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Mathematics at Work 10

CURRICULUM GUIDE

NOVA SCOTIA

Mathematics at Work 10

Implementation Draft
August 2013

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Introduction

Background and Rationale

Mathematics curriculum is shaped by a vision that fosters the development of mathematically literate students who can extend and apply their learning and who are effective participants in society. It is essential that the mathematics curriculum reflect current research in mathematics instruction. To achieve this goal, Western and Northern Canadian Protocol's (WNCP) *The Common Curriculum Framework for Grades 10–12 Mathematics* (2008) has been adopted as the basis for the new mathematics curriculum in Nova Scotia.

The Common Curriculum Framework was developed by the seven ministries of education (Alberta, British Columbia, Manitoba, Northwest Territories, Nunavut, Saskatchewan, and Yukon Territory) in collaboration with teachers, administrators, parents, business representatives, post-secondary educators, and others. The framework identifies beliefs about mathematics, general and specific student outcomes, and performance indicators agreed upon by the seven jurisdictions. The outcomes and performance indicators have been adapted for Nova Scotia. This document is based on both national and international research by the WNCP and the National Council of Teachers of Mathematics (NCTM).

There is an emphasis in the Nova Scotia curriculum on particular key concepts at each grade that will result in greater depth of understanding and, ultimately, stronger student achievement. There is also a greater emphasis on number sense and operations concepts in the early grades to ensure students develop a solid foundation in numeracy.

Purpose

This document provides sets of outcomes and performance indicators to be used as a mandated common base for defining mathematics curriculum expectations. This common base should result in consistent student outcomes in mathematics within the province of Nova Scotia. It should also enable easier transfer for students moving within the province or from any jurisdiction that has adopted the WNCP framework. This document is intended to clearly communicate to all education partners across the province the high expectations for students' mathematical learning.

Program Design and Components

Pathways

The *Common Curriculum Framework for Grades 10–12 Mathematics*, on which the Nova Scotia Mathematics 10–12 curriculum is based, includes pathways and topics rather than strands as in *The Common Curriculum Framework for K–9 Mathematics*. In Nova Scotia, four pathways are available: Mathematics Essentials, Mathematics at Work, Mathematics, and Pre-Calculus.

Each topic area requires that students develop a conceptual knowledge base and skill set that will be useful to whatever pathway they have chosen. The topics covered within a pathway are meant to build upon previous knowledge and to progress from simple to more complex conceptual understandings.

Goals of Pathways

The goals of all four pathways are to provide prerequisite attitudes, knowledge, skills, and understandings for specific post-secondary programs or direct entry into the work force. All four pathways provide students with mathematical understandings and critical-thinking skills. It is the choice of topics through which those understandings and skills are developed that varies among pathways. When choosing a pathway, students should consider their interests, both current and future. Students, parents, and educators are encouraged to research the admission requirements for post-secondary programs of study as they vary by institution and by year.

Design of Pathways

Each pathway is designed to provide students with the mathematical understandings, rigour, and critical-thinking skills that have been identified for specific post-secondary programs of study and for direct entry into the work force.

The content of the Mathematics Essential courses was designed in Nova Scotia to fill a specific need for Nova Scotia students. The content of each of the Mathematics at Work, Mathematics, and Pre-Calculus pathways has been based on the *Western and Northern Canadian Protocol (WNCP) Consultation with Post-Secondary Institutions, Business and Industry Regarding Their Requirements for High School Mathematics: Final Report on Findings* (Alberta Education 2006) and on consultations with mathematics teachers.

MATHEMATICS ESSENTIALS (GRADUATION)

This pathway is designed to provide students with the development of the skills and understandings required in the workplace, as well as those required for everyday life at home and in the community. Students will become better equipped to deal with mathematics in the real world and will become more confident in their mathematical abilities.

MATHEMATICS AT WORK (GRADUATION)

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for entry into some college programs and for direct entry into the work force. Topics include financial mathematics, algebra, geometry, measurement, number, and statistics and probability.

MATHEMATICS (ACADEMIC)

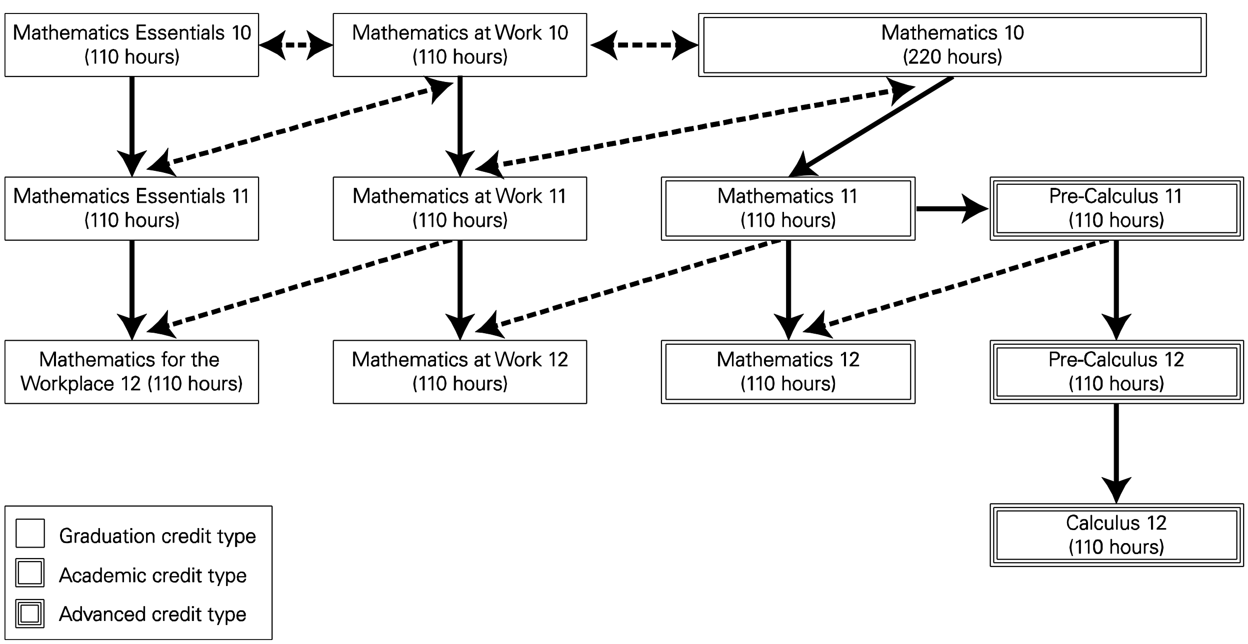
This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for post-secondary studies in programs that require an academic or pre-calculus mathematics credit. Topics include financial mathematics, geometry, measurement, number, logical reasoning, relations and functions, and statistics and probability. *After completion of Mathematics 11, students have the choice of an academic or pre-calculus pathway.*

PRE-CALCULUS (ADVANCED)

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for entry into post-secondary programs that require the study of theoretical calculus. Topics include algebra and number, measurement, relations and functions, trigonometry, and permutations, combinations, and binomial theorem.

Pathways and Courses

The graphic below summarizes the pathways and courses offered.



Instructional Focus

Each pathway in senior high mathematics pathways is arranged by topics. Students should be engaged in making connections among concepts both within and across topics to make mathematical learning experiences meaningful.

Teachers should consider the following points when planning for instruction and assessment.

- The mathematical processes that are identified with the outcome are intended to help teachers select effective pedagogical approaches for the teaching and learning of the outcome.
- All seven mathematical processes must be integrated throughout teaching and learning approaches and should support the intent of the outcomes.
- Wherever possible, meaningful contexts should be used in examples, problems, and projects.
- Instruction should flow from simple to complex and from concrete to abstract.
- The assessment plan for the course should be a balance of assessment for learning and assessment of learning.

The focus of student learning should be on developing a conceptual and procedural understanding of mathematics. Students' conceptual understanding and procedural understanding must be directly related.

Assessment

Ongoing assessment for learning is essential to effective teaching and learning. Research has shown that assessment for learning (formative assessment) practices produce significant and often substantial learning gains, close achievement gaps, and build students' ability to learn new skills (Black & Wiliam 1998; OECD 2006). Student involvement in assessment promotes learning. Timely and effective teacher feedback and student self-assessment allow students to reflect on and articulate their understanding of mathematical concepts and ideas.

Assessment in the classroom includes

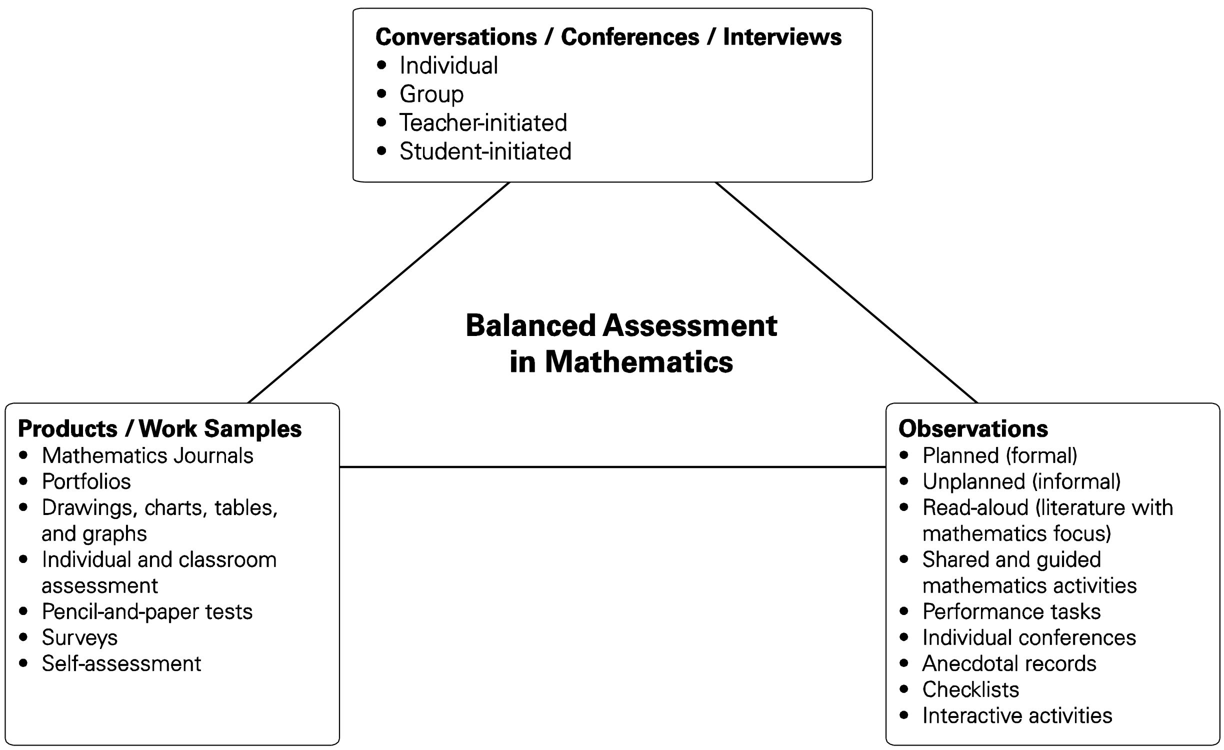
- providing clear goals, targets, and learning outcomes
- using exemplars, rubrics, and models to help clarify outcomes and identify important features of the work
- monitoring progress towards outcomes and providing feedback as necessary
- encouraging self-assessment
- fostering a classroom environment where conversations about learning take place, where students can check their thinking and performance and develop a deeper understanding of their learning

(Davies 2000)

Assessment for learning practices act as the scaffolding for learning, which only then can be measured through assessment of learning (summative assessment). Assessment of learning tracks student progress, informs instructional programming, and aids in decision making. Both forms of assessment are necessary to guide teaching, stimulate learning, and produce achievement gains.

Assessment of student learning should

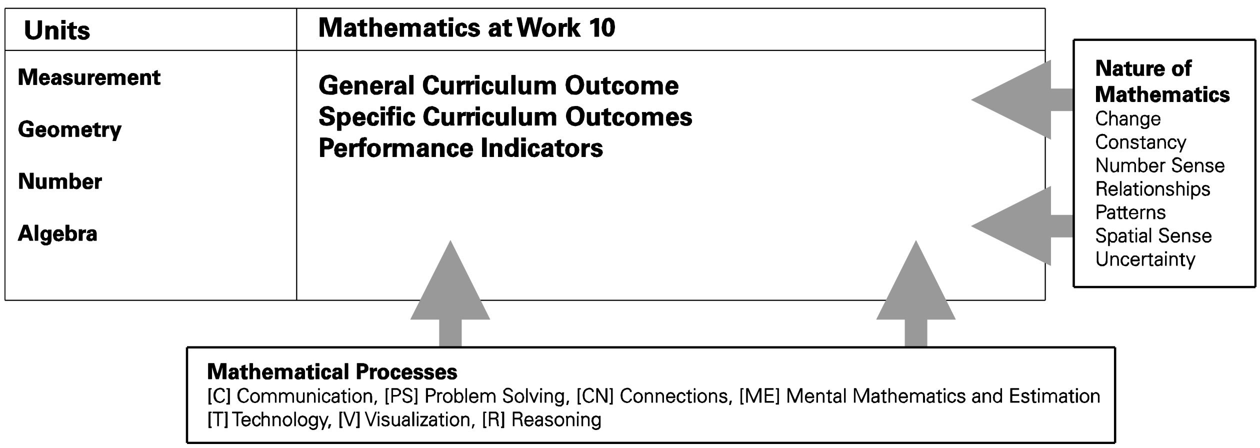
- align with curriculum outcomes
- clearly define criteria for success
- make explicit the expectations for students' performance
- use a wide variety of assessment strategies and tools
- yield useful information to inform instruction



Outcomes

Conceptual Framework for Mathematics at Work 10

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



(Adapted with permission from Western and Northern Canadian Protocol, *The Common Curriculum Framework for K–9 Mathematics*, p. 5. All rights reserved.)

Structure of the Mathematics at Work 10 Curriculum

Units

Mathematics at Work 10 comprises four units:

- Measurement (M) (40–45 hours)
- Geometry (G) (45–50 hours)
- Number (N) (20–25 hours)
- Algebra (A) (integrated throughout)

Outcomes and Performance Indicators

The Nova Scotia curriculum is stated in terms of general curriculum outcomes, specific curriculum outcomes, and performance indicators.

General Curriculum Outcomes (GCOs)

General curriculum outcomes are overarching statements about what students are expected to learn in each strand/sub-strand. The GCO for each strand/sub-strand is the same throughout the pathway.

Measurement (M)

Students will be expected to develop spatial sense through direct and indirect measurement.

Geometry (G)

Students will be expected to develop spatial sense.

Number (N)

Students will be expected to develop number sense and critical thinking skills.

Algebra (A)

Students will be expected to develop algebraic reasoning.

Specific Curriculum Outcomes (SCOs) and Performance Indicators

Specific curriculum outcomes are statements that identify specific concepts and related skills underpinned by the understanding and knowledge attained by students as expected for a given grade.

Performance indicators are samples of how students may demonstrate their performance of the goals of a specific curriculum outcome. The range of samples provided is meant to reflect the scope of the SCO. In the SCOs, the word **including** indicates that any ensuing items *must* be addressed to fully achieve the learning outcome. The phrase **such as** indicates that the ensuing items are provided for clarification only and are *not* requirements that must be addressed to fully achieve the learning outcome. The word **and** used in an outcome indicates that both ideas must be addressed to achieve the learning outcome, although not necessarily at the same time or in the same question.

MEASUREMENT (M)

M01 Students will be expected to demonstrate an understanding of the International System of Units (SI) by
- describing the relationships of the units for length, area, volume, capacity, mass, and temperature
- applying strategies to convert SI units to imperial units

Performance Indicators

(It is intended that this outcome be limited to the base units and the prefixes milli-, centi-, deci-, deca-, hector-, and kilo-.)

M01.01 Explain how the SI system was developed, and explain its relationship to base ten.
M01.02 Identify the base units of measurement in the SI system, and determine the relationship among the related units of each type of measurement.
M01.03 Identify contexts that involve the SI system.
M01.04 Match the prefixes used for SI units of measurement with the powers of ten.
M01.05 Explain, using examples, how and why decimals are used in the SI system.
M01.06 Provide an approximate measurement in SI units for a measurement given in imperial units.

M01.07 Write a given linear measurement expressed in one SI unit in another SI unit.
M01.08 Convert a given measurement from SI to imperial units by using proportional reasoning (including formulas).

M02 Students will be expected to demonstrate an understanding of the imperial system by

- describing the relationships of the units for length, area, volume, capacity, mass, and temperature
- comparing the American and British imperial units for capacity
- applying strategies to convert imperial units to SI units

Performance Indicators

M02.01 Explain how the imperial system was developed.
M02.02 Identify commonly used units in the imperial system, and determine the relationships among the related units.
M02.03 Identify contexts that involve the imperial system.
M02.04 Explain, using examples, how and why fractions are used in the imperial system.
M02.05 Compare the American and British imperial measurement systems.
M02.06 Provide an approximate measure in imperial units for a measurement given in SI.
M02.07 Write a given linear measurement expressed in one imperial unit in another imperial unit.
M02.08 Convert a given measure from imperial to SI units by using proportional reasoning (including formulas).

M03 Students will be expected to solve and verify problems that involve SI and imperial linear measurements, including decimal and fractional measurements.

Performance Indicators

(It is intended that the four arithmetic operations on decimals and fractions be integrated into the problems.)

M03.01 Identify a referent for a given common SI or imperial unit of linear measurement.
M03.02 Estimate a linear measurement, using a referent.
M03.03 Measure inside diameters, outside diameters, lengths, widths of various given objects, and distances, using various measuring instruments.
M03.04 Estimate the dimensions of a given regular 3-D object or 2-D shape, using a referent (e.g., the height of the desk is about three rulers long, so the desk is approximately three feet high).
M03.05 Solve a linear measurement problem including perimeter, circumference, and length + width + height (used in shipping and air travel).
M03.06 Determine the operation that should be used to solve a linear measurement problem.
M03.07 Provide an example of a situation in which a fractional linear measurement would be divided by a fraction.
M03.08 Determine, using a variety of strategies, the midpoint of a linear measurement such as length, width, height, depth, diagonal, and diameter of a 3-D object, and explain the strategies.
M03.09 Determine if a solution to a problem that involves linear measurement is reasonable.

M04 Students will be expected to solve problems that involve SI and imperial area measurements of regular, composite, and irregular 2-D shapes and 3-D objects, including decimal and fractional measurements, and verify the solutions.

Performance Indicators

(It is intended that the four arithmetic operations on decimals and fractions be integrated into the problems.)

M04.01 Identify and compare referents for area measurements in SI and imperial units.
M04.02 Estimate an area measurement, using a referent.
M04.03 Identify a situation where a given SI or imperial area unit would be used.
M04.04 Estimate the area of a given regular, composite, or irregular 2-D shape, using an SI square grid and an imperial square grid.
M04.05 Solve a contextual problem that involves the area of a regular, a composite, or an irregular 2-D shape.
M04.06 Write a given area measurement expressed in one SI unit squared in another SI unit squared.
M04.07 Write a given area measurement expressed in one imperial unit squared in another imperial unit squared.
M04.08 Solve a problem, using formulas for determining the areas of regular, composite, and irregular 2-D shapes, including circles.
M04.09 Solve a problem that involves determining the surface area of 3-D objects, including right cylinders and cones.
M04.10 Explain, using examples, the effect of changing the measurement of one or more dimensions on area and perimeter of rectangles.
M04.11 Determine if a solution to a problem that involves an area measurement is reasonable.

GEOMETRY (G)

G01 Students will be expected to analyze puzzles and games that involve spatial reasoning, using problem-solving strategies.

Performance Indicators

(It is intended that this outcome be integrated throughout the course by using sliding, rotation, construction, deconstruction, and similar puzzles and games.)

G01.01 Determine, explain, and verify a strategy to solve a puzzle or to win a game. For example,
- guess and check
- look for a pattern
- make a systematic list
- draw or model
- eliminate possibilities
- simplify the original problem
- work backward
- develop alternative approaches

G01.02 Identify and correct errors in a solution to a puzzle or in a strategy for winning a game.

G01.03 Create a variation on a puzzle or a game, and describe a strategy for solving the puzzle or winning the game.

G02 Students will be expected to demonstrate an understanding of the Pythagorean theorem by identifying situations that involve right triangles, verifying the formula, applying the formula, and solving problems.

Performance Indicators

G02.01 Explain, using illustrations, why the Pythagorean theorem applies only to right triangles.
G02.02 Verify the Pythagorean theorem, using examples and counterexamples, including drawings, concrete materials, and technology.
G02.03 Describe historical and contemporary applications of the Pythagorean theorem.
G02.04 Determine if a given triangle is a right triangle, using the Pythagorean theorem.
G02.05 Explain why a triangle with the side length ratio of 3:4:5 is a right triangle.
G02.06 Explain how the ratio of 3:4:5 can be used to determine if a corner of a given 3-D object is square (90°) or if a given parallelogram is a rectangle.
G02.07 Solve a problem using the Pythagorean theorem.

G03 Students will be expected to demonstrate an understanding of similarity of convex polygons, including regular and irregular polygons.

Performance Indicators

G03.01 Determine, using angle measurements, if two or more regular or irregular polygons are similar.
G03.02 Determine, using ratios of side lengths, if two or more regular or irregular polygons are similar.
G03.03 Explain why two given polygons are not similar.
G03.04 Explain the relationships between the corresponding sides of two polygons that have corresponding angles of equal measure.
G03.05 Draw a polygon that is similar to a given polygon.
G03.06 Explain why two or more right triangles with a shared acute angle are similar.
G03.07 Solve a contextual problem that involves similarity of polygons.

G04 Students will be expected to demonstrate an understanding of primary trigonometric ratios (sine, cosine, tangent) by applying similarity to right triangles, generalizing patterns from similar right triangles, applying the primary trigonometric ratios, and solving problems.

Performance Indicators

G04.01 Show, for a specified acute angle in a set of similar right triangles, that the ratios of the length of the side opposite to the length of the side adjacent are equal, and generalize a formula for the tangent ratio.

G04.02 Show, for a specified acute angle in a set of similar right triangles, that the ratios of the length of the side opposite to the length of the hypotenuse are equal, and generalize a formula for the sine ratio.
G04.03 Show, for a specified acute angle in a set of similar right triangles, that the ratios of the length of the side adjacent to the length of the hypotenuse are equal, and generalize a formula for the cosine ratio.
G04.04 Identify situations where the trigonometric ratios are used for indirect measurement of angles and lengths.
G04.05 Solve a contextual problem that involves right triangles, using the primary trigonometric ratios.
G04.06 Determine if a solution to a problem that involves primary trigonometric ratios is reasonable.

G05 Students will be expected to solve problems that involve parallel, perpendicular, and transversal lines, and pairs of angles formed between them.

Performance Indicators

G05.01 Sort a set of lines as perpendicular, parallel, or neither, and justify this sorting.
G05.02 Illustrate and describe complementary and supplementary angles.
G05.03 Identify, in a set of angles, adjacent angles that are not complementary or supplementary.
G05.04 Identify and name pairs of angles formed by parallel lines and a transversal, including corresponding angles, vertically opposite angles, alternate interior angles, alternate exterior angles, interior angles on the same side of transversal, and exterior angles on the same side of transversal.
G05.05 Explain and illustrate the relationships of angles formed by parallel lines and a transversal.
G05.06 Explain, using examples, why the angle relationships do not apply when the lines are not parallel.
G05.07 Determine if lines or planes are perpendicular or parallel (e.g., wall perpendicular to floor, and describe the strategy used).
G05.08 Determine the measures of angles involving parallel lines and a transversal, using angle relationships.
G05.09 Solve a contextual problem that involves angles formed by parallel lines and a transversal (including perpendicular transversals).

G06 Students will be expected to demonstrate an understanding of angles, including acute, right, obtuse, straight, and reflex, by drawing, replicating and constructing, bisecting, and solving problems.

Performance Indicators

G06.01 Draw and describe angles with various measures, including acute, right, straight, obtuse, and reflex angles.
G06.02 Identify referents for angles.
G06.03 Sketch a given angle.
G06.04 Estimate the measure of a given angle, using 22.5°, 30°, 45°, 60°, 90°, and 180° as referent angles.
G06.05 Measure, using a protractor, angles in various orientations.

G06.06 Explain and illustrate how angles can be replicated in a variety of ways (e.g., Mira, protractor, compass and straightedge, carpenter's square, dynamic geometry software).
G06.07 Replicate angles in a variety of ways, with and without technology.
G06.08 Bisect an angle, using a variety of methods.
G06.09 Solve a contextual problem that involves angles.

NUMBER (N)

N01 Students will be expected to solve problems that involve unit pricing and currency exchange, using proportional reasoning.

Performance Indicators

N01.01 Compare the unit price of two or more given items.
N01.02 Solve problems that involve determining the best buy, and explain the choice in terms of the cost as well as other factors, such as quality and quantity.
N01.03 Compare, using examples, different sales promotion techniques.
N01.04 Determine the percent increase or decrease for a given original and new price.
N01.05 Solve, using proportional reasoning, a contextual problem that involves currency exchange.
N01.06 Explain the difference between the selling rate and purchasing rate for currency exchange.
N01.07 Explain how to estimate the cost of items in Canadian currency while in a foreign country, and explain why this may be important.
N01.08 Convert between Canadian currency and foreign currencies, using formulas, charts, or tables.

N02 Students will be expected to demonstrate an understanding of income to calculate gross pay and net pay, including wages, salary, contracts, commissions, and piecework.

Performance Indicators

N02.01 Describe, using examples, various methods of earning income.
N02.02 Identify and list jobs that commonly use different methods of earning income (e.g., hourly wage, wage and tips, salary, commission, contract, bonus, shift premiums).
N02.03 Determine in decimal form, from a time schedule, the total time worked in hours and minutes, including time and a half and/or double time.
N02.04 Determine gross pay from given or calculated hours worked when given
- the base hourly wage, with and without tips
- the base hourly wage, plus overtime (time and a half, double time)

N02.05 Determine gross pay for earnings acquired by
- base wage, plus commission
- single commission rate

N02.06 Explain why gross pay and net pay are not the same.
N02.07 Determine the Canadian Pension Plan (CPP), Employment Insurance (EI), and income tax deductions for a given gross pay.
N02.08 Determine net pay when given deductions (e.g., health plans, uniforms, union dues, charitable donations, payroll tax).
N02.09 Investigate, with technology, "what if …" questions related to changes in.
N02.10 Identify and correct errors in a solution to a problem that involves gross or net pay.
N02.11 Describe the advantages and disadvantages for a given method of earning income.

ALGEBRA (A)

A01 Students will be expected to solve problems that require the manipulation and application of formulas related to perimeter, area, the Pythagorean theorem, primary trigonometric ratios, and income.

Performance Indicators

A01.01 Solve a contextual problem that involves the application of a formula that does not require manipulation.
A01.02 Solve a contextual problem that involves the application of a formula that requires manipulation.
A01.03 Explain and verify why different forms of the same formula are equivalent.
A01.04 Describe, using examples, how a given formula is used in a trade or an occupation.
A01.05 Create and solve a contextual problem that involves a formula.
A01.06 Identify and correct errors in a solution to a problem that involves a formula.

Mathematical Processes

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics.

Students are expected to

- communicate in order to learn and express their understanding of mathematics (Communication [C])
- connect mathematical ideas to other concepts in mathematics, to everyday experiences, and to other disciplines (Connections [CN])
- demonstrate fluency with mental mathematics and estimation (Mental Mathematics and Estimation [ME])
- develop and apply new mathematical knowledge through problem solving (Problem Solving [PS])
- develop mathematical reasoning (Reasoning [R])
- select and use technologies as tools for learning and solving problems (Technology [T])
- develop visualization skills to assist in processing information, making connections, and solving problems (Visualization [V])

The Nova Scotia curriculum incorporates these seven interrelated mathematical processes that are intended to permeate teaching and learning. The key to these process standards is presented in a box, as shown below, with each specific outcome within the units.

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics. Communication is important in clarifying, reinforcing, and modifying ideas, knowledge, attitudes, and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics.

Students also need to communicate their learning using mathematical terminology. Communication can help students make connections between and among the different representational modes—contextual, concrete, pictorial, linguistic/verbal, written and symbolic—of mathematical ideas. Students must communicate *daily* about their mathematics learning. This enables them to reflect, to validate, and to clarify their thinking and provides teachers with insight into students' interpretations of mathematical meanings and ideas.

Emerging technologies enable students to engage in communication beyond the traditional classroom to gather data and share mathematical ideas.

Problem Solving [PS]

Problem solving is one of the key processes and foundations within the field of mathematics. Learning through problem solving should be the focus of mathematics at all grade levels. Students develop a true understanding of mathematical concepts and procedures when they solve problems in meaningful contexts.

When students encounter new situations and respond to questions of the type, How would you...? or How could you ...?, the problem-solving approach is being modeled. Students develop their own problem-solving strategies by listening to, discussing, and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. Students should not know the answer immediately. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement. Students will be engaged if the problems relate to their lives, cultures, interests, families, or current events.

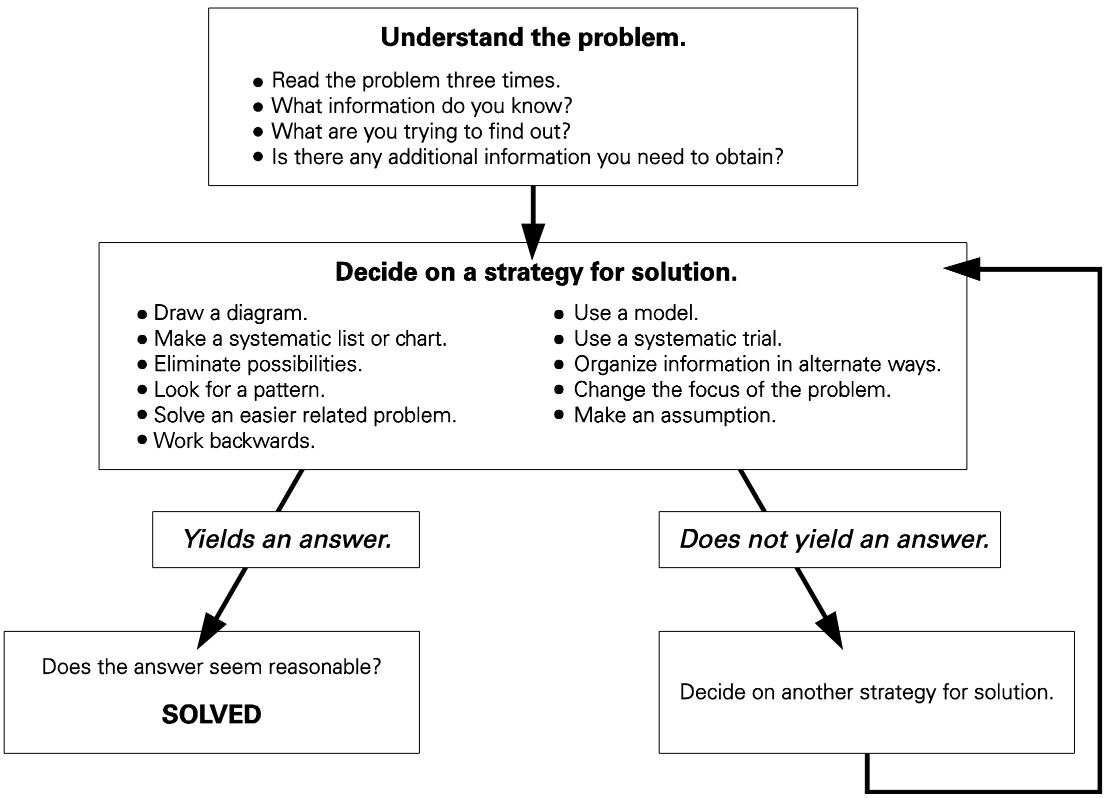
Both conceptual understanding and student engagement are fundamental in molding students' willingness to persevere in future problem-solving tasks. Problems are not just simple computations embedded in a story, nor are they contrived. They are tasks that are rich and open-ended, so there may be more than one way of arriving at a solution or there may be multiple answers. Good problems should allow for every student in the class to demonstrate their knowledge, skill, or understanding. Problem solving can vary from being an individual activity to a class (or beyond) undertaking.

In a mathematics class, there are two distinct types of problem solving: solving contextual problems outside of mathematics and solving mathematical problems. Finding the maximum profit given manufacturing constraints is an example of a contextual problem, while seeking and developing a general formula to solve a quadratic equation is an example of a mathematical problem.

Problem solving can also be considered in terms of engaging students in both inductive- and deductive-reasoning strategies. As students make sense of the problem, they will be creating conjectures and looking for patterns that they may be able to generalize. This part of the problem-solving process often involves inductive reasoning. As students use approaches to solving the problem, they often move into mathematical reasoning that is deductive in nature. It is crucial that students be encouraged to engage in both types of reasoning and be given the opportunity to consider the approaches and strategies used by others in solving similar problems.

Problem solving is a powerful teaching tool that fosters multiple, creative, and innovative solutions. Creating an environment where students openly look for, and engage in finding, a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive mathematical risk-takers.

A possible flow chart to share with students is as follows:



Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to one another or to real-world phenomena, students can begin to view mathematics as useful, relevant, and integrated. Learning mathematics within contexts and making connections relevant to learners can validate past experiences and increase student willingness to participate and be actively engaged. The brain is constantly looking for and making connections.

"Because the learner is constantly searching for connections on many levels, educators need to *orchestrate the experiences* from which learners extract understanding. … *Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching*." (Caine and Caine 1991, 5).

Mathematics should be viewed as an integrated whole rather than as the study of separate strands or units. Connections must also be made between and among the different representational modes—contextual, concrete, pictorial, linguistic/verbal, and symbolic. The process of making connections, in turn, facilitates learning. Concepts and skills should also be connected to everyday situations and other curricular areas. For example, when developing literacy skills students learn to make text-to-world, text-to-text, and text-to-self connections. Students can also make connections to make mathematics come alive through math-to-world, math-to-math, and math-to-self connections.

Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally without the use of external aids.

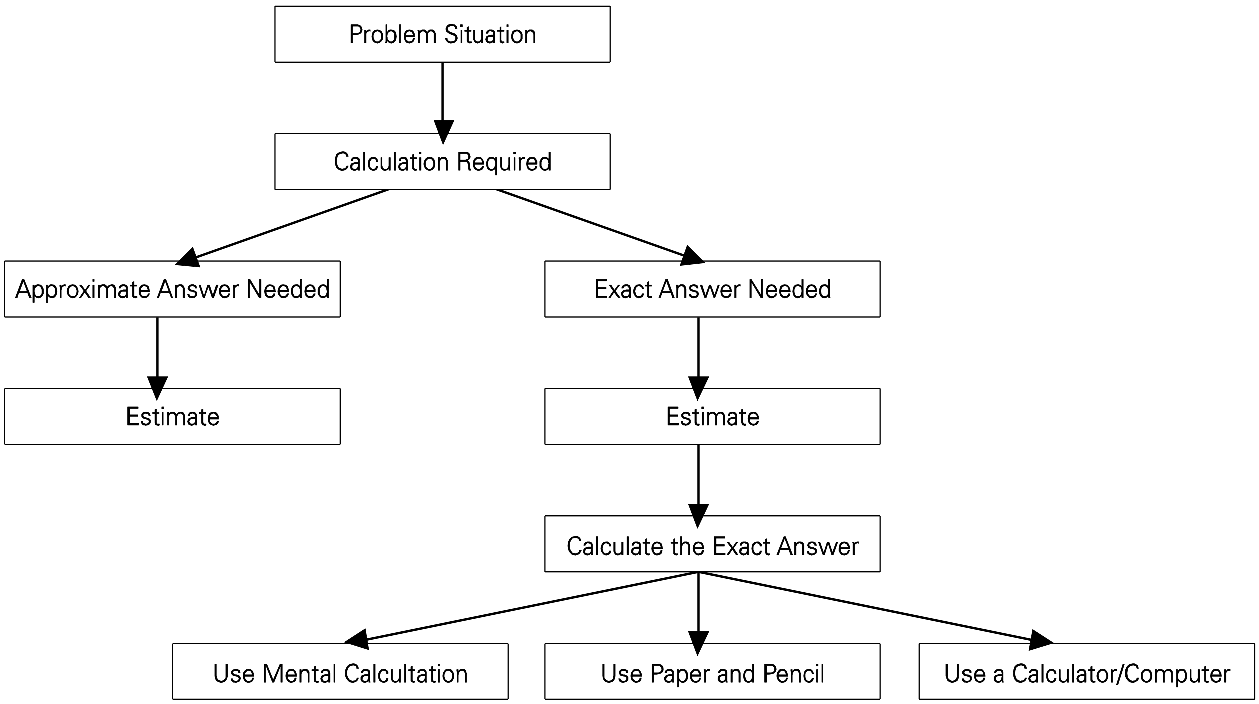
Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy and flexibility.

“Even more important than performing computational procedures or using calculators is the greater facility that students need—more than ever before—with estimation and mental math.” (National Council of Teachers of Mathematics, May 2005).

Students proficient with mental mathematics “become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers, and are more able to use multiple approaches to problem solving.” (Rubenstein 2001) Mental mathematics “provides a cornerstone for all estimation processes, offering a variety of alternative algorithms and nonstandard techniques for finding answers.” (Hope 1988, v)

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when, and what strategy to use when estimating. Estimation is used to make mathematical judgments and develop useful, efficient strategies for dealing with situations in daily life. When estimating, students need to learn which strategy to use and how to use it.

Students need to develop both mental mathematics and estimation skills through context and not in isolation so they are able to apply them to solve problems. Whenever a problem requires a calculation, students should follow the decision-making process as illustrated on the following page.



The skill of estimation requires a sound knowledge of mental mathematics. Both are necessary to many everyday experiences, and students should be provided with frequent opportunities to practise these skills.

Technology [T]

Technology can be effectively used to contribute to and support the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures, and solve problems.

Calculators, computers, and other technologies can be used to

- explore and represent mathematical relationships and patterns in a variety of ways
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- increase the focus on conceptual understanding by decreasing the time spent on repetitive procedures
- reinforce the learning of foundational concepts
- develop personal procedures for mathematical operations
- simulate situations
- develop number and spatial sense
- generate and test inductive conjectures

Technology contributes to a learning environment in which the curiosity of students can lead to rich mathematical discoveries at all grade levels. The use of technology should not replace mathematical understanding. Instead, technology should be used as one of a variety of approaches and tools for creating mathematical understanding.

Visualization [V]

Visualization “involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world.” (Armstrong 1993, 10). The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number, spatial, and measurement sense. Number visualization occurs when students create mental representations of numbers. Being able to create, interpret and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and spatial reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure, when to estimate and to know several estimation strategies. (Shaw and Cliatt 1989, 150)

Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations. It is through visualization that abstract concepts can be understood by the student. Visualization is a foundation to the development of abstract understanding, confidence, and fluency.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking.
Questions that challenge students to think, analyze, and synthesize help them develop an understanding of mathematics. All students need to be challenged to answer questions such as, Why do you believe that’s true/correct? or What would happen if ….

Mathematical experiences provide opportunities for students to engage in inductive and deductive reasoning. Students use inductive reasoning when they explore and record results, analyze observations, make generalizations from patterns, and test these generalizations. Students use deductive reasoning when they reach new conclusions based upon the application of what is already known or assumed to be true.

Mathematics reasoning involves informal thinking, conjecturing, and validating—these help students understand that mathematics makes sense. Students are encouraged to justify, in a variety of ways, their solutions, thinking processes, and hypotheses. In fact, good reasoning is as important as finding correct answers. The thinking skills developed by focusing on reasoning can be used in daily life in a wide variety of contexts and disciplines.

Nature of Mathematics

Mathematics is one way of trying to understand, interpret, and describe our world. There are a number of components that define the nature of mathematics, and these are woven throughout this document. These components include change, constancy, number sense, relationships, patterns, spatial sense, and uncertainty.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics. Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, … can be described as

- skip counting by 2s, starting from 4
- an arithmetic sequence, with first term 4 and a common difference of 2
- a linear function with a discrete domain

(Steen 1990, p. 184).

Students need to learn that new concepts of mathematics as well as changes to previously learned concepts arise from a need to describe and understand something new. Integers, decimals, fractions, irrational numbers, and complex numbers emerge as students engage in exploring new situations that cannot be effectively described or analyzed using whole numbers.

Students best experience change to their understanding of mathematical concepts as a result of mathematical play.

Constancy

Different aspects of constancy are described by the terms **stability**, **conservation**, **equilibrium**, **steady state** and **symmetry** (AAAS–Benchmarks 1993, 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include the following:

- The area of a rectangular region is the same regardless of the methods used to determine the solution.
- The sum of the interior angles of any triangle is 180°.
- The theoretical probability of flipping a coin and getting heads is 0.5.
- Lines with constant slope.

Number Sense

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy. (British Columbia Ministry of Education, 2000, 146) Continuing to foster number sense is fundamental to growth of mathematical understanding.

A true sense of number goes well beyond the skills of simply counting, memorizing facts, and the situational rote use of algorithms. Students with strong number sense are able to judge the reasonableness of a solution, describe relationships between different types of numbers, compare quantities, and work with different representations of the same number to develop a deeper conceptual understanding of mathematics.

Number sense develops when students connect numbers to real-life experiences and use benchmarks and referents. This results in students who are computationally fluent, flexible with numbers, and have intuition about numbers. The evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections.

Relationships

Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects, variables, and concepts. The search for possible relationships involves collecting and analyzing data, analyzing patterns and describing possible relationships visually, symbolically, orally or in written form.

Patterns

Mathematics is about recognizing, describing, and working with numerical and non-numerical patterns. Patterns exist in all of the mathematical topics, and it is through the study of patterns that students can make strong connections between concepts in the same and different topics.

Working with patterns also enables students to make connections beyond mathematics. The ability to analyze patterns contributes to how students understand their environment. Patterns may be represented in concrete, visual, auditory, or symbolic form. Students should develop fluency in moving from one representation to another.

Students need to learn to recognize, extend, create, and apply mathematical patterns. This understanding of patterns allows students to make predictions and justify their reasoning when solving problems. Learning to work with patterns helps develop students' algebraic thinking, which is foundational for working with more abstract mathematics.

Spatial Sense

Spatial sense involves the representation and manipulation of 3-D objects and 2-D shapes. It enables students to reason and interpret among 3-D and 2-D representations.

Spatial sense is developed through a variety of experiences with visual and concrete models. It offers a way to interpret and reflect on the physical environment and its 3-D or 2-D representations.

Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to make predictions about the results of changing these dimensions.

Spatial sense is also critical in students' understanding of the relationship between the equations and graphs of functions and, ultimately, in understanding how both equations and graphs can be used to represent physical situations.

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty. Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty. The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation. Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately. This language must be used effectively and correctly to convey valuable messages.

Curriculum Document Format

This guide presents the mathematics curriculum so that a teacher may readily view the scope of the outcomes that students are expected to achieve during that year. Teachers are encouraged, however, to examine what comes before and what follows after, to better understand how students' learning at a particular grade level is part of a bigger picture of concept and skill development.

The order of presentation in no way assumes or prescribes a preferred order of presentation in the classroom, but simply lays out the specific curriculum outcomes in relation to the overarching general curriculum outcomes.

When a specific curriculum outcome is introduced, it is followed by the mathematical processes and performance indicators for that outcome. A scope and sequence is then provided, which relates the SCO to previous and next grade SCOs. Also for each SCO, there is background information, assessment strategies, suggested instructional strategies, and suggested models and manipulatives, mathematical vocabulary, and resource notes. For each section, the guiding questions should be used to help with unit and lesson preparation.

SCO

Mathematical Processes

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections
[ME] Mental Mathematics and Estimation		
[T] Technology	**[V]** Visualization	**[R]** Reasoning

Performance Indicators

Describes observable indicators of whether students have met the specific outcome.

Scope and Sequence

Previous grade or course SCOs	Current grade or course SCO	Following grade or course SCOs

Background

Describes the “big ideas” to be learned and how they relate to work in previous grade and work in subsequent courses.

Assessment, Teaching, and Learning

Assessment Strategies

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Sample tasks that can be used to determine students’ prior knowledge.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Some suggestions for specific activities and questions that can be used for both instruction and assessment.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcome and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SUGGESTED LEARNING TASKS

Suggestions for general approaches and strategies suggested for teaching this outcome.

Guiding Questions

- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

SUGGESTED MODELS AND MANIPULATIVES

MATHEMATICAL VOCABULARY

Resources/Notes

Contexts for Learning and Teaching

Beliefs about Students and Mathematics Learning

"Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge." (National Council of Teachers of Mathematics 2000, 20).

- The Nova Scotia mathematics curriculum is based upon several key assumptions or beliefs about mathematics learning that have grown out of research and practice. These beliefs include the following:
- Mathematics learning is an active and constructive process.
- Learning is most effective when standards of expectation are made clear with ongoing assessment and feedback.
- Learners are individuals who bring a wide range of prior knowledge and experiences and who learn via various styles and at different rates.
- Leaning is most effective when standards of expectation are made clear with ongoing assessment and feedback.

Students are curious, active learners with individual interests, abilities, and needs. They come to classrooms with varying knowledge, life experiences, and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Students construct their understanding of mathematics by developing meaning based on a variety of learning experiences. This meaning is best constructed when learners encounter mathematical experiences that proceed from simple to complex and from the concrete to the abstract. The use of manipulatives, visuals, and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students. At all levels of understanding, students benefit from working with a variety of materials, tools, and contexts when constructing meaning about new mathematical ideas. Meaningful student discussions also provide essential links among concrete, pictorial, and symbolic representations of mathematics. The learning environment should value, respect, and address all students' experiences and ways of thinking so that students are comfortable taking intellectual risks, asking questions, and posing conjectures. Students need to explore mathematics through solving problems in order to continue developing personal strategies and mathematical literacy. It is important to realize that it is acceptable to solve problems in different ways and that solutions may vary depending upon how the problem is understood.

Goals of Mathematics Education

The main goals of mathematics education are to prepare students to
- use mathematics confidently to solve problems
- communicate and reason mathematically
- appreciate and value mathematics
- make connections between mathematics and its applications
- become mathematically literate adults, using mathematics to contribute to society
- commit themselves to lifelong learning

Students who have met these goals will

- gain understanding and appreciation of the contributions of mathematics as a science, a philosophy, and an art
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical tasks and projects
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity about mathematics and situations involving mathematics

In order to assist students in attaining these goals, teachers are encouraged to develop a classroom atmosphere that fosters conceptual understanding. Students should be encouraged to

- take risks
- think and reflect independently
- share and communicate mathematical understanding
- solve problems in individual and group projects
- pursue greater understanding of mathematics
- appreciate the value of mathematics throughout history

Opportunities for Success

A positive attitude has a profound effect on learning. Environments that create a sense of belonging, encourage risk taking, and provide opportunities for student success help develop and maintain positive attitudes and self-confidence. Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, participate willingly in classroom activities, persist in challenging situations, and engage in reflective practices.

To experience success, students must be taught to set achievable goals and assess their progress as they work toward these goals. Striving toward success and becoming autonomous and responsible learners are ongoing, reflective processes that involve revisiting the setting and assessing of personal goals.

Engaging All Learners

*"*No matter how engagement is defined or which dimension is considered, research confirms this truism of education: *The more engaged you are, the more you will learn."* (Hume 2011, 6)

Student engagement is at the core of learning. This is critical for teachers to take into account when planning and implementing instruction. Effective instruction engages, embraces, and supports all learners through a range of learning experiences. This curriculum is designed to provide learning opportunities that reflect culturally proficient instructional and assessment practices and are equitable, accessible, and inclusive of the multiple facets of diversity represented in today's classrooms.

Engagement in learning occurs when students are provided with opportunities to become more invested in their learning. When teachers know their students as individual learners and as individual people, their students are more likely to be motivated to learn, participate in classroom activities, persist in challenging situations, and engage in reflective practices. Students often become more engaged when teachers demonstrate a genuine belief in each student's potential to learn.

SUPPORTIVE LEARNING ENVIRONMENTS

A supportive and positive learning environment has a profound effect on students' learning. In classrooms where students feel a sense of belonging, are encouraged to actively participate, are challenged without being frustrated, and feel safe and supported to take risks with their learning, students are more likely to experience success. It is realized that not all students will progress at the same pace or be equally positioned in terms of their prior knowledge of and skill with particular concepts and outcomes. Teachers provide all students with equitable access to learning by integrating a variety of instructional approaches and assessment activities that consider all learners and align with the following key principles:

- Instruction must be flexible and offer multiple means of representation.
- Students must have opportunities to express their knowledge and understanding in multiple ways.
- Teachers must provide options for students to engage in learning through multiple ways.

Teachers who know their students well become aware of individual learning differences and infuse this understanding into planned instructional and assessment decisions. They organize learning experiences to accommodate the many ways in which students learn, create meaning, and demonstrate their knowledge and understanding. Teachers use a variety of effective teaching approaches that may include

- providing all students with equitable access to appropriate learning strategies, resources, and technology
- offering a range of ways students can access their prior knowledge to connect with new concepts
- scaffolding instruction and assignments so that individual or groups of students are supported as needed throughout the process of learning
- verbalizing their thinking to model comprehension strategies and new learning
- balancing individual, small-group, and whole-class approaches to learning activities
- involving students in the co-creation of criteria for assessment and evaluation
- providing students with choice in how they demonstrate their understanding according to learning styles and preferences, building on individual strengths, and including a range of difficulty and challenge
- providing frequent and meaningful feedback to students throughout their learning experiences

LEARNING STYLES AND PREFERENCES

The ways in which students make sense of, receive, and process information, demonstrate learning, and interact with peers and their environment both indicate and shape learning preferences, which may vary widely from student to student. Learning preferences are influenced also by the learning context and purpose and by the type and form of information presented or requested. Most students tend to favour one learning style and may have greater success if instruction is designed to provide for multiple learning styles, thus creating more opportunities for all students to access learning. The three most commonly referenced learning styles are

- auditory (such as listening to teacher-presented lessons or discussing with peers)
- kinesthetic (such as using manipulatives or recording print or graphic/visual text)
- visual (such as interpreting information with text and graphics or viewing videos)

While students can be expected to work using all modalities, it is recognized that one or some of these modalities may be more natural to individual students than the others.

A Gender-Inclusive Curriculum

It is important that the curriculum respects the experiences and values of all students and that learning resources and instructional practices are not gender-biased. Teachers promote gender equity and inclusion in their classrooms when they

- articulate equally high expectations for all students
- provide equal opportunity for input and response from all students
- model gender-fair language and respectful listening in their interactions with students

Valuing Diversity: Teaching with Cultural Proficiency

Teachers understand that students represent diverse life and cultural experiences, with individual students bringing different prior knowledge to their learning. Therefore, teachers build upon their knowledge of their students as individuals and respond by using a variety of culturally-proficient instruction and assessment strategies. "Instruction that is embedded in socially meaningful contexts, and tasks that are meaningful and relevant to the lives of students, will engage students in high-level problem-solving and reasoning and enhance students' engagement (Frankenstein 1995; Gutstein 2003; Ladson-Billings 1997; Tate 1995)." (Herzig 2005)

Students with Language, Communication, and Learning Challenges

Today's classrooms include students who have diverse backgrounds, abilities, levels of development, and learning challenges. By observing and interacting with students as they work on assigned activities, teachers can identify areas where students may need additional support to achieve their learning goals. Teachers can then respond with a range of effective instructional strategies. Students who have English as an Additional Language (EAL) may require curriculum outcomes at different levels, or temporary individualized outcomes, particularly in language-based subject areas, while they become more proficient in their English language skills. For students who are experiencing difficulties, it is important that teachers distinguish between students for whom curriculum content is challenging and students for whom language-based issues are at the root of apparent academic difficulties.

Students who Demonstrate Gifted and Talented Behaviours

Some students are academically gifted and talented with specific skill sets or in specific subject areas. Most students who are gifted and talented thrive when challenged by problem-centred, inquiry-based learning and open-ended activities. Teachers may challenge students who are gifted and talented by adjusting the breadth, the depth, and/or the pace of instruction. Learning experiences may be enriched by providing greater choice among activities and offering a range of resources that require increased cognitive demand and higher-level thinking at different levels of complexity and abstraction. For additional information, refer to *Gifted Education and Talent Development* (Nova Scotia Department of Education 2010).

Connections across the Curriculum

The teacher should take advantage of the various opportunities available to integrate mathematics and other subjects. This integration not only serves to show students how mathematics is used in daily life, but it helps strengthen the students' understanding of mathematical concepts and provides them with opportunities to practise mathematical skills. There are many possibilities for integrating mathematics in career education, literacy, music, physical education, science, social studies, technology education, and visual arts.

Measurement
40–45 hours

GCO: Students will be expected to develop spatial sense through direct and indirect measurement.

Specific Curriculum Outcomes

Process Standards Key

[**C**] Communication	[**PS**] Problem Solving	[**CN**] Connections	[**ME**] Mental Mathematics and Estimation
[**T**] Technology	[**V**] Visualization	[**R**] Reasoning	

M01 Students will be expected to demonstrate an understanding of the International System of Units (SI) by
- describing the relationships of the units for length, area, volume, capacity, mass, and temperature
- applying strategies to convert SI units to imperial units [C, CN, ME, V]

M02 Students will be expected to demonstrate an understanding of the imperial system by
- describing the relationships of the units for length, area, volume, capacity, mass, and temperature
- comparing the American and British imperial units for capacity
- applying strategies to convert imperial units to SI units [C, CN, ME, V]

M03 Students will be expected to solve and verify problems that involve SI and imperial linear measurements, including decimal and fractional measurements. [CN, ME, PS, V]

M04 Students will be expected to solve problems that involve SI and imperial area measurements of regular, composite, and irregular 2-D shapes and 3-D objects, including decimal and fractional measurements, and verify the solutions. [ME, PS, R, V]

M01 Students will be expected to demonstrate an understanding of the International System of Units (SI) by ▪ describing the relationships of the units for length, area, volume, capacity, mass, and temperature ▪ applying strategies to convert SI units to imperial units [C, CN, ME, V]			
[**C**] Communication [**T**] Technology	[**PS**] Problem Solving [**V**] Visualization	[**CN**] Connections [**R**] Reasoning	[**ME**] Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have met the corresponding specific curriculum outcome. (**Note:** It is intended that this outcome be limited to the base units and the prefixes milli-, centi-, deci-, deca-, hecto- and kilo-.)

M01.01 Explain how the SI system was developed, and explain its relationship to base ten.
M01.02 Identify the base units of measurement in the SI system, and determine the relationship among the related units of each type of measurement.
M01.03 Identify contexts that involve the SI system.
M01.04 Match the prefixes used for SI units of measurement with the powers of ten.
M01.05 Explain, using examples, how and why decimals are used in the SI system.
M01.06 Provide an approximate measurement in SI units for a measurement given in imperial units.
M01.07 Write a given linear measurement expressed in one SI unit in another SI unit.
M01.08 Convert a given measurement from SI to imperial units by using proportional reasoning (including formulas).

Scope and Sequence

Mathematics 9	Mathematics at Work 10	Mathematics at Work 11
SS02 Students will be expected to determine the surface area of composite 3-D objects to solve problems.	**M01** Students will be expected to demonstrate an understanding of the International System of Units (SI) by ▪ describing the relationships of the units for length, area, volume, capacity, mass, and temperature ▪ applying strategies to convert SI units to imperial units	**M01** Students will be expected to solve problems that involve SI and imperial units in surface area measurements and verify the solutions. **A03** Students will be expected to solve problems by applying proportional reasoning and unit analysis.

Background

Students will explore the origins of the SI system and its history and use in Canada. The metric system was formally developed by France in the 1700s. The system is based on the linear measure of a metre (m), which was defined in terms of the Earth's circumference, and on base ten. For mass, the standard unit is the gram (g), defined as the mass of one cubic centimetre of water. The standard unit for measuring volume or capacity is the litre (L).

In Canada in 1970, with rapidly advancing technology and expanding worldwide trade, the Canadian government adopted a policy for a single, coherent measurement system based on the Système International d'Unités (SI), the latest evolution of the metric system.

Students are familiar with the metric system (the SI system) as a standard for measurement. Students were introduced to the metric system in Mathematics 3 and extended their knowledge in every grade since that time to other units and to conversions within the SI system. Students should be able to identify commonly used SI units, such as centimetres, metres, millilitres, litres, grams, and kilograms.

To fully understand measurement, students will need to be able to distinguish which measures are metric and which are imperial. This outcome focuses on first developing proficiency with the metric system. It is important to take the time to develop this proficiency.

For this outcome, students need to recognize the terminology and abbreviations associated with SI units, such as metre (mm, cm, m, km), litre (mL, L), hectare (ha), degrees Celsius (°C), gram (mg, g, kg).

Students should also be able to identify when SI measures are most commonly used, such as fuel economy (L/100 km), meat or fish (kg), milk or juice (L), cloth (m), boiling point of water (100°C), freezing point of water (0°C), summer air temperature (30°C), carpet lengths (m), volumes of beakers in science class (100 mL, 250 mL). When the relationships among the related units of each type of measurement have been established, students will then be ready to convert between various units in the SI system. Examples of some of these conversions are m to km, mL to L, and g to kg. It is not expected, at this time, that they will do conversions such as cm^2 to m^2. Students have used and will continue to use conversions in science.

Students should also be able to use referents. A referent is an object that can be used to help estimate a measurement. From the earliest introduction to metric units, students have had experience relating non-standard and standard units of measurement. They have used referents to estimate the length of an object in centimetres, metres, and millimetres.

Students will use proportions and proportional reasoning to solve problems that require conversions. A proportion is a statement where two ratios are equivalent.

For example, $\frac{100\text{ cm}}{1\text{ m}} = \frac{350\text{ cm}}{3.5\text{ m}}$.

Students were introduced to proportional reasoning in Mathematics 8 (SCO N05) and in Mathematics 9 continued to work on proportional reasoning through the study of similar polygons. Proper development of proportional reasoning in students means they must become multiplicative thinkers and be able to see and use the multiplicative relationships found within and between the ratios in a problem.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

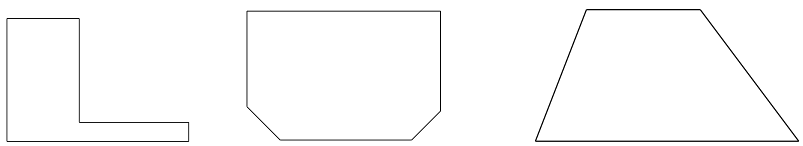
Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Student tasks such as the following could be completed to assist in determining students' prior knowledge.

- Cereal is packed in boxes with a volume of 1000 cm^3. What dimensions should the cereal company choose for the boxes? Explain the reasons for your choice.
- Explain and illustrate how each diagram can be subdivided into two or more simple shapes. Is there more than one solution for each shape?



WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

- Explain how the SI system was developed and explain its relationship to base ten.

- Identify the base units of measurement in the SI system and determine the relationship among the related units of each type of measurement.

- Identify contexts that involve the SI system.

- Match the prefixes used for SI units of measurement with the powers of ten.

- Explain, using examples, how and why decimals are used in the SI system.

- Write a given measurement expressed in one SI unit, in another SI unit.

- Convert the following:
  (a) 25 dam = ___ cm
  (b) 1.7 kg = ___ dg

- Convert the following:
  (a) 42 cm = ___ inches
  (b) 45 km/h = ___ mph
  (c) 26.2 km = ___ miles

- Define the prefixes used in the SI system.

- Jason was asked how many cell phone charging cords it would take to go around the perimeter of the classroom. He measured the length of the cord to be 75 cm. Knowing 100 cm was in 1 m, he did a conversion resulting in 75 cm = 7500 m. Is Jason's answer correct? Why or why not? If the classroom measures 6 m × 5.5 m, how many cords would it take to go around the classroom?

- Fill in the blanks:
  (a) 130 cm = _______ m
  (b) ______ g = 150 mg
  (c) 60 L = ______ mL
  (d) 3.25 km = ______ cm
  (e) ______ g = 0.68 kg

- Measure the mass, volume, capacity, and temperature using different measurement tools (metre sticks, measuring tapes, calipers). Convert the readings to other units that might be appropriate. For example, measuring the thickness of a sheet of paper with calipers will illustrate the usefulness of mm rather than cm or m; measuring the volume of a milk or juice carton will illustrate the usefulness of using L rather than mL.
  Serena used proportional reasoning to do the conversion: 0.78 kg = _____ mg.
  - She wrote: $\frac{10\ 000 \text{ mg}}{1 \text{ kg}} = \frac{? \text{ mg}}{0.78 \text{ kg}}$
  - Has Serena made an error? If so, explain and complete correctly.

FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should inform instruction.

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SUGGESTED LEARNING TASKS

Effective instruction should consist of various strategies.

Guiding Question

- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

Consider the following sample instructional strategies when planning lessons.

- Activate prior knowledge, regarding referents of measurement that students have previously developed (1 m is the approximate distance from the floor to a doorknob, 1 L of milk, room temperature is 21°C, 2 lb. of sugar, 1 kg of salt).

- Opportunities should be provided to explore common referents for SI linear measurements including:

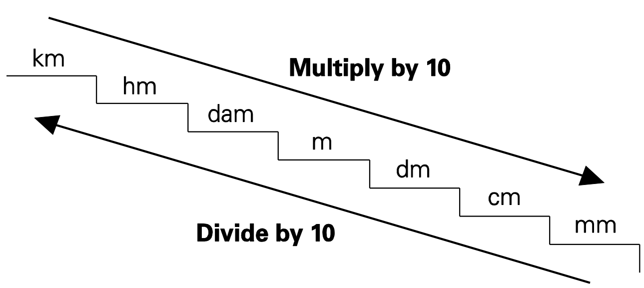
thickness of a dime, thickness of a fingernail	≈ 1 mm
width of a paper clip	≈ 1 cm
distance from a door knob to the floor	≈ 1 m

- Students may relate one kilometre to a distance between two well-known points in their own communities.

- Students should estimate the lengths of objects using referents. The length of a standard white board, for example, is about 3 m because it is approximately three metre sticks long. They should also use various measuring tools to measure lengths in SI units.

- It could be useful for teachers to emphasize the ease of conversion between metric units given that it is developed on the base-ten model. This makes conversions easier with the SI system than the imperial system.

- For length measurements (not area or volume), the following step model may help students visualize converting within the SI measurement system. Each step represents multiplication or division by a factor of 10.



- Reactivate proportional reasoning skills, covered in SCO N01, as a way to complete conversions within the SI system. The following examples illustrate the proportional relationships.

Within

Convert 7.5 cm to inches

$$1\text{ in.} \doteq 2.5\text{ cm}$$

$$\frac{2.5\text{ cm}}{7.5\text{ cm}} = \frac{1\text{ in.}}{x\text{ in.}}$$

$$\times 3 \quad \frac{2.5\text{ cm}}{7.5\text{ cm}} = \frac{1\text{ in.}}{x\text{ in.}} \quad \times 3$$

since $2.5 \times 3 \;\; 7.5$

then $1 \times 3 = 3$

$\therefore\ 7.5\text{ cm} \doteq 3\text{ in.}$

Between

Convert 5 kg to pounds

$$\frac{1\text{ kg}}{5\text{ kg}} = \frac{2.2\text{ lb.}}{x\text{ lb.}}$$

$$\times 2.2$$

$$\frac{1\text{ kg}}{5\text{ kg}} = \frac{2.2\text{ lb.}}{x\text{ lb.}}$$

$$\times 2.2$$

$x = 5 \times 2.2$

$x = 11.0$ lb.

- Some common approximations include the following:

Imperial Measure	SI Approximation
1 inch	≈ 2.5 cm
1 foot	≈ 30 cm
3 feet	≈ 1 m
1 mile	≈ 1.5 km

Suggested Models and Manipulatives

- graduated cylinders
- measuring cups
- scale
- thermometers
- wooden geoblocks

Mathematical Vocabulary

Students need to be comfortable using the following vocabulary.

- Commonly used terminology and abbreviations associated with SI units, such as
  - degrees Celsius (°C)
  - gram (mg, g, kg)
  - hectare (ha)
  - litre (mL, L)
  - metre (mm, cm, m, km)
  - referent

Resources/Notes

Internet

- *Mathematics Learning Commons 7–9, Nova Scotia Virtual School* (Nova Scotia Department of Education and Early Childhood Development 2013):
  - Proportional Reasoning PowerPoint
    http://lrt.ednet.ns.ca/PD/math8support_11/prop_reasoning/01D_proportional_reasoning_ratios.ppt
  - Proportional Reasoning Problems
    http://lrt.ednet.ns.ca/PD/math8support_11/prop_reasoning/01K_question_bank.doc
  - Proportional Reasoning articles:
    http://lrt.ednet.ns.ca/PD/math8support_11/prop_reasoning/01L_problems_encourage_prop_sense.pdf
    http://lrt.ednet.ns.ca/PD/math8support_11/prop_reasoning/01M_multiple_ways_to_solve_proportions.pdf
- Professional Learning, K–12, Newfoundland and Labrador (Professional Learning NL 2013)
  www.k12pl.nl.ca
  The classroom clip demonstrates students estimating linear measurements using referents.
- FPSi, Specialist in French property, Metric Chart (Metric Table) (French Property, Services and Information Ltd. 2008)
  www.france-property-and-information.com/table-of-metric-and-imperial-units.htm?phpMyAdmin=24f3a0e02619b794a6db9c79d8b89c4e
  A list of the most common measures and their relationships to each other.

Print

Math at Work 10 (Etienne et al. 2011)

- Chapter 1: Consumerism and Travel
  - Get Ready
  - Sections 1.1 and 1.3
  - Skill Check
  - Test Yourself
  - Chapter Project
  - Games and Puzzles
- Chapter 2: Measuring Length
  - Get Ready
  - Sections 2.2, 2.3, and 2.4
  - Skill Check
  - Test Yourself
  - Chapter Project
  - Games and Puzzles
- Chapter 3: Measuring Area
  - Get Ready
  - Section 3.2
  - Chapter Project

Notes

<table>
<tr><td colspan="4">M02 Students will be expected to demonstrate an understanding of the imperial system by
▪ describing the relationships of the units for length, area, volume, capacity, mass, and temperature
▪ comparing the American and British imperial units for capacity
▪ applying strategies to convert imperial units to SI units
[C, CN, ME, V]</td></tr>
<tr><td>[C] Communication
[T] Technology</td><td>[PS] Problem Solving
[V] Visualization</td><td>[CN] Connections
[R] Reasoning</td><td>[ME] Mental Mathematics and Estimation</td></tr>
</table>

Performance Indicators

Use the following set of indicators to determine whether students have met the corresponding specific curriculum outcome.

M02.01 Explain how the imperial system was developed.
M02.02 Identify commonly used units in the imperial system, and determine the relationships among the related units.
M02.03 Identify contexts that involve the imperial system.
M02.04 Explain, using examples, how and why fractions are used in the imperial system.
M02.05 Compare the American and British imperial measurement systems.
M02.06 Provide an approximate measure in imperial units for a measurement given in SI units.
M02.07 Write a given linear measurement expressed in one imperial unit in another imperial unit.
M02.08 Convert a given measure from imperial to SI units by using proportional reasoning (including formulas).

Scope and Sequence

Mathematics 9	Mathematics at Work 10	Mathematics at Work 11
SS02 Students will be expected to determine the surface area of composite 3-D objects to solve problems.	**M02** Students will be expected to demonstrate an understanding of the International System of Units (SI) by ▪ describing the relationships of the units for length, area, volume, capacity, mass, and temperature ▪ comparing the American and British imperial units for capacity ▪ applying strategies to convert SI units to imperial units	**M01** Students will be expected to solve problems that involve SI and imperial units in surface area measurements and verify the solutions. **A03** Students will be expected to solve problems by applying proportional reasoning and unit analysis.

Background

The imperial system is a collection of units that were developed at different times to meet different needs. As a result, each group of units has a particular relationship. Because of this fact, the imperial system is not a decimal system, as is the SI system.

Students have been using the metric system in previous grades. This will be their first opportunity to study the imperial system. Students may be familiar with the imperial system for measurements such as distance in miles, height in feet and inches, weight in pounds, and capacity in gallons.

Students will explore the origins of the imperial system and its history and use in Canada. Although Canada officially adopted the metric system in 1970, imperial measures continue to be used extensively. To fully understand measurement, students will need to distinguish which measures are imperial and which are metric. This outcome focuses on developing proficiency with the imperial system and converting units within the imperial system.

For this outcome, students should recognize the terminology and abbreviations associated with imperial measure, such as: foot (ft. or '), inch (in. or "), yard (yd.), miles (mi.), pints (pt.), quarts (qt.), gallons (gal.), teaspoon (tsp.), tablespoon (Tbsp.), pound (lb.), ounces (oz.), degrees Fahrenheit (°F), and acres (ac.).

Students should also be able to identify when imperial measures are most commonly used, such as cooking (tsp., Tbsp., c., lb.), wood products (2" × 4"), fuel economy (mpg.), pant leg length (31" inseam), TV screen size (42"), paper size (8.5" × 11"), photograph size (5" × 7"), a newborn's weight (7 lbs. 6 oz.), floor tile (1 sq. ft.), room temperature (68°F), freezing point of water (32°F), height (5' 5"), healthy body temperature (98.6°F), and lot size (1 ac. house lot).

Students will convert between the commonly used imperial units for linear measure, area, capacity and temperature. They should know some basic conversion equivalencies, such as 12 in. = 1 ft., 3 ft. = 1 yd., 1 c. = 8 fl. oz., 1 lb. = 16 oz., 4 Tbsp. = $\frac{1}{4}$ c. Other conversions need not be memorized, as they are easily accessible.

Since imperial measures are based on traditional measurements rather than a base-ten system, sometimes imperial measures are written in fractional form. For example, inches on a measuring tape are divided into $\frac{1}{2}$ inch, $\frac{1}{16}$ inch, etc. Using the imperial system will provide practice using fractions. As with outcome M01, students will use proportions and proportional reasoning when solving problems. They need to be proficient in seeing and using the multiplicative relationships that are the basis for proportional reasoning.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

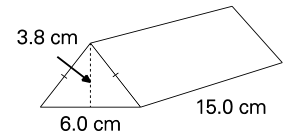
Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

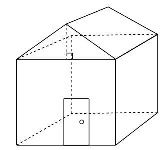
ASSESSING PRIOR KNOWLEDGE

Student tasks such as the following could be completed to assist in determining students' prior knowledge.

- How much packaging is needed to make a box for a fruit bar in the shape shown, to the nearest square centimetre?



- Look at the Robinson family's pool shed.
  (a) What measurements are needed to calculate the surface area of the shed?
  (b) Describe the steps that would be followed to find the total surface area, including the door.



WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

- Explain how the imperial system was developed.

- Identify commonly used units in the imperial system, and determine the relationships among the related units.

- Identify contexts that involve the imperial system.

- Explain, using examples, how and why fractions are used in the imperial system.

- Write a given measurement expressed in one imperial unit, in another imperial unit.

- Convert the following:
  (a) 4 ft. = ___ in.
  (b) 3 mi. = ___ yd.

- Cory is measuring a fishing "haul-up" line for turbot nets. He uses his two outstretched arms as his fathom referent (1 fathom = 6 feet). If he measures 125 fathoms, how many feet and inches has he measured?

- Lead a discussion about why the imperial system is still used in Canada even though it is not the official system.

- When you convert a measurement from a larger unit to a smaller unit, do you expect the number of units to increase or decrease? Why?

- Use flyers or the Internet to investigate products from building supply stores that show the use of imperial units for measurements. Examine what material or objects are measured in the imperial system and which ones are measured in the SI system. Record your findings and identify each measurement as to whether it is for length, area, volume, capacity, mass, or temperature.

- Using a measuring tape with inches and feet, have students measure 10 objects around the classroom to the closest one-eighth of an inch.

- Fill in the blanks:
  (a) 36 in. = ____ ft.
  (b) 6" = ____ '
  (c) 6 ft. = ____ yd.
  (d) ____ in. = 2 ft.

- A baby is born weighing 7 pounds and 8 ounces. The birth announcement in the newspaper said the baby weighed 7.5 lbs. Given this, how many ounces are in a pound?

FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should inform instruction.

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SUGGESTED LEARNING TASKS

Effective instruction should consist of various strategies.

Guiding Question
- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

Consider the following sample instructional strategies when planning lessons.

- Students' prior knowledge for this outcome will vary widely since it is the first introduction to the imperial system. Samples of student's background knowledge could be drawn out of the students through a class discussion of the situations where they have encountered imperial system measures in their lives. The teacher might encourage students to think of any baking they might have done in the past and to recall the units that were used. Similar discussions could take place if students have had the opportunity to measure using a measuring tape or if they had to buy paint. Although some or all of these tasks could have been completed using SI measurement, oftentimes the imperial system is still used.

- From the earliest introduction to metric units, students have had experience relating non-standard and standard units of measurement. They have used referents to estimate the length of an object in centimetres, metres, and millimetres. Opportunities should be provided to explore imperial units using various referents to measure objects. Students should be encouraged to use a referent that represents approximately one unit of measurement.

  Some common referents for linear measurement include the following:

width of a quarter, thickness of a hockey puck	≈ 1 in.
length of a standard floor tile, length of a submarine sandwich	≈ 1 ft.
distance from the tip of the nose to the outstretched fingers	≈ 1 yd.

  Encourage students to select their own referents that make sense to them. For example, to approximate 1 mile, students may choose a distance from two well-known points in their own communities.

- When converting between imperial units, have students use proportional reasoning where appropriate. Spend time on the proper development of proportional reasoning so students will become multiplicative thinkers and be able to see and use the multiplicative relationships found within and between the ratios in the problem.

- Hands-on activities will allow students to be more engaged for this outcome and develop skills for using measurement tools such as a measuring tape.

- Use of fractions can be demonstrated through cooking or construction. For example,
  - halving or doubling a recipe
  - measuring $16\frac{1}{2}$" in construction

SUGGESTED MODELS AND MANIPULATIVES

- calipers
- fraction strips
- measuring cups
- measuring tapes
- thermometer

MATHEMATICAL VOCABULARY

Students need to be comfortable using the following vocabulary.

- Commonly used terminology and abbreviations associated with SI units, such as
  - acres (ac.)
  - degrees Fahrenheit (°F)
  - foot (ft. or '), inch (in. or "), yard (yd.), miles (mi.)
  - pints (pt.), quarts (qt.), gallons (gal.), teaspoon (tsp.), tablespoon (Tbsp.)
  - pound (lb.), ounces (oz.)

Resources/Notes

Print

Math at Work 10 (Etienne et al. 2011)

- Chapter 1: Consumerism and Travel
  - Get Ready
  - Section 1.3
  - Skill Check
  - Test Yourself
  - Chapter Project
  - Games and Puzzles
- Chapter 2: Measuring Length
  - Sections 2.1, 2.3, and 2.4
  - Skill Check
  - Test Yourself
  - Chapter Project
  - Games and Puzzles
- Chapter 3: Measuring Area
  - Chapter Opener
  - Get Ready
  - Section 3.2
  - Chapter Project

Notes

M03 Students will be expected to solve and verify problems that involve SI and imperial linear measurements, including decimal and fractional measurements. [C, CN, ME, V]			
[**C**] Communication	[**PS**] Problem Solving	[**CN**] Connections	[**ME**] Mental Mathematics and Estimation
[**T**] Technology	[**V**] Visualization	[**R**] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have met the corresponding specific curriculum outcome.

M03.01 Identify a referent for a given common SI or imperial unit of linear measurement.
M03.02 Estimate a linear measurement, using a referent.
M03.03 Measure inside diameters, outside diameters, lengths, widths of various given objects, and distances, using various measuring instruments.
M03.04 Estimate the dimensions of a given regular 3-D object or 2-D shape, using a referent (e.g., the height of the desk is about three rulers long, so the desk is approximately three feet high).
M03.05 Solve a linear measurement problem including perimeter, circumference, and length + width + height (used in shipping and air travel).
M03.06 Determine the operation that should be used to solve a linear measurement problem.
M03.07 Provide an example of a situation in which a fractional linear measurement would be divided by a fraction.
M03.08 Determine, using a variety of strategies, the midpoint of a linear measurement such as length, width, height, depth, diagonal, and diameter of a 3-D object, and explain the strategies.
M03.09 Determine if a solution to a problem that involves linear measurement is reasonable.

Scope and Sequence

Mathematics 9	Mathematics at Work 10	Mathematics at Work 11
—	**M03** Students will be expected to solve and verify problems that involve SI and imperial linear measurements, including decimal and fractional measurements.	**M01** Students will be expected to solve problems that involve SI and imperial units in surface area measurements and verify the solutions. **A03** Students will be expected to solve problems by applying proportional reasoning and unit analysis.

Background

Students will compare, estimate, and justify their choice of measurement system and units based on referents such as centimetres and inches.

Students will solve problems that involve linear measure with a variety of tools such as rulers, calipers, and tape measures.

Students will use both the imperial and SI systems and convert between these systems as appropriate to the application. The main focus of this outcome is linear measurement; however, it is also important that students explore how to convert between pounds and kilograms.

Students will apply proportional reasoning to solve, verify, and justify problems involving conversion within or between SI and imperial units.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Assessing Prior Knowledge

Student tasks such as the following could be completed to assist in assessing students' prior knowledge.

- What is the maximum number of boxes measuring 6 cm × 3 cm × 2 cm that can be tightly packed into a box measuring 24 cm × 8 cm × 11 cm? If each of the dimensions of the large packing box doubles, how many smaller boxes will fit?

- Estimate the length, width, and perimeter of the classroom. Then measure the length and width and calculate the perimeter. Compare your estimates and calculations and discuss how you could improve your estimation techniques. Discuss which measurement system you chose to use and why this system was chosen.

- Mentally determine the value of $3 \times 1\frac{1}{2}$ and explain your strategy.

- Mentally determine the value of 204.8 ÷ 4 and explain your strategy.

Whole-Class/Group/Individual Assessment Tasks

Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

- Continue to provide referents for linear measurements, including millimetre, centimetre, metre, kilometre, inch, foot, yard, and mile, and explain the choices.

- Continue to compare SI and imperial units, using referents.

- Estimate a linear measure, using a referent, and explain the process used.

- Justify the choice of units used for determining a measurement in a problem-solving context.

- Solve problems that involve linear measure, using instruments such as rulers, calipers, or tape measures.

- Describe and explain a personal strategy used to determine a linear measurement such as the circumference of a bottle, the length of a curve, or the perimeter of the base of an irregular 3-D object.

- Solve a problem that involves the conversion of units within or between SI and imperial systems.

- Verify, using unit analysis, a conversion within or between SI and imperial systems, and explain the conversion.

- Justify, using mental mathematics, the reasonableness of a solution to a conversion problem.

- Identify objects within the classroom or school that would be approximately
  - 1 cm long
  - 2 m long

- Identify and use an appropriate referent to estimate the length, height, or distance of objects, such as the following:
  (a) classroom wall
  (b) distance from one classroom to another
  (c) perimeter of the cafeteria
  (d) height the clock is off the floor
  (e) diameter of a basketball net
  (f) width of an iPod screen

- Identify and use an appropriate referent to estimate the dimensions of objects, such as the following:
  (a) teacher's desk
  (b) cereal box
  (c) milk can
  (d) soccer field
  (e) school

- Solve the following problems:
  (a) A tire has a radius of 12 inches. What is its circumference?
  (b) The circumference of a CD is 28.26 cm. What is its diameter?
  (c) A rectangular window frame measures 24 inches by 36 inches. If trim for the window costs $2.75 per linear foot, how much will it cost to put trim around the window?
  (d) A box has a height of 30 cm, a width of 20 cm, and a length of 60 cm. Determine if this box can be shipped via Canada Post.

- Compare and contrast the terms **perimeter** and **circumference**. When would it be appropriate to use each?

- A rectangular room has a width 12 feet. If the floor of the room is to be covered with boards of width $3\frac{1}{4}$ inches, how many rows of boards will be needed?

- Given a number of regular and irregular objects,
  (a) estimate the measure of these objects
  (b) use a personal referent to measure these objects
  (c) measure these objects using trundle wheels, metre sticks, rulers, tape measures, and calipers
  (d) convert the measurements from imperial to SI system or vice versa as appropriate

- List real-life examples of how the imperial system is used. As a class, continue to develop referent units of measurement for the more commonly used units of the imperial system. For example, 1 inch = diameter of a quarter, 1 cup = a small cup of coffee, 1 ounce = weight of a pencil, 1 ton = weight of a small passenger car, 1 pound = block of butter.

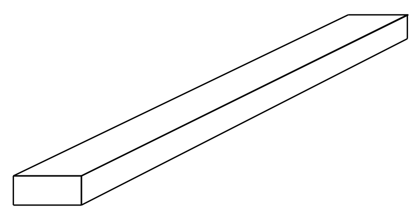
- Body Mass Index (BMI) is calculated in kg/m^2. Calculate the BMI in the following situations:
  (a) Oakley weighs 182 lb. and is 5'10" tall
  (b) Kelsey weighs 145 lb. and is 1.65 m tall
  (c) Marisa weighs 54 kg and is 162 cm tall
  (d) Ashtyn weighs 50.5 kg and is 5'4" tall
  (e) your own BMI

- Isaac bought a second-hand treadmill online. It would register in only miles. Describe a conversion factor that could be used to estimate a conversion from miles to kilometres or vice versa.

- A jet is flying at a height of 28 000 ft. How high is this in metres?

- Estimate each measure:
  (a) the height of a horse in ft.
  (b) the diameter of the dial on an iPod in cm
  (c) the length of a hockey rink in m
  (d) the width of a cell phone in cm
  (e) the length of a new pencil in mm

- A GPS is set in miles. It estimates the distance to a particular destination to be 188 mi.
  (a) How many kilometres is this?
  (b) If the speed is registered as 45 mph, how many km/h is this?
  (c) Given this information, what is the estimated time of arrival (ETA) if it is now 10:20 a.m.
  (d) Does the answer seem reasonable?



- A 2" × 4" stud actually measures $1\frac{1}{2}$" × $3\frac{1}{2}$". To build an interior wall in the basement, the stud would be secured to the ceiling and floor and drywall would be nailed to both of the narrower sides of the stud. How thick is the wall if the drywall measures $\frac{5}{8}$"?

- Create two contextual problems that could be solved by a classmate.

FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should inform instruction.

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SUGGESTED LEARNING TASKS

Effective instruction should consist of various strategies.

Guiding Question
- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

Consider the following sample instructional strategies when planning lessons.

- Set up an investigation where students must estimate and then measure several items using appropriate SI units. Have them convert the measurements to imperial and then verify using an appropriate tool. The items provided for the students should be regular as well as irregular.

- Have students develop a set of referents for both metric and imperial commonly used measures such as metre, gram, inch, and mile, and then use them to estimate the length of an unknown object. Some examples they may discover are listed below.

Metre/Yard	Height of a door knob from the floor
Millimetre	The thickness of a dime
Centimetre	Width of a standard paper clip
Inch	Length between knuckles on an index finger
Kilometre	Distance you could walk comfortably in 12 minutes
Gram	Mass of a jelly bean
Pound/Kilogram	One/two footballs
Litre	Small carton of milk

Millilitre	The liquid you could fit in a base-ten unit cube
Celsius/Fahrenheit	20°C/68°F room temperature
Foot	Foot-long submarine sandwich

- Have students measure, in imperial and in metric, their hand span, foot length, finger length, and stride length and use these as referents to measure the length of the classroom, width of a desk, etc.

SUGGESTED MODELS AND MANIPULATIVES

- graduated cylinders
- measuring cups
- scale
- thermometers
- wooden geoblocks

MATHEMATICAL VOCABULARY

Much of the vocabulary for this outcome will have been seen through the first two outcomes of this unit. However, when conversion charts are used, there will be some SI as well as imperial units encountered that will be new to most students.

Resources/Notes

Videos

- DVL Videos, Multimedia Ednet Web Station, Learning Resources and Technology Services (Province of Nova Scotia 2013)
  - Grade 7–Assess Addition of Decimals Using Make-One Strategy
    http://dvl.ednet.ns.ca/videos/grade-7-assess-addition-decimals-using-make-one-strategy
  - Grade 7–Introduce Make-Zero Strategy for Integers
    http://dvl.ednet.ns.ca/videos/grade-7-introduce-make-zero-strategy-integers
  - Grade 8–Assess Halve/Double Strategy for Percentages
    http://dvl.ednet.ns.ca/videos/grade-8-assess-halvedouble-strategy-percentages
  - Grade 8–Introduce Addition of Fractions Using Make-One Strategy
    http://dvl.ednet.ns.ca/videos/grade-8-introduce-addition-fractions-using-make-one-strategy
  - Grade 8–Reinforce Addition and Subtraction of Fractions by Rearrangement
    http://dvl.ednet.ns.ca/videos/grade-8-reinforce-addition-and-subtraction-fractions-rearrangement

Notes

M04 Students will be expected to solve problems that involve SI and imperial area measurements of regular, composite, and irregular 2-D shapes and 3-D objects, including decimal and fractional measurements, and verify the solutions. [C, CN, ME, V]			
[**C**] Communication [**T**] Technology	[**PS**] Problem Solving [**V**] Visualization	[**CN**] Connections [**R**] Reasoning	[**ME**] Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have met the corresponding specific curriculum outcome.

M04.01 Identify and compare referents for area measurements in SI and imperial units.
M04.02 Estimate an area measurement, using a referent.
M04.03 Identify a situation where a given SI or imperial area unit would be used.
M04.04 Estimate the area of a given regular, composite, or irregular 2-D shape, using an SI square grid and an imperial square grid.
M04.05 Solve a contextual problem that involves the area of a regular, a composite, or an irregular 2-D shape.
M04.06 Write a given area measurement expressed in one SI unit squared, in another SI unit squared.
M04.07 Write a given area measurement expressed in one imperial unit squared in another imperial unit squared.
M04.08 Solve a problem, using formulas for determining the areas of regular, composite, and irregular 2-D shapes, including circles.
M04.09 Solve a problem that involves determining the surface area of 3-D objects, including right cylinders and cones.
M04.10 Explain, using examples, the effect of changing the measurement of one or more dimensions on area and perimeter of rectangles.
M04.11 Determine if a solution to a problem that involves an area measurement is reasonable.

Scope and Sequence

Mathematics 9	Mathematics at Work 10	Mathematics at Work 11
SS02 Students will be expected to determine the surface area of composite 3-D objects to solve problems. **SS04** Students will be expected to draw and interpret scale diagrams of 2-D shapes.	**M04** Students will be expected to solve problems that involve SI and imperial area measurements of regular, composite, and irregular 2-D shapes and 3-D objects, including decimal and fractional measurements, and verify the solutions.	**M01** Students will be expected to solve problems that involve SI and imperial units in surface area measurements and verify the solutions.

Background

The calculation of area was introduced in Mathematics 4. Students in this course should understand and be able to apply formulae to find the area of triangles, parallelograms, and circles. In this outcome, students will determine an appropriate system and unit of measurement to estimate and calculate the area of various 2-D shapes and 3-D objects. For regular polygons, students will be expected to divide the shape into triangles and find the area accordingly.

Students will continue to develop their understanding of both the SI and imperial systems of measurement, and where each is most appropriate, and will be required to solve problems using both systems.

When calculating area, focus should first be on regular and irregular 2-D shapes. A regular shape is defined as having all sides and all angles congruent (e.g., equilateral triangle, square). An irregular shape is one in which all sides and/or angles are not congruent (e.g., rectangle, rhombus, scalene triangle, etc.). Discussion of regular and irregular shapes is appropriate. It is more important, however, that students be given the opportunity to work with area of these various figures rather than being able to distinguish between the two types.

In Mathematics 7, formulae for the area of various quadrilaterals, triangles, and circles were developed and applied (7SS2). Students will now use these formulae to find the area of regular and irregular shapes. Required formulae include the following:

- $A = b \times h$ (parallelogram, rectangle, rhombus, square)
- $A = \frac{1}{2} b \times h$ (triangle)
- $A = \pi r^2$ (circle)

In some situations, the area will be calculated when all measurements are given, requiring no formula manipulation. Alternatively, students may be given the area and one dimension and asked to find the other measurement.

Area formulae can be used to determine such things as the amount of

- plywood or Gyproc needed for a new house
- flooring needed
- paint needed to cover walls
- material needed to make a new dress
- materials being used so there is no wastage

Once students have had exposure to regular and irregular 2-D shapes, they should work with more complex area problems. One suitable example of this would be asking students to calculate the area of a wall with a window removed. They should work with composite shapes, made up of a combination of two or more shapes (e.g., a semicircle on top of a triangle).

There are often several ways to decompose a composite shape. The way in which a shape is decomposed may affect the dimensions used, but not its area. When decomposing a composite shape, encourage students to look for figures they have previously worked with (i.e., rectangles, triangles, and circles).

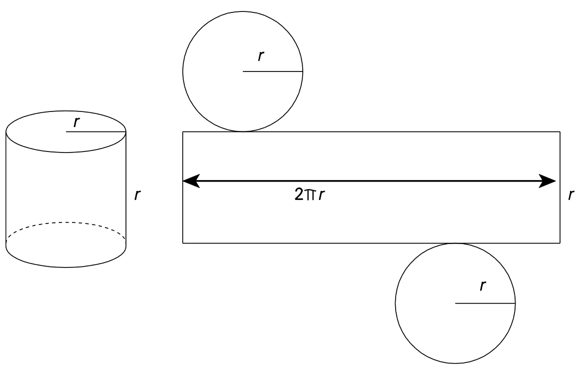
Students should explore changing the dimension(s) of one or two sides of a rectangle to see what effect it will have on the area and perimeter of the rectangle.

Many area problems involve finding surface areas of three-dimensional figures. In Mathematics 8, students worked with surface area of prisms and right cylinders (8M03). The surface area of a cone, however, is new. To find these surface areas, nets (2-D patterns that can be used to construct 3-D figures) should be used. Students should have prior experience with nets and their application to surface area of 3-D objects. In this unit, the focus is on the surface area of rectangular prisms and right cylinders. The surface area of cones will also be explored.

To calculate surface area, students must identify the faces or surfaces, determine the dimensions of each face, and apply appropriate formulae to calculate area. Use of concrete models allows students to visualize the figures and encourages them to use reasoning rather than merely follow procedure. Prior to generalizing formulas and using symbolic representations to calculate surface area, students should use nets of 3-D objects.

The surface area of a prism can be determined from its net, as the net shows all faces making up the object. Working from the net also allows for easy identification of congruent faces, which sometimes avoids the necessity of having to find the areas of each face individually. Students may conclude that the surface area of a rectangular prism can be calculated using the formula $SA = 2lw + 2lh + 2wh$. To ensure students have gained the conceptual understanding of surface area, however, concrete models and nets should be explored before introducing the formula. Some students may never use this formula. Students should always be encouraged to include the units as part of the solution.

Next, consider the right cylinder. The net of a solid cylinder consists of two circles and one rectangle. The curved surface opens up to form a rectangle. A good way to demonstrate this is to unpeel the label on a can to show that it is a rectangle.



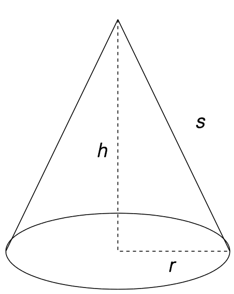
Surface Area = 2 × (area of circle) + area of rectangle

$SA = (2 \times \pi r^2) + (2\pi r \times h)$

$SA = 2\pi r^2 + 2\pi rh$

Another example of a cylinder is a shipping tube for holding blueprints.

Students should also be exposed to situations where calculating the surface area involves only one circle (e.g., a can with the lid removed). Finally, consider the cone. The formula for the surface area of a cone is $A = \pi r^2 + \pi rs$ where r is the radius of the base and s is the slant height of the cone.



If the cone has no top (like a drinking cup), the formula is simply $A = \pi rs$. Discuss examples such as an ice cream cone, the tip of a pen, and a traffic pylon.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Assessing Prior Knowledge

Students tasks such as the following could be completed to assist in assessing students' prior knowledge.

- A store owner wants to make a rectangular area for a special display in one corner of his store. He has 6 m of enclosure rope to block off two sides of the area, using walls for the other two sides. What are the dimensions of the largest area he could rope off?

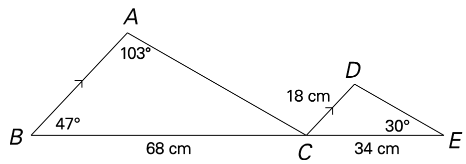
- If you had a length of chicken wire that could bend anywhere, how could you find the largest area you could enclose without measuring? Explain, using different geometric shapes. If you had 16.25 m of the chicken wire, what would the dimensions be?

- The dimensions of five decorative gardens are given below. Which garden has the greatest area?
  (a) Square with sides 10.2 m.
  (b) Rectangle with a length of 15 m and a width of 6.9 m.
  (c) Parallelogram with a base of 14.6 m and a height of 7.2 m.
  (d) Triangle with a base of 16.5 m and a height of 12.4 m.
  (e) Trapezoid with bases of 18.1 m and 10.4 m, and a height of 7.1 m.

- Create a diagram with a lake and the following islands using these directions:
  - A rectangular island, A, with an area of about 100 cm^2.
  - A triangular island, B, with an area of about 18 cm^2.
  - An irregular-shaped island, C, with an area of about 50 cm^2.
  - A circular-shaped island, D, with an area of about 25 cm^2.

- You want to paint one wall of your room. The wall is 7.0 m long and 2.4 m high. It takes one small can of paint to cover 9 m^2 and the paint sells for $3.99 a can.
  (a) What would it cost to purchase the paint?
  (b) What else do you need to consider?
  (c) Make a plan for your trip to the store for supplies for this painting job.

- Determine the length of *AB*.



- The scale on a map is 1:20 000. What is the actual distance between two cities that are 3 cm apart on the map? Give your answer in kilometres.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

- Identify and compare referents for area measurements.

- Estimate an area measurement, using a referent.

- Identify a situation where a given unit would be used.

- Estimate the area of a given regular, composite, or irregular 2-D shape, using a square grid.

- Solve a contextual problem that involves the area of a regular, a composite, or an irregular 2-D shape.

- Write a given area measurement expressed in one unit squared, in another unit squared.

- Solve a problem, using formulas for determining the areas of regular, composite, and irregular 2-D shapes, including circles.

- Use SI and imperial referents to measure the area of the classroom, gymnasium, whiteboard, etc.

- Given different shapes, estimate the area of the shapes using square centimetres and square inches. Overlay the shapes on the cm and inch grid papers to determine the area.

- Soma estimates that the top of her school desk is 60 cm^2. Does this estimate seem reasonable? Explain.

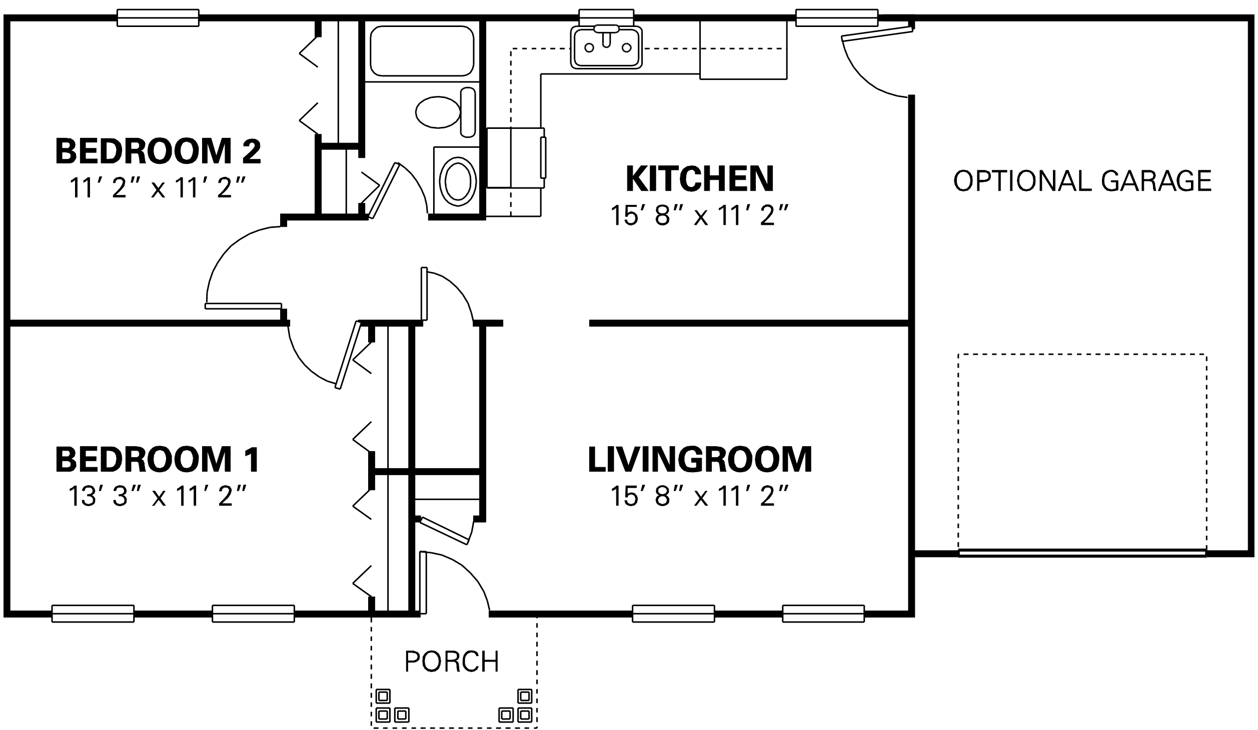
- Sharik is carpeting a bedroom that measures 10 feet by 12 feet. Since there are 3 feet in a yard, he orders 40 yards of carpet. When he is done, he finds he has over 25 yards of carpet left over. Ask students to identify the error Sharik made in his calculations.

- Convert the following:
  (a) 20 cm^2 = ___ mm^2
  (b) 2 yd^2 = ___ in^2
  (c) 4 m^2 = ______ cm^2
  (d) 3 cm^2=______ mm^2

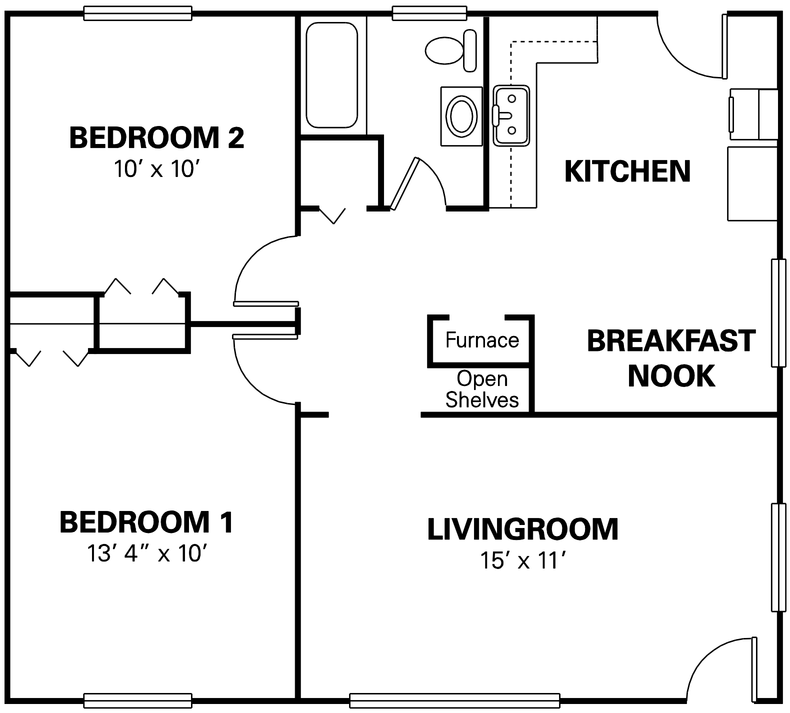
- Give three examples of situations that would usually require calculating area using imperial measurements and three that would require using SI measurements.

- Create two contextual problems that could be solved by a classmate.

- **Project idea:** Assume the role of owner of a contracting company that installs flooring. Develop a proposal for a bid on a new house that is being built. Using the measurements in the floor plans, determine the amount of the bid (including profit) if
  (a) each bedroom is to have carpet ($9.99 per square yard*)
  (b) the living room has hardwood flooring ($6.99 per square foot*)
  (c) the kitchen/dining room has ceramic tile ($3.99 per square foot*)
  * above prices include installation costs
  (See Appendix A.1 for a copyable version of this floorplan.)



- Alternatively, determine the cost of flooring for the living room and bedrooms based on the following floor plan. (See Appendix A.2 for a copyable version of this floor plan.)



- Estimate how many students would fit on one square metre of the classroom floor if each student is standing and needs about one 12 inch × 12 inch tile to stand in. How many students could fit in the entire room if it was empty?

- Using grid chart paper, represent a given composite figure, an irregular figure, or one you created on your own. Illustrate how to calculate the area of the entire figure and then present this to the class.

- Design a logo for a new application (play park, skateboard park, etc.) incorporating 2-D and composite shapes, then calculate the area.

- Imagine you have 50 m of rope.
  (a) What is the smallest area you can enclose if you use only whole numbers for the dimension?
  (b) What is the largest area you can enclose with a whole number dimension?

- You have 50 m of rope and you make a rectangle 1 m × 24 m. How much more rope do you need in order to double the area?

- Calculate the area of the painted surface in the classroom, then calculate how much paint would be needed to do a Classroom Extreme Makeover. (**Note:** This could be extended to the entire school.)

- A loonie ($1 coin) is a regular polygon with 11 sides. It is called a hendecagon. What is the area of the coin if the side length is 7.9 mm and the distance from the centre to the side is 13.3 mm? Express the answer in both mm^2 and cm^2.

- The dial on an iPod has a radius of $\frac{3}{4}$ in. What is the area of the dial?

Follow-up on Assessment

The evidence of learning that is gathered from student participation and work should inform instruction.

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

Suggested Learning Tasks

Effective instruction should consist of various strategies.

Guiding Question
- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

Consider the following sample instructional strategies when planning lessons.

- Provide an opportunity for students to draw 1 cm^2, 1 m^2, 1 in^2, 1 ft^2, and discuss situations where these would be used.

- Students should have the opportunity to solve real-life, authentic problems. For example, photos of architecture or quilts would be useful in introducing area problems involving composite shapes. Students may suggest a process that might involve either subdividing the shape into familiar shapes or extending the figure into a quadrilateral and subtracting the missing area. Encourage different strategies. Compare solutions. Pay particular attention to the written form of the solution. Subsequent steps would include
  - taking needed measurements
  - representing symbolically, substituting into formulas, then computing
  - noting appropriate units

- A Word/Formula Wall would be useful in this section.

SUGGESTED MODELS AND MANUPULATIVES

- graduated cylinders
- measuring cups
- scale
- thermometers
- wooden geoblocks

MATHEMATICAL VOCABULARY

Students need to be comfortable using the following vocabulary.

- 2-D shapes
- 3-D objects
- composite
- irregular
- regular

Resources/Notes

Print

Math at Work 10 (Etienne et al. 2011)

- Chapter 3: Measuring Area
  - Sections 3.1, 3.2, 3.3, and 3.4
  - Skill Check
  - Test Yourself
  - Chapter Project
  - Games and Puzzles
- Chapter 6: Pythagorean Relationship
  - Get Ready
  - Section 6.2
  - Skill Check
  - Test Yourself

Notes

Geometry
40–45 hours

GCO: Students will be expected to develop spatial sense.

Specific Curriculum Outcomes

Process Standards Key

[**C**] Communication	[**PS**] Problem Solving	[**CN**] Connections	[**ME**] Mental Mathematics and Estimation
[**T**] Technology	[**V**] Visualization	[**R**] Reasoning	

G01 Students will be expected to analyze puzzles and games that involve spatial reasoning, using problem-solving strategies. [C, CN, PS, R]

G02 Students will be expected to demonstrate an understanding of the Pythagorean theorem by identifying situations that involve right triangles, verifying the formula, applying the formula, and solving problems. [C, CN, PS, V]

G03 Students will be expected to demonstrate an understanding of similarity of convex polygons, including regular and irregular polygons. [C, CN, PS, V]

G04 Students will be expected to demonstrate an understanding of primary trigonometric ratios (sine, cosine, tangent) by applying similarity to right triangles, generalizing patterns from similar right triangles, applying the primary trigonometric ratios, and solving problems. [CN, PS, R, T, V]

G05 Students will be expected to solve problems that involve parallel, perpendicular, and transversal lines, and pairs of angles formed between them. [C, CN, PS, V]

G06 Students will be expected to demonstrate an understanding of angles, including acute, right, obtuse, straight and reflex, by drawing, replicating and constructing, bisecting, and solving problems. [C, ME, PS, T, V]

G01 Students will be expected to analyze puzzles and games that involve spatial reasoning, using problem-solving strategies. [C, CN, PS, R]			
[**C**] Communication	[**PS**] Problem Solving	[**CN**] Connections	[**ME**] Mental Mathematics and Estimation
[**T**] Technology	[**V**] Visualization	[**R**] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have met the corresponding specific curriculum outcome.

G01.01 Determine, explain, and verify a strategy to solve a puzzle or to win a game. For example,
- guess and check
- look for a pattern
- make a systematic list
- draw or model
- eliminate possibilities
- simplify the original problem
- work backward
- develop alternative approaches

G01.02 Identify and correct errors in a solution to a puzzle or in a strategy for winning a game.

G01.03 Create a variation on a puzzle or a game, and describe a strategy for solving the puzzle or winning the game.

Note: It is intended that this outcome be integrated throughout the course by using sliding, rotation, construction, deconstruction, and similar puzzles and games.

Scope and Sequence

Mathematics 9	Mathematics at Work 10	Mathematics at Work 11
—	**G01** Students will be expected to analyze puzzles and games that involve spatial reasoning, using problem-solving strategies.	**N01** Students will be expected to analyze puzzles and games that involve numerical reasoning, using a variety of problem-solving strategies.

Background

The focus in Mathematics at Work 10 will be on spatial reasoning to solve puzzles and play games, throughout the course. However, it is not enough for students to only do the puzzle or play the game. They should be given a variety of opportunities to analyze the puzzles they solve and the games they play. The goal is to develop their problem-solving abilities using a variety of strategies and to be able to apply these skills to other contexts in mathematics. In Mathematics at Work 11, the focus will shift to numerical reasoning.

An example of one such puzzle/game would be using Pentominoes.

Polyominoes is the general name given to plane shapes made by joining squares together. Note that the squares must be “properly” joined edge to edge so that they meet at the corners.

Each type of polyomino is named according to how many squares are used to make it. So there are monominoes (1 square only), dominoes (2 squares), triominoes (3 squares), tetrominoes (4 squares), pentominoes (5 squares), hexominoes (6 squares), and so on.

Pentominoes (made from 5 squares) are the type of polyomino most worked with. There are only 12 in the set, because shapes that are identical by rotation or reflection are not counted. This means that they are few enough to be handleable, yet quite enough to provide diversity.

There are many websites with puzzles, games, and instructional suggestions involving pentominoes. Another such puzzle, the tangram, would be familiar to many students.

There are many free games and puzzles available for mobile devices that require students to think about symmetry, rotations, reflections, and translations.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

Guiding Questions

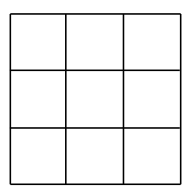
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Student tasks such as the following could be completed to assist in determining students' prior knowledge.

- Students can recall games that they have played that require strategies involving spatial reasoning.

  Tic-tac-toe or counting the number of squares in a design such as this one are examples.

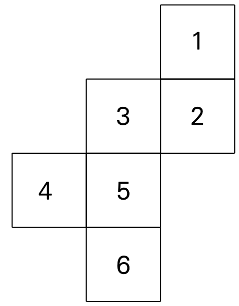


WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

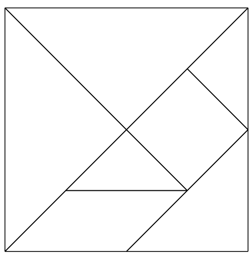
- Using a puzzle or game of their choice, ask students to write about the problem-solving strategies they tried. Which worked well and which did not?
  - Students could write hints for puzzles that they found interesting and then try their hints on another student to determine if they are helpful.

- Explain the rules of the game in your own words. Show your rules to another student. Do they agree with your explanation? Can other people find loopholes in your rules?
- What did you do when you got stuck? Explain through words or diagrams the strategies you tried in solving the puzzle or playing the game.
- What general advice would you give to other students trying to solve the puzzle or play the game?

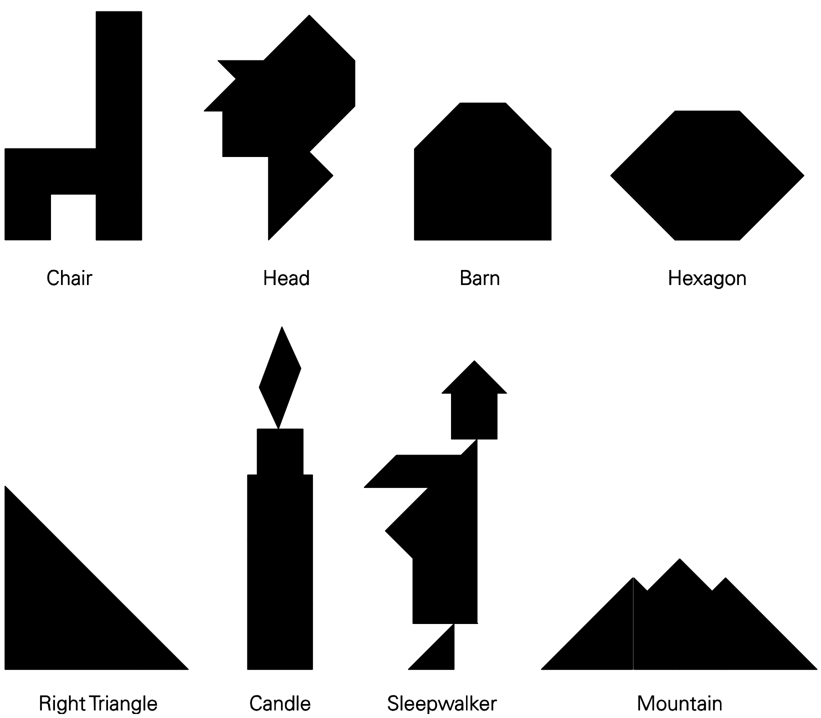


- If the numbered net shown to the right is folded to form a cube, what is the product of the numbers on the four faces sharing an edge with the face number 1?

- The tangram puzzle was invented in China thousands of years ago. The object is to arrange all seven pieces of the tangram (cut from a square as shown below), into various shapes just by looking at the outline of the solution.



  - Try making the shapes shown below, using all seven pieces. Design other puzzles and challenge your classmates to solve them. (See Appendix A.3 for a copyable version of these shapes.)



- Create a variation on a puzzle or a game, and describe a strategy for solving the puzzle or winning the game.

FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should inform instruction.

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
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SUGGESTED LEARNING TASKS

Effective instruction should consist of various strategies.

Guiding Question

- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

Consider the following sample instructional strategies when planning lessons.

- Pentomino Investigation and Exploration
  - Introduce pentominoes by considering a single square region, Mr. Monomino. Students could use an x^2 algebra tile or a colour tile. This monomino has all right angles and is perfectly balanced. Students could discuss its symmetry.
  - Ask students what they might call a shape when two monominos are pushed together. (Their answers might be duomino, binomino, dinomino, or domino.)
  - Ask, can we form another shape with the two square tiles?
  - Ask students what they might call a shape when three monominoes are pushed together. (Their answer will likely be triomonoes.)
  - Ask students to determine how many different or unique triominoes they can generate. Make sure you give them time to work on this. Using grid paper and markers or coloured pencils to explore this can be useful or supply the students with coloured tiles and let them experiment to determine different triominoes. (They will discover that there are only two.) Note that reflections, rotations, and translations are not considered different or unique.

- When four monominoes are pushed together it is called a tetromino. Ask students to determine how many unique tetrominoes they can generate. Again, allow time for students to experiment. (They will discover that there are five.)
- Ask students to discuss what strategies they used to the different tetromino shapes. Some may say that they took the triominoes and just added one square in different locations to each of the two triominoes. The shapes are L, I, Z, T, or square.
- When four monominoes are pushed together it is called a pentomino. Ask students to determine how many unique pentominoes they can generate. Again, allow time for students to experiment. (They will discover that there are twelve.) The shapes are D, F, I, L, N, T, U, V, W, X, Y, and Z.
- **Guiding Questions**
  - Are you certain that each shape you have found is not a reflection or rotation of one of the other shapes you have identified? How can you be certain?
  - Have you identified a systematic approach to moving the tiles to assure you have found all possible combinations? What strategy are you using?
  - Using the following systematic approach, 11 of the 12 pentominoes can be generated. Starting with a straight row of 5 squares, move 1 of the tiles along the length of the remaining 4 to see how many unique pentominoes can be found. Then work with 3 tiles in a row and examine possible locations for the remaining 2 tiles. Which tile would not be found this way?
  - Using the following systematic approach, can all 12 of the pentominoes be generated?— Using the 5 tetrominoes and just adding one square in different locations to each of the five tetromonoes.
- Ask students whether all the pentominoes have the same area and the same perimeter.

- Once the investigation has been completed, have students construct their own Pentomino set using card stock. They are then ready to consider some of the following puzzles/games.
  - **Game #1:** The 12 pentomino pieces fit together to form a rectangle. In fact, there is more than one rectangle that can be created with the 12 pentominoes.
    - Ask students to determine the possible dimensions for these rectangles. (Since the area of the pieces is $12 \times 5 = 60$, the rectangle dimensions will be factors of 60.)
    - Ask students if they can eliminate any of the possible dimensions? (Yes, some of the pieces are units high or wide so the 1×60 and 2×30 can be eliminated)
    - Ask students to form a rectangle with their pieces. (Solutions for the 6×10, 5×12, 4×15 and 3×20 rectangles can be found online.) **Note:** This puzzle takes time to solve, so students may not be able to complete the task in a short period of time. Encourage them to collaborate with others.
  - **Game #2:** Players take turns drawing pentominoes in pencil. Co-operative thinking is allowed! When students are satisfied that they have drawn a pentomino that is not already on the grid, they can colour it, using a different color for each pentomino. Remind students that two pentominoes can share a square and that each of the 12 pentominoes may be used only once. **Caution:** Watch out for slides, flips, and turns!
    - The game ends when there is no room available for another pentomino.
    - Have students record the number of pentominoes created by their team.
    - Invite students to play another game using any of the blank pentomino playing grids. The goal is to beat their team score.
    - Invite students to continue playing additional games, adjusting the strategy so that the goal is to maximize their score.

    - > When students feel that they have obtained a strategy that will work consistently, ask them to share that strategy with the class.
  - **Game #3:** Take the T-pentomino. Have students use nine of the remaining pentominoes to make an enlarged scale model of the T-pentomino.
    - > What is the area of the enlarged model?
    - > How does this area compare with the area of the T-pentomino?
    - > What are the lengths of the each of the sides of the enlarged model?
    - > How do these lengths compare with the corresponding sides of the T-pentomino?
    - > Repeat this process for the W-pentomino and the X-pentomino.
    - > What other pentominoes can students triplicate?
    - > What strategy can students use to determine which pentominoes can be triplicated?

- Students will benefit from solving puzzles and playing games if they take time to reflect on their experiences. Ask them to choose one of the puzzles or games they have worked on and write about it. Students may find it easier to record their thoughts if they talk about what they are thinking as they work through a puzzle, while a partner takes notes.

- Choose a variety of games or puzzles for students to play online, with pencil and paper, or using models that require a variety of strategies to solve.

- Have students create a game or puzzle to challenge other students.

- Puzzles and discussions of strategies should be spread throughout the semester.

SUGGESTED MODELS AND MANIPULATIVES

- dice
- puzzles
- various games

MATHEMATICAL VOCABULARY

Students need to be comfortable using the following vocabulary.

- rotation
- special reasoning
- strategy
- systematic list
- visualization

Resources/Notes

Internet

There are numerous games and puzzles available on the internet. What follows are just a few suggestions of spatial games and puzzles, available for free online. Many can also be done on paper, with models, or acted out.

Note: Teachers must always confirm the validity of the site prior to directing students to it.

- Math Is Fun (MathIsFun.com 2013)
  www.mathsisfun.com
  - Towers of Hanoi: www.mathsisfun.com/games/towerofhanoi.html
  - Tic-Tac-Toe: www.mathsisfun.com/games/tic-tac-toe.html
  - Dots and Boxes Game: www.mathsisfun.com/games/dots-and-boxes.html
  - Four In A Line: www.mathsisfun.com/games/connect4.html
- Calculation Nation (National Council of Teachers of Mathematics 2013)
  http://calculationnation.nctm.org/Games/ (requires log-in)
  - **neXtu:** In this game, players alternate claiming shapes on a tessellation with shapes of greater value than the adjoining shape.
  - **Flip-n-Slide:** This is a game in which triangles are translated and reflected to capture ladybugs. It is spatially challenging to envision the final position of the triangle.
- NRICH: enriching mathematics (University of Cambridge 2012)
  http://nrich.maths.org
  - **Sprouts:** A game for two players, and can be played with a paper and a pencil. The rules are simple, but the strategy can be complex. The site discusses the game, its history, and strategies for solving the puzzle. (http://nrich.maths.org/1208)
  - **Junior Frogs:** A well-known puzzle in which frogs and toads must change places in as few turns as possible. (http://nrich.maths.org/6282)
- Jill Britton (personal site) (Britton 2013)
  http://britton.disted.camosun.bc.ca/nim.htm
  - **Nim:** An ancient game that can be played online, on paper, or using sticks. The winner is the player to not pick up the last stick (Common Nim) or to pick up the last stick (Straight Nim).
- **Cool Math-Games.com** (Coolmath.com, Inc. 2013)
  www.coolmath-games.com
  - **B-Cubed:** is a game where students must pass over each block prior to reaching the final red block.
  - Other spatial games on this site include Aristetris, Bloxorz, and Pigstacks.
- Various other games that can be found online are as follows:
  - Blockus
  - Rushhour
  - Chess
  - Blockers
  - Towers of Hanoi
  - Board Games
  - Free Flow

Print

Math at Work 10 (Etienne et al. 2011)

- Chapter 1: Consumerism and Travel
  - 1.3 Puzzler
  - Games and Puzzles
- Chapter 2: Measuring Length
  - 2.2 Puzzler
  - 2.4 Puzzler
  - Games and Puzzles
- Chapter 3: Measuring Area
  - 3.4 Puzzler
  - Games and Puzzles
- Chapter 4: Getting Paid for Your Work
  - Games and Puzzles
- Chapter 5: All About Angles
  - 5.4 Puzzler
  - Games and Puzzles

- Chapter 6: Pythagorean Relationship
  - Games and Puzzles
- Chapter 7: Trigonometry
  - 7.4 Puzzler
  - Games and Puzzles

Notes

G02 Students will be expected to demonstrate an understanding of the Pythagorean theorem by identifying situations that involve right triangles, verifying the formula, applying the formula, and solving problems. [C, CN, PS, V]

[**C**] Communication	[**PS**] Problem Solving	[**CN**] Connections	[**ME**] Mental Mathematics and Estimation
[**T**] Technology	[**V**] Visualization	[**R**] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have met the corresponding specific curriculum outcome.

G02.01 Explain, using illustrations, why the Pythagorean theorem applies only to right triangles.
G02.02 Verify the Pythagorean theorem, using examples and counterexamples, including drawings, concrete materials, and technology.
G02.03 Describe historical and contemporary applications of the Pythagorean theorem.
G02.04 Determine if a given triangle is a right triangle, using the Pythagorean theorem.
G02.05 Explain why a triangle with the side length ratio of 3:4:5 is a right triangle.
G02.06 Explain how the ratio of 3:4:5 can be used to determine if a corner of a given 3-D object is square (90°) or if a given parallelogram is a rectangle.
G02.07 Solve a problem, using the Pythagorean theorem.

Scope and Sequence

Mathematics 9	Mathematics at Work 10	Mathematics at Work 11
M01 Students will be expected to solve problems and justify the solution strategy using circle properties including the following: ▪ The perpendicular from the centre of a circle to a chord bisects the chord. ▪ The measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc. ▪ The inscribed angles subtended by the same arc are congruent. ▪ A tangent to a circle is perpendicular to the radius at the point of tangency. **G01** Students will be expected to determine the surface area of composite 3-D objects to solve problems. **G03** Students will be expected to draw and interpret scale diagrams of 2-D shapes.	**G02** Students will be expected to demonstrate an understanding of the Pythagorean theorem by identifying situations that involve right triangles, verifying the formula, applying the formula, and solving problems.	**G01** Students will be expected to solve problems that involve two and three right triangles.

Background

Students were introduced to the Pythagorean theorem in Mathematics 8. They should know that the theorem only applies to right triangles and should be able to solve problems that involve Pythagorean triples.

Students will further explore the development, conditions, and practical applications of the Pythagorean theorem.

Pythagoras of Samos, c. 560–480 BC, was a Greek philosopher who is credited with providing the first proof of the Pythagorean relationship. It states that the area of the square on the hypotenuse of a right triangle is equal to the sum of the areas of the squares on the other two sides of the triangle. The conventional formula for the Pythagorean relationship, $c^2 = a^2 + b^2$, should be developed through investigations. It is also important for students to recognize that the Pythagorean relationship can be labelled differently from the conventional *a-b-c* notation. Using this notation, the hypotenuse, or the longest side, is c and two shorter sides, or legs, are a and b.

A Pythagorean triple is any set of three whole numbers a, b, and c, for which $a^2 + b^2 = c^2$. It is believed that the Egyptians and other ancient cultures used a 3-4-5 rule ($a = 3$, $b = 4$, $c = 5$) in construction to ensure buildings were square. The 3-4-5 rule allowed them a quick method of establishing a right angle. This method is still used today in construction as well as many other trades.

In presenting diagrams of right triangles, it is important to give diagrams of the triangles in various orientations. Students should recognize the hypotenuse as being the side opposite the right angle, regardless of the orientation of the figure. Whenever a triangle has a right angle and two known side lengths, the Pythagorean relationship should be recognized by students. In addition to being provided with situations that involve finding the length of the hypotenuse, students should also be given situations where the hypotenuse and one side is known and the other side is to be found. Also, it is important for students to realize that they can use the Pythagorean relationship when only one side is known if the right triangle is isosceles. Finally, students should be able to use the Pythagorean relationship to determine if three given side lengths are, or are not, the sides of a right triangle. There are many opportunities to use the Pythagorean relationship to solve problems, such as determining the height of a building or finding the shortest distance across a rectangular field.

Students need to be provided with opportunities to model and explain the Pythagorean theorem concretely, pictorially, and symbolically:

- *Concretely* by cutting up areas represented by a^2 and b^2, and fitting the two areas onto c^2.
- *Pictorially* using grid paper or technology.
- *Symbolically* by confirming that a right triangle is formed by showing that $a^2 + b^2 = c^2$.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

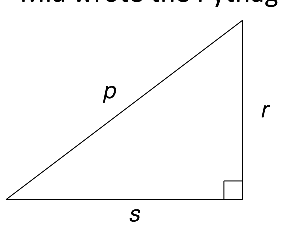
Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

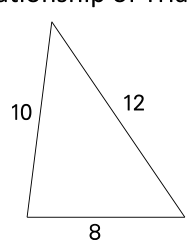
ASSESSING PRIOR KNOWLEDGE

Student tasks such as the following could be completed to assist in determining students' prior knowledge.

- Explain the difference between the square of a number and the square root of a number.
- Draw a 6 cm, 8 cm, 10 cm right triangle on grid paper. Explain the Pythagorean relationship using models or drawings on the grid paper and then show the relationship symbolically.
- Solve:
  - For safety reasons a construction company established the following rule. When placing a ladder against the side of a building, the distance of the base of the ladder from the wall should be at least one third of the length of the ladder. Can an 8 m ladder reach a 7 m window when this rule is followed?
  - An airplane is flying at an elevation of 5000 m. The airport is 3 kilometres away from a point directly below the airplane on the ground. How far is the airplane from the airport?
  - The dimensions of a rectangular frame are 10 cm × 24 cm. A carpenter wants to put a diagonal brace between two opposite corners of the frame. How long should the brace be?
  - What is the largest television you can put in this space?
- Explain how you can determine whether or not a triangle is a right triangle if you know that it has side lengths of 7 cm, 11 cm, and 15 cm.
- Determine whether each of the following student's work is correct and explain your thinking.
  - Farren wrote the Pythagorean relationship of Triangle A as $r^2 = p^2 + s^2$.
  - Mia wrote the Pythagorean relationship of Triangle B as $10^2 + 8^2 = 12^2$.



Triangle A



Triangle B

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

- Design a question that involves the application of the Pythagorean theorem.
- Craig is laying out forms for the cement footings of a house. The house is to be 36 feet by 48 feet. Ask students to determine how long the diagonal of the rectangle should be to ensure there will be right angles at the corners of the house.
- Verify that a quilting square is actually square.
- Draw triangles other than right triangles. Measure the side lengths and check to see if the Pythagorean theorem works for these non-right triangles.
- Determine whether each triangle with sides of given lengths is a right triangle.
  (a) 9 cm, 12 cm, 15 cm
  (b) 16 mm, 29 mm, 18 mm
  (c) 9 m, 7 m, 13 m
- Anja is building a garage on a floor that measures 18 feet by 24 feet.
  (a) Calculate the length of the diagonal of the rectangular floor.
  (b) Anja measures the length of the diagonal to be 29.5 feet. Are the angles at the corners of the garage right angles? Explain.
- Carpenters often use a 3-4-5 triangle to determine if corners are square (90°). Explain why this works.
- Alain has a rectangular garden in his backyard. He measures one side of the garden as 7 m and the diagonal as 11 m. What is the length of the other side of his garden? (Hint: draw a diagram.)
- The dimensions of a rectangular frame are 30 cm × 50 cm. A carpenter wants to put a diagonal brace between two opposite corners of the frame. How long should the brace be?
- Khanna is giving her boyfriend a hockey stick as a gift. She wants to wrap it in a box so that he can't guess what it is. The hockey stick is 63 inches long. She goes to the furniture store. They have several boxes:
  - A TV box that is 52" × 10".
  - A coffee table box that is 48" × 48".
  - Which one should she pick? Why?

FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should inform instruction.

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?

- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

Suggested Learning Tasks

Effective instruction should consist of various strategies.

Guiding Question
- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

Consider the following sample instructional strategies when planning lessons.

- Explore the multiple ways of proving the Pythagorean theorem concretely, pictorially and symbolically. Consider using online models as well as historical examples (e.g., Egyptian).

- Use counter-examples so that students can discover the Pythagorean theorem works only with 90° angles.

- Ask students if a triangle with dimensions 10 cm × 12 cm × 15 cm is a right triangle. Have them justify their answer.

- Invite students to prove that 15–20–25 are the sides of a right angle triangle. Ask how they know which side is the hypotenuse.

- Provide students with a piece of string and a marker. They should make 11 equally-spaced marks that separate the string into 12 equal lengths. Teachers could explain that the ancient Egyptians would have used a similar cord to ensure they had right angles when they laid out the boundaries of their fields.
  (a) Ask students how the ancient Egyptians would have used this cord to ensure they had a right angle.
  (b) Students can determine what the lengths of the sides of the triangles would have been.

- Have students sketch and then solve the following problem: A 13 m-long wheelchair ramp, leading to an entrance, is being constructed. If the top of the ramp is 4.5 m off the ground, determine the length of the ramp sitting on the driveway.

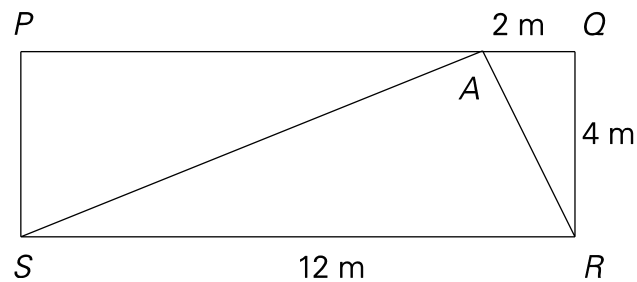
- Tell students, Maria was given a triangle and asked whether it was a right triangle. She felt it was, so she set out to prove it by measuring the sides and using Pythagorean theorem. Her work is shown below. Ask, did she prove this was a right triangle? Why or why not?

  $c^2 = a^2 + b^2$
  $10^2 = 6^2 + 9^2$
  $100 = 36 + 81$
  $100 = 117$

- Tell students, a ship leaves port and sails 20 km north and then 13 km east. Ask, How far is the ship from port?

- Tell students, the rectangle *PQRS* represents the floor of a room.

  Artina stands at point A. Ask students to calculate her distance from
  (a) the corner *R* of the room
  (b) the corner *S* of the room



- Tell students, in a flyer, a TV is listed as being 55 inches. This distance is the diagonal distance across the screen. If the screen measures 28 inches in height, will the TV fit on my TV stand, which is 48 inches wide?

- Ask students which of the following objects require right angles. This can be done using Thumbs Up/Down, Yes/No cards or interactive response systems.
  (a) door
  (b) TVs
  (c) picture frames
  (d) sheds
  (e) tire alignment
  (f) swimming pools
  (g) wall junctions
  (h) columns

Suggested Models and Manipulatives

- measuring tapes
- metre sticks
- rope/string
- wooden geo-blocks

Mathematical Vocabulary

Students need to be comfortable using the following vocabulary.

- hypotenuse
- leg of a triangle
- Pythagorean theorem
- right triangle

Resources/Notes

Internet

Current sites include a Youtube video that uses origami to investigate the Pythagorean theorem, and a teacher's site that has examples from a variety of ancient texts, including a medieval European military handbook, an Egyptian mathematical papyrus (300 BC), a Mesopotamian clay table (1900–1600), an Indian mathematician and astronomer (1150 BC), and a Chinese text (200 BC).

Mathematics 115, Homework Assignment #4 (Beery 2002)
http://bulldog2.redlands.edu/fac/beery/math115/day4.htm

Note: Teachers must always check the current validity of an online site prior to directing students to it.

Print

Math at Work 10 (Etienne et al. 2011)

- Chapter 6: Pythagorean Relationship
  - Sections 6.1, 6.2, and 6.3
  - Skill Check
  - Test Yourself
  - Chapter Project
  - Games and Puzzles
- Chapter 7: Trigonometry
  - Get Ready
  - Sections 7.1 and 7.3
  - 7.4 Puzzler
  - Test Yourself
  - Chapter Project

Notes

G03 Students will be expected to demonstrate an understanding of similarity of convex polygons, including regular and irregular polygons. [C, CN, PS, V]			
[**C**] Communication	[**PS**] Problem Solving	[**CN**] Connections	[**ME**] Mental Mathematics and Estimation
[**T**] Technology	[**V**] Visualization	[**R**] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have met the corresponding specific curriculum outcome.

G03.01 Determine, using angle measurements, if two or more regular or irregular polygons are similar.
G03.02 Determine, using ratios of side lengths, if two or more regular or irregular polygons are similar.
G03.03 Explain why two given polygons are not similar.
G03.04 Explain the relationships between the corresponding sides of two polygons that have corresponding angles of equal measure.
G03.05 Draw a polygon that is similar to a given polygon.
G03.06 Explain why two or more right triangles with a shared acute angle are similar.
G03.07 Solve a contextual problem that involves similarity of polygons.

Scope and Sequence

Mathematics 9	Mathematics at Work 10	Mathematics at Work 11
G02 Students will be expected to demonstrate an understanding of similarity of polygons.	**G03** Students will be expected to demonstrate an understanding of similarity of convex polygons, including regular and irregular polygons.	**G02** Students will be expected to solve problems that involve scale.

Background

In Mathematics 9 students were first introduced to what conditions makes two polygons similar. They matched up similar polygons and explained why they were similar, drew a polygon similar to a given polygon, and used the properties of polygons to solve a problem.

In this outcome, the focus is on the similarity of convex polygons. A line drawn through a convex polygon will intersect the polygon exactly twice, as shown in Figure 3A. A convex polygon can also be identified as a polygon where all diagonals are completely within the interior of the polygon as shown in Figure 3B.

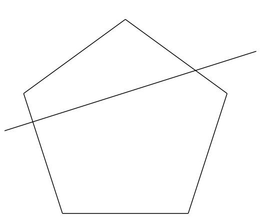


Figure 3A

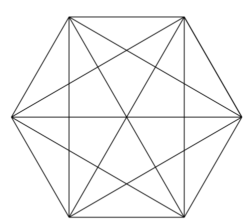


Figure 3B

A concave polygon is a polygon where at least one diagonal will be entirely or partially outside the polygon as shown in Figure 3C.

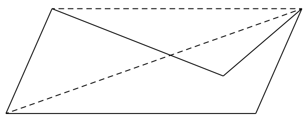


Figure 3C

Two polygons are similar when

- the corresponding angles are congruent

and

- the ratios of corresponding sides are equal

This makes the corresponding sides proportional.

The symbol for “is similar to” is ~.

If two polygons have only one of these two facts, then they are not *necessarily* similar, as can be seen in the following diagrams.

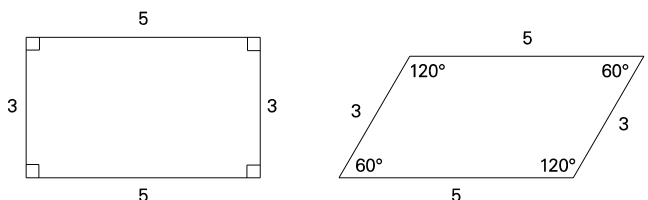


Figure 3D

As shown in Figure 3D, even though the ratios of the corresponding sides are equal, the polygons are not similar.

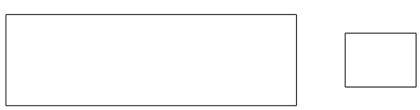


Figure 3E

Figure 3E shows that even though the corresponding angles are congruent, the polygons are not similar.

If polygons are similar, corresponding side lengths are all enlarged or reduced by the same factor. Triangles, however, are a special case of similar polygons. In triangles (and this applies *only* to triangles), it is not necessary to have *both* conditions.

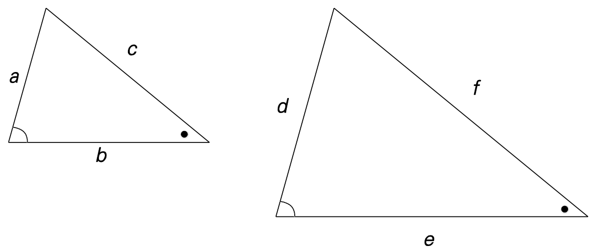
Two triangles are similar if

- the corresponding angles are congruent

or

- the corresponding sides are proportional

In Mathematics 9, students would have expressed this as "the ratios of their corresponding sides are congruent" or the sides are proportional.



Therefore, in the triangles above, $\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$.

An alternative test for similarity in triangles involves comparing side lengths within each shape. That is, if the ratio between two side lengths on one shape is the same as the ratio between the two corresponding side lengths on the other shape, the figures are similar.

Therefore, $\frac{a}{b} = \frac{d}{e}$, $\frac{c}{b} = \frac{f}{e}$, and $\frac{a}{c} = \frac{d}{f}$.

Indirect measurement is one example of the applications of similar triangles. The concept of similarity is very useful in measuring the heights of inaccessible objects such as buildings, trees and mountains. The shadowing technique works very well outdoors on sunny days. This hands-on project gives students an opportunity to go outside of the classroom and take measurements. Students use an object perpendicular to the ground, a metre stick and their shadows to determine the height of the object. Using the fact that the sun's rays are parallel, students can set up a proportion with similar triangles.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Student tasks such as the following could be completed to assist in determining students' prior knowledge.

- One triangle has two 50° angles. Another triangle has a 50° angle and an 80° angle. Could the triangles be similar? Explain your thinking.

- A software program offers these preset paper sizes for printing:
  - A4 (210 mm × 297 mm)
  - A5 (148 mm × 210 mm)
  - B5 (182 mm × 257 mm)

  Use scale factors to determine if the paper sizes are similar.

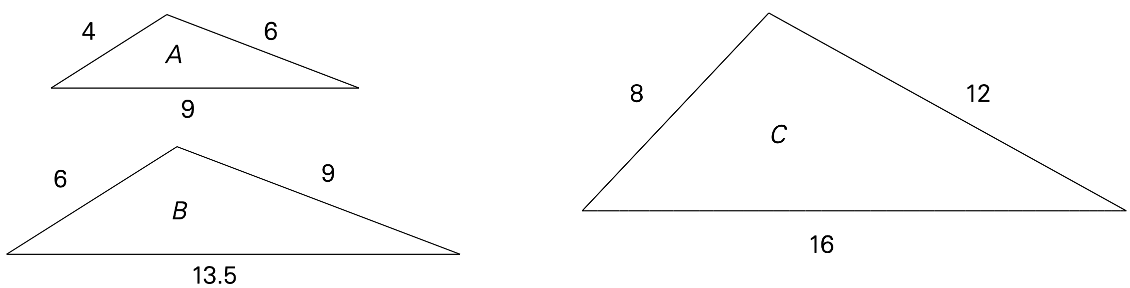
- Design a logo that includes geometric shapes.
  (a) Decide on the dimensions of an enlargement of the logo that would fit on a banner or billboard.
  (b) Determine the scale factor.
  (c) Create a business card using the logo by repeating the process for a reduction.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

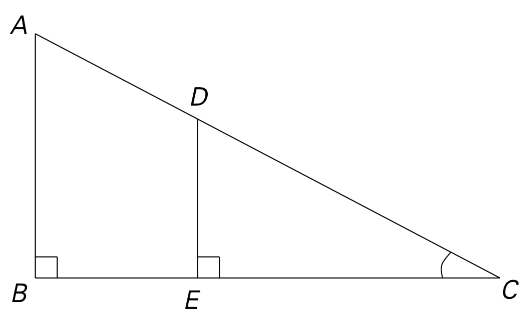
Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

- Answer the following:

  (a) Determine if either triangle *B* or *C* is similar to triangle *A*.



  (b) Answer the questions that follow based on the diagram given.

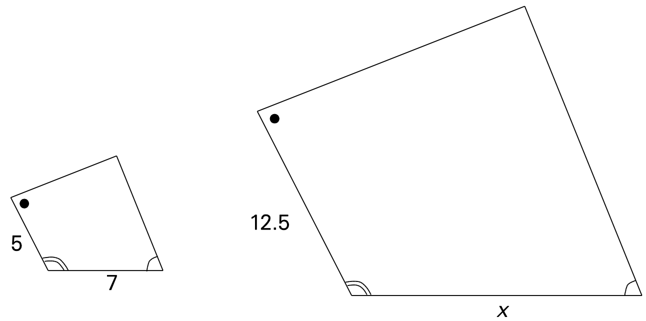


  (i) Which triangles are similar?
  (ii) Measure the sides and determine the ratios of
  > $\frac{AB}{DE}, \frac{AC}{DC}$
  > $\frac{AB}{BC}, \frac{DE}{EC}$
  > $\frac{BC}{EC}, \frac{AC}{DC}$

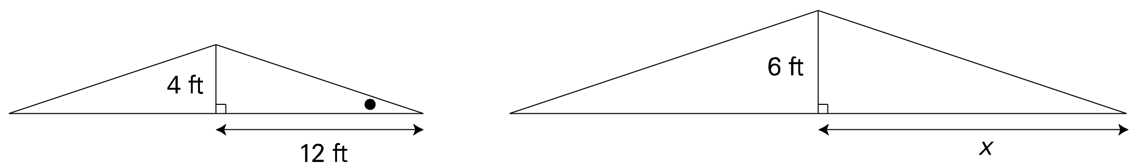
  What do you notice about the values?
  (iii) If *AB* = 9 cm, *DE* = 6 cm, and *EC* = 8, what is the length of *BC*?

- Two triangles are similar. The side lengths of the smaller triangle are 3 cm, 4 cm, and 5 cm respectively. Describe how you can determine possible side lengths of the larger triangle.

- Answer the following:
  (a) The following quadrilaterals are similar. Determine the value of *x*.



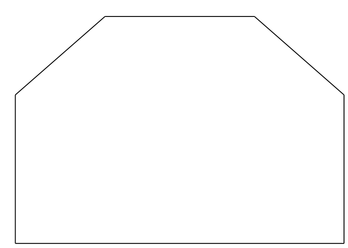
  (b) Omar wants to build a roof truss that is "4 on 12" (see diagram below). If the roof truss height changes to 6 feet, how wide will the new roof truss be?



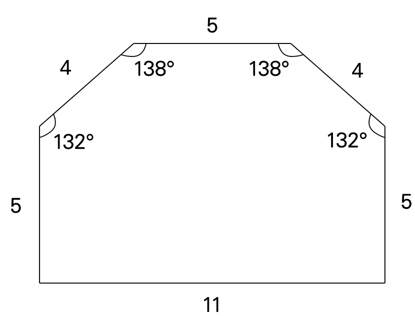
- You have been given a triangle. Compare your triangle with one of your classmate's. Determine if and justify why the triangles are or are not similar.

- Create a poster to advertise a field trip to see the Cape Forchu Lighthouse in Yarmouth. The owners want you to enlarge a photo of the lighthouse that is 3.5 inches wide and 5 inches long. Determine how long the poster will be if the enlargement is to be 16 inches wide.

- The diagram to the right represents the top view of a patio. Choose one of the following parallel tasks.

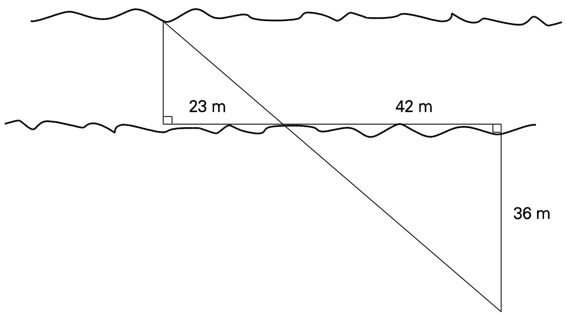


  (a) Measure the angles and sides and reproduce the drawing using a scale factor of 2.
  (b) Given the same diagram with the angle measures, reproduce the drawing using a scale factor of 2.



- Create sets of similar triangles. Put all the class triangles together and ask students to sort them by similarity.

- The two triangles in the following diagram are similar. Determine the width of the river. Round off to answer to one decimal place.



FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should inform instruction.

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SUGGESTED LEARNING TASKS

Effective instruction should consist of various strategies.

Guiding Question
- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

Consider the following sample instructional strategies when planning lessons.

- Some students may benefit from tracing two similar triangles to compare angles and sides or to use tick marks or dashes to identify the corresponding angles and sides. This may help in setting up the ratios.

- Some students may find it helpful to redraw a diagram containing two triangles as two separate triangles. Students may also find it helpful to redraw the triangles so that the orientations are the same.

- Support students' understanding of comparison of corresponding angles with concrete materials.

- Promote reasoning strategies by having students prove that polygons have equal corresponding angles and proportional sides. Polygons of differing levels of difficulty can be presented to extend student's thinking.

- Construct polygons on grid paper, and then copy the same polygon onto larger or smaller grid paper to create a similar figure.

- Facilitate constructions with technology by using the tools for enlarging and reducing figures (e.g., on a computer or an overhead projector).

- Emphasize the importance of modelling the process when students present their reasoning.

SUGGESTED MODELS AND MANIPULATIVES

- geo-strips
- power polygons
- rulers

MATHEMATICAL VOCABULARY

Students need to be comfortable using the following vocabulary.

- adjacent side
- concave polygon
- convex polygon
- corresponding sides
- hypotenuse
- irregular polygon
- opposite side
- regular polygon
- similar figure

Resources/Notes

Internet

- Polygon Playground (Petti 2013)
  www.mathcats.com/explore/polygonplayground.html
  Explore polygons of several shapes and sizes.

Print

Math at Work 10 (Etienne et al. 2011)

- Chapter 7: Trigonometry
  - Chapter Opener
  - Section 7.1
  - Skill Check
  - Test Yourself

Notes

G04 Students will be expected to demonstrate an understanding of primary trigonometric ratios (sine, cosine, tangent) by applying similarity to right triangles, generalizing patterns from similar right triangles, applying the primary trigonometric ratios, solving problems. [CN, PS, R, T, V]
[**C**] Communication [**PS**] Problem Solving [**CN**] Connections [**ME**] Mental Mathematics and Estimation [**T**] Technology [**V**] Visualization [**R**] Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have met the corresponding specific curriculum outcome.

G04.01 Show, for a specified acute angle in a set of similar right triangles, that the ratios of the length of the side opposite to the length of the side adjacent are equal, and generalize a formula for the tangent ratio.

G04.02 Show, for a specified acute angle in a set of similar right triangles, that the ratios of the length of the side opposite to the length of the hypotenuse are equal, and generalize a formula for the sine ratio.

G04.03 Show, for a specified acute angle in a set of similar right triangles, that the ratios of the length of the side adjacent to the length of the hypotenuse are equal, and generalize a formula for the cosine ratio.

G04.04 Identify situations where the trigonometric ratios are used for indirect measurement of angles and lengths.

G04.05 Solve a contextual problem that involves right triangles, using the primary trigonometric ratios.

G04.06 Determine if a solution to a problem that involves primary trigonometric ratios is reasonable.

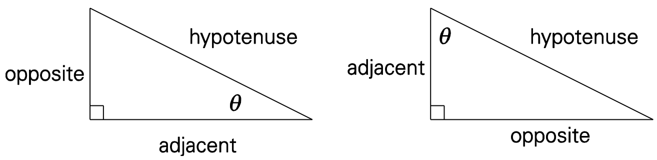
Scope and Sequence

Mathematics 9	Mathematics at Work 10	Mathematics at Work 11
G03 Students will be expected to draw and interpret scale diagrams of 2-D shapes.	**G04** Students will be expected to demonstrate an understanding of primary trigonometric ratios (sine, cosine, tangent) by applying similarity to right triangles, generalizing patterns from similar right triangles, applying the primary trigonometric ratios, solving problems.	**G01** Students will be expected to solve problems that involve two and three right triangles.

Background

This is the first time that students have seen trigonometric ratios in the mathematics curriculum. Their exploration will be limited to the primary trigonometric ratios with respect to right angle triangles.

The term **trigonometry** comes from two Greek words, trigon and metron, meaning “triangle measurement.” A trigonometric ratio is a ratio of the lengths of two sides of a right triangle. One of the difficulties students sometimes have when working with trigonometric ratios is correctly identifying the opposite and adjacent sides in relation to the reference angle. They should be exposed to right triangles with the reference angle in various locations, so that they recognize that the opposite and adjacent sides are relevant to the reference angle. Angles are often labelled using Greek letters, such as theta (θ).



Students should realize that there are three possible pairs of sides with respect to the reference angle, θ: opposite and hypotenuse, opposite and adjacent, and adjacent and hypotenuse. A trigonometric ratio is a ratio of the measures of two sides of a right triangle. The three primary trigonometric ratios are tangent, sine, and cosine. The short form for the tangent ratio of angle *A* is tan *A*. It is defined as

$$\tan A = \frac{\text{length of side opposite } \angle A}{\text{length of side adjacent to } \angle A}.$$

The short form for the sine ratio of angle *A* is sin *A*. It is defined as $\sin A = \dfrac{\text{length of side opposite } \angle A}{\text{length of hypotenuse}}$

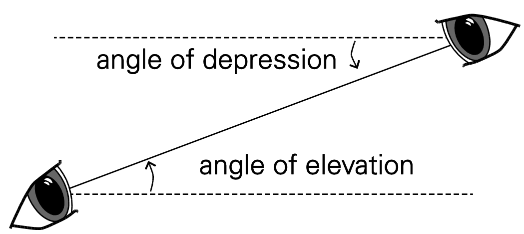
The short form for the cosine ratio of angle *A* is cos *A*. It is defined as

$$\cos A = \frac{\text{length of side adjacent to } \angle A}{\text{length of hypotenuse}}.$$

The line of sight is an invisible line from one person or object to another person or object. Some applications of trigonometry involve an angle of elevation or an angle of depression.

An angle of elevation is the angle formed by the horizontal line and the line of sight when a person is looking at an object above them.

An angle of depression is the angle formed by the horizontal line and the line of sight when a person is looking at an object below them.



Students will also need to be proficient at rearranging a formula for a given variable. This is one of the applications from SCO A01.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Assessing Prior Knowledge

Students tasks such as the following could be completed to assist in assessing students' prior knowledge.

- Solve such questions as $23 = \frac{x}{7}$ and $89 = \frac{534}{x}$.
- Explain why two or more right triangles with a shared acute angle are similar.
- Trace two similar triangles, comparing their angles and sides. Identify all corresponding angles and sides.

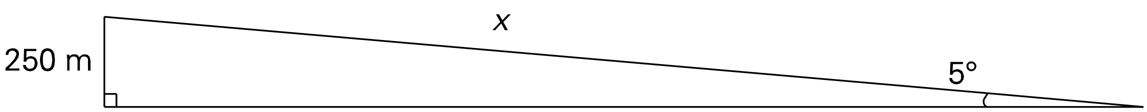
Whole-Class/Group/Individual Assessment Tasks

Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

- Show, for a specified acute angle in a set of similar right triangles, that the ratios of the length of the side opposite to the length of the side adjacent are equal, and generalize a formula for the tangent ratio.
- Show, for a specified acute angle in a set of similar right triangles, that the ratios of the length of the side opposite to the length of the hypotenuse are equal, and generalize a formula for the sine ratio.
- Show, for a specified acute angle in a set of similar right triangles, that the ratios of the length of the side adjacent to the length of the hypotenuse are equal, and generalize a formula for the cosine ratio.
- Identify situations where the trigonometric ratios are used to determine measurement of angles and lengths indirectly (e.g., determining the height of a flagpole or a tree from the ground).
- Solve a contextual problem using the primary trigonometric ratios that involves right triangles.
- Determine whether a solution to a problem that involves primary trigonometric ratios is reasonable.
- Given a right triangle with the right angle and reference angle labelled, label the sides as opposite, adjacent and hypotenuse.
- Draw a right triangle, mark the right angle, and place a bingo chip on one of the acute angles. Challenge your partner to place sticky notes that read “opposite,” “adjacent,” and “hypotenuse” in the proper place.
- Create your own mnemonic for the primary trigonometric ratios. Share your mnemonics, and take a class vote for the preferred one.

- A guy wire 6 m-long is holding up a telephone pole. The guy wire makes an angle of 70° with the ground. At what height on the telephone pole is the guy wire attached?

- An airplane is approaching the Halifax Stanfield International Airport as represented by the diagram below. Find the line of sight distance from the airplane to the terminal.

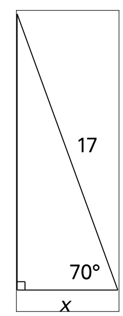


- A ramp 4 m long is being built to reach a loading dock that is 1.5 m in height. What is the measure of the angle between the ramp and the ground?

- Evaluate each of the following trigonometric ratios to four decimal places.

  (a) sin 57°
  (b) cos 23°

- Using the diagram to the right, find the length of side *x*. Round off the answer to one decimal place.



- A pilot starts his takeoff and climbs steadily at an angle of 12.2°. Determine the distance, along the flight path, that the plane has travelled when it is at an altitude of 5.4 km. Express the answer to one decimal place.

FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should inform instruction.

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SUGGESTED LEARNING TASKS

Effective instruction should consist of various strategies.

Guiding Question

- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

Consider the following sample instructional strategies when planning lessons.

- Remind students that a diagram showing all the given information should be the first step in the solution of any trigonometric problem. Their sketches need not be accurate, but reasonable representations of the given situation can help them decide on a strategy.

- Some students have difficulty identifying the opposite and adjacent sides for the angle under consideration. Remind them that the hypotenuse is the longest side in a right triangle. Then, the adjacent side will be the leg of the right triangle that is next to the angle under consideration and the right angle, and the opposite side will be the leg opposite the angle under consideration.

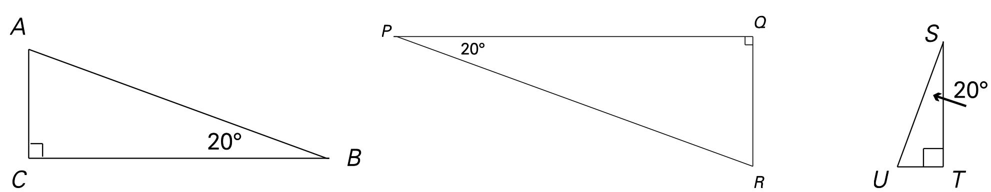
- Allow students to discover the trigonometry ratios through angle and side length measurements, and calculation of ratio values for triangles with common angle measurements.

- Use the angle of elevation and the angle of depression to help create contextual problems for this outcome.

- Using clinometers, have students go outside to find the height of tall buildings or trees using trigonometric ratios.

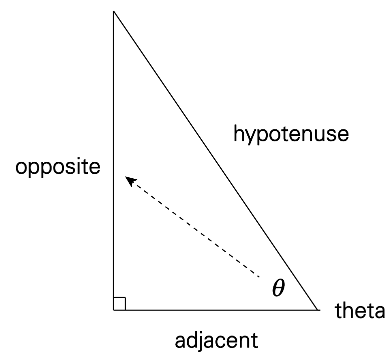
- Work with students to create diagrams that are labelled correctly from word problems (a skill that many students struggle with).

- Allow students to discover the trigonometric ratios through completing the following (or a similar chart) for three similar triangles with a common angle measurement.

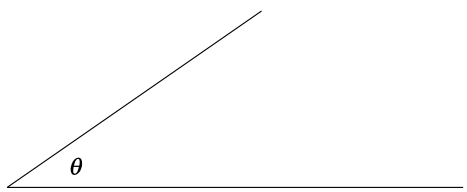


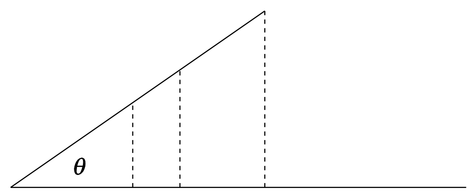
Triangle	For 20° Angle	Length of Opposite	Length of Adjacent	Length of Hypotenuse	$\frac{O}{H}$	$\frac{A}{H}$	$\frac{O}{A}$
ABC	*B*						
PQR	*P*						
STU	*S*						
				Mean Values			

- Create 5 large right-angled triangles on the floor with masking tape. Then divide your class into 5 groups. As the students stand around the triangles, each member of the group receives one of the following words or symbols: “opposite,” “adjacent,” “hypotenuse,” “theta,” “θ” (symbol for theta), and a 5 cm x 5 cm square.
  - Have students identify the type of triangle on the floor by indicating the right angle with the square and then label the hypotenuse.
  - Have the person holding the symbol for theta place it in one of the other angles of the triangle. The word “theta” is then placed above it as reinforcement for the new terminology.
  - The “opposite” person then goes to theta and walks across the triangle to get to the “opposite” side and places it accordingly.
  - Adjacent is then placed on the side next to theta.
  - Finally, students are instructed to move theta to the other angle in the triangle. At that point opposite and adjacent must be relocated but hypotenuse remains in place.



- Give students an angle of 30°. Label the angle as theta (θ) and have them construct a series of right-angled triangles at various distances from the angle.





  Students should then measure the sides of each right-angled triangle and fill out the chart below for each triangle.

Length of Opposite	**Length of Adjacent**	**Length of Hypotenuse**	**Sine**	**Cosine**	**Tangent**

- If the data from all students can be entered into an Excel file, the sine, cosine, and tangent values can be automatically calculated, and students will see that their sine, cosine, and tangent values are all very similar (discuss why they may differ), if not identical, to their classmates. This will lead to the discussion of the Trigonometric Table and how it can be used to determine that the value of theta is 30°.
  - Verify if the following two formulas are equivalent.

$$\sin \theta = \frac{\text{length of opposite}}{\text{length of hypotenuse}} \text{ and length of hypotenuse} = \frac{\text{length of opposite}}{\sin \theta}$$

- Create a contextual problem that could be solved using the following formula:

$$\cos \theta = \frac{\text{length of adjacent}}{\text{length of hypotenuse}}.$$

SUGGESTED MODELS AND MANUPULATIVES

- clinometers
- geo-strips
- power polygons
- ramps for wheelchairs (if available)

MATHEMATICAL VOCABULARY

Students need to be comfortable using the following vocabulary.

- opposite side, adjacent side, hypotenuse
- primary trigonometric ratios
  - cosine ratio
  - sine ratio
  - tangent ratio

Resources/Notes

Internet

Trigonometry Applications with Right Triangles and Ratios (SliderMath 2013)
http://slidermath.com/rpoly/Trigapps.shtml
Students can play the game found at this site to practise word problems that require the use of sine, cosine, and tangent.

Print

Math at Work 10 (Etienne et al. 2011)
- Chapter 7: Trigonometry
  - Sections 7.2, 7.3, and 7.4
  - 7.4 Puzzler
  - Skill Check
  - Test Yourself
  - Chapter Project
  - Games and Puzzles

Notes

G05 Students will be expected to solve problems that involve parallel, perpendicular, and transversal lines, and pairs of angles formed between them. [C, CN, PS, V]			
[**C**] Communication [**T**] Technology	[**PS**] Problem Solving [**V**] Visualization	[**CN**] Connections [**R**] Reasoning	[**ME**] Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have met the corresponding specific curriculum outcome.

G05.01 Sort a set of lines as perpendicular, parallel or neither, and justify this sorting.
G05.02 Illustrate and describe complementary and supplementary angles.
G05.03 Identify, in a set of angles, adjacent angles that are not complementary or supplementary.
G05.04 Identify and name pairs of angles formed by parallel lines and a transversal, including corresponding angles, vertically opposite angles, alternate interior angles, alternate exterior angles, interior angles on same side of transversal, and exterior angles on same side of transversal.
G05.05 Explain and illustrate the relationships of angles formed by parallel lines and a transversal.
G05.06 Explain, using examples, why the angle relationships do not apply when the lines are not parallel.
G05.07 Determine if lines or planes are perpendicular or parallel, and describe the strategy used.
G05.08 Determine the measures of angles involving parallel lines and a transversal, using angle relationships.
G05.09 Solve a contextual problem that involves angles formed by parallel lines and a transversal (including perpendicular transversals).

Scope and Sequence

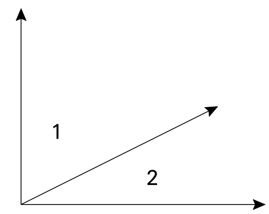
Mathematics 9	**Mathematics at Work 10**	**Mathematics at Work 11**
M01 Students will be expected to solve problems and justify the solution strategy using circle properties including the following: ▪ The perpendicular from the centre of a circle to a chord bisects the chord. ▪ The measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc. ▪ The inscribed angles subtended by the same arc are congruent. ▪ A tangent to a circle is perpendicular to the radius at the point of tangency.	**G05** Students will be expected to solve problems that involve parallel, perpendicular, and transversal lines, and pairs of angles formed between them.	**G01** Students will be expected to solve problems that involve two and three right triangles.

Background

Students have explored the concepts of parallel and perpendicular lines in previous grades. Students will now extend this learning to explore and categorize the relationship between two lines as parallel, perpendicular, or neither, and to determine the relationships between the angles formed in various contexts.

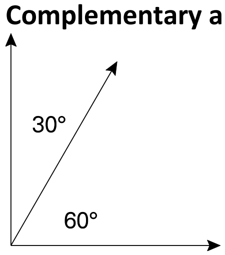
Students will be introduced to the concepts of complementary angles (totaling 90°), and supplementary angles (totaling 180°).

Adjacent angles are any two angles that share a common vertex and a common ray separating the two angles.



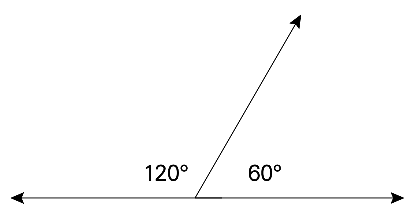
Complementary angles and **supplementary angles** are new terms for students. Two angels are complementary if the sum of their angles equals 90°, whereas two angles are supplementary if the sum of their angles equals 180°.

Complementary angle



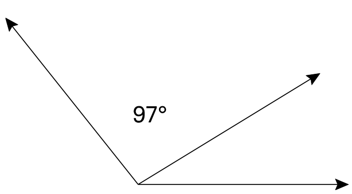
Adjacent and complementary

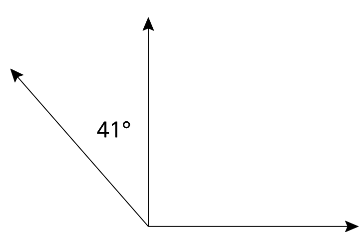
Supplementary angles



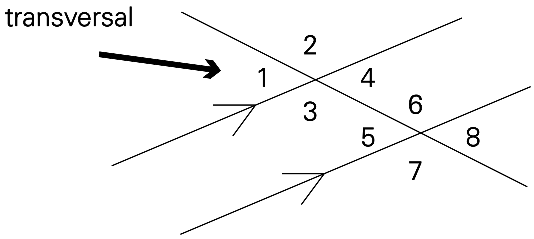
Adjacent and supplementary

Some adjacent angles are neither complementary nor supplementary.





Transversals of parallel lines and the resulting angle relationships will be investigated. Pairs of angles in this context can be congruent or supplementary.



With reference to the figure above, the following angles are congruent to each other:

∠1, ∠4, ∠5, and ∠8
∠2, ∠3, ∠6, and ∠7

This includes congruence between pairs of
- *vertically opposite angles*: ∠1 and ∠4, ∠2 and ∠3, ∠5 and ∠8, ∠6 and ∠7
- *corresponding angles*: ∠1 and ∠5, ∠3 and ∠7, ∠2 and ∠6, ∠4 and ∠8
- *alternate interior angles*: ∠3 and ∠6, ∠4 and ∠5
- *alternate exterior angles*: ∠1 and ∠8, ∠2 and ∠7

With reference to the figure above, the following pairs of angles are supplementary.
- *interior angles on the same side of the transversal*: ∠3 and ∠5, ∠4 and ∠6
- *exterior angles on the same side of the transversal*: ∠2 and ∠8, ∠1 and ∠7

These angle relationships hold when lines are parallel (below, Figure 1), however, when lines are not parallel (below, Figure 2), the angle relationships do not apply, except for the congruency of vertically opposite angles.

This can be explained by recognizing that if the lines are not parallel, they will eventually meet, forming a triangle with the transversal. In the example shown below, angles 3 and 5 are no longer supplementary as they are now two angles in a triangle and ∠3 + ∠5 + ∠9 = 180°. Although opposite angles remain congruent for any two lines crossing, the other angle relationships no longer apply.

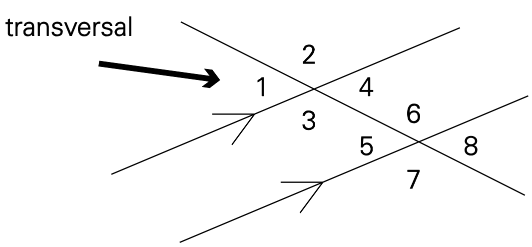


Figure 1

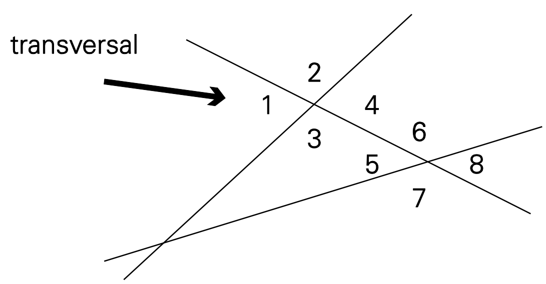


Figure 2

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Assessing Prior Knowledge

Students tasks such as the following could be completed to assist in assessing students' prior knowledge.

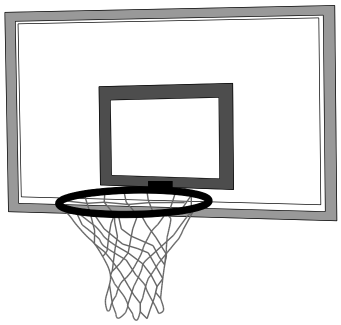
- Using a map (e.g., a city in Nova Scotia), find examples of parallel lines and perpendicular lines.
- Using geo-strips, construct various examples of parallel and perpendicular lines.
- Record examples of what appears to be parallel and perpendicular lines around the classroom.

Whole-Class/Group/Individual Assessment Tasks

Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

- Sort a set of lines as perpendicular, parallel, or neither, and justify this sorting.
- Illustrate and describe complementary and supplementary angles.
- Identify, in a set of angles, adjacent angles that are not complementary or supplementary.
- Identify and name pairs of angles formed by parallel lines and a transversal, including corresponding angles, vertically opposite angles, alternate interior angles, alternate exterior angles, interior angles on the same side as the transversal, and exterior angles on the same side as the transversal.
- Explain, using examples, why the angle relationships do not apply when the lines are not parallel.
- Determine the measures of angles involving parallel lines and a transversal, using angle relationships.
- Solve a contextual problem that involves angles formed by parallel lines and a transversal (including perpendicular transversals).

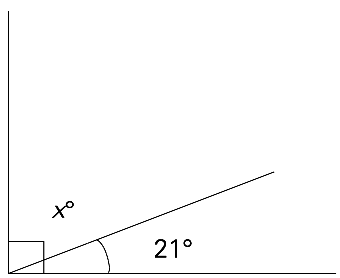
- Make a drawing or bring in pictures from magazines or newspapers of examples of parallel and perpendicular lines in the world. One example of parallel and perpendicular lines can be seen on a basketball backboard.



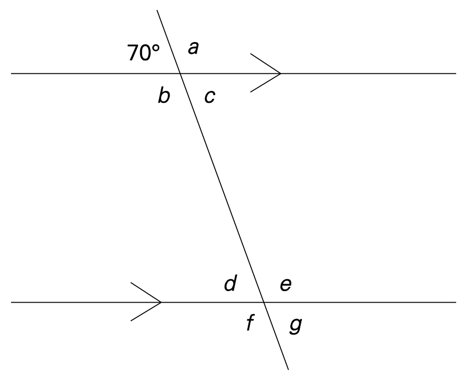
- Design and build a popsicle stick bridge. Sort the lines by colouring parallel sticks blue, colour sticks that are perpendicular to each other red, and colour all others with random colours.

- Determine the missing angles in the diagrams below.

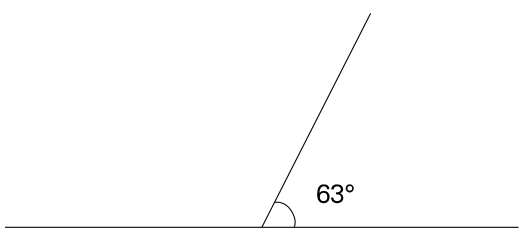
(a)



(b)

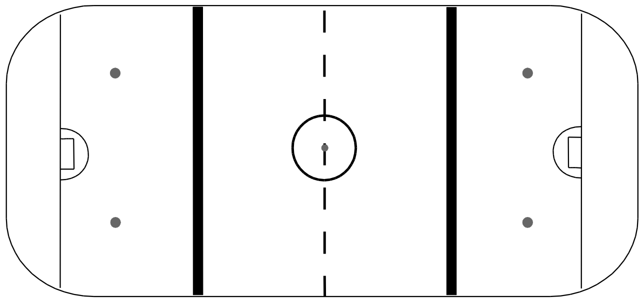


(c)



- Sue is using a compound mitre saw to cut boards for a tree house. The largest angle indicated on the saw is 45°. Explain how she could cut a 60° angle.

If a community group were building a hockey rink, explain how they could ensure that the blue lines (shown below) would be parallel.



FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should inform instruction.

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SUGGESTED LEARNING TASKS

Effective instruction should consist of various strategies.

Guiding Question
- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

Consider the following sample instructional strategies when planning lessons.

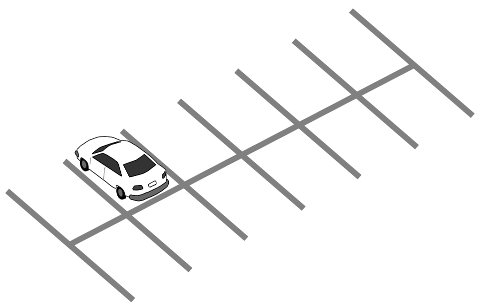
- Review with students the various pairs of angles such as vertically opposite, corresponding, and alternate interior and alternate exterior angles.

- Have students create a legend of rules for angles formed when a transversal intersects parallel lines.

- Have students create their own question with parallel lines and a transversal and then have them solve it.

- To identify parallel lines, students could draw a transversal, and use the angle properties to confirm the lines are parallel.

- Students could investigate these angles using a parking lot. When workers paint lines for a parking lot, they aim to paint lines that are parallel to each other. The lines in a parking lot, therefore, provide an ideal illustration of the relationships between angles created by parallel lines and a transversal. Using chalk, students can discuss and mark the different types of angles in the school's parking lot. They can then measure the angles to determine which angles are equal and which are supplementary.



- Use tape to create two parallel lines with a transversal on the floor. Students should work in pairs. Student *A* stands on an angle. Student *B* picks a card to direct Student *A* to move to an adjacent angle, vertically opposite angle, etc. Students switch roles after 10 moves. Alternatively, this activity could be done with the lines drawn on paper using coloured bingo chips to represent each type of angle.

- Students should also be given a set of non-parallel lines intersected by a transversal and be asked to measure each angle. They should discover that the same angle relationships (corresponding, alternate interior, alternate exterior, interior angles on the same side of the transversal, and exterior angles on the same side of the transversal) do not exist.

- Students should determine, with justification, whether a set of lines are perpendicular or parallel. To determine if lines are perpendicular, students could use a protractor, a carpenter's square, or Pythagorean triples. For example, if a flooring installer wants to ensure that the corner of a room is square, a 3-4-5 Pythagorean triple could be used.

Suggested Models and Manupulatives

- geo-strips
- carpenter's square
- protractor

Mathematical Vocabulary

Students need to be comfortable using the following vocabulary.

- alternate exterior angles
- alternate interior angles
- complementary angles
- corresponding angles
- parallel lines
- perpendicular lines
- same side exterior angles
- same side interior angles
- supplementary angles
- transversal
- vertically opposite angles

Resources/Notes

Internet

- MathsZone: Interactive Maths Games and Activities “Math Games–Angles” (Barrow 2013)
  http://resources.woodlands-junior.kent.sch.uk/maths/shapes/angles.html
  This website offers a variety of games that use angles.
- Interactivate, “Angles.” (CSERD 2013)
  shodor.org/interactive/activities/Angles
  Students can work with angles formed by parallel lines and a transversal at this interactive angle site.

Print

Math at Work 10 (Etienne et al. 2011)
- Chapter 5: All About Angles
  - Sections 5.3, and 5.4
  - 5.4 Puzzler
  - Skill Check
  - Test Yourself
  - Chapter Project
  - Games and Puzzles

Notes

G06 Students will be expected to demonstrate an understanding of angles, including acute, right, obtuse, straight, and reflex, by drawing, replicating and constructing, bisecting, and solving problems. [C, ME, PS, T, V]			
[**C**] Communication	[**PS**] Problem Solving	[**CN**] Connections	[**ME**] Mental Mathematics and Estimation
[**T**] Technology	[**V**] Visualization	[**R**] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have met the corresponding specific curriculum outcome.

G06.01 Draw and describe angles with various measures, including acute, right, straight, obtuse, and reflex angles.
G06.02 Identify referents for angles.
G06.03 Sketch a given angle.
G06.04 Estimate the measure of a given angle, using 22.5°, 30°, 45°, 60°, 90°, and 180° as referent angles.
G06.05 Measure, using a protractor, angles in various orientations.
G06.06 Explain and illustrate how angles can be replicated in a variety of ways (e.g., Mira, protractor, compass and straightedge, carpenter's square, dynamic geometry software).
G06.07 Replicate angles in a variety of ways, with and without technology.
G06.08 Bisect an angle, using a variety of methods.
G06.09 Solve a contextual problem that involves angles.

Scope and Sequence

Mathematics 9	**Mathematics at Work 10**	**Mathematics at Work 11**
—	**G06** Students will be expected to demonstrate an understanding of angles, including acute, right, obtuse, straight, and reflex, by drawing, replicating and constructing, bisecting, and solving problems.	**G01** Students will be expected to solve problems that involve two and three right triangles.

Background

Students have studied the concept of angles since Mathematics 6. In Mathematics 7, students performed geometric constructions including perpendicular line segments, parallel line segments, perpendicular bisectors, and angle bisectors. Review of these terms may be necessary.

Students will explore angles from a variety of perspectives. They will sketch, describe, and estimate angles with an understanding of the referent angles—30, 45, 60, 90, and 180 degrees. Students will also use a variety of tools to measure, replicate, and bisect angles.

Students worked with angles and referents in Mathematics 6 (6SS1). Referent angles are commonly-used angle measurements such as 22.5°, 30°, 45°, 60°, 90°, and 180°. Students should use these angle measures as a point of reference to estimate or "eyeball" unknown angle measurements. Students should be able to estimate, for example, an angle that is approximately 100° or estimate the angle they hold their pencil when used for writing.

The values of these referent angles should be compared to real-life situations. Most corners on cabinets, for example, are 90°. Cabinets on an angle in the corner are typically set at 45°. Discuss with students that each corner has two angles. A 90° angle is made up of two 45° angles, and a 45° angles is made up of two 22.5° angles. When installing crown mouldings, 22.5° angles are often used. When installing baseboards, 45° angles are common.

Students should be able to sketch the approximate measure of various angles using their knowledge of referent angles. For example, they should be able to sketch a 50° angle because they can estimate it is between 45° and 60°.

These skills will be applied to contextual problems (e.g., in navigation, orientation, and construction).

Students will be required to use a variety of tools. Teachers should be aware that there are various software programs that will assist with constructions.

In addition to standard dimensions in length, the angle is one of the most common home construction calculations. Framing, roofing, and basic woodworking all depend on exact angle measurements and cuts. The angle at which a ball is kicked is an important factor in whether or not a goal results. The required angle measurement of a wheelchair ramp is also an important calculation.

Students will also construct angles accurately. This can be done using a variety of techniques such as a compass and straightedge, a protractor, a set square, or a rafter angle square. Teachers could illustrate how to construct a 30° angle on a piece of wood before it is to be cut with the carpenter's tool, a rafter angle square.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

Guiding Questions
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Students tasks such as the following could be completed to assist in assessing students' prior knowledge.

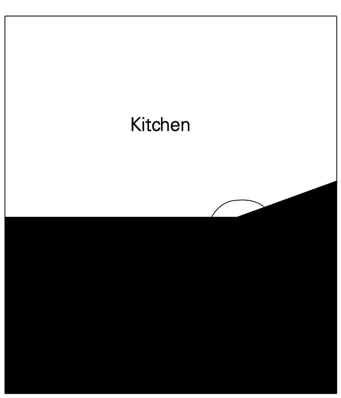
- Describe examples of parallel line segments, perpendicular line segments, perpendicular bisectors, and angle bisectors in the environment.

- Identify line segments on a given diagram that are parallel or perpendicular.
- Draw a line segment perpendicular to another line segment and explain why they are perpendicular.
- Draw a line segment parallel to another line segment and explain why they are parallel.
- Draw the bisector of a given angle using more than one method and verify that the resulting angles are equal.
- Draw the perpendicular bisector of a line segment using more than one method and verify the construction.

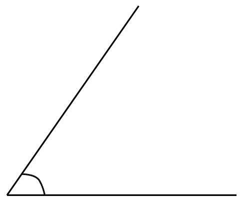
WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

- Draw, describe and identify angles with various measures, including acute, right, straight, obtuse, and reflex angles.
- Estimate the measure of a given angle, using 30°, 45°, 60°, 90° and 180° as referent angles.
- Sketch an angle, based on referent angles.
- Measure, using a protractor, angles in various orientations.
- Explain and illustrate how angles can be replicated in a variety of ways—Mira, protractor, compass and straightedge, carpenter's square, and dynamic geometry software.
- Replicate angles in a variety of ways, with and without technology.
- Bisect an angle, using a variety of methods.
- Solve a contextual problem that involves angles.
- The floor plan represents a kitchen and living room. Jamie is going to retile the kitchen floor. Determine the measure of the angle indicated below.

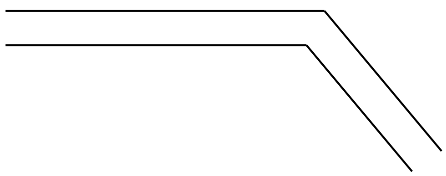


- Estimate the measure of the angle below using referent angles as a guide.



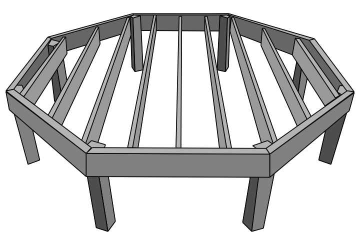
- Saige is building a patio and one of the corners is going to look like the diagram below. She will need to bisect the angle to make the correct cut.

  (a) At what angle should she cut the boards?



  (b) Saige needs to copy the angle of this corner to use on the next corner. Explain and demonstrate how she can replicate the angle.

- A carpenter is building an octagon-shaped deck.



  (a) Determine the angles at which the boards must be cut.
  (b) Bisect one of the angles and check to see if it matches.

Complete the following paper-folding activity.

(a) Construct a 60° angle on a loose-leaf sheet of paper and label the rays as *A* and *B*.
(b) Fold the paper from the vertex so that ray *A* is folded exactly onto ray *B*.
(c) Draw the angle bisector by tracing the paper along the fold.
(d) Measure each angle to verify they are equal.

FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should inform instruction.

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SUGGESTED LEARNING TASKS

Effective instruction should consist of various strategies.

Guiding Question

- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

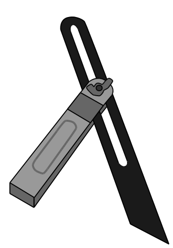
Consider the following sample instructional strategies when planning lessons.

- As an introduction to this unit, students should be given the opportunity to measure angles using a protractor. Students could be given various angles and asked to measure them accurately or they can measure the angles of objects within the classroom. Students will categorize angles according to their measure. The terms **acute**, **right**, **straight**, **obtuse**, and **reflex** should be defined.

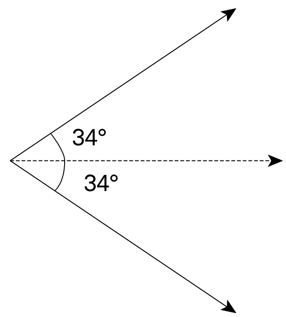
- Students should then independently draw a variety of angles and measure them. Encourage students to exchange their angles with other students so they can compare their answers and discuss how close the angle measurements actually were.

- Remind students that degrees can both represent a scale that can be used to measure angles or temperatures.

- Students may use various methods to make an exact copy of, or replicate, an angle. A Mira can be used, for example, to copy the reflection of an angle. Another tool is a T-bevel (see diagram), which is an adjustable carpenter’s tool used to copy angles.



- Angles can also be bisected using various methods. Students can use a protractor, a compass, a Mira, paper folding, or a carpenter's square to bisect the angle, or divide it into two equal parts.



- Discuss with students examples of real-life and workplace situations where it is necessary to bisect angles. One example occurs when a carpenter installs mouldings in a corner (not always restricted to a right angle). The mouldings must be cut at an angle so that the two pieces fit together tightly. The carpenter is creating an angle bisector of the corner angle.

- The compass tool in SMART Notebook Math Tools is useful for bisecting and replicating angles.

SUGGESTED MODELS AND MANUPULATIVES

- geo-strips

MATHEMATICAL VOCABULARY

Students need to be comfortable using the following vocabulary.

- acute angle
- angle
- angle bisector
- bisect
- obtuse angle
- reflex angle
- straight angle
- vertex

Resources/Notes

Internet

- WonderHowTo, "Tools and Equipment: How to Use a rafter-angle square." (WonderHowTo 2013)
  www.wonderhowto.com/howto-rafter-angle-square-789221/
  This video shows how to use a rafter-angle square for measuring and drawing angles.
- These links offer angle resources for Smart Notebook use.
  - What's My Angle? "Investigating Angles and the Use of Protractors." (Robinson 2000)
    www.amblesideprimary.com/ambleweb/mentalmaths/protractor.html
  - MCISD, "Angles" (Wikispace) (Tangient LLC 2013)
    http://smartmeasurement.wikispaces.com/Angles
  - The Carnival Game (Alberta Education 2003)
    www.learnalberta.ca/content/mec/flash/index.html?url=Data/5/A/A5A2.swf

- Math Playground, "Measuring Angles." (MathPlayground.com 2013)
  www.mathplayground.com/measuringangles.html
  Students can use an interactive protractor to measure angles.
- Maths, "Angles Game." (Innovations Learning 2013)
  innovationslearning.co.uk/subjects/maths/activities/year6/angles/games.asp
  Students can play the Angles Game.

Print

Math at Work 10 (Etienne et al. 2011)

- Chapter 5: All About Angles
  - Chapter Opener
  - Get Ready
  - Sections 5.1 and 5.2
  - Skill Check
  - Test Yourself
  - Chapter Project
  - Games and Puzzles
- Chapter 6: Pythagorean Relationship
  - Chapter Opener
  - Get Ready
  - Sections 6.1 and 6.3
  - Skill Check
  - Test Yourself
  - Chapter Project
- Chapter 7: Trigonometry
  - Get Ready
  - Section 7.1
  - 7.4 Puzzler

Notes

Number
20–25 hours

GCO: Students will be expected to develop number sense and critical thinking skills.

Specific Curriculum Outcomes

Process Standards Key

[**C**] Communication	[**PS**] Problem Solving	[**CN**] Connections	[**ME**] Mental Mathematics and Estimation
[**T**] Technology	[**V**] Visualization	[**R**] Reasoning	

N01 Students will be expected to solve problems that involve unit pricing and currency exchange, using proportional reasoning. [CN, ME, PS, R]

N02 Students will be expected to demonstrate an understanding of income and calculate gross pay and net pay, including wages, salary, contracts, commissions, and piecework. [C, CN, R, T]

N01 Students will be expected to solve problems that involve unit pricing and currency exchange, using proportional reasoning. [CN, ME, PS, R]			
[**C**] Communication	[**PS**] Problem Solving	[**CN**] Connections	[**ME**] Mental Mathematics and Estimation
[**T**] Technology	[**V**] Visualization	[**R**] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have met the corresponding specific curriculum outcome.

N01.01 Compare the unit price of two or more given items.
N01.02 Solve problems that involve determining the best buy, and explain the choice in terms of the cost as well as other factors, such as quality and quantity.
N01.03 Compare, using examples, different sales promotion techniques.
N01.04 Determine the percent increase or decrease for a given original and new price.
N01.05 Solve, using proportional reasoning, a contextual problem that involves currency exchange.
N01.06 Explain the difference between the selling rate and purchasing rate for currency exchange.
N01.07 Explain how to estimate the cost of items in Canadian currency while in a foreign country, and explain why this may be important.
N01.08 Convert between Canadian currency and foreign currencies, using formulas, charts, or tables.

Scope and Sequence

Mathematics 9	**Mathematics at Work 10**	**Mathematics at Work 11**
N03 Students will be expected to demonstrate an understanding of rational numbers by comparing and ordering rational numbers, and solving problems that involve arithmetic operations on rational numbers. **G03** Students will be expected to draw and interpret scale diagrams of 2-D shapes.	**N01** Students will be expected to solve problems that involve unit pricing and currency exchange, using proportional reasoning.	**N02** Students will be expected to solve problems that involve personal budgets. **N03** Students will be expected to demonstrate an understanding of compound interest. **A03** Students will be expected to solve problems by applying proportional reasoning and unit analysis. **M01** Students will be expected to solve problems that involve the application of rates. **M02** Students will be expected to solve problems that involve scale diagrams, using proportional reasoning.

Background

Students are introduced to proportional reasoning in Mathematics 8 (N05), and in Mathematics 9 they continue to work on proportional reasoning through the study of similar polygons. For students to experience proper development of proportional reasoning, they must become multiplicative thinkers and be able to see and use the multiplicative relationships found within and between the ratios in the problem.

Students will extend their skills with proportional reasoning to everyday situations such as shopping, calculating taxes, and currency exchange. Teachers should work from simpler to more complex examples as students increase their proficiency.

Proportional reasoning will be used in estimating and calculating the unit price. Estimation and proportional reasoning are skills that have been identified as weaknesses in our adult population, but they are critical components of financial mathematics.

For students to become financially knowledgeable consumers, they must be able to estimate and/or calculate total cost, taking into account discounts and additional costs such as taxes and shipping. The students must also take into account other factors such as ethical implications, product quality, and practicality before making a purchase.

On a more global level, this topic will allow students to explore the use of ratio to estimate or calculate a currency value based on fluctuating currency rates.

Selling rate and purchasing (buying) rate are terms related to currency exchange. Selling rate is the rate at which a bank sells money to the consumer. The purchasing rate is the rate at which a bank buys money from the consumer. It should be noted that the selling and purchasing rates are not the same and can change at any time.

Consider the following situation: Filipe decides to travel to Japan. He converts C$500 to yen and receives 44030 yen in cash. Just minutes after completing this conversion, Filipe finds out that his trip has been cancelled and he returns to the bank to change the 44030 yen back into Canadian dollars. He receives C$462.84. This transaction cost Filipe C$37.16. A bank has two rates for exchanging cash—a buying rate (in this case 0.010512) and a selling rate (in this case 0.011356). If Filipe had bought non-cash, such as traveller's cheques, the rates he would have received would have been more favourable. His C$500 would have purchased 44185 yen, and returning his 44185 yen would have him receiving C$471.32 (Cost to him would have been C$28.68).

A bank explains this difference in cash or non-cash rates as follows: "Exchange rates applied to cash transactions include shipping and handling charges, making the exchange rate for cash less favourable than the non-cash rate. Non-cash rates are applied to paper instruments such as cheques, drafts, and traveller's cheques. Non-cash rates are also applied to incoming and outgoing wire payments. These instruments are easier to manage and incur less time and cost for processing than cash transactions. Therefore, a more favourable rate is applied to non-cash instruments."

These specific rates can be found online. Students should understand that it saves money if they convert their Canadian dollars to the local currency of their travel destination before they leave Canada. Most banks, foreign exchange kiosks, and hotels in other countries charge commission or service charges for converting your Canadian dollars to their local currency.

The following table represents a currency unit when compared to the Canadian dollar. Since exchange rates frequently fluctuate, it is important that students understand how to read a table such as this one. The complete table can be found on the Royal Bank of Canada website at http://www.rbcroyalbank.com/cgi-bin/travel/currency-converter.pl.

Country	Currency	Bank Buy Rate	Bank Sell Rate
United States	US Dollars (USD)	0.9965	1.0535
European Union	Euros (EUR)	1.2956	1.4061
Great Britain	Pounds Sterling (GBP)	1.5008	1.6045
Switzerland	Swiss Francs (CHF)	1.0605	1.1513
Japan	Yen (JPY)	0.010512	0.011356
Australia	Australian Dollars (AUD)	0.9958	1.1245
New Zealand	New Zealand Dollars (NZD)	0.8110	0.9161
Denmark	Danish Kroners (DKK)	0.1735	0.1912
Norway	Norwegian Kroners (NOK)	0.1729	0.1913
Sweden	Swedish Kroners (SEK)	0.1524	0.1686
Bahrain	Bahraini Dinar (BHD)	2.4623	2.9518
Barbados	Barbados Dollars (BBD)	0.4712	0.5564
Belize	Belize Dollar (BZD)	0.4701	0.5613
Bermuda	Bermuda Dollars (BMD)	0.8481	1.0571
Brazil	Brazilian Real (BRL)	0.4764	0.5688

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Assessing Prior Knowledge

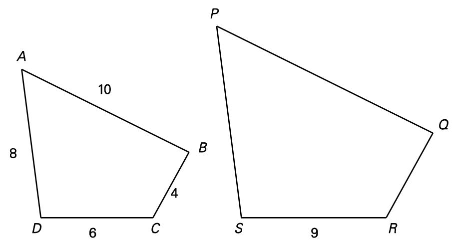
Student tasks such as the following could be completed to assist in determining students' prior knowledge.

- Order these rational numbers from least to greatest.

$$\frac{7}{5}, -0.95, \frac{3}{4}, 1.52, 0.777..., -\frac{11}{10}$$

- A case of 12 cans of juice costs \$4.80. Samuel wants to determine the cost of each can. Explain how Samuel can do this.

- If the following polygons are similar, determine the measure of all missing sides.



WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

- Felix can make 3 dozen tea biscuits with $2\frac{1}{4}$ cups of flour. How many cups of flour will he need to make 9 dozen tea biscuits?

- I travelled 40 km in 80 minutes. How far did I travel in one hour?

- Four grey garden tiles are used for every 3 red ones. If you use 210 tiles, how many are grey and how many are red?

- There are 3 oranges for every 1 apple in the bowl. How many oranges will there be if there are 5 apples? How many apples and oranges are there if the bowl has 20 pieces of fruit in it?

- When mixing paint, which combination will result in the bluer shade of green—2 parts blue with 3 parts yellow, or 3 parts blue with 5 parts yellow? Explain why.

- Mathieu ate $\frac{2}{3}$ of a box of chocolates. That left only 16 chocolates for his brother Michael. How many chocolates were in the box at the start?

- The regular price of a pair of shoes is \$140. During a sale, the store gives a discount of 35%. What will the price of the shoes be?

- One store advertises a “buy one sweater, get one for half price” sale. A second store advertises a 20% off sale on all sweaters. Rachel decides to buy two \$28 sweaters. What would be her cost per sweater at each of the two stores?

- A 12-oz. bottle of barbecue sauce costs \$1.54. A 16-oz. bottle of barbecue sauce costs \$1.99. Which is the better buy?

- Anya decides to adapt the following five-star recipe for holiday punch to make enough for 50 people. Morgan decides to use the same recipe to make enough for 15 people.
  (a) How much of each ingredient does Anya need to purchase?
  (b) How much of each ingredient does Morgan need to purchase?

  Original recipe makes 20 servings

  4 cups cranberry juice cocktail
  2 cups orange juice
  1 (2 L) bottle ginger ale
  8 cups prepared lemonade
  1 (4 ounce) jar maraschino cherries
  1 orange, sliced in rounds

- Seth went to the cafeteria at lunch to purchase milk. The cafeteria had two different sizes available. The 250-mL milk costs \$0.45 while the 500-mL milk costs \$0.80.
  (a) Which milk is the better buy?
  (b) What other factors may influence his decision?

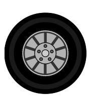
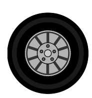
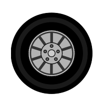
- First estimate your answer, then solve for *x*. Compare the estimated and calculated answers to check if your answer is correct.

  (a) $\dfrac{268}{5 \text{ rolls}} = \dfrac{x}{1 \text{ roll}}$

  (b) $\dfrac{x}{300} = \dfrac{1}{1.56}$

  (c) A box of 12 pencils costs \$1.69. How much does one pencil cost?

-

Tires for All Inc.	Best Value Tires Inc.	Treads-R-Us
		
Regular \$118 Now 30% off!	Regular \$118 Buy one, get 2nd half-off!	Regular \$118 Buy 3, Get one free!

  You need 4 new winter tires. Which company has the better deal? Discuss the promotional techniques used by each company. (**Note:** Use real examples of sales and promotions that are found in your community in local flyers.)

- A particular brand of house paint is available as 4-L cans of paint for \$42.95 and 250 mL for \$7.50. The first option (4 L) requires the purchase of primer, while the second option (250 mL) includes the primer. Which option would you have chosen and what conditions would affect your choice? (**Note:** This is an open question, and each option has its merits. Students should be given the opportunity to explore both options and the merits of each, depending on the situation.)

- You have purchased an iPod Touch for \$225. The original price was \$300. What percentage discount did you receive?

- Chelsea bought stock in a company for $25. Two weeks later she sold it for $60. What was the percent increase in value?

- Jordan bought an adapter for his computer that cost $29.99. Two weeks later he noticed that the same adapter was priced at $19.99. What was the percent decrease in price?

- Frank decided to buy an iPod for $297. He scratched a discount card at the checkout and got $50 off. What is the percent decrease in cost?

- How are percent increase and percent decrease alike and how are they different? Include examples in your explanation.

- The original price of a car was $19 295. The sale price of the same car was $17 995. Khalid calculated the percent decrease to be 93%. Did Khalid make an error? Explain why or why not.

- Define the terms **selling rate** and **purchasing rate**, and then explain the difference between the two using an appropriate example.

- Juan is planning a trip to Florida. He uses C$300 to buy US dollars at the bank at the current daily rate of 0.9084. Later that day his trip gets cancelled so he changes his money back to Canadian dollars at the rate of 1.0361. Ask students to determine how much money he lost and explain why he would not get exactly $300 back.

- For a school exchange trip to Europe, your parents have given you C$500 as spending money.
  (a) At the bank, you exchanged this for Euros, at a rate of 1 EUR = 1.35 CAD. How many Euros will you receive? At this exchange rate, what is the value of C$1 in Euros?
  (b) On return you have €50 left, and you exchange these Euros back into Canadian dollars at a rate of 1 EUR = 1.27 CAD. How much will you have lost (paid to the bank) for the exchange of this €50 back and forth?
  Note: Actual currency rates can be found online and used in place of rates given.

- Prior to your trip to Mexico, you exchanged some Canadian dollars for pesos at an exchange rate of 1 CAD = 12.35 MXN. You buy a burrito for 30 pesos. Approximately how much has this cost you in Canadian dollars?

- Use currency exchange rates from the Internet or the newspaper to answer the following:
  (a) Stephanie is travelling to the Philippines on vacation.
    (i) What is the name of the currency used in the Philippines?
    (ii) What is the exchange rate of that currency in Canadian dollars?
    (iii) If Stephanie goes into a bank and purchases, in cash, 4500 units of Philippine currency, how many Canadian dollars would this cost her?
    (iv) If Stephanie's trip were cancelled and she returned to the bank to return the Philippine currency, how many Canadian dollars would she get back?
    (v) How much did this exchange process cost Stephanie? How much would it have cost her if she had obtained traveller's cheques rather than cash?
  Note: Groups of students could also be assigned different countries.

- Chantelle gets a job in Malaysia where the currency is the Ringitt (RM). She has the option to get paid C$60 000 per year, or RM 210 000. Ask students to look up today's exchange rates on the Internet to determine the best option for Chantelle.

- Explore the daily exchange rate over the last 30 days and determine how the rates may have been influenced by current events or other factors.

- Answer the following: (You will need to look up exchange rates).
  (a) While in the United States, you wish to purchase a laptop computer for US$385. If the selling rate of the US dollar compared to the Canadian dollar is 1.0375, estimate the cost of the laptop in Canadian funds.
  (b) Alivia is in Mexico bargaining with a local seller. The cloth she wants to buy costs 85 pesos. If the exact value of 1 Canadian dollar is 12.3 pesos, what is a good estimate (in Canadian dollars) for 85 pesos? Why is estimating quickly useful in a situation like this?
  (c) Compare shopping prices for a similar product on Canadian and American store websites. Consider which site is more economical for making online purchases based on currency exchange rates.

  Extension: Include shipping, duties, brokerage fees, and tax rates.

- A company manufactures and sells a product for $15 (before taxes) here in Canada. If the cost of shipping and exporting it to Europe is $1 per item, determine the equivalent price in Euros for this item when it is sold in Europe. Use the rate of 1 EUR = 1.35 CAD.

- A company is buying desk calendars for their employees. They can buy them in packages of 10 for $32, packages of 15 for $45, or packages of 25 for $70. The company is buying desk calendars for 70 employees. Which combination of packages should be purchased to minimize the cost to the company?

- Liam has decided to order new kitchen doorknobs for his apartment buildings. He has found one supplier who sells them in groups of 20 doorknobs for $66. Another supplier sells them in groups of 50 for $155. He has found a third supplier who is willing to sell individual doorknobs for $4.50 each. If Liam plans to purchase 185 doorknobs, determine what combination of purchases would result in the best buy.

FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should inform instruction.

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SUGGESTED LEARNING TASKS

Effective instruction should consist of various strategies.

Guiding Question
- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

Consider the following sample instructional strategies when planning lessons.

- Activate prior knowledge of proportions and percentage seen in previous years. Determine if students are able to think multiplicatively to solve problems.

- Estimation should be emphasized before a student calculates the answer, to help the student predict whether or not the answer is reasonable.

- Encourage solving problems mentally where possible.

- Develop proportional reasoning through a progression of questions. Start with simple numbers (whole number examples that are easy to double, triple, etc.) to establish the understanding before moving to other numbers such as fractions and decimals as examples.

- "How much for one of something → two of something → four of something" using examples from real life.

- Use flyers, catalogues, and websites to provide real-life examples. Bring in products—such as yogurt, cereal, granola bars, and vitamins—so students can visually make comparisons.

- Check currency rates as a class and then talk about fluctuations that occur on a daily or longer term basis.

- In a class discussion, ask students to explain how fluctuations in the exchange rates of different countries could affect the import and export business.

- Discuss various sales promotion techniques that stores use to help sell items. Stores often sell different quantities of the same product at different prices (e.g., soft drinks sold at 4 for $5 as opposed to 1 for $1.49). Promotions, such as "buy one get one free"/discounted, also encourage consumers to shop in a particular store. Students must realize, however, that to effectively compare the prices of two or more items they must use the same units. For example, deli meat sold at $2 per 100 g may seem less expensive than $20 per kilogram. Using the conversion factor of 1000 g = 1 kg, students should realize that deli meat at $2 per 100 g is equivalent to $20 per kilogram. Although students have explored the relationship between metric units of measurement, it may be necessary to revisit the following conversions:

  1000 g = 1 kg   100 cm = 1 m   1000 mL = 1 L

- After comparing unit prices, discuss other factors that may influence the choice for a "best buy." Students should be reminded that more is not always better. Engage students in a discussion about this. Buying large quantities of items that have a cheaper unit price is not helpful if the consumer ends up wasting some of the product because it was not fully used or has expired. Other factors, such as the travelling distance from stores and the quality of one product over another, must also be considered. Students should realize that decisions to buy an item should not be based on price alone. Present students with a choice such as the following:

  – Mustard is sold in a 2-bottle package for $2.49 and a 12-bottle package for $12.99. Which package has the lower unit price? How much would you save by buying a 12-bottle package rather than six 2-bottle packages? When deciding which package size is the better buy for you, what should you consider in addition to unit price?

- Ask students to collect flyers to compare various products. Ask them to create their own problem related to comparing unit price and finding the best buy. After doing the comparison, students should determine other factors that could influence their decision to purchase that item.

- Students could work in centres, with each centre containing similar items in different sizes with prices given, such as soup, cans of juice, dog food, and shampoo. In their journals, students could list the item they would buy and why it is the better deal or the better purchase.

- Gather empty containers of dish detergent, including various different sizes. Ask students to discuss factors that contribute to deciding which size to purchase. They should be encouraged to consider environmental concerns, such as packaging, use of water, and concentration of chemicals.

- Students could compare a jumbo-size and a regular-size liquid laundry detergent. Ask them to determine what constitutes one use for each size. To visualize how much more the jumbo size contains, they could measure out each serving (empty containers with water could be used). They should then determine the cost per usage.

- Ask small groups of students to research the cost of lumber at the local hardware store(s) to determine
  (a) the store that offers the best buy on one piece of 2" × 4" × 8'
  (b) the best rate per foot of a piece of
    (i) 2" × 4" × 8'   (ii) 2" × 4" × 10'   (iii) 2" × 4" × 12'

- Students will explore currencies in countries around the world and should recognize the importance of understanding currency rates, especially when travelling and buying and selling goods in different countries. Discuss with students some of the different systems of currencies in a variety of countries. Some samples are provided on below.

  Currency by Country

Canada	dollar	C$
United States	dollar	US$
Germany	Euro	€
England	pound	£
Japan	yen	¥
Denmark	Krone	kr
Thai	Baht	฿
South Korean	Won	₩
Poland	Zloty	zł

- Engage students in a discussion about businesses that import or export materials and how the fluctuation in the Canadian dollar can affect these businesses.

- Problems involving currency exchange provide opportunities for discussion. Often, for example, both Canadian and American prices are listed on magazines and books. Discuss whether or not customers would benefit from choosing which price to pay. Travellers to countries that use a different currency from their home country's currency can exchange their money to make purchases while they are travelling. The exchange rate may determine how much Canadian travellers will buy in a foreign country. Before items are bought in a foreign country, students need to be aware of what the item actually costs in their own currency to ensure they are not paying more than they would at home. Estimation can help students compare foreign prices to Canadian prices. Consider the following example:
  - When Kalie was vacationing in France, she wanted to purchase a print of the Eiffel Tower costing 190 Euros. What would be the cost in Canadian dollars if the exchange rate were 1.644814?

Exact Value	**Estimated Value**
1 Euro = C$1.644814	1 Euro = C$1.6
190 Euros = C$312.51	200 Euros = C$320

- An engaging activity for students involves setting up an international store with food items from different countries (e.g., a can of olives from Greece, lettuce from the United States, bananas from Chile). The items should be labelled with the cost in the currency of the source country. Each group chooses from provided recipes and selects the required ingredients. They calculate the cost, in Canadian dollars, of the completed dish. As an extension, they could determine the cost per serving.

Suggested Models and Manipulatives

- coins from various countries
- consumer items
- newspapers
- sales flyers
- various-sized containers

Mathematical Vocabulary

Students need to be comfortable using the following vocabulary.

- buying rate
- currency
- exchange rate
- percentage decrease
- percentage increase
- purchasing rate
- selling rate
- unit price

Resources/Notes

Internet

- Bank of Canada (Bank of Canada 2013)
  www.bank-banque-canada.ca/en/rates/converter.html
  For the current Canadian exchange rates, go to the daily currency converter at the Bank of Canada's website.
- Canadian Bankers Association, "Banks and Financial Literacy" (Canadian Bankers Association 2013)
  www.cba.ca/en/component/content/category/79-banks-and-financial-literacy
  Financial Literacy Information (Banks)
- Financial Consumer Agency of Canada. *The City: A Financial Life Skills Resource* (Government of Canada 2013)
  www.fcac-acfc.gc.ca/eng/education/index-eng.asp
- *Mathematics Learning Commons 7–9, Nova Scotia Virtual School* (Nova Scotia Department of Education and Early Childhood Development 2013):
  - Proportional Reasoning PowerPoint
    http://lrt.ednet.ns.ca/PD/math8support_11/prop_reasoning/01D_proportional_reasoning_ratios.ppt
  - Proportional Reasoning Problems
    http://lrt.ednet.ns.ca/PD/math8support_11/prop_reasoning/01K_question_bank.doc
  - Proportional Reasoning articles:
    http://lrt.ednet.ns.ca/PD/math8support_11/prop_reasoning/01L_problems_encourage_prop_sense.pdf
    http://lrt.ednet.ns.ca/PD/math8support_11/prop_reasoning/01M_multiple_ways_to_solve_proportions.pdf
- Practical Money Skills Canada: Financial Literacy for Everyone, "For Educators"
  Lesson Plans: Choices & Decisions (Visa 2013)
  http://practicalmoneyskills.ca/foreducators/lesson plans
- RBC Royal Bank (Royal Bank of Canada 2013)
  www.rbcroyalbank.com
  Lists both the buy and sell rates.
- XE, Currency Encyclopedia (XE 2013)
  http://xe.com/currency
  Provides rates and information for every currency.

Print

Math at Work 10 (Etienne et al. 2011)

- Chapter 1: Consumerism and Travel
  - Chapter Opener
  - Sections 1.1 and 1.2
  - Skill Check,
  - Test Yourself
  - Chapter Project
- Chapter 2: Measuring Length
  - Get Ready
  - Section 2.3

Notes

N02 Students will be expected to demonstrate an understanding of income to calculate gross pay and net pay, including wages, salary, contracts, commissions, and piecework. [C, CN, R, T]

[**C**] Communication	[**PS**] Problem Solving	[**CN**] Connections	[**ME**] Mental Mathematics and Estimation
[**T**] Technology	[**V**] Visualization	[**R**] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have met the corresponding specific curriculum outcome.

N02.01 Describe, using examples, various methods of earning income.

N02.02 Identify and list jobs that commonly use different methods of earning income (e.g., hourly wage, wage and tips, salary, commission, contract, bonus, shift premiums).

N02.03 Determine in decimal form, from a time schedule, the total time worked in hours and minutes, including time and a half and/or double time.

N02.04 Determine gross pay from given or calculated hours worked when given
- the base hourly wage, with and without tips
- the base hourly wage, plus overtime (time and a half, double time)

N02.05 Determine gross pay for earnings acquired by
- base wage, plus commission
- single commission rate

N02.06 Explain why gross pay and net pay are not the same.

N02.07 Determine the Canadian Pension Plan (CPP), Employment Insurance (EI), and income tax deductions for a given gross pay.

N02.08 Determine net pay when given deductions (e.g., health plans, uniforms, union dues, charitable donations, payroll tax).

N02.09 Investigate, with technology, “what if …” questions related to changes in income.

N02.10 Identify and correct errors in a solution to a problem that involves gross or net pay.

N02.11 Describe the advantages and disadvantages for a given method of earning income.

Scope and Sequence

Mathematics 9	Mathematics at Work 10	Mathematics at Work 11
N03 Students will be expected to demonstrate an understanding of rational numbers by comparing and ordering rational numbers and solving problems that involve arithmetic operations on rational numbers. **N04** Students will be expected to explain and apply the order of operations, including exponents, with and without technology.	**N02** Students will be expected to demonstrate an understanding of income to calculate gross pay and net pay, including wages, salary, contracts, commissions, and piecework.	**N02** Students will be expected to solve problems that involve personal budgets. **N04** Students will be expected to demonstrate an understanding of financial institution services used to access and manage finances.

Background

Increasingly, students are working in part-time jobs. In addition to providing an income, work experiences enhance resumés, college applications, and future job applications. In this unit, students will be introduced to the various methods of income payment, deductions, and calculations involving gross and net pay. They will also be presented with flawed solutions involving gross or net pay that require them to identify and correct mistakes.

Students will gain an understanding of income, how it can be earned, and what the advantages and disadvantages of various ways of earning an income might be.

Income is the money received within a specified time frame, usually in return for work completed. This can be in the form of an hourly wage in which a worker is paid at a set rate per hour, a wage as piecework in which a worker is paid a fixed “piece rate” for each unit produced or job completed (such as planting trees, completing a translation job), or a salary which is paid regularly by an employer to an employee, and may be specified in an employment contract.

Working on commission involves an employee performing a service or making a sale for a business and being paid a percentage of the money received by their employer for each service performed or sale made. Workers can work entirely on commission or work for a base salary and receive a commission over and above their salary. Additional pay can also be received if an employee works overtime or on holidays, or extra pay can come in the form of tips, bonuses, or shift premiums.

Applying SCO A01, students will use formulae to calculate income, as well as gross and net pay. They will determine which deductions are required and which are optional depending on circumstances. They will understand that gross pay is what you make before any deductions. Net pay is the actual “take-home” pay after taxes, health benefits, Canada Pension Plan (CPP), Employment Insurance (EI), and other deductions are taken into account.

Additional Details

- There are many methods of earning income.

- Various combinations of these methods of earning income, such as hourly wage and tips, are also common. Students usually begin work with jobs that earn an hourly wage, wage and tips, or a salary.

- Employees sometimes have to work extra hours in addition to their regular hours. Overtime usually begins when they work beyond 40 hours in a workweek. Overtime pay must be received for those additional hours. Overtime pay is typically 1.5 times the employee’s regular rate of pay. Students may be familiar with this as time and a half. For example, if regular pay is $12 an hour, then the overtime rate is $18 an hour (12 × 1.5) for every hour worked beyond 40 hours in each week. Other employees, in professions such as nursing, receive a shift premium. In this case, they receive an extra amount of money per hour because they work non-standard hours.

- Gross pay is the total amount earned before any deductions are taken out. There are three types of fixed gross earnings:
  - wages (rate × hours worked)
  - salaries (set amount)
  - bonuses (discretionary)

Students should
- be able to calculate weekly (52 pay periods), bi-weekly (26 pay periods), or monthly (12 pay periods) gross wages given the hourly rate of pay or the annual income (The gross pay calculation is straightforward when the number of hours worked in the pay period and the hourly pay rate are known. Gross pay calculations for salaried employees may require more focus. The total annual pay is divided by the number of pay periods per year.)
- recognize the difference between hourly gross pay and salaried gross pay through exploration of situations such as the following:

Hourly Gross Pay	**Salaried Gross Pay**
An employee works 20 hours a week at $12/h. If he or she is paid weekly, the gross pay for each of the 52 pay periods is $240.	If an employee's annual salary is $30 000 and he or she is paid biweekly, the gross pay for each of the 26 pay periods is $1153.85.

Gross pay is the total of all earnings including regular pay and overtime pay. Net pay is the gross pay minus any deductions. Students may be familiar with this as "take-home" pay. Mandatory government deductions are CPP, EI, and income tax. The employee's share will be deducted from his or her paycheque and the employer's share will be a cost to the company. Students will calculate these deductions given the appropriate percentage rates found on the Canada Revenue Agency website.

Note: Teachers should visit the CRA website yearly for the updated rates for Employment Insurance, Canada Pension Plan, and tax tables.

Employment Insurance offers financial assistance for some people who lose their jobs through no fault of their own. The number of hours or weeks an employee needs to qualify for EI is based on where he or she lives and the unemployment rate in his or her economic region at the time he or she files the claim. In Nova Scotia, most people will need between 420 and 665 insurable hours of work in the last 52 weeks in order to qualify for EI.

EI is a fund into which employees and employers pay. Employers pay 1.4 times the employee's rate. The greater a worker's earnings, the greater the deductions and EI payments will be, if collected. EI contributions on all eligible earnings will continue throughout the year until the maximum contribution levels are reached. How fast an employee reaches that figure, or if he or she reaches it at all, depends on how much the worker earns.

Canada Pension Plan protects families against income loss due to retirement, disability, or death. Both employees and employers contribute a portion to the Canada Pension Plan. The employer matches the contributions made by the employee. For CPP, there are yearly maximum contribution amounts (for example, $2163.15 for 2010) and once these are reached during the calendar year the contributions will cease. In 2012, the CPP contribution rate was 4.95% of any gross earnings above $3500 and the maximum rate for pensionable earnings was $47 200.

CPP = (Earnings – $3500) × 0.0495

Students should be exposed to situations where
- income is below the minimum contribution level
- income is between the minimum and maximum contribution rates
- income is higher than the maximum contribution level

For example,

- Kyle earned $3280 through a summer job, $220 less than the minimum contribution level. As a result, he did not have to contribute to the CPP in 2012. If Kyle's employer deducted any CPP contributions, they would have been refunded to him when he filed his tax return.

- Quentin is employed as a photographer. His annual salary is $45 000. This figure is between the minimum contribution level of $3500 and the 2010 maximum rate of $47 200. His maximum contribution for 2012 was ($45 000 – $3500) x 0.0495 = $2054.25.

- Hana is employed as a dental hygienist. Her annual salary is $68 700. This figure is higher than the maximum pensionable earnings of $47 200 in 2012. She will make her contributions of $130.79 biweekly; when her total deductions for the year reaches the maximum, however, she will see an increase in her net pay as there will no longer be CPP deductions. Beginning with the new year, CPP contributions will recommence until she reaches the maximum level again.

Income tax is a type of deduction used to help pay for everything from maintaining law and order to funding our health service. Most students will be interested in actual deductions for different ranges of income, as well as different methods of payment. Federal and provincial/territorial tax rates vary depending on the employee's taxable income. Taxable income is the gross income less a variety of deductions. Canada, for example, is recognized for its effective health care system. Employees still opt, however, to buy extended coverage from their place of work to cover unforeseen expenses such as vision care, dental care, prescription drugs, and accidental death. Once all the deductions have been totalled, the net pay will then be calculated. This can be demonstrated using a sample pay stub.

Students will next investigate commission earnings and earnings for piecework and contract work. They should explore various methods of calculating regular pay, including commission only, salary plus commission, and wages plus tips. It might be worth noting that, although it is illegal in Canada to have someone working for tips only, in many places in the United States servers do not receive a minimum wage. Instead, they take home their tips only.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Student tasks such as the following could be completed to assist in determining students' prior knowledge.

- Ask students to add brackets where required to make this a true statement.
  13.5 + 4 ÷ 0.75 + (8.1) = 1.9

- Some people are saying that the answer to the skill-testing question (3 × 50) + 20 ÷ 5 is 154 and some say the answer is 34. Which answer is correct and why?

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

- Calculate and compare wage situations involving minimum wage rates, regular pay, overtime pay, gratuities, piecework, straight commission, salary and commission, salary plus quota, and graduated commission.

- Two restaurants have offered Jamir a job. Mario's pays $8/h, and tips average $24 daily. Teppan's pays $5.50/h, and tips average $35 daily. If Jamir works 30 hours weekly, spread over four days, how much would she earn at each restaurant?

- Identify and calculate various payroll deductions, including income tax, CPP, EI, medical benefits, union and professional dues, and life insurance premiums.

- Estimate, calculate, and compare gross and net pay for various wage or salary earners in your community.

- You have a summer job at a local restaurant as a server. The owner presents three choices for your income:
  (a) $14 per hour (no tips)
  (b) $10 per hour (plus tips)
  (c) salary of $320 per week
  Which option would you choose and why? What aspects of the job should you consider before choosing an option?

- Crystal is working at a fish plant for the summer. She makes $12 per hour, and she earns a shift premium of $2 per hour for hours worked between 12 a.m. and 8 a.m. She gets 1.5 times the regular pay for overtime hours worked above 40 hours per week. Her weekly time schedule is shown below:

Monday	12 p.m.–6 p.m.
Tuesday	8 a.m.–4:30 p.m.
Wednesday	6 a.m.–2 p.m.
Thursday	12 a.m.–10 a.m.
Friday	10 p.m.–4 a.m.
Saturday	10 p.m.–4 a.m.
Sunday	4 p.m.–11 p.m.

  (a) calculate Crystal's regular hours
  (b) calculate Crystal's premium hours
  (c) calculate Crystal's overtime hours
  (d) calculate Crystal's gross pay

- Design your own time schedule and create a problem for other students to answer.

- Choose a job that suits your skills and interests. Research the rate of pay in Nova Scotia on the jobbank.ca website (http://jobbank.ca) and post the information on the job wall that your teacher has set up for that purpose.

- Oliver is paid \$12.50 per hour for 40 hours per week. If he works more than 40 hours per week, he makes time and a half. If Oliver worked 52 hours this week, what would his gross earnings be?

- Xena is a hairstylist who works for a base hourly wage of \$12. She works 35 hours per week and receives \$160.50 in tips for the week. Calculate her gross pay.

- Joshua is a waiter at the local restaurant and is paid an hourly rate of \$10 plus tips. Joshua earns 6% of the tips received in a shift. During his shift on Tuesday, \$1500 in total was received in tips. Calculate Joshua's gross pay for Tuesday.

- Saleem works 48 hours per week at an hourly rate of \$16 per hour. After 40 hours he receives time and a half. Saleem calculated his income using the following method.

  Step 1: Regular pay = 40 hr. × \$16 = \$640
  Step 2: Overtime hours = 48 – 40 = 8 hr.
  Step 3: Overtime pay = 8 hr. × \$16 = \$128
  Step 4: Gross pay = regular pay + overtime pay
  Step 5: Gross pay = \$640 + \$128 = \$768

  Is Saleem's gross pay correct? If not, identify the step in which the error occurred and determine the correct gross pay.

- Paxton 's biweekly gross salary is \$2800. He has to pay the following deductions:

  EI: 1.73%
  CPP: 4.95%
  Income Tax: 25%

  (a) Calculate each deduction.
  (b) Determine Jeff's net pay.

- Lesley earns \$11.50 per hour. She works 35 hours a week. Her weekly deductions are as follows:

  EI: \$9.06
  CPP: \$14.41
  Income Tax: \$49.10
  Company Pension Plan: \$10.77
  Health Plan: \$4.85

Determine her
(a) gross pay
(b) total deductions
(c) net pay

- Describe, in your own words, how a higher gross income affects deductions.

- Kadeem wants to move into an apartment and is wondering how much he can afford to pay for rent. Offer him advice on whether he should consider his gross income or his net income. Explain.

- In the role of a business owner, use the Payroll Deductions Online Calculator (see Resources/Notes, p. 134) to determine the CPP, EI, and tax deductions for an employee.

- Working with the job that your teacher has assigned you (and your partner):
  (a) Calculate the gross annual salary.
  (b) Determine which federal tax bracket it fits into and calculate the federal income tax deduction.
  (c) Repeat (b) for Nova Scotia tax.
  (d) Calculate EI and CPP deductions.
  (e) Complete a blank T4 form.
  (f) Draw five cards from a deck of cards containing other considerations, such as childcare, charitable donations, rent, RRSP, tuition amounts, tips, student loan payments, moving expenses, transportation, and dependents. Complete the income tax including your five drawn considerations.

- Daija's biweekly gross salary is $2400. She has to pay the following deductions:

  EI: 1.73%
  CPP: 4.95%
  Income Tax: 25%

  Daija used the following steps to calculate her net income.

  Step 1: EI = $2400 × 0.0173 = $41.52
  Step 2: CPP = $2400 × 0.0495 = $118.80
  Step 3: Tax = $2400 × 0.25 = $600
  Step 4: Total deductions = $760.32
  Step 5: Net income = $2400.00 – $760.32 = $1639.68

  Is Daija's net pay correct? If not, identify the step in which the error occurred and calculate Daija's correct net pay.

- Research the business and classified ads section of a newspaper or the government's job bank and find a job that pays by
  (a) a salary
  (b) an hourly wage
  (c) a straight commission
  (d) a salary plus commission
  (e) piecework

- Share job descriptions with other students in your class and discuss which jobs are of interest to you. From the figures you have acquired on your ads, calculate the yearly, monthly and weekly gross wages available for each job. Describe the advantages and disadvantages of one of the methods of earning income you have researched.

- Adair works at Sears selling appliances. His base salary is $300/week and he makes 5% on his sales. During the month of August, he sold appliances worth $120 000. What is his gross pay for that particular month?

- Lan fishes with his grandfather in the summertime. He is paid 6% commission on the amount of catch landed. If $7500 worth of fish is landed, what is his gross pay?

- Monica's monthly gross income is $1916.67
  (a) Calculate her Employment Insurance (EI) payment using the formula given.
  (b) Calculate her monthly Canada Pension Plan (CPP) payment using the formula given.

  Note: Check rates for EI and CPP, as they change yearly, and provide students with the appropriate formula.

- Brogan is a sales clerk in a bicycle shop. He is paid $11.25/hour for a 37.5 hour week, plus a commission of 6% of his sales for the week. In one week Brogan's sales were $2319.75.
  (a) Calculate Brogan's gross pay for the week.
  (b) What was his average hourly wage for that week?
  (c) What would Brogan's sales for the week have to be for him to earn a total of $700 in one week?

- Complete the following time schedule for each employee:

Employee Hours:					Regular Hours: 8					Overtime Rate: 1.5		
Name	Start	Lunch Out	Lunch In	End	Total Hours	Worked Hours	Regular Hours	Over-time Hours	$/hr.	Reg $	Over $	Total
Ryan	9:00	12:00	13:00	18:00	9:00	8:00	8:00	0:00	$11.00	$88.00	$0.00	$88.00
Sheila	8:30	11:30	12:30	18:30					$11.00			
Katelyn	8:45	12:30	13:15	16:30					$12.25			
Simon	22:00	0:30	1:00	9:00					$11.50			

- Claudette has been offered two jobs, one of which offers an annual salary of $37 500, and the other that requires working a 40-hour week for 50 weeks of the year at $18.25/hr.
  (a) Calculate the weekly pay for each option.
  (b) Which option gives Claudette the greatest gross income?

- As a waitress, Carla earns $7.25/hour for a 40-hour week and shares 25% of her tips with other employees. In one week, her tips were $318. What was Carla's gross pay for the week?

- Aadesh is paid $8.50/hr. for a 37.5-hour week and earns double-time for overtime.
  (a) Calculate Aadesh's gross pay if his total over-time was 4.5 hours.
  (b) Determine CPP, EI, and Income Tax deduction amounts
  (c) Calculate net pay if the other deductions total $14.73.

- Identify and correct the error made when solving the following problem:
  A real estate agent earns 2.4% on the sale of a house. The last house she sold earned her a commission of $4128. What was the selling price of the house?

  C = PR
  $4128 = P × 2.4%
  P = $4128 × 2.4%
  P = $9907.20

  The selling price of the house is $9907.20.

FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should inform instruction.

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SUGGESTED LEARNING TASKS

Effective instruction should consist of various strategies.

Guiding Question

- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

Consider the following sample instructional strategies when planning lessons.

- Students should be provided with a time schedule and asked to compute regular, overtime and shift premium hours. They should be exposed to situations in which an employee works a fraction of an hour (e.g., 15 minutes, 30 minutes, or 45 minutes). In such cases, students should convert minutes to hours in decimal form. They should recognize, for example, that 15 minutes is 0.25 of an hour.

When considering an example such as the following, students must be careful not to include overtime hours twice.

– Alyson works at a local donut shop. Her regular pay is $10 per hour and she earns a shift premium of $1 per hour for hours worked between 12 a.m. and 8 a.m. Alyson gets 1.5 times the regular rate of pay for overtime hours worked above 40 hours per week. Alyson's time schedule is shown below:

Monday	8 a.m.–4 p.m.
Tuesday	8 a.m.–8:00 p.m.
Wednesday	12 a.m.–8 a.m.
Thursday	12 a.m.–5:30 a.m.
Friday	6 a.m.–12:15 p.m.
Saturday	Holiday
Sunday	8 a.m.–4 p.m.

Ask students to
(a) Calculate Alyson's regular hours.
(b) Calculate Alyson's premium hours.
(c) Calculate Alyson's overtime hours.
(d) Calculate Alyson's gross pay for the week.

Note: An extension of this activity would be to ask students to calculate the gross pay ($531.75). It may be a good idea to encourage students to compute overtime hours first. Otherwise, because overtime hours can occur during the regular work day, the tendency could be to include the overtime hours with the regular hours as well.

- A discussion of terminology related to income calculations is necessary before doing the calculations. A pre-assessment of what students know about gross pay, pay periods, deductions, and the types of deductions may be beneficial here. Students could be given an admit card with a job and three options of how to be paid. They choose the best method and justify their choice.

- Technology (e.g., calculator, online payroll calculator, tax programs, spreadsheets) should be used to compare various income rates. Students should examine how changes in rate of pay, number of hours worked, increases in income, or decreases in deductions impact net income.

- Engaging students in error analysis heightens awareness of common errors. Along with providing the correct solutions, students should be able to identify incorrect solutions, including why errors might have occurred and how they can be corrected. Questions requiring error analysis can be effective tools to assess students' understanding of gross and net pay calculations because it requires a deeper understanding than simply "doing the problem." Analyzing errors is a good way to focus discussion on "How did you get that?" rather than being limited to "Is my answer right?" This reinforces the idea that the process of determining the solution is as important as the solution itself.

- Ask students if anyone has had the experience of being surprised when he or she received his or her first paycheque and realized how much money had been deducted. Students need to be aware of the deductions their employers take from their cheques and by how much this will reduce their gross income. Some typical deductions are Employment Insurance (EI), Canada Pension Plan (CPP), pension, union dues, benefits, and income tax.

- In 2012, the EI premium rate was 1.73% of gross earnings and the maximum annual employee premium was $747.36. Illustrate with a pay stub an example of EI being deducted, and one where it is paid up.

- Students should examine federal and provincial tax deduction tables and discuss why provincial and federal amounts are different.

- Students should also discuss optional deductions, such as company health and pension plans, union fees.

- Students should brainstorm, research, and discuss possible advantages and disadvantages of various income earning methods. Some suggestions follow. However, this list is not intended to be exhaustive.

Method	**Advantage**	**Disadvantage**
Hourly Wage	guaranteed income for hours worked	reduced hours during slow periods
Tips	additional income beyond regular salary	job may not pay well
Piecework	more money if you work faster	may ignore safety standards to work faster
Salary	income continues during slow sales periods	work overtime without extra income
Commission	increased income during good sales periods	decreased income during slow sales periods
Contract Work	guaranteed contract income	decreased yearly income if job takes longer than expected or expenses are greater than expected

At the end of this section

- have students generate a list of jobs that interest them and then have them work in teams to categorize the jobs by payment method (salary, hourly wage, commission)
- (as a class) discuss other payment methods and the advantages and disadvantages of each method
- provide students with a scenario and have them complete a pay stub, filling in all the important information (EI, CPP, gross pay, net pay, etc.)

SUGGESTED MODELS AND MANIPULATIVES

- Payroll deduction tables (income tax, EI, CPP, etc.)

MATHEMATICAL VOCABULARY

Students need to be comfortable using the following vocabulary.

- contract
- CPP
- deductions
- EI
- gross pay
- hourly wage
- income
- net pay
- piecework
- salary
- wage

Resources/Notes

Internet

- Canada Revenue Agency (Government of Canada 2013)
  www.cra-arc.gc.ca/menu-eng.html
- Canada Revenue Agency, "Payroll Deductions On-line Calculator" (Government of Canada 2013)
  www.cra-arc.gc.ca/esrvc-srvce/tx/bsnss/pdoc-eng.html
  PDOC calculates federal and provincial payroll deductions for provinces and territories.
- PayScale, "Get the Right Salary Data for You." (PayScale Inc. 2013)
  www.payscale.com
  Students can visit this site to determine hourly wages and gross annual income for jobs in various Canadian cities.
- XE, Currency Encyclopedia (XE 2013)
  http://xe.com/currency
  Provides rates and information for every currency.

Print

Math at Work 10 (Etienne et al. 2011)

- Chapter 4: Getting Paid for Your Work
  - Chapter Opener
  - Sections 4.1, 4.2, and 4.3
  - Skill Check
  - Test Yourself
  - Chapter Project

Notes

Algebra
(integrated throughout)

GCO: Students will be expected to develop algebraic reasoning.

Specific Curriculum Outcomes

Process Standards Key

[**C**] Communication	[**PS**] Problem Solving	[**CN**] Connections	[**ME**] Mental Mathematics and Estimation
[**T**] Technology	[**V**] Visualization	[**R**] Reasoning	

A01 Students will be expected to solve problems that require the manipulation and application of formulas related to perimeter, area, the Pythagorean theorem, primary trigonometric ratios, and income. [C, CN, ME, PS, R]

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[**C**] Communication	[**PS**] Problem Solving	[**CN**] Connections	[**ME**] Mental Mathematics and Estimation
[**T**] Technology	[**V**] Visualization	[**R**] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have met the corresponding specific curriculum outcome.

A01.01 Solve a contextual problem that involves the application of a formula that does not require manipulation.
A01.02 Solve a contextual problem that involves the application of a formula that requires manipulation.
A01.03 Explain and verify why different forms of the same formula are equivalent.
A01.04 Describe, using examples, how a given formula is used in a trade or an occupation.
A01.05 Create and solve a contextual problem that involves a formula.
A01.06 Identify and correct errors in a solution to a problem that involves a formula.

Scope and Sequence

Mathematics 9	Mathematics at Work 10	Mathematics at Work 11
PR03 Students will be expected to model and solve problems using linear equations of the form: ▪ $ax = b$ ▪ $\frac{x}{a} = b$, $a \neq 0$ ▪ $ax + b = c$ ▪ $\frac{x}{a} + b = c$, $a \neq 0$ ▪ $ax = b + cx$ ▪ $a(x + b) = c$ ▪ $ax + b = cx + d$ ▪ $a(bx + c) = d(ex + f)$ ▪ $\frac{a}{x} = b$, $x \neq 0$ where a, b, c, d, e and f are rational numbers. **G01** Students will be expected to determine the surface area of composite 3-D objects to solve problems.	**A01** Students will be expected to solve problems that require the manipulation and application of formulas related to perimeter, area, the Pythagorean theorem, primary trigonometric ratios, and income.	**A01** Students will be expected to solve problems that require the manipulation and application of formulas related to ▪ volume and capacity ▪ surface area ▪ slope and rate change ▪ simple interest ▪ finance charges **G03** Students will be expected to solve problems that involve the cosine law and the sine law, including the ambiguous case.

Background

Students have been modelling and solving various forms of linear equations since Mathematics 7 and have practised manipulating these equations to solve for an unknown variable.

This skill should be further developed and the outcome addressed throughout this course as students apply formulas in a variety of contexts such as income, currency exchange, perimeter, area, volume, capacity, the Pythagorean theorem, and trigonometric ratios.

This is not an outcome that can be taught in isolation but must be integrated as a foundational concept within each of the units in the course.

Examples of formulas that students will be expected to manipulate are as follows:

Concept	Formula
Converting Fahrenheit temperature to Celsius	$\mathrm{C} = \frac{5}{9}(\mathrm{F} - 32)$
Converting Celsius temperature to Fahrenheit	$\mathrm{F} = \frac{9}{5}(\mathrm{C} + 32)$
Perimeter of a rectangle	$P = 2l + 2w$ or $P = 2(l + w)$
Circumference of a circle	$C = 2\pi r$ or $C = \pi d$
Area of a rectangle	$A = lw$
Area of a triangle	$A = \frac{bh}{2}$
Area of a circle	$A = \pi r^2$
Surface area of a rectangular prism	$SA = 2(lw + lh + wh)$
Surface area of a cylinder	$SA = 2\pi r^2 + 2\pi rh$
Pythagorean theorem	$c^2 = a^2 + b^2$
Sine of angle A	$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$
Cosine of angle A	$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$
Tangent of angle A	$\tan A = \frac{\text{opposite}}{\text{adjacent}}$
Gross salary	$S = (\text{hourly rate}) \times (\text{hours worked})$

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Student tasks such as the following could be completed to assist in determining students' prior knowledge. Again, these should not be completed in isolation but as part of the other units of this course.

- Model the solution of a given linear equation, using concrete or pictorial representations, and record the process.
- Determine, by substitution, whether a given rational number is a solution to a given linear equation.
- Solve a given linear equation symbolically.
- Identify and correct an error in a given incorrect solution of a linear equation.
- Represent a given problem using a linear equation.
- Solve a given problem using a linear equation and record the process.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

- **Journal:** In everyday life, when do you think you will be working with perimeter? Do you think you will need to estimate or calculate more frequently? Explain your reasoning.

- Given different shapes, estimate the area of the shapes using square centimetres and square inches. Overlay the shapes on the centimetre and inch grid papers to determine the area.

- Given the area of the top of an engine piston is 25.43 cm^2, determine the diameter of the piston.

- How could you ensure a wall is perpendicular (square) to the floor if the only tool available is a measuring tape.

- Mattayo is laying out forms for the cement footings of a house. The house is to be 36 feet by 48 feet. Determine how long the diagonal of the rectangle should be to ensure there will be right angles at the corners of the house.

- Verify that a quilting square is actually square.

- Yurii is building a garage on a floor that measures 18 feet by 24 feet.
  (a) Calculate the length of the diagonal of the rectangular floor.
  (b) Yurii measures the length of the diagonal to be 29.5 feet. Are the angles at the corners of the garage right angles? Explain.

- Answer the following:
  (a) Rhys is paid $12.50 per hour for 40 hours a week. If he works more than 40 hours per week, he makes time and a half. If Rhys worked 52 hours this week, what would be his gross earnings?
  (b) Hannah is a hairstylist who works for a base hourly wage of $12. She works 35 hours per week and receives $160.50 in tips for the week. Calculate her gross pay.

- Answer the following:
  (a) A tire has a radius of 12 inches. What is its circumference?
  (b) The circumference of a CD is 28.26 cm. What is its diameter?
  (c) A rectangular window frame measures 24 inches by 36 inches. If trim for the window costs $2.75 per linear foot, how much will it cost to put trim around the window?
  (d) A box has a height of 30 cm, a width of 20 cm, and a length of 60 cm. Determine if this box can be shipped via Canada Post.
  (e) Compare and contrast the terms **perimeter** and **circumference**. When would it be appropriate to use each?

FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should inform instruction.

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SUGGESTED LEARNING TASKS

Effective instruction should consist of various strategies.

Guiding Question
- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

Consider the following sample instructional strategies when planning lessons.

SUGGESTED MODELS AND MANIPULATIVES

- calculator

MATHEMATICAL VOCABULARY

Students need to be comfortable using the following vocabulary.

- area
- circumference
- formula
- linear equation
- perimeter
- substitution
- verify

Resources/Notes

Print

Math at Work 10 (Etienne et al. 2011)

- Chapter 2: Measuring Length
  - Sections 2.1–2.2, 2.4
  - 2.4 Puzzler
  - Skill Check
  - Test Yourself
  - Games and Puzzles
- Chapter 3: Measuring Area
  - Sections 3.1, 3.2, 3.3, and 3.4
  - Skill Check
  - Test Yourself
  - Chapter Project
  - Games and Puzzles
- Chapter 4: Getting Paid for Your Work
  - Sections 4.1 and 4.2
  - Skill Check
  - Test Yourself
  - Chapter Project
- Chapter 6: Pythagorean Relationship
  - Sections 6.2 and 6.3
  - Skill Check
  - Test Yourself
  - Chapter Project
- Chapter 7: Trigonometry
  - Get Ready
  - Sections 7.1, 7.2, 7.3, and 7.4
  - Skill Check
  - Test Yourself
  - Chapter Project
  - Games and Puzzles

Notes

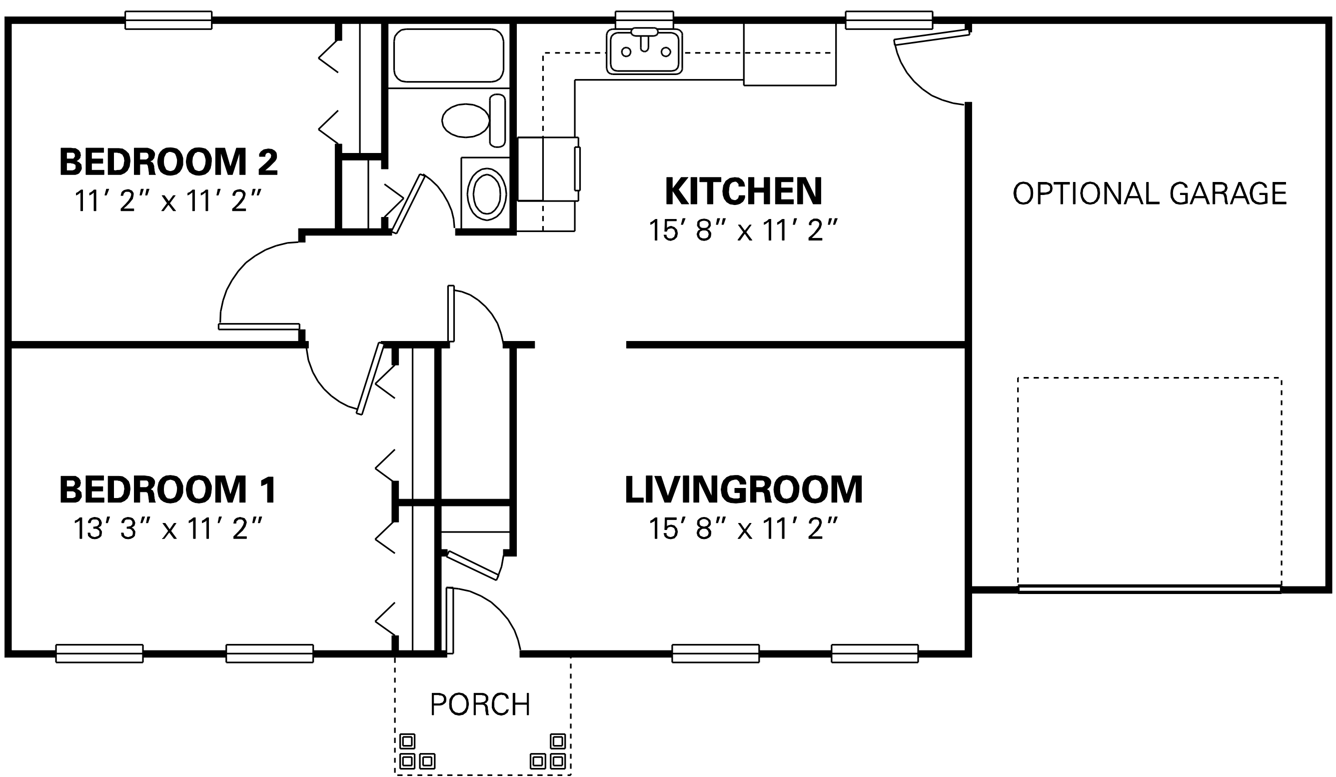
Appendices

Appendix A: Copyable Pages

A.1 Flooring Installation Project A

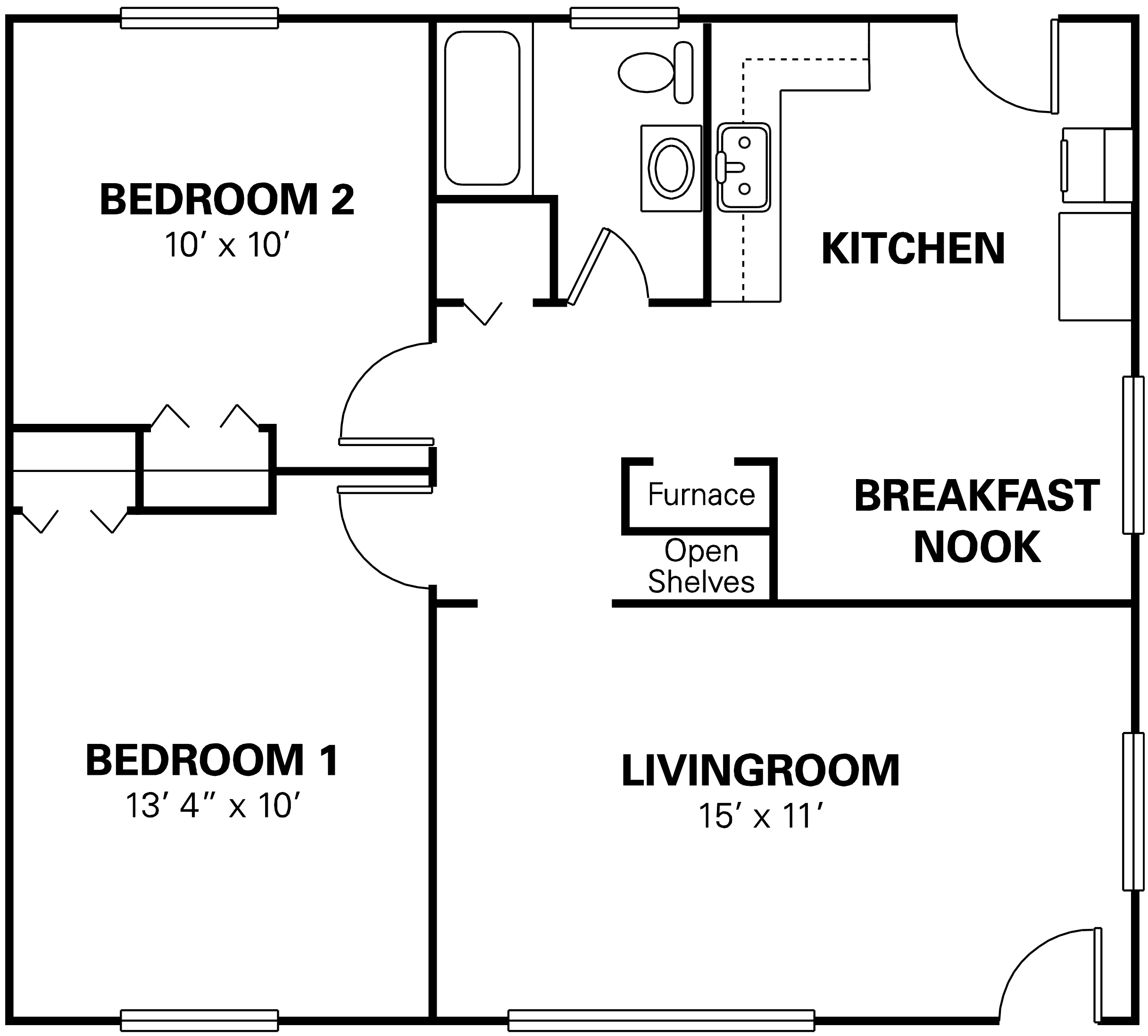
Project idea: Assume the role of owner of a contracting company that installs flooring. Develop a proposal for a bid on a new house that is being built. Using the measurements in the floor plans, determine the amount of the bid (including profit) if
(a) each bedroom is to have carpet ($9.99 per square yard*)
(b) the living room has hardwood flooring ($6.99 per square foot*)
(c) the kitchen/dining room has ceramic tile ($3.99 per square foot*)

* above prices include installation costs

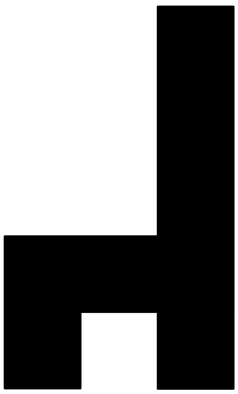


A.2 Flooring Installation Project B

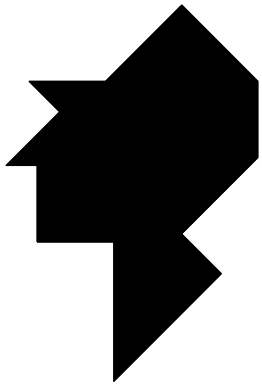
Alternatively, determine the cost of flooring for the living room and bedrooms based on the following floor plan.



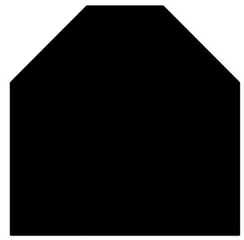
A.3 Tangram Puzzles



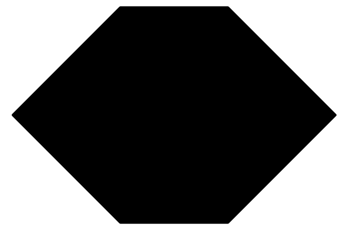
Chair



Head

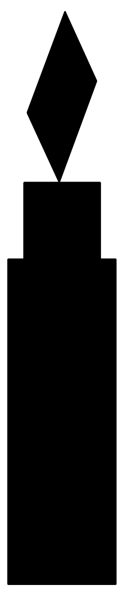


Barn

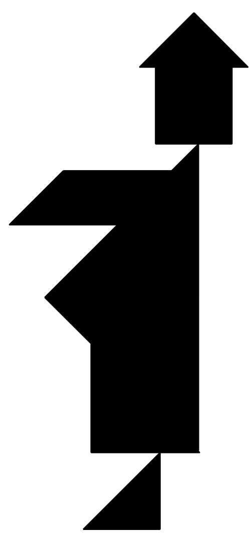


Hexagon

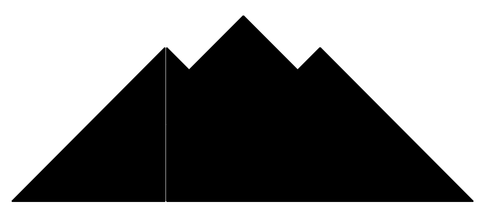
Right Triangle



Candle



Sleepwalker



Mountain

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