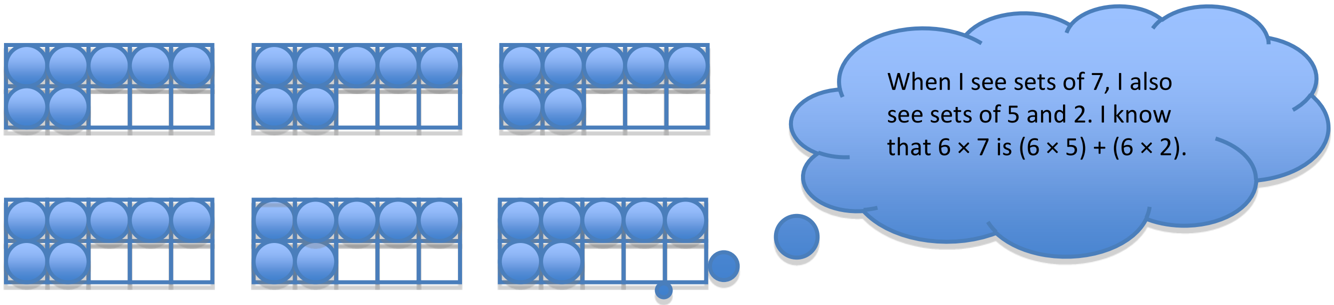
N06.05 Model and solve a given multiplication problem using an array, and record the process.

N06.06 Determine the product of two given numbers using a personal strategy, and record the process symbolically.

Performance Indicator Background

N06.01 Students should be able to explain the distributive property using pictures words and symbols. This property is important for recall of certain facts as well as for multi-digit calculations. Students might for example, explain why the distributive property allows them to develop a meaningful strategy for remembering the sevens facts. Consider the diagrams below.

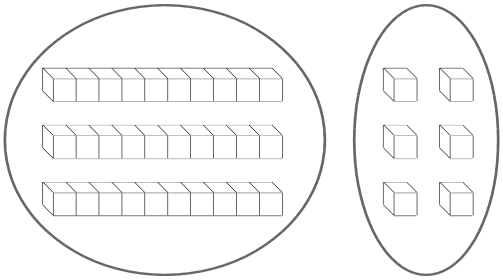
Students should be able to use this diagram to explain why any multiple of 7 can be seen as a multiple of (5 + 2).



Students should also be able to explain that the array representing 4 × 8, can also be used to show two other arrays—4 × 5 and 4 × 3.

XXXXX|XXX   4 × 8 = (4 × 5) + (4 × 3)
XXXXX|XXX   = 20 + 12
XXXXX|XXX   = 32
XXXXX|XXX

Students should be able to extend their understanding of the distributive property to multi-digit multiplication. They should be able to explain that to multiply 3 × 12 they can multiply (3 × 10) + (3 × 2), and they should be able to demonstrate this with pictures and models.



3 sets of 10   3 sets of 2

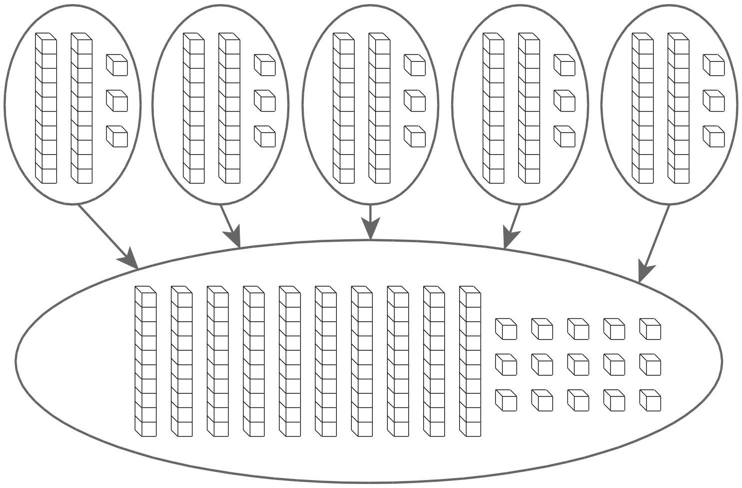
3 × 12
= 3 × (10 + 2)
= (3 × 10) + (3 × 2)
= 30 + 6
= 36

N06.02 and **N06.05** In Mathematics 3, students worked with small numbers (up to 5 × 5). In Mathematics 4, students should extend their understanding of multiplication to include a one-digit by a one-, two-, or three-digit number. Students should draw on their conceptual knowledge to use models and pictures in developing strategies for more complex multiplication tasks. Students should always start with multiplication tasks that can be built using concrete materials, and use these models to develop a strategy for performing the multiplication operations. Students should be encouraged to share a variety of approaches and should discuss the effectiveness of each approach in different contextual situations.

In Mathematics 3, students learned that multiplication can be modelled with sets, arrays, area models, and number lines. They should use these models in determining products in Mathematics 4 as well. For example, a student may model 5 × 23 using base-ten materials to develop a strategy based on the distributive property. Another student might choose to use an area model strategy. Examples of each are given below.

SET MODEL

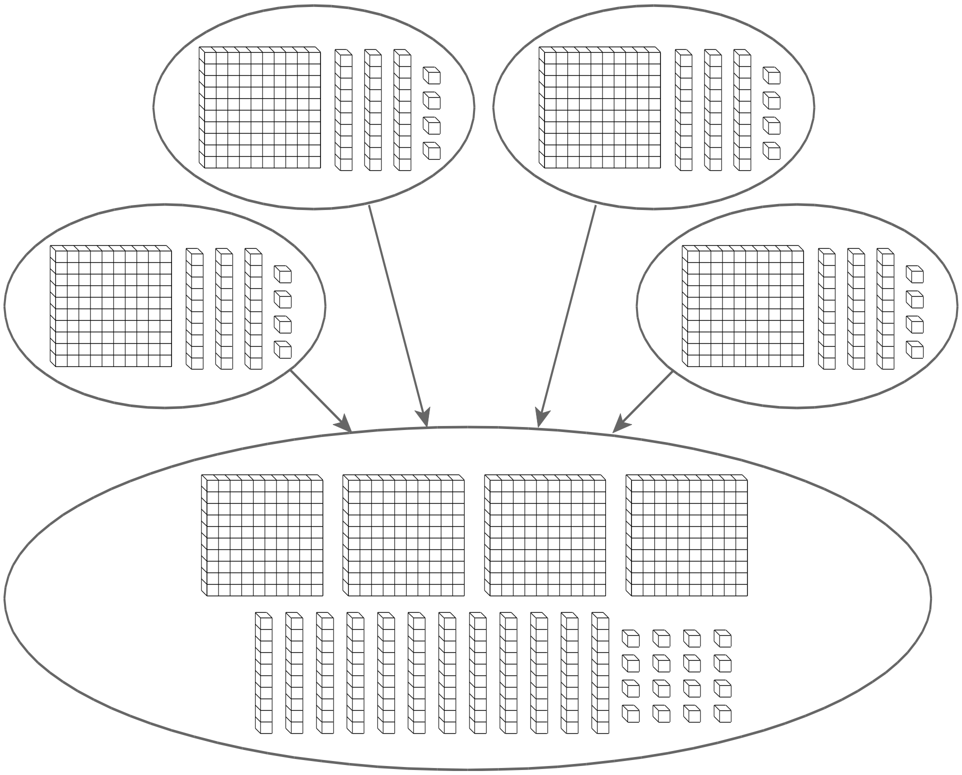
5 × 23



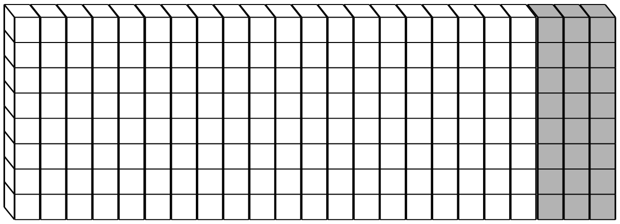
5 × 20 = 100
5 × 3 = 15
100 + 15 = 115
5 × 23 = 115

4 × 134

A student might also use base-ten materials to build 4 × 134 by building 4 sets of 134 and regrouping to get the product 532.



AREA MODEL



5 rows of 23
5 × 20 = 100
5 × 3 = 15
5 × 23 = 115

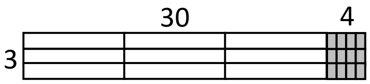
The teacher should ensure that students recognize that multiplication and division are two ways of looking at the same situation—this is very clear when they examine models or pictures. Some students might think, What do I multiply 3 by to get 18? when asked to find 18 ÷ 3. Other students might imagine the area model and think, How many will be in each row if I organize 18 objects into 3 rows?

N06.06 It is expected that, by the end of the year, students will be able to symbolically multiply one-, two- and three-digit numbers by a one-digit multiplier using reliable, accurate, and efficient strategies. While some of these strategies may have emerged directly from students work with base-ten blocks, other strategies should be modelled by students using the base-ten blocks to help understand the logic behind them. Students should be able to explain the strategy used and whether the solution is reasonable based on their prior estimate. Through the sharing of strategies, students will be exposed to a variety of possible multiplication strategies, and each student will adopt ones that he or she understands well and has made his or her own. That is why these strategies are often referred to as personal strategies. The most appropriate strategy used may vary depending on the student and the numbers involved in the problem.

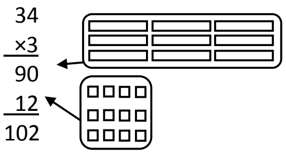
Personal strategies make sense to students and are as valid as the traditional algorithm. Therefore, emphasis should be on students' algorithms rather than on the traditional algorithm. The paper-and-pencil recording of students' personal strategies should reflect their thinking and must be reliable, accurate, and efficient. Most important is that students can justify how and why an algorithm works. Students should be encouraged to refine their strategies to increase their efficiency and teachers should monitor each student's symbolic recording of the strategy to ensure that the recording is accurate, mathematically correct, organized, and efficient. In particular, teachers should monitor student algorithms to ensure that the equal sign is correctly used.

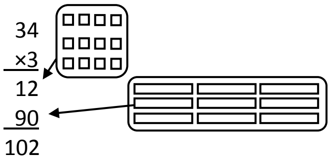
Examples of personal strategies and their symbolic recordings are shown below.

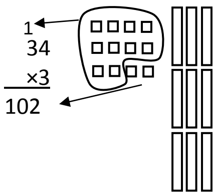
Algorithms for 3 × 34:



3 × 34 =
3 × 30 = 90
3 × 4 = 12
90 + 12 = 102







N06.03 Equal groups where the result is unknown is the typical type of multiplication task used in school and is associated with repeated addition. The equal groups structure may be modelled with sets, arrays (or area models), and linear or measurement models (such as number lines). A student may use a set model when they need to find how many apples they will need to make eight bags of apples with a dozen apples in each bag for the school fundraiser (multiplication). A student might use an array model to determine how many chairs would be needed if 9 rows of 24 chairs were being set up for the concert. A student might use a number line when they need to know the distance they will travel if they make 12 jumps of 3 metres. Students should develop flexibility with modelling multiplication using these various representations.

Additionally, multiplication is used in comparison situations. Multiplicative comparisons lay the groundwork for proportional reasoning. With comparison, we may have a situation where we want to determine the size of a result, given the initial amount, and the multiplier. Models used for comparison may involve sets, arrays, and linear models as well; however, students should be encouraged to build both sets. For example, if a student is asked to build a tower that is eight blocks tall and then a tower that is four times as tall as the first tower, it would be appropriate to use a linear model such as linking cubes to show both towers. Students might also model this on a number line showing a distance of eight units repeated four times.

The third structure is the combinations structure, which provides the foundation for later work in probability. Combinations have only two substructures—finding the product given the size of the two sets or finding the size of one set given the product and the other set. Commonly used models for three combinations are tables. For example, if we know that Mike has three choices of lunch and four choices for a beverage, we can determine the number of possible lunches he can have using a tree diagram or a table.

	Milk	Orange juice	Water	Apple juice
Sandwich				
Salad				
Soup				

Equal Groups	Comparison	Combinations
Result Unknown (Given the number of groups and the size of the group, find the result.) A bag holds 8 carrots. If you have 5 bags of carrots, how many carrots do you have? 5 × 8 = ? There are 5 rows of chairs in the library. Each row has 9 chairs in it. How many chairs are in the library? 5 × 9 = ? A grasshopper jumps 9 cm in a single jump. If the grasshopper jumps 6 times, what distance will it have travelled? 9 × 6 = ?	**Result Unknown** (Given the initial amount and the multiplier, find the result.) Kylie ate 5 apples last week. Her brother ate twice as many apples. How many apples did her brother eat last week? 5 × 2 = ?	**Result Unknown** (Given the size of the two sets, find the result.) Khaled has 3 pairs of pants and 5 shirts. How many different outfits can he make? 3 × 5 = ?
Size of a Group Unknown (Given the result and the number of equal groups, find the size of the group.) (partition division) You have 32 chairs. You need to put them in 8 rows. How many chairs will be in each row? 32 ÷ 8 = ? or 8 × ? = 32	**Multiplier Unknown** (Given the result and the initial amount, find the multiplier.) A frog jumped 2 metres. A kangaroo jumped 12 metres. How many times farther did the kangaroo jump? 12 ÷ 2 = ? or 2 × ? = 12	**One Set Unknown** (Given the result and one of the sets, find the other set.) Chika likes to eat yogurt with berries for recess. Chika has 5 different kinds of berries that she adds to her yogurt. If she can make 15 different yogurt with berries snacks, how many different kinds of yogurt does she use to make her snacks? 15 ÷ 5 = ? or 5 × ? = 15
Number of Equal Groups Unknown (Given the result and the size of the set find the number of groups.) (measurement division) You have 27 photographs. You want to put 3 photographs on each page of your photo album. How many pages will you fill? 27 ÷ 3 = ? or 3 × ? = 27	**Initial Unknown** (Given the result and the multiplier find the initial amount.) Katy collected 45 cans for recycling. That was 5 times as many as cans as Beth collected. How many cans did Beth collect for recycling? 45 ÷ 5 = ? or 5 × ? = 45	

N06.04 Students should be expected to estimate products (limited to one-, two-, or three-digit by one-digit numerals) using a personal strategy. After the multiplication facts and the related strategies are reviewed, or at the same time, these facts should be extended to the tens and hundreds multiplied by one-digit numbers.

A simple estimation strategy is to round one of the factors to a multiple of 10 or 100. Then, combine the single non-zero digits as if they were single-digit multiplication facts and attach the appropriate number of zeros to the result. Students, however, should be encouraged to approach these questions as modelled in the examples provided below, so the place value of the answers are known before any multiplication is undertaken. It would be beneficial to connect these products to groups of base-ten blocks. For example, 6 groups of 3 small cubes, 6 groups of 3 rods, or 6 groups of 3 flats all result in 18 blocks, whether they are ones, tens, or hundreds.

Examples:

- To estimate 3 × 73, I know that 73 is close to 70. So, to find my estimate, I think 3 × 70. I know my estimate will be tens and the number of those tens is 3 × 7 or 21. My estimate is 21 tens or 210.
- To estimate 6 × 378, I know that 378 is close to 400. So, to find my estimate, I think 6 × 400. I know my estimate will be hundreds and the number of those hundreds is 6 × 4 or 24. My estimate is 24 hundreds or 2400.

SCO N07 Students will be expected to demonstrate an understanding of division (one-digit divisor and up to two-digit dividend) to solve problems by

- using personal strategies for dividing, with and without concrete materials
- estimating quotients
- relating division to multiplication

[C, CN, ME, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

N07.01 Model the division of two given numbers without a remainder, limited to a one-digit divisor and up to a two-digit dividend, using concrete or visual representations, and record the process pictorially and symbolically.

N07.02 Model the division of two given numbers with a remainder, limited to a one-digit divisor and up to a two-digit dividend, using concrete or visual representations, and record the process pictorially and symbolically. (It is not intended that remainders be expressed as decimals or fractions.)

N07.03 Solve a given division problem, using a personal strategy, and record the process symbolically.

N07.04 Create and solve division word problems involving a one- or two-digit dividend, and record the process pictorially and symbolically.

N07.05 Estimate a quotient using a personal strategy (e.g., 86 ÷ 4 is close to 80 ÷ 4 or close to 80 ÷ 5).

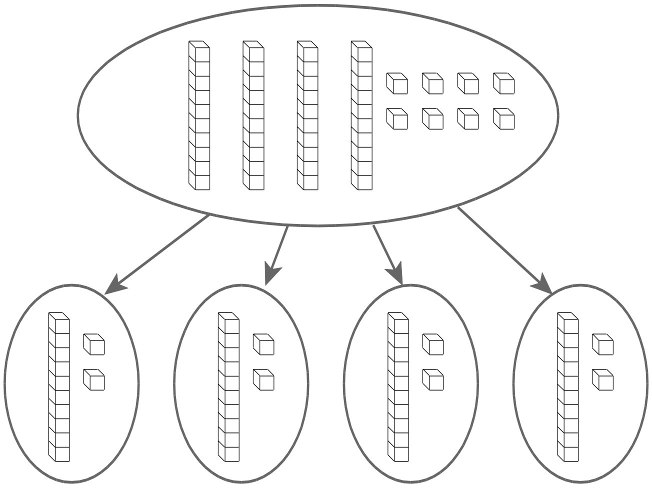
N07.06 Solve a given division problem by relating division to multiplication (e.g., for 80 ÷ 4, we know that 4 × 20 = 80, so 80 ÷ 4 = 20).

Performance Indicator Background

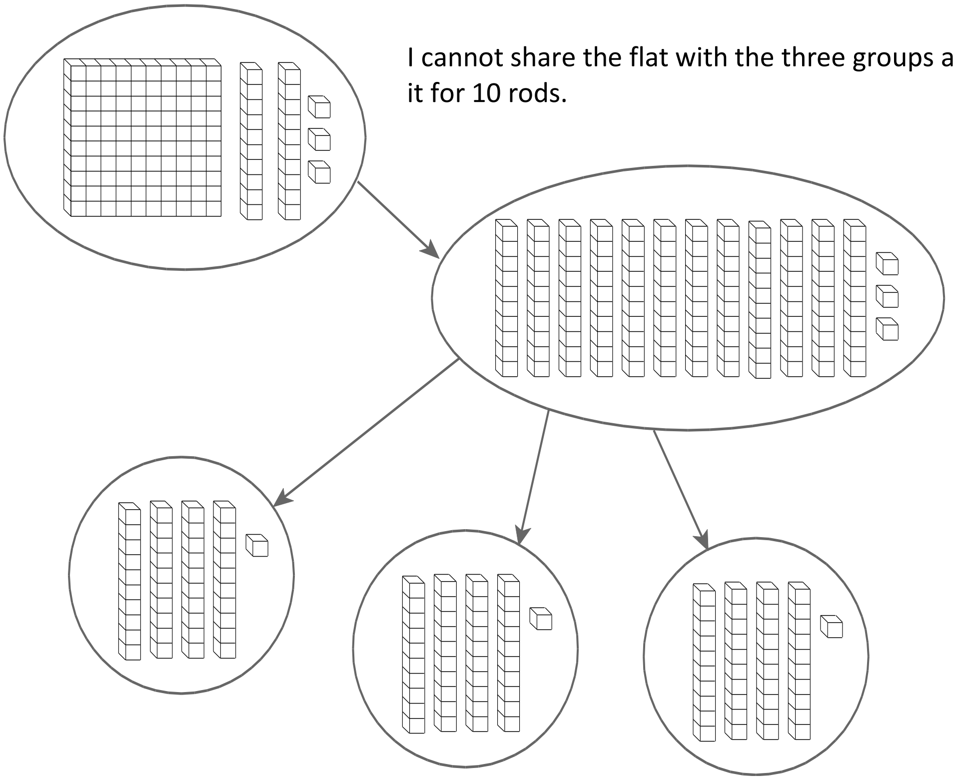
N07.01 and **N07.02** In Mathematics 3, students began to develop an understanding of the meaning of division working with single-digit numbers (limited to division related to multiplication facts up to 5 × 5). In Mathematics 4, they will extend this understanding to up to two-digit dividends and single-digit divisors. Students should draw on their conceptual knowledge to use models and pictures in developing strategies for more complex division tasks.

In Mathematics 3, students learned that division can be modelled with sets, arrays, area models, and number lines. They should use these models in determining products and quotients in Mathematics 4 as well. For example, a student may model 5 × 23 using base-ten materials to develop a strategy based on the distributive property. Another student might choose to use an area model strategy. Examples of each are given below.

Students should be able to use base-ten blocks to model the solution to a problem requiring them to identify how many in each group (partitioning or sharing), and then they should record a picture of their work. For example, There are 48 pencils. They are shared equally by 4 students. How many pencils does each student get?



I have 123 books. I want to place them on 3 shelves on a bookcase. How many books will a put on each shelf?

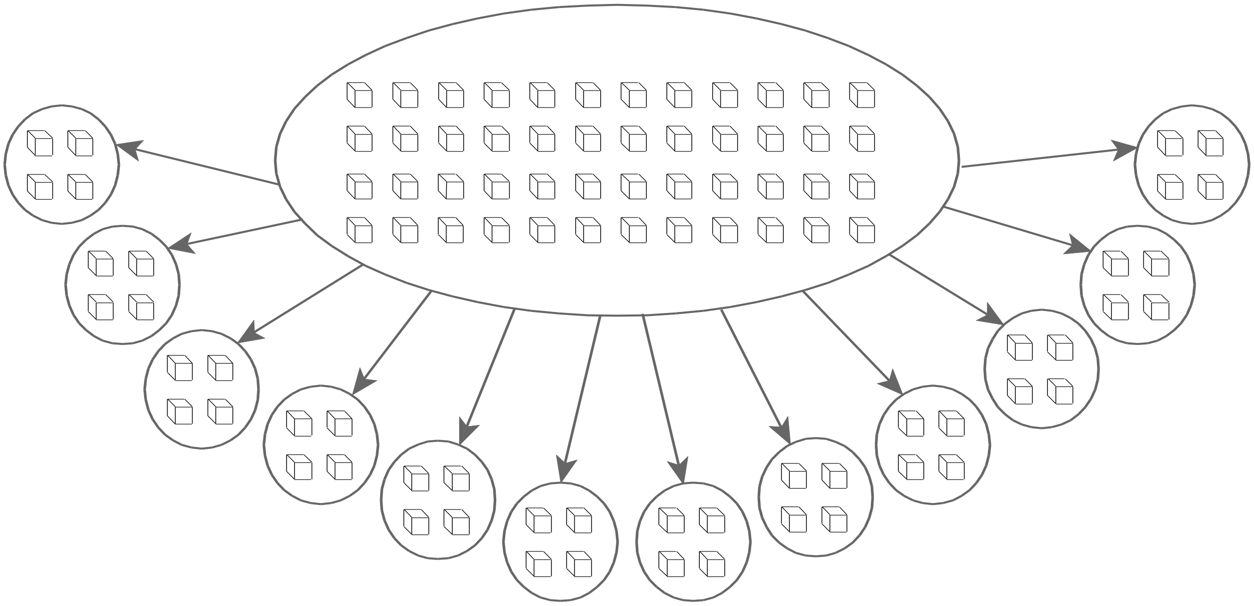


I cannot share the flat with the three groups as a whole flat, so I trade it for 10 rods.

Now I have 12 rods to share, so I can put 4 in each group. I then have 3 little cubes to share and I can put one in each group, so each group has 41.

Students should also be able to use base-ten blocks to model the solution to a problem requiring them to identify how many groups (repeated subtraction), and then they should record a picture of their work.

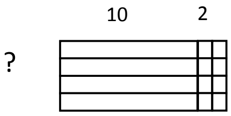
There are 48 cubes on the table. I need to put 4 in each container. How many containers will I need?



Students may also use the area model with a missing dimension to show division’s relationship with multiplication.

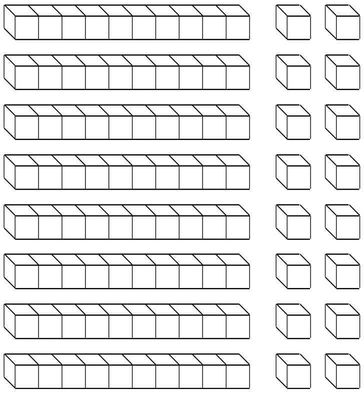
There are 48 chairs in the gym. The chairs are arranged in rows of 12. How many rows of chairs will be in the gym?

The area is 48.
I put 12 in each row.
I made 4 rows.

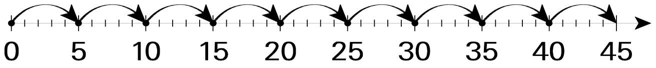


Students can also use area models or arrays to divide, the solution being the missing dimension. For example, 96 ÷ 12 can mean, “If I organize 96 objects into rows of 12 objects, how many rows will there be?” the solution is given by the model below.

Area is 96.
I used 8 rows of 12 to make it.
$8 \times 10 + 8 \times 2 = 80 + 16 = 96$
$96 \div 12 = 8$

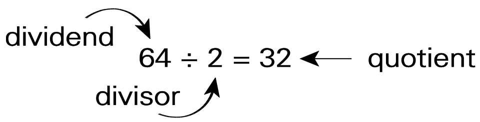


A kangaroo can travel a distance of 5 metres each time it hops. If a kangaroo hopped a total of 45 metres, how many times did it hop?



N07.02 Students should use concrete models to develop division strategies. While students have learned that division can mean fair sharing / partitioning (finding the size of a group) or measurement division (finding the number of groups of equal size), it is often helpful when dividing by a small number to use the fair-sharing model. For example, to divide 123 by 3, it is easier to think of this as sharing 123 in 3 equal groups, rather than sharing it in groups of 3.

Providing students with base-ten materials allows them to solve the problems and discuss the concept of remainders. Teachers can work with the students to demonstrate ways of documenting their thinking. Students should understand that the **remainder** (the number of units left over) must be less than the **divisor**. Models help to clarify this idea. In Mathematics 4, students are expected to express remainders as a digit and not as a fraction or decimal (e.g., a remainder of 7 is written as R7). Students also need to know that the answer for a division sentence is the **quotient** and the number to be divided is the **dividend**.



Students should understand that when solving division problems, remainders are handled differently depending on the context. They should recognize when a remainder is significant for decision making. For example, the remainder

- needs to be ignored (When you want to know how many $2 notebooks can be bought with $11, the answer if 5, since there is not enough money to buy 6 notebooks.)
- needs to be rounded up (When you want to know how many four-passenger cars are needs to transport 27 children, the answer is 7 since you cannot leave anyone behind.)
- must be addressed specifically (When 91 students are to be transported in 3 buses, there may be 30 students on two buses and 31 on the other because you cannot leave anyone behind.)
- is best described as a fraction (When 4 children share 9 oranges, each gets 2 oranges and $\frac{1}{4}$ of the remaining orange.) (Remainders expressed as fractions will be addressed in Mathematic 5.)

N07.03 It is expected that, by the end of the year, students will be able to symbolically divide one- and two-digit numbers by a one-digit divisor using reliable, accurate, and efficient strategies. While some of these strategies may have emerged directly from students' work with base-ten blocks, other strategies should be modelled by students using the base-ten blocks to help understand the logic behind them. Students should be able to explain the strategy used and whether the solution is reasonable based on their prior estimate. Through the sharing of strategies, students will be exposed to a variety of possible division strategies, and each student will adopt ones that he or she understands well and has made his or her own. That is why these strategies are often referred to as "personal strategies." The most appropriate strategy used may vary depending on the student and the numbers involved in the problem.

Personal strategies make sense to students and are as valid as the traditional algorithm. Therefore, emphasis should be on students' algorithms rather than on the traditional algorithm. The paper-and-pencil recording of students' personal strategies should reflect their thinking and must be reliable, accurate, and efficient. Most important is that students can justify how and why an algorithm works. Students should be encouraged to refine their strategies to increase their efficiency and teachers should monitor each student's symbolic recording of the strategy to ensure that the recording is accurate, mathematically correct, organized, and efficient.

Examples of personal strategies and their symbolic recordings are shown below.

- I had to solve 63 ÷ 3. I took 6 rods and 3 small cubes, and I needed to divide them into three groups. So, I put 1 rod in each of the three groups. So I used up 30. That left me with 3 rods and 3 small cubes. So, I put 1 more rod in each of the three groups. So I used up another 30. That left me with 3 small cubes. I put 1 small cube in each group. So 63 divided by 3 is 21.

$$\begin{array}{rl} 3\overline{)63} & \\ \underline{-30} & \underline{(10)} \\ 33 & \\ \underline{-30} & \underline{(10)} \\ 3 & \\ \underline{-3} & \underline{(1)} \\ 0 & \end{array}$$

63 ÷ 3 = 21

- I had to divide 63 into three groups. I broke 63 up into 60 and 3. I knew that 60 divided into 3 groups would give me 20 in each group. Then, I worked with the 3. I knew I could put 1 in each group. So, I had 20 and 1 in each group. So, 63 divided by 3 is 21.

$$\begin{array}{rl} 3\overline{)63} & \\ \underline{-60} & \underline{(20)} \\ 3 & \\ \underline{-3} & \underline{(1)} \\ 0 & \end{array}$$

- I had to divide 63 into 3 groups. I broke 63 up into 6 tens and 3 ones. I knew that 6 tens divided into 3 groups would give me 2 tens in each group. Then, I worked with the 3 ones. I knew I could put 1 one in each group. So, I had 2 tens and 1 one in each group. So, 63 divided by 3 is 21.

$$\begin{array}{r} 21 \\ 3\overline{)63} \\ \underline{-60} \\ 3 \\ \underline{-3} \\ 0 \end{array}$$

- I had to divide 269 into 4 groups. I broke 269 up into 200 + 40 + 29. I started with 200 because I knew that 4 groups of 50 are 200. Then, I knew that 4 groups of 10 are 40. So, that left me with 29. I knew that 4 × 7 is 28, and I would have 1 left over.

$269 \div 4 = ?$

$200 \div 4 = 50$

$40 \div 4 = 10$

$29 \div 4 = 7$ remainder 1

$269 \div 4 = 67$ R. 1

or

$$\begin{array}{rl} 4\overline{)269} & \\ \underline{-200} & \underline{(50)} \\ 69 & \\ \underline{-40} & \underline{(10)} \\ 29 & \\ \underline{-28} & \underline{(7)} \\ 1 & \end{array}$$

N07.04 Students must be able to create and solve division story problems that reflect different meanings for division. These meanings include equal groups, comparison, and combinations.

The equal groups structure involves determining the size of a group (partition division) or the number of equal groups (measurement division). The “equal groups” structure may be modelled with sets, arrays (or area models), and linear or measurement models such as number lines.

EQUAL GROUPS: PARTITION DIVISION

A student may use a set model to determine the size of a group. For example, Bill has 30 apples. He wants to share them equally among his 5 friends. How many apples will each friend receive?

A student might use an array model to determine the size of a group. For example, How many chairs should you put in each row if you need to organize 86 chairs into 6 rows?

EQUAL GROUPS: MEASUREMENT DIVISION

A student might use a number line when they need to know how many jumps it will take to cover a 24 m distance if they can jump 3 m each time.

Additionally, division is used in comparison situations. Models used for comparison may involve sets, arrays, and linear models as well.

COMPARISON: COMPARING SETS

A student might be asked to compare sets. For example, Kathy collected 45 juice cans for the recycle bin. That was three times as many juice cans as Kerry collected. How many juice cans did Kerry collect? To model this, a student might build a set of 45. Then, they begin with another set of 45 and divide it into three equal groups, taking one of the three groups to show that Kerry collected 15 juice cans.

COMPARISON: COMPARISON RATE/MULTIPLIER

A student might also be asked to determine the comparison rate or multiplier for two given sets or models. For example, if the height of the school is 6 m and the height of an office building in the city is

72 m, how many times as tall is the office building than the school? A student might model this using base-ten materials or a number line.

COMBINATIONS

The third structure is the combinations structure, which provides the foundation for later work in probability. For division, students may be asked to find the size of one set given the product and the other set. For example, Kevin says he has 18 outfits made up of pants and shirts. He has 3 pairs of pants. How many shirts must he have?

N07.05 Students should habitually estimate answers before attempting pencil-and-paper, or calculator, computations in order to be alert to the reasonableness of answers. Usually, these need only be "ball-park" estimates, especially when using a calculator where typical input errors result in place-value mistakes that can be detected from those "ball-park" estimates. There are also many instances in life when an estimate is all that is needed, such estimates should be as close to the actual answer as possible.

The language of estimation should be used throughout estimation lessons. Some of the common words and phrases are *about*, *just about*, *between*, *a little more than*, *a little less than*, *close*, *close to*, and *near*.

It is also important for students to hear and see a variety of contexts for each estimation strategy, so they are able to transfer the use of estimation and strategies to situations found in their daily lives.

The strategy for rounding in division questions with single-digit divisors is to round the dividends to compatibles with the divisors.

Examples:

- To estimate 65 ÷ 6, think, Round 65 to 60, a compatible with 6 in division, so 60 ÷ 6 gives an estimate of 10.
- To estimate 87 ÷ 3, think, Round 87 to 90, a compatible with 3 in division, so 90 ÷ 3 gives an estimate of 30.
- To estimate 79 ÷ 2, think, Round 79 to 80, a compatible with 2 in division, so 80 ÷ 2 gives an estimate of 40.

SCO N08 Students will be expected to demonstrate an understanding of fractions less than or equal to one by using concrete, pictorial, and symbolic representations to
- name and record fractions for the parts of one whole or a set
- compare and order fractions
- model and explain that for different wholes, two identical fractions may not represent the same quantity
- provide examples of where fractions are used

[C, CN, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

N08.01 Represent a given fraction of one whole object, region, or a set using concrete materials.
N08.02 Identify a fraction from its given concrete representation.
N08.03 Name and record the shaded and non-shaded parts of a given whole object, region, or set.
N08.04 Represent a given fraction pictorially by shading parts of a given whole object, region, or set.
N08.05 Explain how denominators can be used to compare two given unit fractions with a numerator of 1.
N08.06 Order a given set of fractions that have the same numerator, and explain the ordering.
N08.07 Order a given set of fractions that have the same denominator, and explain the ordering.
N08.08 Identify which of the benchmarks 0, $\frac{1}{2}$, or 1 is closer to a given fraction.
N08.09 Name fractions between two given benchmarks on a number line.
N08.10 Order a given set of fractions by placing them on a number line with given benchmarks.
N08.11 Provide examples of instances when two identical fractions may not represent the same quantity.
N08.12 Provide, from everyday contexts, an example of a fraction that represents part of a set and an example of a fraction that represents part of one whole.

Performance Indicator Background

N08.01, **N08.02**, **N08.03**, **N08.04**, and **N08.12** Students were introduced to fractions in Mathematics 3. In Mathematics 4, students must continue to have meaningful experiences to support this concept. Presenting fractions in contexts makes them much more meaningful to students. The emphasis is placed on student construction and understanding of fractions concretely, pictorially, and symbolically.

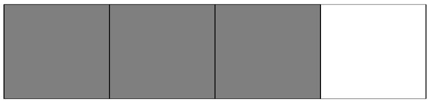
Concrete materials must be used to develop fractional concepts adequately, therefore a variety of materials are effective. Pattern blocks are very useful models. Using pattern blocks as concrete representations for either fractions of a whole or fractions of a set can help students make connections between the two models. For example,

- The triangle is $\frac{1}{3}$ of the trapezoid (fractions of a whole).
- The triangle is $\frac{1}{4}$ of a set of 4 blocks (made up of a triangle, two squares, and a rhombus) (fractions of a set).

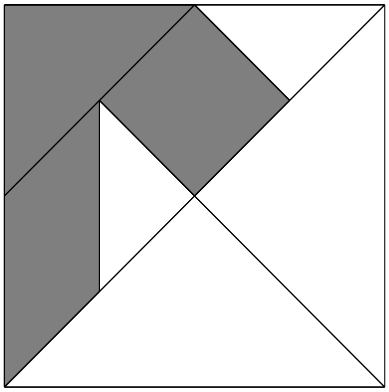
While there are several different meanings to fractions, in Mathematics 4 students will understand fractions as part of a whole (area or region model), part of a group (set model), and part of measures.

PART OF ONE WHOLE

This is when one unit is partitioned into equal parts. The sharing of food items or a piece of paper is commonplace to students. The more opportunities they have to partition fairly, the better their visual concept will be for fractions. The emphasis should be on equal parts or fair shares. Students should understand that while the parts are equal in area they do not need to be identical; this can be a misconception. A tangram set demonstrates this idea clearly where the square, the medium-sized triangle and the parallelogram all have equivalent area but are not identical in shape. It is important that the representation of the whole, one whole or one, is clear so students understand which region they are taking apart; this concept is essential for comparing fractions.



$\frac{3}{4}$ of the bar is shaded.

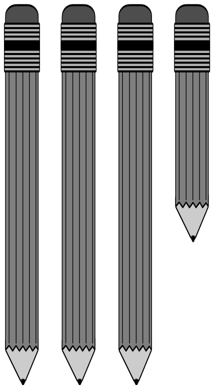


The parallelogram, the square, and the medium sized triangle each represent $\frac{1}{8}$ of the whole region.

PART OF A SET

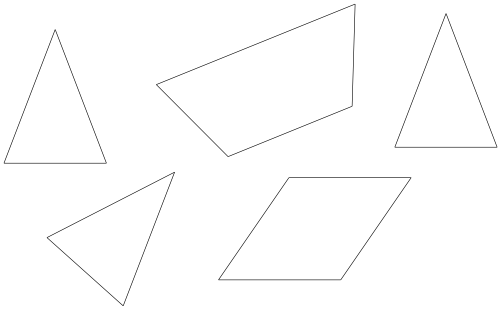
In Mathematics 3, students worked only with wholes or regions. They have had no experience working with parts of sets. Because the part of a set model is new to Mathematics 4, opportunities should be provided to allow this concept to be developed carefully. An important point relating to fractions of a set is that the equal parts into which one whole is divided are equal but do not have to be identical. Students may be easily confused by sets that contain different items or are different shapes. For example, in a group of 8 people, there may be 5 children, 2 women, and 1 man. It is still possible to identify the fraction of the set represented by the women as $\frac{2}{8}$, by the children as $\frac{5}{8}$, or the man as $\frac{1}{8}$.

The model below involves finding fair shares of a set of objects. This generally involves partitioning one at a time to each person to ensure that the sharing is fair. This concept of fractions requires students to view the total number as one unit; therefore, initial experiences should be with sets that are contained such as a box of 10 pencils, a package of 8 erasers, a carton of eggs, etc. This helps to solidify this concept. Students can connect this concept of fractions with division or fair sharing; for example, if 15 books are shared equally with 3 children each person gets $\frac{1}{3}$ of the original pile of books. In this case, students might not get exactly the same books but they each get $\frac{1}{3}$ of the set of books. This points to the idea that a group of objects do not need to be identical to be shared.



part of a set

$\frac{3}{4}$ (part of the pencils that are long)



$\frac{3}{5}$ of a set

(part of the shapes that are triangles)

The meaning for part of one whole can be extended to the part of a set meaning in certain situations. For example, when sharing a pizza that has been cut into 8 equal pieces, students can see that one-half also means 4 of the 8 pieces. Also, we can say $\frac{1}{4}$ of the pizza is eaten or $\frac{2}{8}$ of the slices are eaten.

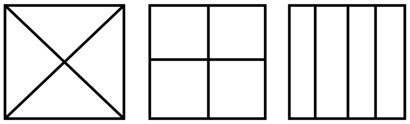
Tangrams can also be used for this meaning of fractions: $\frac{5}{7}$ of the Tangram pieces are triangles; $\frac{6}{8}$ area of the whole region is made up of triangles.

PART OF MEASURE

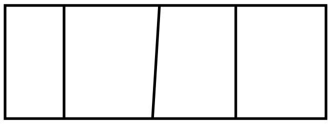
Fractions of measure, at this level, involve the measure of length such as finding a fractional unit on a number line. Fraction strips, Cuisenaire rods, number lines, line segments, and rulers are used for length.

Fractions are students’ first experiences in which a number represents something more than a count. Students should investigate the more common fraction families such as halves, thirds, fourths, fifths, sixths, eighths, tenths, and twelfths. Provide opportunities to explore other fractions in problem situations and in literature. They will need experiences with a variety of materials, including, among others, fraction pieces (Fraction Factory / Cuisenaire rods) geo-boards, counters, coloured tiles, pattern blocks, egg cartons, grid paper, folded papers, and circle pieces. Take caution to be flexible using the manipulatives, such as not having the same piece representing one whole. Provide many and varied opportunities for students to estimate fractional quantities and to explore the idea of simple fractions in meaningful situations. Initial experiences should be with part-of-a-whole situations then making connections to the other fraction models. It is also helpful when examining a situation involving a fraction to show the related fraction (e.g., if one-third of a pie is eaten, then two-thirds of that pie remains). Informal experiences will help students see that when wholes are divided into a greater number of fair shares, the shares are smaller; this will help later when comparing fractions.

Students should see that there are many ways to make the same fractional part. Using pattern blocks where the hexagon is designated as one whole, students could find how many different ways they can make $\frac{1}{2}$, $\frac{1}{3}$, etc. Or using a square, find how many different ways to make $\frac{1}{4}$. This can help with the understanding of equivalence.



It is important that students see and represent non-examples of the area model for fractions. Each piece in the rectangledoes not represent $\frac{1}{4}$ area of the whole.



Continue to use language such as “1 of 3 equal parts” and help students connect the language with its symbol $\frac{1}{3}$. Point out to students that $\frac{1}{4}$ may be read either “one-fourth” or “one-quarter.” The money application of four quarters makes a whole dollar can be connected to this use of the word one-quarter. To assist with clarity of meaning, always write fractions with a horizontal bar.

Note: Students should not work in any formal way with adding or subtracting fractions. However, it makes sense to capitalize on students’ intuitive knowledge that

- one-half and one-half are one whole
- one-fourth and one-fourth are two-fourths or one-half
- one-eighth and one-eighth are two-eighths
- one-tenth and two-tenths are three-tenths

When working with tenths, a connection to decimal computations should be made.

N08.05, **N08.06**, **N08.07**, **N08.08**, **N08.09**, and **N08.10** By providing everyday contexts in which the region representing one whole varies in size, students’ thinking is stimulated to generalize that when comparing fractions, the whole must be the same size for each fraction.

Students should begin to use a variety of conceptual methods to compare two fractions, cognizant of the fact that fractions are a part of the whole. These methods include the following:

- Comparing the two numerators when they have the same denominator (e.g., If an item is cut into 6 equal pieces, 2 of those pieces are less than 5 of them.)
- Comparing the two denominators when they have the same numerators (e.g., If 3 people share one item, they will each get more than if 4 people share this item.)
- Comparing both to benchmarks such as 0, $\frac{1}{2}$, or 1 (e.g., $\frac{2}{5} < \frac{4}{7}$ because $\frac{2}{5}$ is less than $\frac{1}{2}$, while $\frac{4}{7}$ is more than $\frac{1}{2}$. This method lends itself well to measurement.)

When comparing like numerators, students may make a common error because of their experience comparing whole numbers. In comparing fractions such as $\frac{3}{6}$ and $\frac{3}{7}$, they might think that $\frac{3}{7}$ is greater than $\frac{3}{6}$ because 7 is greater than 6. Students will need to spend considerable time on activities and discussions that help them to develop number sense for fractions. Contexts such as the pizza model work well. Ask students, Which would you rather have, a piece of pizza divided into 6 equal parts or a piece of the same pizza divided into 7 equal parts? Presenting students with a variety of comparison questions allows them the opportunity to select an appropriate method for comparing fractions and explain why they chose that method. Invite students to create problems for others to solve.

N08.08 and **N08.09** The most important reference points for fractions are 0, $\frac{1}{2}$, and 1, and are referred to as benchmarks. Comparing fractions to these three benchmarks can provide students with a lot of information. Understanding why a fraction is close to 0, $\frac{1}{2}$, or 1 is a good beginning for fraction number sense. It begins to focus on the relative size of fractions in an important, yet similar manner.

Invite three students to represent the three benchmarks by holding a skipping rope at the beginning, middle, and end. Two students, each holding one end, represent endpoints 0 and 1, and ask a third student to stand in the middle to represent $\frac{1}{2}$. Give several students fraction cards and ask them to stand in front of the person representing the benchmark closest to their fraction (e.g., A student might say, "$\frac{2}{10}$ is closer to 0, so I'll stand in front of Amy who is holding the zero end of the rope.").

Identifying which of the benchmarks, 0, $\frac{1}{2}$, or 1, is closer to a given fraction can be done by using the following strategies:

- with paper fraction strips
  - Provide students with fraction strips showing halves and other fractions, such as thirds, quarters, fifths, and tenths. Using fraction strips, have students order two fractions by comparing each fraction to one-half, such as one-quarter and two-thirds or three-fifths and eight-tenths. Through discussion, have students generalize that some fractions can be ordered by deciding if they are greater than or less than one-half.
- by looking at the denominator and numerator
  - Ask students to explain how they would know if a fraction was greater than or less than one-half without using the paper fraction strips. Guide students to explore and conclude, on their own, that if the numerator is less than half the denominator, then the fraction is less than one-half. Similarly, if the numerator is greater than half the denominator, then the fraction is greater than one-half. If the numerator is half the denominator, then the fraction shows another name for one-half.

Ask students to name fractions between two given benchmarks on a number line. For example, when asked to name a fraction between 0 and $\frac{1}{2}$, encourage students to think of as many possibilities as they can, using a set of fractions with like denominators, ($\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}$) unlike denominators ($\frac{1}{3}, \frac{3}{10}, \frac{5}{12}$, etc.)

N08.10 To place the fractions on a number line using benchmarks of 0, $\frac{1}{2}$, and 1, students must also make estimates of fraction size in addition to simply ordering the fractions. When ordering fractions, fractions strips can be placed against a number line to help mark the fractions. A good introduction to this concept is to provide students with fraction strips of fourths, eighths, twelfths, and sixteenths and ask students to identify the fractions that are equal to $\frac{1}{2}$. This benchmark is the most familiar with students as they frequently share things into two equal groups. Students can then extend their understanding by ordering other fractions by using words such as “closer to,” or “less than” half. Consider using an overhead transparency cut into fraction strips resembling student sets. Using an overhead projector to order fractions will help confirm students’ individual responses to ordering of their own fraction strips.

N08.11 It is important for students to be able to explain why two identical fractions do not represent the same amount (when the wholes are different sizes). By providing everyday contexts in which one whole (the region) varies in size, student thinking is generalized, and they conclude that when comparing fractions, the whole must be the same size for each fraction. For example, ask students whether halves are always the same. Discuss student responses and demonstrate by cutting different kinds of fruit, such as a strawberry, a watermelon, and an orange, in half. Discuss that the halves are different sizes even though they all represent one-half of a piece of fruit.

Students should be given many opportunities to compare fractions using multiple representations. Students should be able to explain why two fraction models might look different but both represent the same fraction. Students should be encouraged to discuss their reasoning in determining equality or inequality and should be able to justify any such statements made about fractions.

“A key idea about fractions that students must come to understand is that a fraction does not say anything about the size of the whole or the size of the parts. A fraction tells us only about the relationship between the part and the whole. ” (Van De Walle 2006, p. 267) Consider this example: Both Alex and Jennifer attend a pizza party. They decide that they both want $\frac{1}{4}$ of a pizza. They go to different areas to pick up their pizza. Alex takes $\frac{1}{4}$ of a pepperoni pizza, and Jennifer takes $\frac{1}{4}$ of a veggie pizza. When they meet back at their table, they realize that they do not have the same amount of pizza, but that Jennifer’s is larger. They come to the realization that Jennifer’s slice came from a larger pizza, and that they did not check the size of the wholes before selecting their choice. Van de Walle (2006, p. 267) refers to this as the “pizza fallacy” in that whenever two or more fractions are discussed in the same context, the correct assumption (the one that Jennifer and Mark made) is that the fractions are all parts of the same size whole. It is important for students to be able to explain why two identical fractions do not represent the same amount (when the wholes are different sizes).

SCO N09 Students will be expected to describe and represent decimals (tenths and hundredths) concretely, pictorially, and symbolically.
[C, CN, R, V]

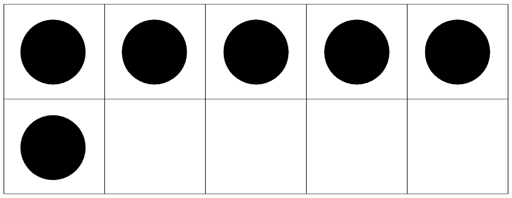
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

N09.01 Write the decimal for a given concrete or pictorial representation of part of a set, part of a region, or part of a unit of measure.
N09.02 Represent a given decimal using concrete materials or a pictorial representation.
N09.03 Explain the meaning of each digit in a given decimal.
N09.04 Represent a given decimal using money values (dimes and pennies).
N09.05 Record a given money value using decimals.
N09.06 Provide examples of everyday contexts in which tenths and hundredths are used.
N09.07 Model, using manipulatives or pictures, that a given tenth can be expressed as a hundredth (e.g., 0.9 is equivalent to 0.90, or 9 dimes is equivalent to 90 pennies).
N09.08 Read decimal numbers correctly.

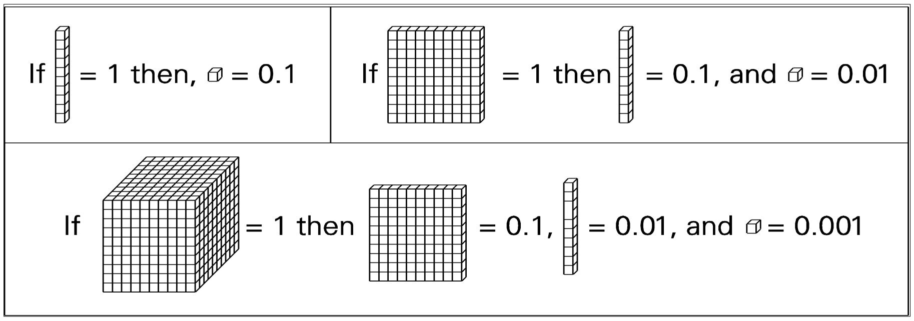
Performance Indicator Background

N09.01 and **N09.02** Students must have ample opportunities to represent and describe decimals concretely, pictorially, contextually, verbally, and symbolically. Students can begin work with decimal tenths using ten frames. In this case, the whole ten-frame represents one and each block of the frame represents one tenth.



If the ten-frame represents 1, this represents 0.6 or six-tenths.

Base-ten blocks and/or base-ten fraction squares are also great models to use for decimals. Like fractions, students should recognize that some decimals can represent part of one whole (e.g., 0.3 is three-tenths of one-whole) or a mixed number (e.g., 2.5 is two and five-tenths). To help students develop the concept of tenths and hundredths, it is essential to clearly establish the whole that they are, or will be, dividing into ten equal parts. As with fractions, flexibility with representing the “one” should be encouraged. Students should also be working with decimals that are more than one.

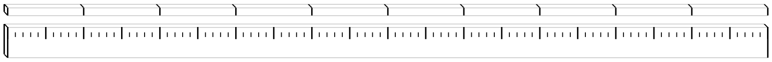


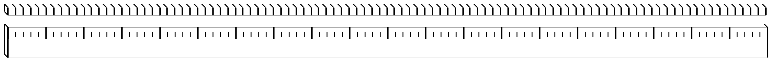
If ▯ = 1 then, ▫ = 0.1

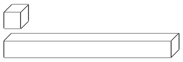
If ▦ = 1 then ▯ = 0.1, and ▫ = 0.01

If ▩ = 1 then ▦ = 0.1, ▯ = 0.01, and ▫ = 0.001

Metre sticks can also be used as models for decimal numbers. Assuming that the metre is one whole or one, tenths may be modelled by lining up base-ten rods along the metre stick. Hundredths may be modelled by lining up small base-ten cubes along the metre stick. The centimetre would be one-hundredth of a metre stick and one-tenth of the rod.



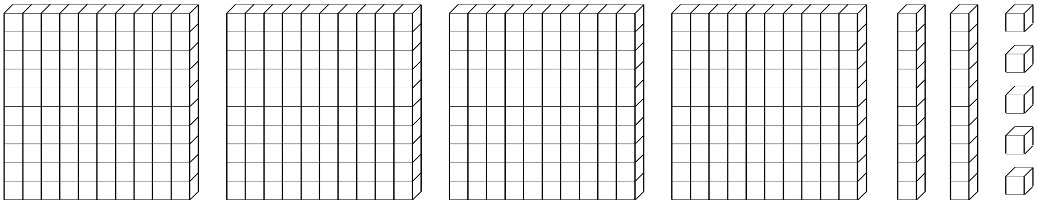


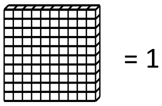


Students should experience skip-counting with decimals by tenths and hundredths. For example, by recognizing the pattern one-tenth, two-tenths, three-tenths, and so forth, students will see ten-tenths as one. It is important to emphasize that this is skip-counting using the pattern, and that decimals cannot be used to count on because there is not a unique decimal that “comes next” after any given decimal.

N09.03 Students should recognize that decimals extend the place-value system. Students should use prior knowledge of the patterns associated with place value to explain how tenths and hundredths fit into the place-value system. Students should identify that the first place to the right of the decimal point is tenths because ten of these would make one whole (the place to the left of it). Similarly, the second decimal place is hundredths because it would take ten of these to make one tenth, and ten-tenths to make the whole, thus one hundred of these will make one whole.

Students should represent decimals using concrete materials such as base-ten materials and use these models to demonstrate the place value of decimals. The value of a decimal number is the product of the face value of the digit and its place value in the base-ten system, e.g., 4.25 = (4 × 1) + (2 × 0.1) + (5 × 0.01). This can be represented with base-ten material as shown below.





Students should recognize and work with the idea that the value of a digit varies depending on its position or place in a numeral. Students should recognize the value represented by each digit in a number as well as what the number means as a whole. The digit “2” in 2.3 represents 2 ones whereas the digit “2” in 3.2 represents 2 tenths. Students should be able to explain the meaning of the digits, including numerals with all digits the same (e.g., for the numeral 2.22, the first digit represents 2 ones the second digit 2 tenths, and the third digit 2 hundredths).

It is important to spend time developing a good understanding of the meaning and use of zero in numbers. Students need many experiences using base-ten materials to model numbers with zeros as digits. Teachers should ask students to write the numerals for numbers such as seven and five-hundredths; ninety and two-tenths; or zero and five-hundredths. When writing a number, such as seven and five-hundredths, in its symbolic form using digits, the digit 0 is a place holder. If the digit 0 was not used, the number would be recorded as 7.5, and you would mistakenly think that the 5 represented five-tenths instead of five-hundredths. Students need many experiences using base-ten materials to make connections with the symbols for numbers with zeros as digits.

N09.04 and **N09.05** Money provides a real-life context for working with decimals and is one that is likely familiar to students. It should be noted that whereas ten-frames, base-ten blocks, metre sticks, decimal squares, and grids are proportional models, money is a non-proportional model for decimal numbers. Although ten dimes have an agreed upon value of one dollar, each dime is not one-tenth the physical size of a loonie. Caution should be used when introducing this non-proportional model.

When representing a given decimal using money values, the dollar represents one whole, dimes represent one-tenth of the whole, and pennies represent one-hundredth of the whole.

N09.06 Teaching decimals through meaningful contexts such as those below will strengthen students' understanding.

- fingers and toes
- items packaged in tens, such as pencils, pens, or sticks of gum
- food that can be shared among ten people
- metre stick, with the metre representing one whole and centimetres representing hundredths
- scores and times for sporting events, for example, the hundred metre dash was completed in 13.9 sec.
- statistics for athletes (e.g., points per game—NBA player Chris Paul averages 11.8 assists per game or NHL player Sidney Crosby averages 1.5 points per game)
- gas prices on signs shown to the nearest tenth (e.g., 123.9 cents per litre)

N09.07 Students should be able to read and interpret decimal numbers in more than one way. For example, if representing sixty-hundredths with base-ten blocks, (with the flat representing 1), students might use either 60 small cubes or 6 rods, as both have a value of sixty-hundredths. If using money, students should understand that 60 pennies is equivalent to 6 dimes. Students should also understand that sixty-hundredths may be represented symbolically as 0.60, or as 0.6.

N09.08 Students should be able to read decimal numbers in print and record the numeric form of decimals upon hearing them orally, seeing them written out in words, or when presented with concrete or pictorial models. When reading numbers, the word and is reserved for the decimal. For example, 5.32 is read as five and thirty-two hundredths, not as five point three two, or five decimal thirty-two. Students should also have experience reading numbers in several ways. For example, 1.83 may be read as 1 and 83 hundredths, but might also be read as 1 and 8 tenths, 3 hundredths, or as 18 tenths, 3 hundredths.

Although teachers will model the correct reading of whole numbers and decimal numbers and will use the word **and** only for the decimal point (e.g., 16.8 is read as sixteen and eight-tenths, while 1235 is read as one thousand two hundred thirty-five), it is also important to acknowledge that in everyday use people often read numbers in ways that are not mathematically accurate, such as reading 0.34 as zero decimal thirty-four or zero point thirty-four.

SCO N10 Students will be expected to relate decimals to fractions and fractions to decimals (to hundredths).
[C, CN, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

N10.01 Express, orally and symbolically, a given fraction with a denominator of 10 or 100 as a decimal.

N10.02 Read decimals as fractions (e.g., 0.5 is zero and five tenths).

N10.03 Express, orally and symbolically, a given decimal in fraction form.

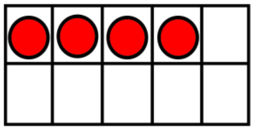
N10.04 Express a given pictorial or concrete representation as a fraction or decimal (e.g., 15 shaded squares on a hundredth grid can be expressed as 0.15 or $\frac{15}{100}$).

N10.05 Express, orally and symbolically, the decimal equivalent for a given fraction (e.g., $\frac{50}{100}$ can be expressed as 0.50).

Performance Indicator Background

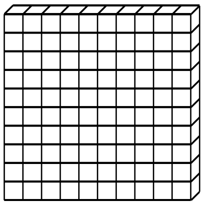
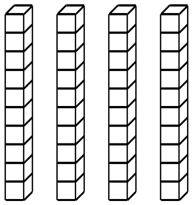
N10.01, **N10.02**, **N10.03**, **N10.04**, and **N10.05** Connecting fractions to decimals should begin with the tenths, and later move to hundredths. Using hundred grids and base-ten blocks are excellent ways to concretely and pictorially connect fractions to decimals and decimals to fractions.

Ten-frames will be familiar to students from their early work with whole numbers, but these can be used to model fractions as well. If the entire ten-frame represents one whole, then each space in the ten frame represents one-tenth. Students will see that the model shown can be read as "four-tenths," which they should connect with two different symbolic representations: $\frac{4}{10}$ or 0.4. From exploring representations such as these, students should be able to recognize that fractions with a denominator of 10 can be written as decimals as well.

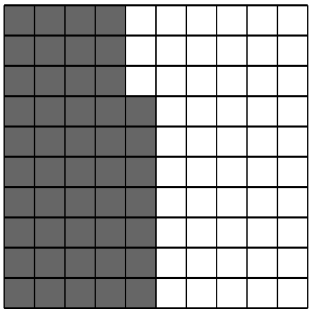


Students should also use models such as base-ten materials and hundredths grids to model fractions with denominators of 10 or 100 or decimals to the hundredths. Students should use these concrete models to explain why these models also represent a fraction with a denominator that is a power of ten or as a decimal. See examples in the figure below.

BASE-TEN BLOCKS

If  represents 1, then , represents four-tenths which may be written as 0.4 or $\frac{4}{10}$.

HUNDRED GRID OR DECIMAL SQUARE

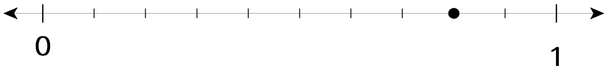


This represents forty-seven hundredths, which may be written as 0.47 or $\frac{47}{100}$.

METRE STICK

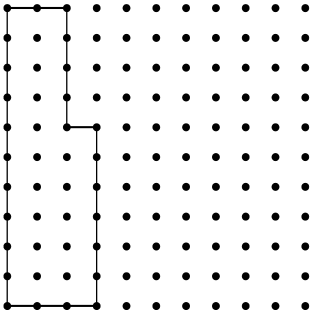
0.4 = $\frac{4}{10}$

NUMBER LINE



$\frac{8}{10}$

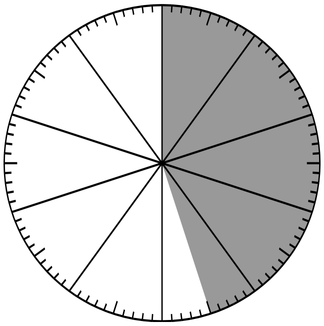
10 × 10 GEO-BOARD



= 0.26 (26 hundredths)

= $\frac{26}{100}$

HUNDREDTHS CIRCLES



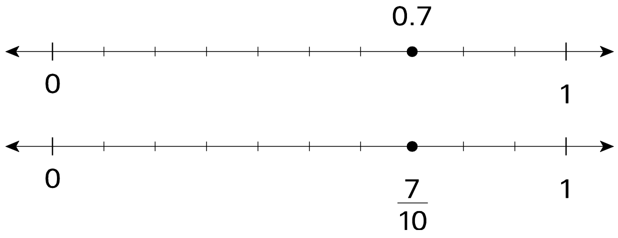
0.45 (45 hundredths)

$= \frac{45}{100}$

MONEY

= 0.23

DOUBLE NUMBER LINES



SCO N11 Students will be expected to demonstrate an understanding of addition and subtraction of decimals (limited to hundredths) by

- estimating sums and differences
- using mental mathematics strategies to solve problems
- using personal strategies to determine sums and differences

[C, ME, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

N11.01 Predict sums and differences of decimals, using estimation strategies.
N11.02 Solve problems, including money problems, which involve addition and subtraction of decimals (limited to hundredths), using personal strategies.
N11.03 Ask students to determine which problems do not require an exact solution.
N11.04 Determine the approximate solution of a given problem not requiring an exact answer.
N11.05 Count back change for a given purchase.
N11.06 Determine an exact solution using mental computation strategies.

Performance Indicator Background

N11.01 and **N11.04** Students should use a variety of estimation strategies including the following:

- Compatible numbers (e.g., 0.72 + 0.23 are close to 0.75 and 0.25, which are compatible numbers, so the sum of the decimal numbers must be close to 1.)
- Front-end addition (e.g., 32.3 + 24.5 + 14.1; A student might think, 30 + 20 + 10 is 60 and the ones and tenths clustered together make about another 10 for a total of 70.)
- Front-end subtraction (e.g., 1.92 – 0.7; A student might think, 19 tenths – 7 tenths is 12 tenths and 2 hundredths more is 1.22).
- Rounding (e.g., 4.39 + 5.2 is approximately 4 + 5 for an estimate of 9.)

Students should also apply mental computation strategies including the following:

- Compatible numbers (e.g., 3.55 + 6.45 or $3 and $6 would be $9 while 55 cents and 45 cents would make another dollar, for a sum of $10 or 10.)
- Front-end strategy (e.g., 7.69 – 2.45—A student might think there’s no regrouping needed. So 7 ones subtract 2 ones is 5 ones, 6 tenths subtract 4 tenths is 2 tenths and 9 hundredths subtract 5 hundredths is 4 hundredths, so the difference would be 5.24.)
- Compensate (e.g., $4.99 + $1.98 + $0.99 could be calculated by finding the sum of $5 + $2 + $1 which is $8 and then subtracting 0.04 or 4 cents. The sum would be $7.96.)
- Counting on / counting back (e.g., $2 – 1.48—A student might think, 2 more pennies would make $1.50 and 50 cents more makes $2 so the difference (change) is 52 cents.)
- Renaming (Think of 3.2 + 0.9 as 32 tenths + 9 tenths; 32 tenths – 9 tenths is 21 tenths; 21 tenths is the same as 2.1.)

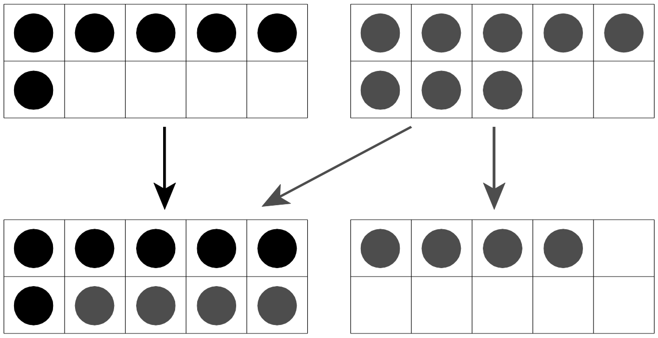
N11.02 Students should be presented with addition and subtraction story problems of all structures.

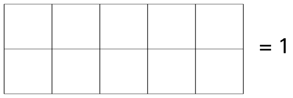
- Join (result, change, and start unknown)
- Separate (result, change, and start unknown)
- Part-part-whole (part and whole unknown)
- Compare (difference, smaller, and larger unknown)

Join story problems all have an action that causes an increase, while separate story problems have an action that causes a decrease. Part-part-whole story problems, on the other hand, do not involve any actions, and compare story problems involve relationships between quantities rather than actions.

In addition to solving story problems, students should also be asked to create story problems for different representations of these structures. The number sentences students will generate depend upon how they think about the problem.

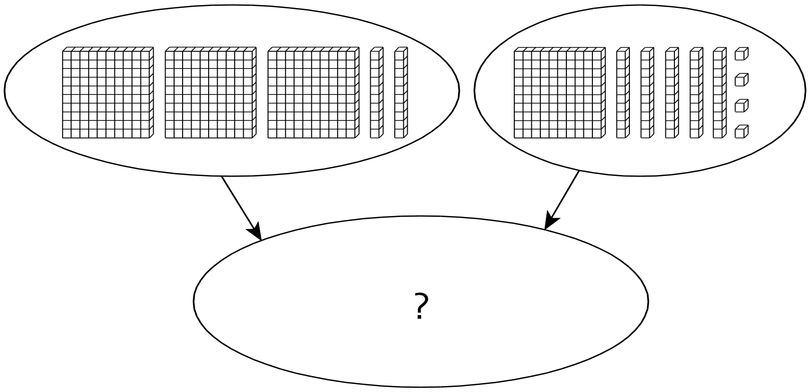
N11.02 and **N11.06** Students could use ten-frames to begin operations with decimal tenths, for example 0.6 + 0.8 = 1.4 could be represented as





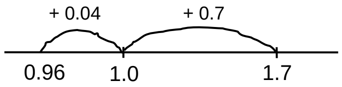
Through experience with concrete materials, students should recognize that adding or subtracting tenths (e.g., 3 tenths and 4 tenths are 7 tenths) is analogous to adding or subtracting quantities of other items (e.g., 3 apples and 4 apples are 7 apples). The same is true with hundredths. Rather than simply telling students to line up decimals vertically, or suggesting that they add zeroes, they should be directed to think about what each digit represents and what parts go together. For example, to solve 1.62 + 0.3, a student might think, 1 whole, 9 (6 + 3) tenths, and 2 hundredths, or 1.92.

Base-ten blocks and hundredths grids are useful models to explore these concepts. If a flat represents one whole unit, then 3.2 + 1.54 would be modelled as follows:

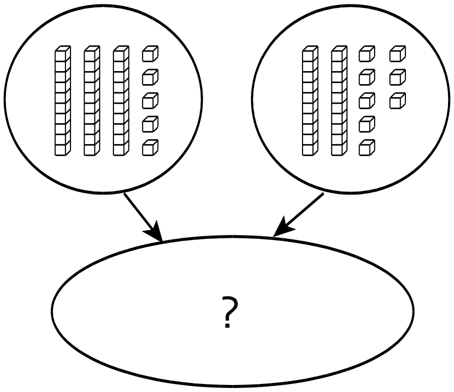


The sum would be represented by the joining of the sets, so students should again be directed to think about what each digit represents and what parts go together. Thus, the student might think, 3 ones and 1 one is 4 ones, 2 tenths and 5 tenths is 7 tenths, and 4 hundredths. The sum is 3.74. These amounts could also be represented by shading hundredths grids (three whole grids and two-tenths of a fourth one and one whole grid and fifty-four hundredths of a second one).

Number lines and open number lines can also be useful to model addition and subtraction. For example, to find the answer for 1.7 – 0.96, students could add on 0.04 to make 1 and then add on 0.7 to make 1.7.

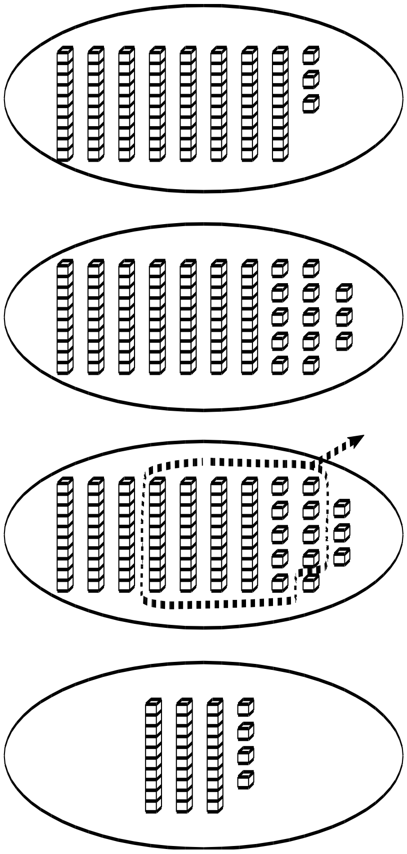


Emphasis should be placed on simple decimals that can be easily built with the concrete materials, as this is simply a developmental concept in this grade. For example, students could explore adding tenths before moving on to hundredths. Students should experience questions where regrouping is needed so that they have the opportunity to explain what happens when this occurs. For example, if the rod represents one whole, then students could be asked to combine the two sets below to determine the solution for 3.5 + 2.8.



Students should also have experiences with subtraction of tenths and hundredths with and without regrouping and should get solutions through modelling.

For example, if the flat represents one whole, then students could be asked to determine the solution for 0.83 – 0.49.



N11.03 and **N11.04** Most students understand that an exact sum or difference is not always required in real life and an estimate is often sufficient. This is particularly true when adding or subtracting decimal numbers that represent money and distances. When adding and subtracting decimal numbers, students should always estimate first as this requires them to focus on the relationships between numbers and the effect of number operations, rather than simply applying a rule to compute.

By providing students with many opportunities to estimate sums and differences in meaningful contexts, students will learn to assess which strategy works best, based on the decimal numbers with which they are working. Students should also recognize the usefulness of these strategies in everyday life, and in doing so, further develop their number sense.

When estimating, students will often use mental computation strategies. A number of these strategies were explored in outcome N03 and can be also be used in the context of decimals. Students may choose to use strategies such as,

- compatible numbers (e.g., \$0.72 + \$0.23 is close to \$0.75 and \$0.25 for an estimate of \$1)
- front-end addition (e.g., 32.3 + 24.5 may be thought of as 30 + 20 for an estimate of 50)
- front-end subtraction (e.g., 4.47 – 3.48 may be thought of as 3 ones minus 3 ones for an estimate of 1
- rounding (e.g., \$4.09 + \$5.99 is about \$4 + \$6 for an estimate of \$10)

N11.05 Focus on the value of estimating to determine how much change one would receive after a purchase, as well as determining the exact amount of change. Students will have many opportunities to calculate mentally with decimals, with the goal of arriving at an estimate and being able to explain why the answer is reasonable.

To determine an exact answer, students may choose to count on to calculate change and may use a number line to help them record the jumps taken when counting on. The number line in this case may be an open number line. For example, if students were to use a number line to help calculate change from \$20 for a purchase of \$18.65, it may look like this:

To help students further develop their own personal strategies for calculating change, and to reinforce the strategy of counting on, provide students with many opportunities to count back change for given purchases.

Patterns and Relations

SCO PR01 Students will be expected to identify and describe patterns found in tables and charts, including a multiplication chart.
[C, CN, PS, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

PR01.01 Identify and describe a variety of patterns in a multiplication chart.
PR01.02 Determine the missing element(s) in a given table or chart.
PR01.03 Identify the error(s) in a given table or chart.
PR01.04 Describe the pattern found in a given table or chart.

Performance Indicator Background

PR01.01, **PR01.02**, and **PR01.03** Mathematics 4 is a key year for students to develop fluency with multiplication facts. It is important for students to explore patterns in multiplication to help support this fluency. Students should find and explain patterns that occur in the multiplication grid. It is important that students understand they can use these patterns to determine other products or quotients.

×	0	1	2	3	4	5	6	7	8	9
9	0	9	18	27	36	45	54	63	72	81

Students should explore the following patterns in the multiplication table and explain why they work

- The product is an even number when two even numbers are multiplied.
- The product is an odd number when two odd numbers are multiplied
- The product is an even number when an odd and even number are multiplied.
- The numbers in each row and column increase by the same amount.
- The numbers in each row increase by an amount that is one greater than the increase in the previous row.
- The square numbers (1, 4, 9, 16, …) are found on the left to right diagonal and the numbers on the left to right diagonal increase by 1, 3, 5, ...
- The row for products of 4 is double the row for products of 2, the row for products of 6 is double the row for products of 3.
- When you add the corresponding products for 2 and 3, you get the products for 5 (e.g., 2 × 4 (8) plus 3 × 4 (12) is the same as 5 × 4 (20)).
- When you choose any 4 numbers that form a square on the grid, and multiply the numbers on the diagonal, the product is the same (e.g., 2 × 6 = 3 × 4).
- The grid is symmetrical (i.e., numbers under the left to right diagonal are reflections of the numbers over this diagonal).

In learning multiplication, students might explore the square numbers (along the diagonal of the multiplication chart) and notice that the representations of these numbers are, in fact, squares.

Students should be familiar with tables that list either all of the multiplication facts or some portion of them. For example, the three times table might be shown as follows:

×	0	1	2	3	4	5	6	7	8	9
3	0	3	6	9	?	15	18	21	24	27

Students can be encouraged to identify missing elements or errors in a table or chart.

PR01.04 An addition table can be used to explore patterns.

0	1	2	3
1	2	3	4
2	3	4	5
3	4	5	6

Students may observe that the number in any line is one more than in the preceding line, since one addend is increased by one and the other is not changed. Other patterns include the following:

- Only even numbers are located on the main diagonal (upper left to lower right), so the sum of a number with itself is always even.
- The numbers increase by 1 across a row, since 1 more is added for each step right.
- All of the 8s are on one diagonal line, since each time an addend is 1 greater, the other must be 1 less.
- There are three 2s, four 3s, five 4s, ... on the table.
- The diagonals of any four numbers that form a square will have the same sum.

Students should also explore the many patterns in the hundred chart. The hundred chart is a useful model to provide opportunities for students to find and describe a variety of patterns as well as identify missing elements and errors. Students should use vocabulary, such as vertical, horizontal, diagonal, row, and column to help describe patterns.

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

Students may discover patterns such as the following:

- The numbers increase by 1 across a row, since 1 more is added for each step right.
- The tens' digit increases by 1 as you go down a column.
- If you select any four numbers on the chart that form a square, and you add the two numbers on the diagonal, such as 59 + 68 and 58 + 69, the sum will be the same.
- There is a pattern when skip-counting by a particular number (by 2s, 3s, 4s, 5s, 9s, 10s, 25s, 50s, 100s).

Provide several hundred charts so students can explore place value and other patterns from 1 to 100, 101 to 200, up to 999. On these charts, use coloured counters to cover numbers forming a pattern and encourage students to explore the place-value representation of the covered numbers.

SCO PR02 Students will be expected to translate among different representations of a pattern (a table, a chart, or concrete materials).
[C, CN, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

PR02.01 Create a table or chart from a given concrete representation of a pattern.
PR02.02 Create a concrete representation of a given pattern displayed in a table or chart.
PR02.03 Translate between pictorial, contextual, and concrete representations of a pattern.
PR02.04 Explain why the same relationship exists between the pattern in a table and its concrete representation.

Performance Indicator Background

PR02.01, **PR02.02**, **PR02.03**, and **PR02.04** Students should be able to translate between representations of a given pattern. That is, when given a pattern made with concrete materials, students should create a table or a chart. Conversely, when given a pattern displayed in a table or chart, students should reproduce the pattern using concrete materials.

Students should be provided with ample opportunities to construct patterns with concrete materials, and then to create a table or chart to represent the same patterns. Students should be asked to describe what is happening in each representation of the pattern, thus enabling them to see that the relationships discovered exist in a variety of forms.

SCO PR03 Students will be expected to represent, describe, and extend patterns and relationships, using charts and tables, to solve problems.
[C, CN, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

PR03.01 Translate the information in a given problem into a table or chart.
PR03.02 Identify, describe, and extend the patterns in a table or chart to solve a given problem.

Performance Indicator Background

PR03.01 Students should be presented with a variety of problems to solve. Students should be encouraged to use concrete materials or grid paper to draw pictures to represent the problem. They should then arrange the data in charts or tables to see if a pattern exists. If students determine that a pattern exists, they may use or extend that pattern to solve the problem. Students should be encouraged to create problems that can be solved by making charts to find patterns.

PR03.02 Students should have opportunities, through problem solving, to make connections between physical patterns and information displayed in a table or a chart. Meaningful, real-life situations should be regularly provided to ensure that students have sufficient practice to extend patterns found in a table in order to solve problems.

SCO PR04 Students will be expected to identify and explain mathematical relationships, using charts and diagrams, to solve problems.
[CN, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

PR04.01 Complete a given Carroll diagram to solve a problem.
PR04.02 Determine where new elements belong in a given Carroll diagram.
PR04.03 Solve a given problem using a Carroll diagram.
PR04.04 Identify a sorting rule for a given Venn diagram.
PR04.05 Describe the relationship shown in a given Venn diagram when the circles overlap, when one circle is contained in the other, and when the circles are separate.
PR04.06 Determine where new elements belong in a given Venn diagram.
PR04.07 Solve a given problem by using a chart or diagram to identify mathematical relationships.

Performance Indicator Background

PR04.01, **PR04.02**, and **PR04.03** “Carroll diagrams are tables that work much like Venn diagrams, for the purpose of cross-classification. Two attributes are being used for sorting, with one attribute of each characteristic being the focus. For example, shapes might be sorted by whether they are round or not, and whether they are blue or not. A table is created with four cells to show the four possible combinations of these two attributes. Either the items themselves or the count of how many items of each type are put in the cells.” (Small 2008, p. 521). In a Carroll diagram, numbers or objects are either categorized as having an attribute or not having an attribute.

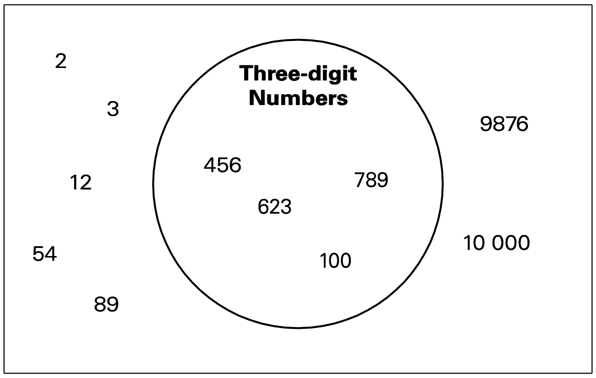
Carroll Diagram

	Even	Odd
Numbers less than 1000	892, 44, 240	39, 491, 999
Numbers more than 1000	7354, 6608	3421, 6507

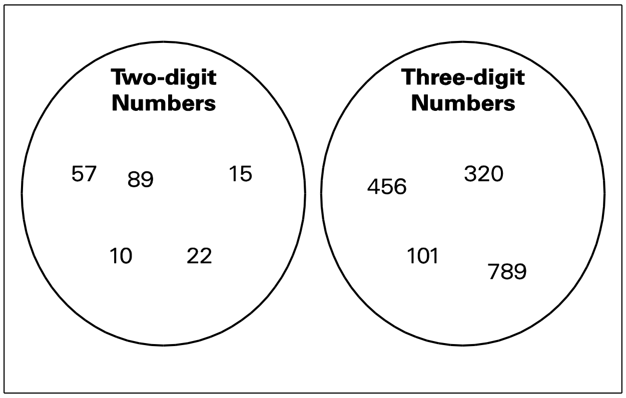
PR04.04, **PR04.05**, and **PR04.06** A Venn diagram is typically drawn with one, two, or three circles. At this grade level, we will use Venn diagrams with one and/or two circles. “Note that the circles of a Venn diagram do not have to overlap. They can be two discrete circles if the attributes are exclusive, for example, if one were used to contain girls and one contained boys. They could also be separate circles if the items involved in the sort do not exhibit the same characteristics, even if they have the potential to do so.” (Small 2008, p. 521).

Venn diagrams may take the forms below:

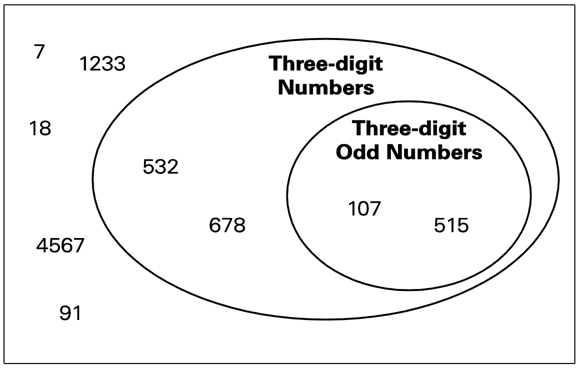
One Circle—If only one attribute is being used to sort the objects, then there would be only one circle in the Venn diagram.



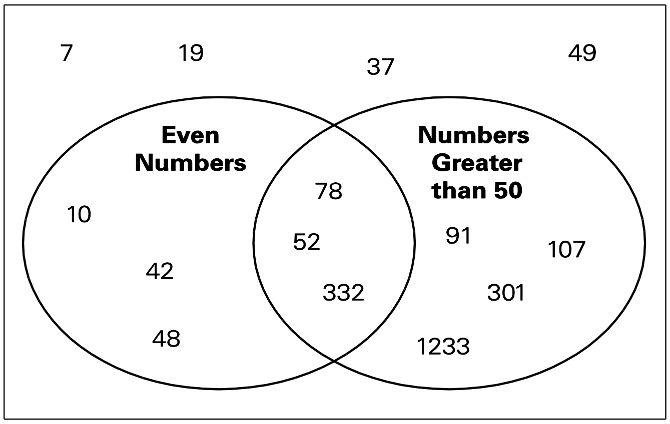
Two Circles—Two separate circles are used when the items being sorted do not share common attributes.



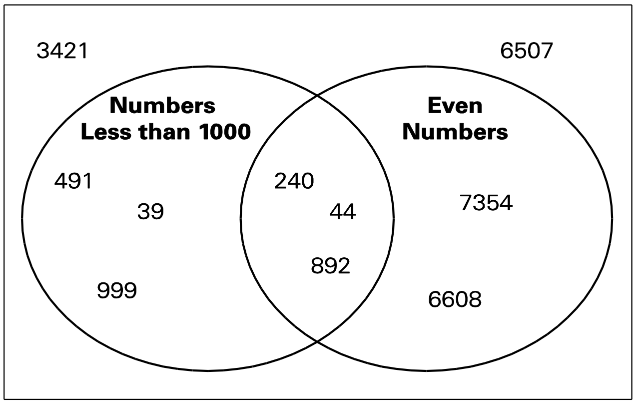
Two Circles—One circle is contained within another circle when the inner circle is a subset of the outer circle.



Two Overlapping Circles—Two overlapping circles are used when the items being sorted share common attributes.



Introduce the notion of cross-classification using Venn diagrams by setting out two hoops, side by side. Label each hoop with a sorting rule. Ensure the sorting rules and objects to be sorted lend themselves to cross-classification.



SCO PR05 Students will be expected to express a given problem as an equation in which a symbol is used to represent an unknown number.
[CN, PS, R]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

PR05.01 Explain the purpose of the symbol in a given addition, subtraction, multiplication, or division equation with one unknown (e.g., 36 ÷ ⍰ = 6).
PR05.02 Express a given pictorial or concrete representation of an equation in symbolic form.
PR05.03 Identify the unknown in a problem; represent the problem with an equation; and solve the problem concretely, pictorially, and/or symbolically.
PR05.04 Create a problem in context for a given equation with one unknown.

Performance Indicator Background

PR05.01 In Mathematics 3, students have explored open sentences primarily with addition and subtraction of whole numbers. At this level, the four basic operations for whole numbers will be used. Emphasis should be placed on open sentences involving the facts and simple multi-digit calculations. Students should be given the opportunity to create and solve a variety of open sentences.

Although open multiplication sentences with missing factors—particularly with a missing first factor—may be more challenging for students to solve than open sentences with the product missing; students need experience with all types to develop the facility needed for division.

Provide students with opportunities to practise stating the meaning for open sentences, such as

- 4 × ____ = 24 may be read as, Four sets of how many make 24?
- _____ × 5 = 15 may be read as, How many sets of 5 equal 15?
- 3 × 6 = ____ may be read as, Three sets of six results in a product of ____?

Students should explain and apply procedures to solve some of these open sentences. For example, students might note that if 4 sets of something makes 24 in all, then by sharing 24 in 4 sets, they can determine how much is in each set. Thus 24 ÷ 4 = 6 gives the solution to 4 × Δ = 24.

PR05.02 The use of equations is an efficient way to present information. In order for students to have a clear understanding of equations, they need to be able to use mathematical symbols such as the operational symbols (+, –, x, ÷) and relational symbols (=, <, >) with understanding. Students frequently see the equal sign as an operator symbol and believe, erroneously, that it means, “Here is the answer.” However, the equal signs demonstrates that there are two equivalent ways to show a number. Students should understand that there are many ways to represent the same number, sum, product, difference, or quotient. The equal sign is used to represent these equivalencies; just as the other relational symbols (<, >) are used to show which number is greater or lesser.

PR05.03 One way to clarify the role of the equal sign as a relational symbol is to present open sentences in a variety of forms. Open sentences are often presented to students in the form of $a + b = \Delta$. Students should have experience seeing the unknown in a variety of positions. As well, they should see operational symbols on both sides of the equal sign. Students can be shown the parallel between sentences such as 5 + Δ = 8, 8 = 5 + Δ, 8 – Δ = 5, and 8 – 5 = Δ. Students should be familiar with the use of open sentences for all operations with simple numbers. They should be extending their understanding to more complex situations (e.g., 14 × 3 = Δ; 14 sets of 3 are how many? 14 × Δ = 42; 14 sets of how many are 42? 42 ÷ 3 = Δ; 42 divided into groups of 3 are how many groups?)

Connect the concrete, pictorial, and symbolic representation consistently as the students develop and demonstrate understanding of equations. Provide students with a variety of story problems and ask them to record appropriate equations to represent the situations.

PR05.04 Creating a context for solving open sentences is essential for developing an understanding of open frames. Students need to read equations for meaning and write equations when given the meanings. While mathematical symbols are important, students need to be able to interpret the symbols using words and mathematical contexts, such as story problems. Similarly, students need to be able to interpret story problems with the language of mathematics and record their understanding using mathematical symbols.

Use everyday contexts for problems that are meaningful to students. This will support students in translating the meaning of the problem into an appropriate equation using the symbol to represent the unknown number.

SCO PR06 Students will be expected to solve one-step equations involving a symbol to represent an unknown number.
[C, CN, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

PR06.01 Represent and solve a given one-step equation concretely, pictorially, or symbolically.
PR06.02 Solve a given one-step equation using guess and test.
PR06.03 Describe, orally, the meaning of a given one-step equation with one unknown.
PR06.04 Solve a given equation when the unknown is on the left or right side of the equation.
PR06.05 Represent and solve a given addition or subtraction problems involving a “part-part-whole” or comparison context using a symbol to represent the unknown.
PR06.06 Represent and solve a given multiplication or division problem involving equal grouping or partitioning (equal sharing) using symbols to represent the unknown.
PR06.07 Solve equations using a symbol to represent the unknown.

Performance Indicator Background

Outcome **PR05** and **PR06** are closely related. Please refer to the performance indicator background for PR05 for additional information.

PR06.07 Students should use reasoning and problem-solving strategies to determine missing values in open sentences of the form $a + b = c$ where one of a, b, or c is missing. Students should be able to explain the strategy they have used (counting on, think addition, etc.).

$a + b = \square$	(e.g., $6 + 3 = \square$)	Join—Result Unknown
$a + \square = c$	(e.g., $2 + \square = 8$)	Join—Change Unknown
$\square + b = c$	(e.g., $\square + 4 = 5$)	Join—Initial Unknown
$c - a = \square$	(e.g., $7 - 2 = \square$)	Separate—Result Unknown
$c - \square = b$	(e.g., $4 - \square = 2$)	Separate—Change Unknown
$\square - a = b$	(e.g., $\square - 8 = 1$)	Separate—Initial Unknown

Similarly, students should be given the opportunity to determine a missing multiplicand in a product involving the fact families to 10 × 10. Students should use models and pictures to determine the missing values. For example if given 4 × ____ = 24, students could use tiles to build an array with 24 counters that has four rows. Or a student might be asked 12 ÷ ____ = 6 and use 12 counters to make groups of 6 and see that there would be two groups. Students should see a variety of open sentences for multiplication and division, but these should only be explored developmentally and only after considerable work with developing the meaning of multiplication and division has occurred.

$a \times b = \square$	(e.g., $6 \times 3 = \square$)
$a \times \square = c$	(e.g., $2 \times \square = 8$)
$\square \times b = c$	(e.g., $\square \times 4 = 20$)
$c \div a = \square$	(e.g., $14 \div 2 = \square$)

$c \div \square = b$ (e.g., $12 \div \square = 2$)
$\square \div a = b$ (e.g., $\square \div 4 = 3$)

Students should recognize and be able to explain that the missing dimension is the missing multiplicand. It is important to note that students are not expected to be proficient at open sentences for multiplication in this grade, but should be given opportunities to solve these problems through building; as such, it is best to work with small manageable numbers.

Measurement

SCO M01 Students will be expected to read and record time using digital and analog clocks, including 24-hour clocks. [C, CN, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

M01.01 State the number of hours in a day.
M01.02 Express the time orally and numerically from a 12-hour analog clock.
M01.03 Express the time orally and numerically from a 24-hour analog clock.
M01.04 Express the time orally and numerically from a 12-hour digital clock.
M01.05 Express time orally and numerically from a 24-hour digital clock.
M01.06 Describe time orally as “minutes to” or “minutes after” the hour.
M01.07 Explain the meaning of a.m. and p.m., and provide an example of an activity that occurs during the a.m., and another that occurs during the p.m.

Performance Indicator Background

M01.02 and **M01.06** To read analog clocks students should begin by focusing on the position of the hour hand. Van de Walle and Lovin (2006 p. 244) suggest using a one-handed clock to help students understand and read analog clocks.

Students need to be very attentive to the length of the hands, as well as the numbers, on an analog clock. Students should learn that the minute and hour hands on an analog clock are different lengths. Students should also understand that whether they read the number as an hour or as minutes will depend on which hand is pointing to the number. They should also understand that the minute hand is pointing to the 6 for _:30 and at 12 for the _:00.

Students need to be aware that the hour hand moves during the course of the hour, and should be able to describe what happens to the minute hand as the hour hand moves from one hour to another. They should also recognize when the time is at _:30, the hour hand is halfway between the two numbers. Often students do not position the hour hand to reflect the number of minutes after the hour. Discuss with students how the hour hand moves over the course of each hour. Note such things as “a little past three o’clock” or “halfway between six and seven o’clock” or “about four o’clock.” It is essential to provide many opportunities for students to manipulate the hands of an analog clock to help them visualize how the hour hand moves in relation to the minute hand. As time-telling skills develop, suggest to students that they look first to the hour hand to predict an approximate time and then to use the minute hand for precision.

Students need to learn how to express time using a 12-hour analog clock. After having experiences telling time on the hour, and to the hour, students should be given opportunity to read time that falls between the hours. This is often more challenging for students; therefore, it is important to take time to analyze with students how the analog clock shows the passage of the standard units of time. Building students’ conceptual understanding of analog time helps students make sense of time-related terms (e.g., 8:15 may be read as “quarter past 8”). Expressing time orally offers an excellent opportunity for students to use their bodily-kinesthetic to visualize and solve problems relating to time on an analog clock. Introduce the terms **half past**, **quarter after**, and **quarter to** using an analog clock. Provide an open space to create movement where children can sit on the floor and arrange themselves to represent the numbers and hands on a clock. Once they have physically arranged themselves, ask them to show various times on a clock.

If time falls more than halfway through the hour, it may be read either as a number of minutes to the hour, or a number of minutes after the previous hour. Time after the hour and time before the hour should be addressed. The time 8:45 might be read as eight forty-five, forty-five minutes past eight, quarter to nine, or fifteen minutes to nine.

Students should not be limited to reading time to the nearest five minutes if they show good understanding. They can be encouraged to read time to the minutes. Using a clock that shows not only the numbers from 1–12 but also minute amounts from 5–55 beside the numbers from 1–11 may be useful. As students establish a comfort level with skip-counting by five, this will support them in reading time to the nearest five minutes. This provides students with an opportunity to relate the numbers on a clock to time. Students may have already been introduced to the “clock facts” as one mental mathematics strategy for learning multiplication of five facts. This would be an appropriate time for teachers to make students aware that there are five minutes between numbers on a clock. The long hand on the 2 represents 10 minutes, so two one-minute spaces past the 2 is 12 minutes.

M01.04 and **M01.05** Students find it easier to read times from a digital clock, but it is important to talk about the meaning of the times they are reading. Using an analog and a digital clock together may help with this. Reading a clock, or telling time is more about reading an instrument. Time is a measurement; however, it encompasses duration. For students to make sense of the concept of time, they need to understand that time, as a measurement, is about how long an event takes from beginning to end. This is called elapsed time. Elapsed time can be found only by counting the hours and minutes between the start and end times. Much of the learning that students are to attain can be assessed on an ongoing basis through daily conversations and activities in which time is naturally included.

Throughout the school day, students can learn the duration of long and short events that can be measured in seconds, minutes, and hours. Students should read times on various clocks to provide information about relevant situations, such as

- comparing start and end times to determine how much time has passed
- estimating how long before an event begins
- planning events
- reading schedules

M01.03 and **M01.05** It is important for students to learn about the 24-hour time system. The 24-hour clock is a system used for telling time in which the day runs from midnight to midnight and is numbered from 0 to 23. It is often used when it is very important that times are not confused. The 24-hour clock eliminates uncertainty as there is only one 11:32, for instance, during the day. Student may have encountered everyday life situations where the 24-hour time system is used if they have travelled on flights, trains, or ferries. It is also used in the practise of medicine because it helps prevent ambiguity about important events in a patient’s medical history. The 24-hour system is sometimes referred to as military time. Once students are comfortable reading a 24-hour clock, they may observe that subtracting 12 is a convenient way to tell the more familiar 12-hour time. In the 24-hour notation, a time of day is written in the form hh:mm (e.g., 22:30, where the 22 means 22 full hours have passed since midnight and 30 full minutes have passed since the last full hour). Note that digital time is always expressed with four digits, with a 0 being placed at the beginning of times less than 10. For example, 8:00 a.m. is expressed as 08:00 hours.

M01.07 “A good way to investigate a.m. and p.m. is to use a full-day timeline.”(Small 2008, p. 447) This helps students become familiar with using a.m. and p.m. notations correctly. It also helps students who confuse a.m. and p.m..

SCO M02 Students will be expected to read and record calendar dates in a variety of formats.
[C, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

M02.01 Write dates in a variety of formats (e.g., yyyy/mm/dd, dd/mm/yyyy, March 21, 2014, dd/mm/yy).
M02.02 Relate dates written in the format yyyy/mm/dd to dates on a calendar.
M02.03 Identify possible interpretations of a given date (e.g., 06/03/04).

Performance Indicator Background

M02.01 and **M02.02** Writing dates in a numeric representation is faster than writing words and is often in use today. Students should explore newspapers, receipts, school registrations, forms, calendars, and other documents to investigate the varieties of ways in which dates are written.

M02.03 Students should understand that the different forms for dates may result in different interpretations of those dates. For examples, 06/03/04 might mean March 6, 2004, June 3, 2004, or March 4, 2006. For this reason, Canada has adopted a standard notation for recording dates.

SCO M03 Students will be expected to demonstrate an understanding of area of regular and irregular 2-D shapes by

- recognizing that area is measured in square units
- selecting and justifying referents for the units square centimetre (cm^2) or square metre (m^2)
- estimating area using referents for cm^2 or m^2
- determining and recording area (cm^2 or m^2)
- constructing different rectangles for a given area (cm^2 or m^2) in order to demonstrate that many different rectangles may have the same area

[C, CN, ME, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

M03.01 Describe area as the measure of surface recorded in square units.
M03.02 Identify and explain why the square is the most efficient unit for measuring area.
M03.03 Provide a referent for a square centimetre, and explain the choice.
M03.04 Provide a referent for a square metre, and explain the choice.
M03.05 Determine which standard square unit is represented by a given referent.
M03.06 Estimate the area of a given 2-D shape using personal referents.
M03.07 Determine the area of a regular 2-D shape, and explain the strategy.
M03.08 Determine the area of an irregular 2-D shape, and explain the strategy.
M03.09 Construct a rectangle for a given area.
M03.10 Demonstrate that many rectangles are possible for a given area by drawing at least two different rectangles for the same given area.

Performance Indicator Background

M03.01 Length is a one-dimensional measurement, whereas area describes how many square units are required to measure two-dimensional surfaces. Sometimes square units refer to the space inside a region (e.g., inside the perimeter, such as the area of a field. Other times, square units measure how much it takes to cover a region, such as the number of tiles needed to cover a floor. Area is most often expressed in square units, such as square centimetres (cm^2) and square metres (m^2).

Area measures are often thought of as being flat, and at this time students will mainly investigate the area of flat surfaces. Be aware, however, that there are instances in our environment when this might not necessarily be the case (e.g., farm acreage or a golf course that might include hills). Students should understand that area tells about the space an object takes up on a flat surface. Students should learn that the area of a shape is preserved (i.e., the area of a shape does not change if it is cut up and rearranged to make a different shape). Students may not understand that an area that is rearranged into different shapes will still have the same area measurement as the original shape.

As students explore area, reinforce the importance of naming the measurement unit each time a measurement is said because the units communicate how big the measurement is. Without the unit, there is no way of knowing what the numbers mean. It is also important that students learn that the units used to measure the area of an object or to compare the areas of two objects must be the same size.

Comparison activities should be designed to help students discover that it is necessary to apply the same unit of measure when comparing two different areas. A common misconception is for students to rely on numbers alone, without considering the size of the units.

Initially, students may explore measuring area with different types of non-standard units, and then transition into using standard units to measure area. By allowing students to make choices of which unit to use to measure an area, they will be afforded the opportunity to discuss and compare the effectiveness of each area unit. They should consider questions, including the following:

- Which area unit gives a more accurate measure?
- Which area unit is easier to count?
- Why would leaving gaps not give an accurate measurement?
- Why would overlapping not give an accurate measurement?

Review linear units (centimetre and metre) used to find the perimeter of 2-D shapes. Connect the need for standard units in finding perimeter to the need for standard units in finding area. Standard units are used so that everyone can understand the size of the unit. "The first standard unit students encounter is the square centimetre. A square centimetre is an area equivalent to the area of a square with a side length of 1 cm." (Small 2008, p. 395) The symbol is written 1 cm^2 and is read as one square centimetre and not one centimetre squared.

Small (2008) suggests a sequence for the introduction of square centimetres. "Initially square centimetres are introduced in a concrete way, using materials such as centimetre cubes (base ten units), and then pictorially, using centimetre grid paper and transparent centimetre grids. The cubes and grids provide a good transition from non-standard units because students can simply cover a shape with cubes and count to measure the area without focusing on the fact that it is a standard unit." (Small 2008, p. 395)

Students need to learn that it is important to state the square unit of measure, usually square centimetres or square metres, when recording and reporting area measurements in standard units. It is recommended that the use of words precede the use of the abbreviated form of the measure in order to facilitate conceptual understanding. Students will understand that an area measuring 14 square centimetres can be written as 14 cm^2.

M03.02 To help students identify the square as the most efficient unit for measuring area, invite them to measure a rectangle using coins that obviously do not fit tightly together. Students will see that there are spaces between the coins that are not covered and are, therefore, not counted in the measurement. Consequently, they end up with an inaccurate measurement. Students will come to understand that any object that fills a space can be used, but squares are most commonly used because squares fit together on any side, do not leave gaps, fit no matter which way they are turned, and because they make rows that are easy to count. It is important to point out, however, than any units that fit together with no spaces in between and with no overlaps can also be used.

M03.03, **M03.04**, and **M03.05** Referents are familiar objects that students can refer to or visualize to help them develop a strong understanding of a unit of measure. Students should suggest a suitable referent for 1 cm^2 and 1 m^2. For example, students could use one face of a small base-ten cube as a benchmark for one square centimetre and then suggest a personal referent for 1 cm^2, which might be the surface of the nail on their smallest finger. Students could also build a square metre out of four metre sticks or with other materials. With this model as their referent, students should be able to estimate how many square metres make up the area of such things as the classroom floor.

M03.06 Personal referents can be used to help students estimate area, and students should be expected to use their personal referents for 1 cm^2 and 1 m^2 to estimate the area of shapes. Various techniques can be used to estimate area. These include using a referent for a single unit of measure and then iterating it to obtain an estimate, or using chunking (i.e., estimating the area of a smaller portion of a shape and using this estimate to estimate the entire area of the shape).
Once students have developed personal referents for standard units to measure area, they should be provided with ongoing opportunities to apply their understanding to problem-solving situations. Encourage students to explore different strategies to solve problems.

M03.07 Students will be introduced to standard measures for area including the square centimetre and square metre. Students can use square centimetres transparent grids as a way to cover objects to determine their area and should be encouraged to determine “easy ways” to count the area connecting to multiplication concepts and by drawing on their knowledge of multiplication (although they will not be expected to give a formula). For example, a student may note that a given rectangle measures 4 rows of 6 square centimetres or 24 square centimetres in all.

Students should also be provided with the opportunity to build a square metre using materials such as rolled newspaper and tape and should use this square metre to measure areas such as the classroom floor or a part of the playground. Students should develop approaches for determining the area of both geometric shapes with which they are familiar and irregular shapes (i.e., approximating the area of a lake in an aerial photo).

M03.08 It is essential to provide students with opportunities to measure irregular shapes, since the real-life applications of area measurement apply to all two-dimensional shapes or regions, not just rectangular ones. The measurement of irregular two-dimensional shapes can be explored in the following ways:

- Drawing on centimetre grid paper or centimetre dot paper
- Laying an acetate centimetre grid over shapes
- Using geo-boards
- Cutting and reassembling
- Using pentominoes

M03.09 and **M03.10** Students should examine the areas of different rectangles to develop an understanding that a given area can result in rectangles with different dimensions. This should be done through the use of concrete materials and exploration. For example, students could be given a number of colour tiles and asked to arrange the tiles to create as many different rectangles as possible. They can then examine the dimensions of each rectangle to discover that many different rectangles may have the same area. Students can also use centimetre grid paper to draw various rectangles with a given area.

Geometry

SCO G01 Students will be expected to describe and construct rectangular and triangular prisms.
[C, CN, R, V]

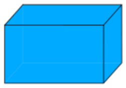
[C] Communication **[PS]** Problem Solving **[CN]** Connections **[ME]** Mental Mathematics and Estimation
[T] Technology **[V]** Visualization **[R]** Reasoning

Performance Indicators

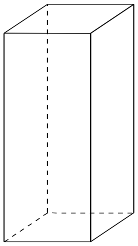
G01.01 Identify and name common attributes of rectangular prisms from given sets of rectangular prisms.
G01.02 Identify and name common attributes of triangular prisms from given sets of triangular prisms.
G01.03 Sort a given set of right rectangular and triangular prisms, using the shape of the base.
G01.04 Construct and describe a model of a rectangular and a triangular prism, using materials such as pattern blocks or modelling clay.
G01.05 Construct rectangular prisms from their nets.
G01.06 Construct triangular prisms from their nets.
G01.07 Identify examples of rectangular and triangular prisms found in the environment.

Performance Indicator Background

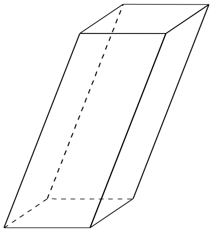
G01.01 and **G01.02** Prisms are special 3-D shapes that have two congruent and parallel polygons as bases. Two shapes are congruent when they have the same size and shape. Shapes are parallel when the shapes are the same distance apart, such as the shelves on a bookcase. The shape of the base is used to name the prism. For example, a rectangular prism has two congruent and parallel rectangles as bases, and a triangular prism has two congruent and parallel triangles as bases.



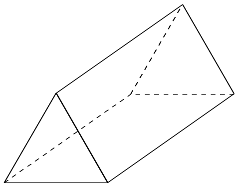
All prisms used in Mathematics 4 are right prisms. For clarification purposes, a prism is “right” if the faces form a right angle with the bases (or we can say “are perpendicular with the bases”). Below are examples of prisms illustrating the difference.



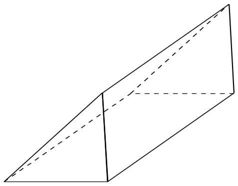
Right
Prism



Non-Right
Prism



Right
Triangle
Prism



Non-Right
Triangle
Prism

All prisms have **faces**, two of which are customarily referred to as **bases**. These two bases may take the shape of any polygon. For clarification purposes, prisms can be thought of as having two names. The first name refers to the shape of the bases and a second name, which is **prism** (e.g., triangular prism, rectangular prism).

Some students may be keen to identify other prisms such as hexagonal prisms or square prisms (square prisms fall into the category of rectangular prisms because a square is a rectangle). However, in Mathematics 4, exploration is focused on **rectangular prisms** and **triangular prisms** only.

There is a developmental sequence associated with how students think and reason geometrically. Many students in Mathematics 4 are beginning to develop more sophisticated abilities to identify and name 3-D objects. As levels of geometric thinking develop, students will notice more attributes of three dimensional objects. These attributes are the components that go together to make up the form—**edges**, **vertices**, and **faces** (two which are the **bases**). In the process of identifying and naming attributes of prisms, it may necessary to review and encourage students to use appropriate vocabulary such as number of faces, number of edges, number of vertices, or shapes of the faces/bases. A rectangular prism has 6 faces, 12 edges, and 8 vertices. (Note that all square prisms can be called rectangular prisms because a square is a rectangle.) A triangular prism has 5 faces, 9 edges, and 6 vertices. Allow each student to manipulate concrete models of 3-D shapes so that they are able to touch and count each of the faces, vertices, and edges.

One way to familiarize students with right rectangular and triangular prisms is to have several 3-D objects in front of pairs of students for them to examine as you call out clues about the properties of the object. For example, This 3-D object has 8 vertices. As you give clues, have the children figure out which 3-D object you are thinking of. While some students may be able to think of the objects visually, it is best to give all students (or pairs of students) the concrete objects to help them. Showing models of triangular pyramids, square pyramids, and rectangular pyramids next to the corresponding prisms will help students see the similar way in which prisms and pyramids are named by their bases.

G01.03 Sorting requires students to attend to specific attributes of objects. Give students a variety of 3-D prisms (real-life or commercially made models). Ask students to sort the prisms according to a given attribute, such as shape of the base.

Provide 3-D objects (real-life or commercially made models) such as spheres, cones, cylinders, pyramids, (students will be familiar with these from Mathematics 3) as well as rectangular prisms and triangular prisms. Place two hula hoops on the floor to represent a large scale Venn diagram. Provide labels and have students sort the objects according to triangular prisms, rectangular prisms, and other. As students place their object on the diagram, have them explain, to the class, why they placed objects in certain places.

G01.04 Constructing prisms can take many forms. One way is to make concrete models. Pattern blocks are very good for this, but many teacher-made materials can be used. While the pattern block pieces are prisms, they have been treated as 2-D shapes; however, stacking a number of triangles or squares from the pattern blocks would provide examples of different prisms. This stacking would help students conceptualize the uniform nature of prisms. Students may also build rectangle-based prisms with interlocking cubes or with other commercially available materials, such as geo-frames.

Another type of model is a skeleton. This is a model showing only the edges and vertices of a 3-D shape. Students can make skeletal models for prisms using rolled newspapers and tape, straws and string, toothpicks and miniature marshmallows, and small balls of modelling clay or straws with pieces of pipe cleaners. Display a variety of 3-D objects for students and invite students to build skeletons of those prisms. Some students may need to touch the edges and vertices in order to construct a skeleton. The process of making a skeleton helps students visualize the object and remember its properties.

G01.05 and **G01.06** Students should be given copies of **nets** of **rectangular** and **triangular** prisms to cut out and fold up. They should be encouraged to unfold them and examine the 2-D shapes that are connected to make each net. Encourage students to visualize the nets folding up and unfolding. In addition to cutting out and assembling prepared nets, it is expected that students will create their own nets for rectangular and triangular prisms. They will also consider the various possibilities for these nets. Ask students to trace on paper the various **faces** of a given prism to make its net. Ask students to cut out the net and fold it up around the shape to see if it works. Ask them to record their net on grid paper. Then, ask them to cut one of the faces off and investigate the possible places it could be reattached to make a new net. Have them record each new net on grid paper.

Students should make the “footprints” for prisms and then compare these “footprints” (faces) and share their discoveries. One way to become familiar with prisms is to observe, touch, and manipulate 3-D models of these shapes. Another way is to build them from 2-D plans, called nets. Students should be provided with nets for triangular, square, and rectangular prisms. They should then cut them out and fold and tape them into the 3-D models.

G01.07 Display a set of right rectangular prisms on a table (vary the set by including different sizes of triangular and rectangular prisms, cubes, and prisms positioned in different orientations). Invite students go around the room or the school and find objects that match the shapes. Ask individual students to present their findings. Ask how their “found” object is alike or different from prisms on the display table. Listen to student responses and encourage them to name the common attributes.

Students should have the opportunity to explore various rectangular and triangular prisms and explain how they are similar and how they are different. Students should have a great deal of practice touching, examining, and talking about these objects. Looking for real-world examples of these prisms would also help students construct strong visual images of these objects. These experiences will support students in sorting prisms.

SCO G02 Students will be expected to demonstrate an understanding of congruency, concretely and pictorially.
[CN, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

G02.01 Determine if two given 2-D shapes are congruent, and explain the strategy used.
G02.02 Create a shape that is congruent to a given 2-D shape, and explain why the two shapes are congruent.
G02.03 Identify congruent 2-D shapes from a given set of shapes shown in different positions in space.

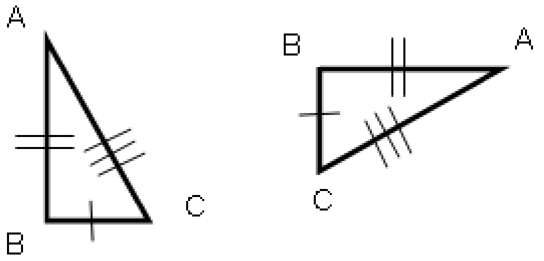
Performance Indicator Background

G02.01 "Two 2-D shapes are congruent if they are identical in shape and size (i.e., if one is an exact duplicate of the other). Students sometimes do not understand the difference between the math term **congruent** and the everyday term **the same**. It is important to recognize that the term **congruent** applies only to size and shape. Thus, figures can be different colours, or oriented in different ways, and they will still be congruent as long as they are the same shape and the same size." (Small 2008, p. 316)

Ask students to explain the strategy they used to determine whether the shapes were congruent. Suggest that they trace and cut out the shape they created and then superimpose them on the given shape to prove congruency.

G02.02 Students should be given a 2-D shape and be asked to create a shape that is congruent. It is important for students to test for congruency because shapes in different orientations may not appear to be congruent, even when they are.

G02.03 Students should label corresponding vertices and colour-code corresponding sides of congruent pairs of 2-D shapes that they created or are presented to them. Instead of colour-coding the corresponding sides, the students may wish to use markings on the sides as shown below. Include examples that have the congruent shapes in different orientations as shown in the diagram.



Encourage students to justify that they have identified the corresponding sides and vertices correctly by tracing one shape complete with the markings and superimposing it on the other congruent shape. The labelled vertices and colour-coded or marked sides should match.

SCO G03 Students will be expected to demonstrate an understanding of line symmetry by
- identifying symmetrical 2-D shapes
- creating symmetrical 2-D shapes
- drawing one or more lines of symmetry in a 2-D shape

[C, CN, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

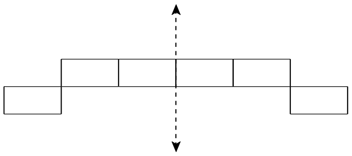
G03.01 Identify the characteristics of given symmetrical and non-symmetrical 2-D shapes.
G03.02 Sort a given set of 2-D shapes as symmetrical and non-symmetrical.
G03.03 Complete a symmetrical 2-D shape, given one-half the shape and its line of symmetry, and explain the process.
G03.04 Identify lines of symmetry of a given set of 2-D shapes, and explain why each shape is symmetrical.
G03.05 Determine whether or not a given 2-D shape is symmetrical by using an image reflector or by folding and superimposing.
G03.06 Create a symmetrical shape with and without manipulatives and explain the process.
G03.07 Provide examples of symmetrical shapes found in the environment, and identify the line(s) of symmetry.
G03.08 Sort a given set of 2-D shapes as those that have no lines of symmetry, one line of symmetry, or more than one line of symmetry.
G03.09 Explain connections between congruence and symmetry using 2-D shapes.

Performance Indicator Background

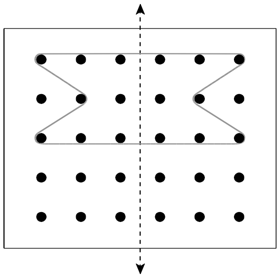
G03.01 Knowledge of congruency underpins the understanding of symmetry. Teachers should model the word **congruent** however, students may describe the concept of congruency without always using the explicit term **congruent** (equal parts; same size and shape).

Congruency and symmetry can be used to determine what makes some shapes alike and different. Any symmetrical shape can be divided into two congruent parts along the line of symmetry; however, not every composite shape made up of two congruent figures is symmetrical.

Students should become familiar with the terms **symmetry**, and **lines of symmetry**. A 2-D shape has **line symmetry** when it can be divided or folded so that the two parts match exactly. We refer to a fold line as a **line of symmetry**. Any given line of symmetry divides a figure into equal halves (relates to Outcome N08 for fractions). It may also be said that each of the halves are mirror images of each other. Some texts may refer to line symmetry as reflective symmetry or mirror symmetry.



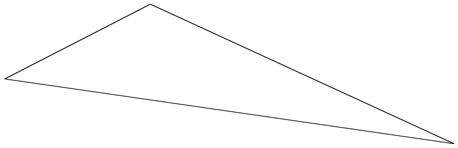
symmetrical tile model



symmetrical shape on a geo-board

Students should also use paper folding and cutting to determine lines of symmetry that divide shapes into two congruent parts. Students should be encouraged to predict what shapes will be created by performing a cut or fold of a shape and should investigate what will happen if the shape is folded or cut in a different way. Students might also explore folding and cutting shapes in ways that are non-symmetrical and make predictions related to these actions.

Students might also explore shapes that cannot be folded exactly in half such as the triangle shown below. Such explorations lead nicely into conversations about line symmetry. Students will see that shapes that have line symmetry can be folded exactly in half and shapes that cannot be folded in half do not have line symmetry.

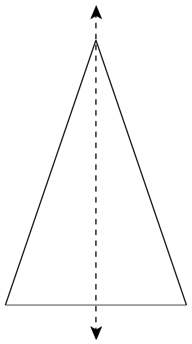


G03.02 This is an opportunity for students to learn that there is more than one way to determine whether or not a shape is symmetrical. Provide students with Power Polygons or other manipulatives, as well as Miras, paper, and scissors. Have students decide and record whether each given shape is symmetrical or not. (Ask students how they might use folding as a strategy when using hard plastic shapes. They will quickly conclude that to solve this problem, they might trace and cut the shape from paper so that it is foldable.)

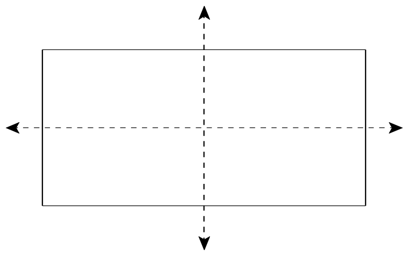
G03.03, **G03.05**, and **G03.06** Students have learned about symmetry and now should be provided with opportunities to create their own symmetrical 2-D drawings. Miras (transparent mirrors) are very helpful to students when investigating symmetry. They are useful because they are both transparent and reflective. If a shape is symmetrical along the line where the Mira has been placed, the image on one side of the shape will fall right on top of the other side of the shape.

Display a simple shape and tell students that it is half of a symmetrical picture. Ask what the whole shape looks like. Allow some time for students to offer suggestions. Then introduce the Mira as one tool to assist them in completing the symmetrical shape. Is there more than one possibility depending on where the Mira is positioned? Give students a Mira and provide them with drawings of half shapes that have a dotted line representing the line of symmetry. Next, have them place the Mira on the dotted line and trace the reflection to complete a symmetrical design.

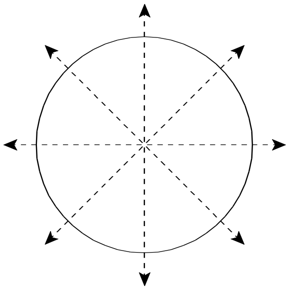
G03.04 Students should be using basic reasoning to justify whether a shape has or does not have line symmetry. When asked if a shape has line symmetry students might respond with statements such as, This square does because I can fold it in half, or This triangle doesn’t because I can’t fold it in half. Through exploration of folding and cutting many 2-D shapes, students will see that some shapes can be cut into two congruent parts in only one way, while other shapes have more ways to cut them into two congruent parts. The main focus should be on recognizing that shapes that can be folded into two congruent parts are special; they have line symmetry.



One way to fold



Two ways to fold



Many ways to fold (in fact an infinite number of ways)

G03.07 Symmetry is quite common in the world. It is relevant for students to learn what asymmetry [not having symmetry] looks like and to identify examples of symmetrical shapes in the world. Use everyday contexts to introduce congruence and symmetry, drawing upon the students' prior experiences in the real world.

Encourage students to visualize the matching halves of things in the environment that are not conducive to folding or testing with a Mira. Observing objects in the environment may require that students look at the 2-D faces of 3-D objects. When looking around the environment, consider the following:

- Where can you find examples of symmetry in your environment? In texts? In visual media?
- Why do different shapes have different numbers of lines of symmetry? Why do some shapes have no lines of symmetry?
- Why can a line of symmetry not divide a 2-D shape into thirds?

Statistics and Probability

SCO SP01 Students will be expected to demonstrate an understanding of many-to-one correspondence. [C, R, T, V]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

SP01.01 Compare graphs in which the same data has been displayed using one-to-one and many-to-one correspondences, and explain how they are the same and different.

SP01.02 Explain why many-to-one correspondence is sometimes used rather than one-to-one correspondence.

SP01.03 Find examples of graphs in print and electronic media, such as newspapers, magazines, and the Internet, in which many-to-one correspondence is used; and describe the correspondence used.

Performance Indicator Background

SP01.01 As students compare their own graphs and those from other sources, they should examine how the graphs are similar and different. Students should discuss why they think the interval or correspondence was chosen and what other scales may have also been used. Deciding on what scale to use requires students to apply their knowledge of multiplication, and therefore, it is very helpful for students to have a good knowledge of these facts.

SP01.02 When students are creating bar graphs and pictographs, it is important to allow opportunities for them to decide which scales to use for their graphs. These decisions are based on the data being used. By choosing a scale, an interval and a correspondence will be identified. Suggest that students create a graph that shows the most popular authors, movies, types of food, etc., of class members. Ask some students to create a bar graph that shows the collected data using a scale of 2 and other groups use a scale of 1, 3, 4, and 5. Ask students to explain which scale was the most appropriate to display the data.

SP01.03 As students compare given graphs from various sources, they should examine how the graphs are similar and different. Students should discuss why they think the particular correspondence was chosen and what other correspondence may have also been used. Deciding what scale to use allows students to apply their knowledge of multiplication, and therefore, it is very helpful for students to have a good knowledge of basic facts. As students begin to work with greater amounts of data, it becomes inconvenient to draw a symbol to represent every piece of data. Using a scale allows a single symbol to represent a number of items, a situation referred to as many-to-one correspondence.

SCO SP02 Students will be expected to construct and interpret pictographs and bar graphs involving many-to-one correspondence to draw conclusions.
[C, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

SP02.01 Identify an interval and correspondence for displaying a given set of data in a graph, and justify the choice.
SP02.02 Create and label (with categories, title, and legend) a pictograph to display a given set of data, using many-to-one correspondence, and justify the choice of correspondence used.
SP02.03 Create and label (with axes and title) a bar graph to display a given set of data, using many-to-one correspondence, and justify the choice of interval used.
SP02.04 Answer a given question, using a given graph in which data is displayed using many-to-one correspondence.

Performance Indicator Background

SP02.01 Students need to be given many opportunities to explore what scale is most appropriate for their set of data. For example, if they want to display a graph to show their marble collection and they have 36 blue, 24 red, and 42 clear, students may decide to draw a pictograph where each symbol represents 2 marbles or one where each symbol represents 6 marbles.

In cases where the numbers are all less than 20, it is usually more appropriate to use a one-to-one correspondence. For larger numbers, however, students may find it better to use intervals (increments) of 10, 25, 100, or 1000 based on the data being graphed. Students should discuss their data displays and be able to explain why they chose the scale they did.

Students would not be expected to use the term **interval** in their explanations, but may justify their choice by telling how they **skip counted**. It is important for students to ensure that the interval in their data display is consistent. For example, if they are creating a bar graph that has a scale with an interval of 2, all of the numbers need to increase by 2 (2, 4, 6, 8, 10, 12, … and not 2, 4, 6, 7, 8, 9, 10, 12, …). Depending on the data and the scale that is selected, it may become necessary to create partial symbols and bars that fall between numbers.

SP02.02 and **SP02.03** Graphs prepared by students are visual communication tools. As such, student-constructed data displays must tell a story. The graph relates the complete story on its own without reference to a written explanation, a table of values, or any other device. A title and labels on the axes of the graph provide essential information to support the story. This is true whether the student draws the graph or constructs it using technology. There are a variety of software and spreadsheet programs available for students to display data quickly and easily. Commercial programs are available through the Nova Scotia School Book Bureau. Excel spreadsheets can be constructed using Microsoft Office tools.

SP02.04 Students should be developing the ability to obtain information from bar graphs. When looking at graphs based on contextual information, students should be able to make observations about what they see on the graph (e.g., There are almost as many people who like apples as those who like bananas, or The graph shows that 6 more students bring lunch than those who do not). Students should display some understanding of the data shown in given graphs through comparing different parts of the

information on the graph or by making observations about the data in general. They could also tell a story or draw a picture about the information as they see it presented in the given graph.
Students should have regular opportunities to examine graphs in order to interpret the information displayed, draw conclusions about the data, look for patterns, make predictions, pose questions, and solve problems. Students should make observations about their own graphs. For example, students may use bar graphs to demonstrate plant growth. When the bar graphs are constructed, students could make observations about the growth of their plants, such as the periods of greatest, least, and no growths and how long it took to reach half of the maximum growth recorded. Students should analyze and interpret two or more graphs using the same data. For example, students might compare and contrast bar graphs and pictographs.

Students should also have opportunities to read and interpret graphs found in other sources. Many newspapers use a variety of graphs in their articles and presentations. These can be sources of graphs for discussion and to show how graphs are used in the world around us. Census at Schools, a project of Statistics Canada, also has a wide range of data displays appropriate for students.

Students may benefit from examining graphs with missing labels and posing or answering questions about the graphs. Possible activities include answering given questions, at least some of which are impossible with a label missing, writing an explanation of the graph, that will underscore the communication problem of missing labels, critiquing the graph, suggesting improvements that need to be made, and providing suggestions for missing labels so the graph tells a complete story.

Questioning should be ongoing throughout tasks to encourage students to interpret the data presented and to draw inferences. It is important to ask questions that go beyond simplistic reading of a graph. Both literal questions and inferential questions should be posed. For example,

- How many … ?
- How many more/less than … ?
- Order from least to greatest/greatest to least …
- Based on the information presented in the graph, what other conclusions can you make?
- Why do you think ... ?

Invite students to discuss what kinds of information they can get from reading given bar graphs and pictographs that display the use of many-to-one correspondence.

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