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July 2014

Mathematics 6

CURRICULUM GUIDE

NOVA SCOTIA

Mathematics 6

Implementation Draft
July 2014

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Mathematics 6, Implementation Draft

Prepared by the Nova Scotia Department of Education and Early Childhood Development

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Contents

Introduction

Background and Rationale

Mathematics curriculum is shaped by a vision that fosters the development of mathematically literate students who can extend and apply their learning and who are effective participants in society. It is essential that the mathematics curriculum reflect current research in mathematics instruction. To achieve this goal, Western and Northern Canadian Protocol's (WNCP) *The Common Curriculum Framework for K–9 Mathematics* (2006) has been adopted as the basis for the new mathematics curriculum in Nova Scotia.

The Common Curriculum Framework (WNCP 2006) was developed by the seven ministries of education (Alberta, British Columbia, Manitoba, Northwest Territories, Nunavut, Saskatchewan, and Yukon Territory) in collaboration with teachers, administrators, parents, business representatives, post-secondary educators, and others. The framework identifies beliefs about mathematics, general and specific student outcomes, and performance indicators agreed upon by the seven jurisdictions. The outcomes and performance indicators have been adapted for Nova Scotia. This document is based on both national and international research by the WNCP and the National Council of Teachers of Mathematics (NCTM).

There is an emphasis in the Nova Scotia curriculum on particular key concepts at each grade that will result in greater depth of understanding and, ultimately, stronger student achievement. There is also a greater emphasis on number sense and operations concepts in the early grades to ensure students develop a solid foundation in numeracy.

Purpose

This document provides sets of outcomes and performance indicators to be used as a mandated common base for defining mathematics curriculum expectations. This common base should result in consistent student outcomes in mathematics within the province of Nova Scotia. It should also enable easier transfer for students moving within the province or from any jurisdiction that has adopted the WNCP framework. This document is intended to clearly communicate to all education partners across the province the high expectations for students' mathematical learning.

Program Design and Components

Assessment

Ongoing assessment for learning is essential to effective teaching and learning. Research has shown that assessment for learning (formative assessment) practices produce significant and often substantial learning gains, close achievement gaps, and build students' ability to learn new skills (Black & Wiliam 1998; OECD 2006). Student involvement in assessment promotes learning. Timely and effective teacher feedback and student self-assessment allow students to reflect on and articulate their understanding of mathematical concepts and ideas.

Assessment in the classroom includes

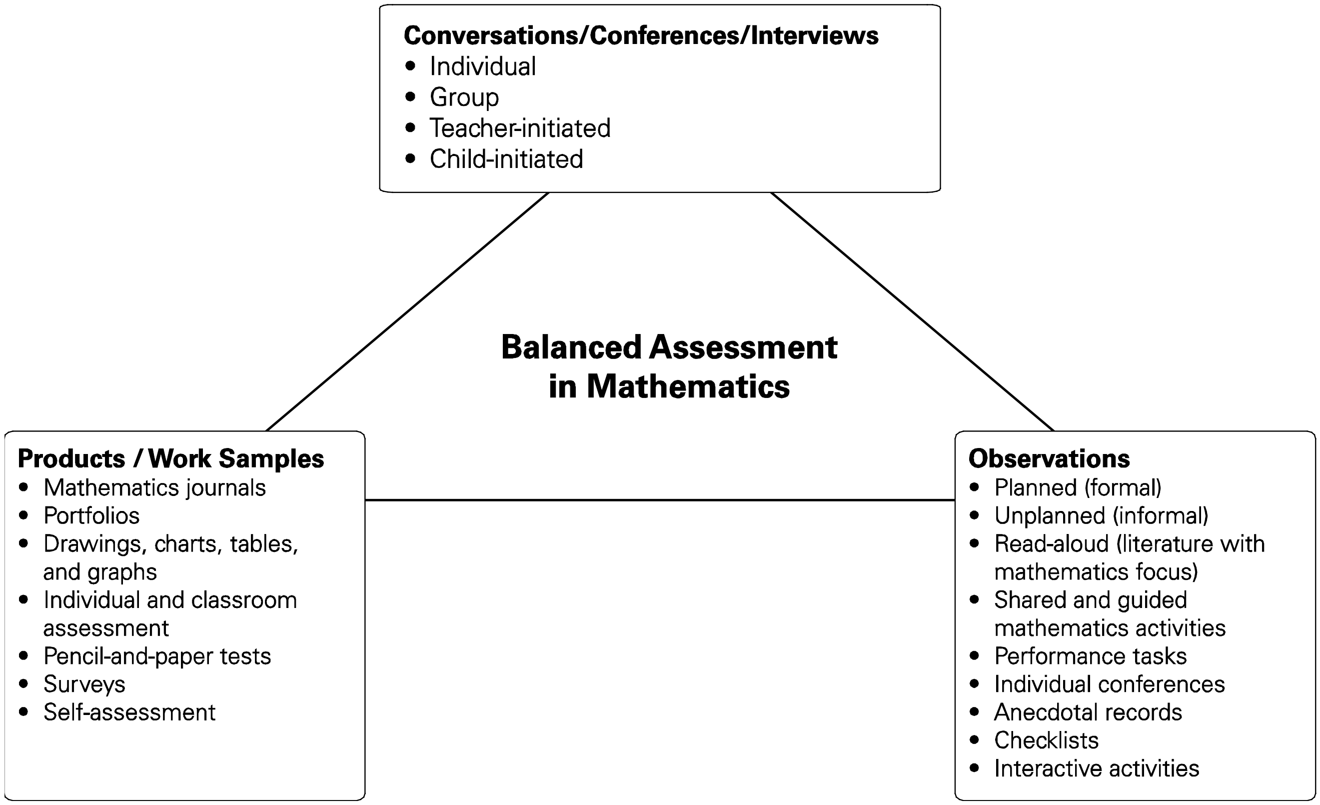
- providing clear goals, targets, and learning outcomes
- using exemplars, rubrics, and models to help clarify outcomes and identify important features of the work
- monitoring progress towards outcomes and providing feedback as necessary
- encouraging self-assessment
- fostering a classroom environment where conversations about learning take place, where students can check their thinking and performance and develop a deeper understanding of their learning

(Davies 2000)

Assessment for learning practices act as the scaffolding for learning, which only then can be measured through assessment of learning (summative assessment). Assessment of learning tracks student progress, informs instructional programming, and aids in decision making. Both forms of assessment are necessary to guide teaching, stimulate learning, and produce achievement gains.

Assessment of student learning should

- align with curriculum outcomes
- clearly define criteria for success
- make explicit the expectations for students' performance
- use a wide variety of assessment strategies and tools
- yield useful information to inform instruction



Time to Learn for Mathematics

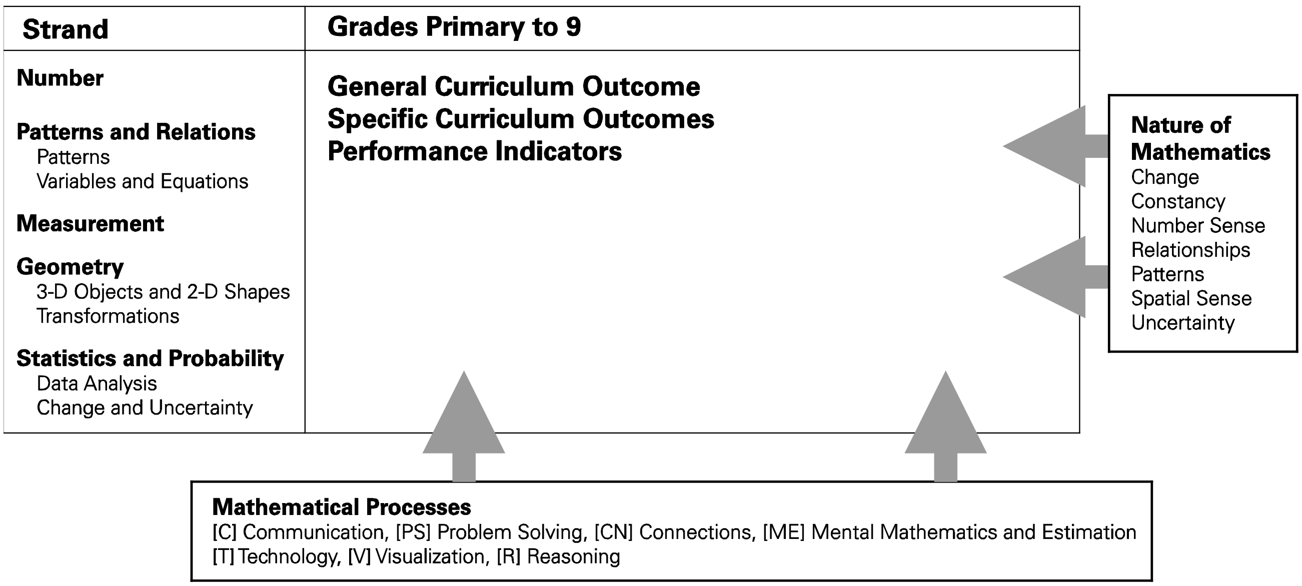
The *Time to Learn Strategy: Guidelines for Instructional Time, Grades Primary*–6 (Nova Scotia Department of Education 2002) includes time for mathematics instruction in the "Required Each Day" section. In order to support a constructivist approach to teaching through problem solving, it is highly recommended that the 45 minutes required daily in grades primary–2 and the 60 minutes required daily for grades 3–6 mathematics instruction be provided in an uninterrupted block of time.

Time to Learn guidelines can be found at
www.ednet.ns.ca/files/ps-policies/semestering.pdf
www.ednet.ns.ca/files/ps-policies/instructional_time_guidelines_p-6.pdf

Outcomes

Conceptual Framework for Mathematics Primary–9

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



(Adapted with permission from Western and Northern Canadian Protocol, *The Common Curriculum Framework for K–9 Mathematics*, 2006, p. 5. All rights reserved.)

Structure of the Mathematics Curriculum

Strands

The learning outcomes in the Nova Scotia Framework are organized into five strands across grades primary to 9.

- Number (N)
- Patterns and Relations (PR)
- Measurement (M)
- Geometry (G)
- Statistics and Probability (SP)

General Curriculum Outcomes (GCO)

Some strands are further subdivided into sub-strands. There is one general outcome (GCO) per sub-strand. GCOs are overarching statements about what students are expected to learn in each strand/sub-strand. The general curriculum outcome for each strand/sub-strand is the same throughout the grades.

Number (N)

GCO: Students will be expected to demonstrate number sense.

Patterns and Relations (PR)

Patterns
GCO: Students will be expected to use patterns to describe the world and solve problems.

Variables and Equations
GCO: Students will be expected to represent algebraic expressions in multiple ways.

Measurement (M)

GCO: Students will be expected to use direct and indirect measure to solve problems.

Geometry (G)

3-D Objects and 2-D Shapes
GCO: Students will be expected to describe the characteristics of 3-D objects and 2-D shapes and analyze the relationships among them.

Transformations
GCO: Students will be expected to describe and analyze position and motion of objects and shapes.

Statistics and Probability (SP)

Data Analysis
GCO: Students will be expected to collect, display, and analyze data to solve problems.

Chance and Uncertainty
GCO: Students will be expected to use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

Specific Curriculum Outcomes (SCOs) and Performance Indicators

Specific curriculum outcomes (SCOs) are statements that identify the specific conceptual understanding, related skills, and knowledge students are expected to attain by the end of a given grade.

Performance indicators are statements that identify specific expectations of the depth, breadth, and expectations for the outcome. Teachers use these statements to determine whether students have achieved the corresponding specific curriculum outcome.

Process Standards Key

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

NUMBER (N)

N01 Students will be expected to demonstrate an understanding of place value for numbers greater than one million and less than one-thousandth. [C, CN, R, T]

Performance Indicators

N01.01 Explain how the pattern of the place-value system (e.g., the repetition of ones, tens, and hundreds) makes it possible to read and write numerals for numbers of any magnitude.
N01.02 Describe the pattern of adjacent place positions moving from right to left and from left to right.
N01.03 Represent a given numeral using a place-value chart.
N01.04 Explain the meaning of each digit in a given numeral.
N01.05 Read a given numeral in several ways.
N01.06 Record, in standard form, numbers expressed orally, concretely, pictorially, or symbolically as expressions, in decimal notation, and in expanded notation, using proper spacing without commas.
N01.07 Express a given numeral in expanded notation and/or in decimal notation.
N01.08 Represent a given number using expressions.
N01.09 Represent a given number in a variety of ways, and explain how they are equivalent.
N01.10 Read and write given numerals in words.
N01.11 Compare and order numbers in a variety of ways.
N01.12 Establish personal referents for large numbers.
N01.13 Provide examples of where large whole numbers and small decimal numbers are used.

N02 Students will be expected to solve problems involving whole numbers and decimal numbers. [ME, PS, T]

Performance Indicators

N02.01 Determine whether technology, mental mathematics, or paper-and-pencil calculation is appropriate to solve a given problem and explain why.
N02.02 Identify which operation is necessary to solve a given problem and solve it.
N02.03 Determine the reasonableness of an answer.
N02.04 Estimate the solution and solve a given problem using an appropriate method (technology, mental mathematics, or paper-and-pencil calculation).
N02.05 Create problems involving large numbers and decimal numbers.

N02.06 Use technology, mental mathematics, or paper-and-pencil calculation to solve problems involving the addition, subtraction, multiplication, and division of whole numbers.
N02.07 Use technology, mental mathematics, or paper-and-pencil calculation to solve problems involving the addition and subtraction of decimal numbers.

N03 Students will be expected to demonstrate an understanding of factors and multiples by

- determining multiples and factors of numbers less than 100
- identifying prime and composite numbers
- solving problems using multiples and factors [PS, R, V]

Performance Indicators

N03.01 Identify multiples for a given number and explain the strategy used to identify them.
N03.02 Determine all the whole number factors of a given number using arrays.
N03.03 Identify the factors for a given number and explain the strategy used (e.g., concrete or visual representations, repeated division by prime numbers, or factor trees).
N03.04 Provide an example of a prime number, and explain why it is a prime number.
N03.05 Provide an example of a composite number, and explain why it is a composite number.
N03.06 Sort a given set of numbers as prime and composite.
N03.07 Solve a given problem involving factors or multiples.
N03.08 Explain why 0 and 1 are neither prime nor composite.

N04 Students will be expected to relate improper fractions to mixed numbers and mixed numbers to improper fractions. [CN, ME, R, V]

Performance Indicators

N04.01 Demonstrate, using models, that a given improper fraction represents a number greater than 1.
N04.02 Express improper fractions as mixed numbers.
N04.03 Express mixed numbers as improper fractions.
N04.04 Place a given set of fractions, including mixed numbers and improper fractions, on a number line, and explain strategies used to determine position.
N04.05 Represent a given improper fraction using concrete, pictorial, and symbolic forms.
N04.06 Represent a given mixed number using concrete, pictorial, and symbolic forms.

N05 Students will be expected to demonstrate an understanding of ratio, concretely, pictorially, and symbolically. [C, CN, PS, R, V]

Performance Indicators

N05.01 Represent a given ratio concretely and pictorially.
N05.02 Write a ratio from a given concrete or pictorial representation.
N05.03 Express a given ratio in multiple forms, such as “three to five,” 3:5, 3 to 5, or $\frac{3}{5}$.
N05.04 Identify and describe ratios from real-life contexts and record them symbolically.
N05.05 Explain the part-whole and part-part ratios of a set (e.g., For a group of three girls and five boys, explain the ratios 3:5, 3:8, and 5:8.).
N05.06 Solve a given problem involving ratio.
N05.07 Verify that two ratios are or are not equivalent using concrete materials.

N06 Students will be expected to demonstrate an understanding of percent (limited to whole numbers) concretely, pictorially, and symbolically. [C, CN, PS, R, V]

Performance Indicators

N06.01 Explain that “percent” means “out of 100.”
N06.02 Explain that percent is a ratio out of 100.
N06.03 Represent a given percent concretely and pictorially.
N06.04 Record the percent displayed in a given concrete or pictorial representation.
N06.05 Express a given percent as a fraction and a decimal.
N06.06 Identify and describe percent from real-life contexts, and record them symbolically.
N06.07 Solve a given percent problem involving benchmarks of 25%, 50%, 75%, and 100%.

N07 Students will be expected to demonstrate an understanding of integers contextually, concretely, pictorially, and symbolically. [C, CN, R, V]

Performance Indicators

N07.01 Extend a given number line by adding numbers less than 0 and explain the pattern on each side of 0.
N07.02 Place given integers on a number line and explain how integers are ordered.
N07.03 Describe contexts in which integers are used (e.g., on a thermometer).
N07.04 Compare two integers; represent their relationship using the symbols <, >, and =; and verify using a number line.
N07.05 Order given integers in ascending or descending order.

N08 Students will be expected to demonstrate an understanding of multiplication and division of decimals (one-digit whole number multipliers and one-digit natural number divisors). [C, CN, ME, PS, R, V]

Performance Indicators

N08.01 Model the multiplication and division of decimals using concrete and visual representations.
N08.02 Predict products and quotients of decimals using estimation strategies.
N08.03 Place the decimal point in a product using front-end estimation (e.g., For 15.205 × 4, think 15 × 4, so the product is greater than 60.).
N08.04 Place the decimal point in a quotient using front-end estimation (e.g., For \$25.83 ÷ 4, think \$24 ÷ 4, so the quotient is greater than \$6.).
N08.05 Use estimation to correct errors of decimal point placement in a given product or quotient without using paper and pencil.
N08.06 Create and solve story problems that involve multiplication and division of decimals using multipliers from 0 to 9 and divisors from 1 to 9.
N08.07 Solve a given problem, using a personal strategy, and record the process symbolically.

N09 Students will be expected to explain and apply the order of operations, excluding exponents, with and without technology (limited to whole numbers). [CN, ME, PS, T]

Performance Indicators

N09.01 Demonstrate and explain, with examples, why there is a need to have a standardized order of operations.
N09.02 Apply the order of operations to solve multi-step problems with or without technology (e.g., computer, calculator).

PATTERNS AND RELATIONS (PR)

PR01 Students will be expected to demonstrate an understanding of the relationships within tables of values to solve problems. [C, CN, ME, PS, R, V]

Performance Indicators

PR01.01 Generate values in one column of a table of values, given values in the other column, and a pattern rule.
PR01.02 State, using mathematical language, the relationship in a given table of values.
PR01.03 Create a concrete or pictorial representation of the relationship shown in a table of values.
PR01.04 Predict the value of an unknown term using the relationship in a table of values, and verify the prediction.
PR01.05 Formulate a rule to describe the relationship between two columns of numbers in a table of values.
PR01.06 Identify missing terms in a given table of values.
PR01.07 Identify errors in a given table of values.
PR01.08 Describe the pattern within each column of a given table of values.
PR01.09 Create a table of values to record and reveal a pattern to solve a given problem.

PR02 Students will be expected to represent and describe patterns and relationships, using graphs and tables. [C, CN, PS, R]

Performance Indicators

PR02.01 Translate a pattern to a table of values, and graph the table of values (limited to linear graphs with discrete elements).
PR02.02 Create a table of values from a given pattern or a given graph.
PR02.03 Describe, using everyday language, orally or in writing, the relationship shown on a graph.

PR03 Students will be expected to represent generalizations arising from number relationships using equations with letter variables. [C, CN, PS, R, V]

Performance Indicators

PR03.01 Write and explain the formula for finding the perimeter of any regular polygon.
PR03.02 Write and explain the formula for finding the area of any given rectangle.
PR03.03 Develop and justify equations using letter variables that illustrate the commutative property of addition and multiplication (e.g., $a + b = b + a$ or $a \times b = b \times a$).
PR03.04 Describe the relationship in a given table using a mathematical expression.
PR03.05 Represent a pattern rule using a simple mathematical expression, such as $4d$ or $2n + 1$.

PR04 Students will be expected to demonstrate and explain the meaning of preservation of equality concretely, pictorially, and symbolically. [C, CN, PS, R, V]

Performance Indicators

PR04.01 Model the preservation of equality for addition using concrete materials, such as a balance, or using pictorial representations, and orally explain the process.

PR04.02 Model the preservation of equality for subtraction using concrete materials, such as a balance, or using pictorial representations, and orally explain the process.

PR04.03 Model the preservation of equality for multiplication using concrete materials, such as a balance, or using pictorial representations, and orally explain the process.

PR04.04 Model the preservation of equality for division using concrete materials, such as a balance, or using pictorial representations, and orally explain the process.

PR04.05 Write equivalent forms of a given equation by applying the preservation of equality and verify using concrete materials (e.g., $3b = 12$ is the same as $3b + 5 = 12 + 5$ or $2r = 7$ is the same as $3(2r) = 3(7)$).

Measurement (M)

M01 Students will be expected to demonstrate an understanding of angles by

- identifying examples of angles in the environment
- classifying angles according to their measure
- estimating the measure of angles using 45°, 90°, and 180° as reference angles
- determining angle measures in degrees
- drawing and labelling angles when the measure is specified

[C, CN, ME, V]

Performance Indicators

M01.01 Identify examples of angles found in the environment.

M01.02 Classify a given set of angles according to their measure (e.g., acute, right, obtuse, straight, reflex).

M01.03 Sketch 45°, 90°, and 180° angles without the use of a protractor, and describe the relationship among them.

M01.04 Estimate the measure of an angle using 45°, 90°, and 180° as reference angles.

M01.05 Measure, using a protractor, given angles in various orientations.

M01.06 Draw and label a specified angle in various orientations using a protractor.

M01.07 Describe the measure of an angle as the measure of rotation of one of its sides.

M01.08 Describe the measure of angles as the measure of an interior angle of a polygon.

M02 Students will be expected to demonstrate that the sum of interior angles is 180° in a triangle and 360° in a quadrilateral. [C, R]

Performance Indicators

M02.01 Explain, using models, that the sum of the interior angles of a triangle is the same for all triangles.

M02.02 Explain, using models, that the sum of the interior angles of a quadrilateral is the same for all quadrilaterals.

M03 Students will be expected to develop and apply a formula for determining the
- perimeter of polygons
- area of rectangles
- volume of right rectangular prisms

[C, CN, PS, R, V]

Performance Indicators

M03.01 Explain, using models, how the perimeter of any polygon can be determined.
M03.02 Generalize a rule (formula) for determining the perimeter of polygons.
M03.03 Explain, using models, how the area of any rectangle can be determined.
M03.04 Generalize a rule (formula) for determining the area of rectangles.
M03.05 Explain, using models, how the volume of any rectangular prism can be determined.
M03.06 Generalize a rule (formula) for determining the volume of rectangular prisms.
M03.07 Solve a given problem involving the perimeter of polygons, the area of rectangles, and/or the volume of right rectangular prisms.

GEOMETRY (G)

G01 Students will be expected to construct and compare triangles, including scalene, isosceles, equilateral, right, obtuse, or acute in different orientations. [C, PS, R, V]

Performance Indicators

G01.01 Sort a given set of triangles according to the length of the sides.
G01.02 Sort a given set of triangles according to the measures of the interior angles.
G01.03 Identify the characteristics of a given set of triangles according to their sides and/or their interior angles.
G01.04 Sort a given set of triangles and explain the sorting rule.
G01.05 Draw a specified triangle.
G01.06 Replicate a given triangle in a different orientation and show that the two are congruent.

G02 Students will be expected to describe and compare the sides and angles of regular and irregular polygons. [C, PS, R, V]

Performance Indicators

G02.01 Sort a given set of 2-D shapes into polygons and non-polygons and explain the sorting rule.
G02.02 Demonstrate congruence (sides to sides and angles to angles) in a regular polygon by superimposing.
G02.03 Demonstrate congruence (sides to sides and angles to angles) in a regular polygon by measuring.
G02.04 Demonstrate that the sides of a regular polygon are the same length and that the angles of a regular polygon are the same measure.
G02.05 Sort a given set of polygons as regular or irregular and justify the sorting.
G02.06 Identify and describe regular and irregular polygons in the environment.

G03 Students will be expected to perform a combination of translation(s), rotation(s), and/or reflection(s) on a single 2-D shape, with and without technology, and draw and describe the image. [C, CN, PS, T, V]

Performance Indicators

G03.01 Demonstrate that a 2-D shape and its transformation image are congruent.
G03.02 Model a given set of successive translations, successive rotations, or successive reflections of a 2-D shape.
G03.03 Model a given combination of two different types of transformations of a 2-D shape.
G03.04 Draw and describe a 2-D shape and its image, given a combination of transformations.
G03.05 Describe the transformations performed on a 2-D shape to produce a given image.
G03.06 Model a given set of successive transformations (translation, rotation, or reflection) of a 2-D shape.
G03.07 Perform and record one or more transformations of a 2-D shape that will result in a given image.

G04 Students will be expected to perform a combination of successive transformations of 2-D shapes to create a design and identify and describe the transformations. [C, CN, T, V]

Performance Indicators

G04.01 Analyze a given design created by transforming one or more 2-D shapes, and identify the original shape and the transformations used to create the design.
G04.02 Create a design using one or more 2-D shapes and describe the transformations used.
G04.03 Describe why a shape may or may not tessellate.
G04.04 Create a tessellation and describe how tessellations are used in the real world.

G05 Students will be expected to identify and plot points in the first quadrant of a Cartesian plane using whole number ordered pairs. [C, CN, V]

Performance Indicators

G05.01 Label the axes of the first quadrant of a Cartesian plane and identify the origin.
G05.02 Plot a point in the first quadrant of a Cartesian plane given its ordered pair.
G05.03 Match points in the first quadrant of a Cartesian plane with their corresponding ordered pair.
G05.04 Plot points in the first quadrant of a Cartesian plane with intervals of 1, 2, 5, or 10 on its axes, given whole number ordered pairs.
G05.05 Draw shapes or designs in the first quadrant of a Cartesian plane, using given ordered pairs.
G05.06 Determine the distance between points along horizontal and vertical lines in the first quadrant of a Cartesian plane.
G05.07 Draw shapes or designs in the first quadrant of a Cartesian plane, and identify the points used to produce them.

G06 Students will be expected to perform and describe single transformations of a 2-D shape in the first quadrant of a Cartesian plane (limited to whole number vertices). [C, CN, PS, T, V]

Performance Indicators

G06.01 Identify the coordinates of the vertices of a given 2-D shape (limited to the first quadrant of a Cartesian plane).
G06.02 Perform a transformation on a given 2-D shape, and identify the coordinates of the vertices of the image (limited to the first quadrant).
G06.03 Describe the positional change of the vertices of a given 2-D shape to the corresponding vertices of its image as a result of a transformation (limited to first quadrant).

Statistics and Probability (SP)

SP01 Students will be expected to create, label, and interpret line graphs to draw conclusions. [C, CN, PS, R, V]

Performance Indicators

SP01.01 Determine the common attributes (title, axes, and intervals) of line graphs by comparing a given set of line graphs.
SP01.02 Determine whether a given set of data can be represented by a line graph (continuous data) or a series of points (discrete data) and explain why.
SP01.03 Create a line graph from a given table of values or a set of data.
SP01.04 Interpret a given line graph to draw conclusions.

SP02 Students will be expected to select, justify, and use appropriate methods of collecting data, including questionnaires, experiments, databases, and electronic media. [C, PS, T]

Performance Indicators

SP02.01 Select a method for collecting data to answer a given question, and justify the choice.
SP02.02 Design and administer a questionnaire for collecting data to answer a given question, and record the results.
SP02.03 Answer a given question by performing an experiment, recording the results, and drawing a conclusion.
SP02.04 Explain when it is appropriate to use a database as a source data.
SP02.05 Gather data for a given question by using electronic media, including selecting data from databases.

SP03 Students will be expected to graph collected data and analyze the graph to solve problems. [C, CN, PS]

Performance Indicators

SP03.01 Determine an appropriate type of graph for displaying a set of collected data and justify the choice of graph.
SP03.02 Solve a given problem by graphing data and interpreting the resulting graph.

SP04 Students will be expected to demonstrate an understanding of probability by
- identifying all possible outcomes of a probability experiment
- differentiating between experimental and theoretical probability
- determining the theoretical probability of outcomes in a probability experiment
- determining the experimental probability of outcomes in a probability experiment
- comparing experimental results with the theoretical probability for an experiment

[C, ME, PS, T]

Performance Indicators

SP04.01 List the possible outcomes of a probability experiment, such as
- tossing a coin
- rolling a die with a given number of sides
- spinning a spinner with a given number of sectors

SP04.02 Determine the theoretical probability of an outcome occurring for a given probability experiment.

SP04.03 Predict the probability of a given outcome occurring for a given probability experiment by using theoretical probability.

SP04.04 Conduct a probability experiment, with or without technology, and compare the experimental results to the theoretical probability.

SP04.05 Explain that as the number of trials in a probability experiment increases, the experimental probability approaches the theoretical probability of a particular outcome.

SP04.06 Distinguish between theoretical probability and experimental probability, and explain the differences.

Mathematical Processes

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics.

Students are expected to
- communicate in order to learn and express their understanding of mathematics (Communication [C])
- develop and apply new mathematical knowledge through problem solving (Problem Solving [PS])
- connect mathematical ideas to other concepts in mathematics, to everyday experiences, and to other disciplines (Connections [CN])
- demonstrate fluency with mental mathematics and estimation (Mental Mathematics and Estimation [ME])
- select and use technologies as tools for learning and solving problems (Technology [T])
- develop visualization skills to assist in processing information, making connections, and solving problems (Visualization [V])
- develop mathematical reasoning (Reasoning [R])

The Nova Scotia curriculum incorporates these seven interrelated mathematical processes that are intended to permeate teaching and learning. The key to these process standards is presented in a box, as shown below, with each specific curriculum outcome within the strands.

Process Standards Key

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics. Communication is important in clarifying, reinforcing, and modifying ideas, knowledge, attitudes, and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics.

Students also need to communicate their learning using mathematical terminology. Communication can help students make connections between and among the different representational modes—contextual, concrete, pictorial, linguistic/verbal, and symbolic—of mathematical ideas. Students must communicate *daily* about their mathematics learning. This enables them to reflect, to validate, and to clarify their thinking and provides teachers with insight into students' interpretations of mathematical meanings and ideas.

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, How would you ...? or How could you ...? the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

In order for a task to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement, perseverance, and collaboration.

Problem solving is also a powerful teaching tool that fosters multiple, creative, and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive, mathematical risk takers.

When students are exposed to a wide variety of problems in all areas of mathematics, they explore various methods for solving and verifying problems. In addition, they are challenged to find multiple solutions for problems and to create their own problem.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to one another or to real-world phenomena, students can begin to view mathematics as useful, relevant, and integrated. Learning mathematics within contexts and making connections relevant to learners can validate past experiences and increase student willingness to participate and be actively engaged. The brain is constantly looking for and making connections.

"Because the learner is constantly searching for connections on many levels, educators need to *orchestrate the experiences* from which learners extract understanding. … *Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching*." (Caine and Caine 1991, 5).

Mathematics should be viewed as an integrated whole rather than as the study of separate strands or units. Connections must also be made between and among the different representational modes—contextual, concrete, pictorial, linguistic/verbal, and symbolic. The process of making connections, in turn, facilitates learning. Concepts and skills should also be connected to everyday situations and other curricular areas. For example, when developing literacy skills students learn to make text-to-world, text-to-text, and text-to-self connections. Students can also make connections to make mathematics come alive through math-to-world, math-to-math, and math-to-self connections.

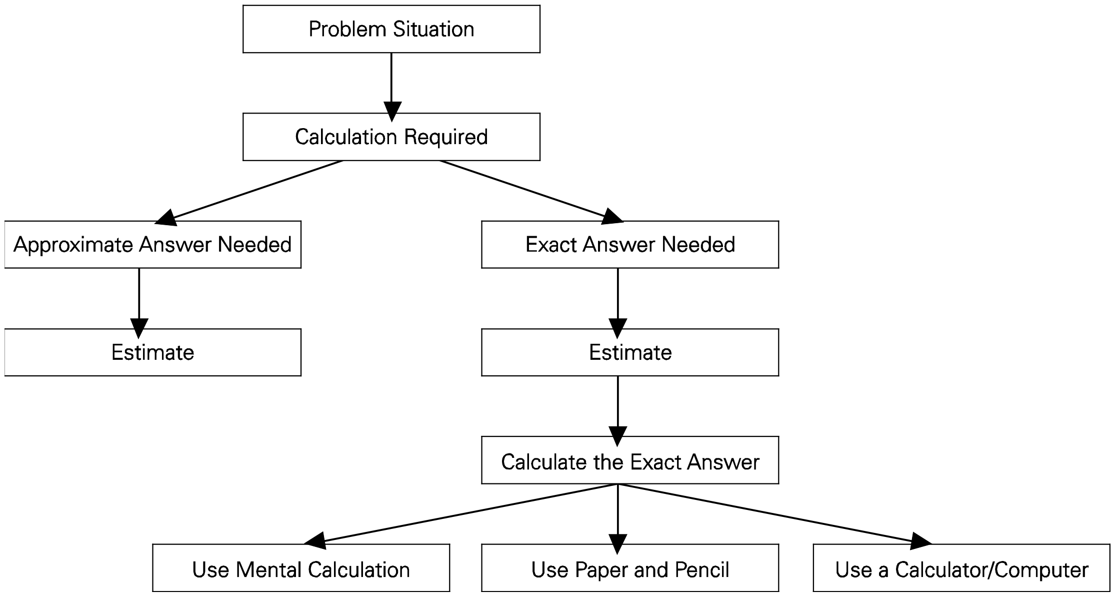
Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally without the use of external aids. Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy, and flexibility. "Even more important than performing computational procedures or using calculators is the greater facility that students need—more than ever before—with estimation and mental math." (National Council of Teachers of Mathematics, May 2005).

Students proficient with mental mathematics "become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers, and are more able to use multiple approaches to problem solving." (Rubenstein 2001) Mental mathematics "provides a cornerstone for all estimation processes, offering a variety of alternative algorithms and nonstandard techniques for finding answers." (Hope 1988, v)

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when, and what strategy to use when estimating. Estimation is used to make mathematical judgments and develop useful, efficient strategies for dealing with situations in daily life.

Students need to develop both mental mathematics and estimation skills through context and not in isolation so they are able to apply them to solve problems. Whenever a problem requires a calculation, students should follow the decision-making process as illustrated on the following page.



The skill of estimation requires a sound knowledge of mental mathematics. Both are necessary to many everyday experiences, and students should be provided with frequent opportunities to practise these skills.

Technology [T]

Technology can be effectively used to contribute to and support the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures, and solve problems. Information and communication technology best improves learning when it is accessible, flexible, responsive, participatory, and integrated thoroughly into all public school programs.

Technology can be used to

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- decrease the time spent on computations when other mathematical learning is the focus
- reinforce the learning of basic facts and test properties
- develop personal procedures for mathematical operations
- create geometric displays
- simulate situations
- develop number sense

The use of calculators is recommended to enhance problem solving, to encourage discovery of number patterns, and to reinforce conceptual development and numerical relationships. Calculators do not, however, replace the development of number concepts and skills. Carefully chosen computer software can provide interesting problem-solving situations and applications.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. While technology can be used in grades primary–3 to enrich learning, it is expected that students will achieve all outcomes without the use of technology. *The Integration of Information and Communication Technology Within the Curriculum*, (Revision 2014) (P–6) can be found online in several locations including: http://lrt.ednet.ns.ca.

Visualization [V]

Visualization "involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world." (Armstrong 1999). The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number, spatial, and measurement sense. Number visualization occurs when students create mental representations of numbers. These mental images are needed to develop concepts and understand procedures. Images and explanations help students clarify their understanding of mathematical ideas in all strands.

Being able to create, interpret, and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure, when to estimate and to know several estimation strategies. (Shaw and Cliatt 1989)

Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics. Mathematical experiences in and out of the classroom provide opportunities for inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyze observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Mathematics reasoning involves informal thinking, conjecturing, and validating—these help students understand that mathematics makes sense. Students are encouraged to justify, in a variety of ways, their solutions, thinking processes, and hypotheses. In fact, good reasoning is as important as finding correct answers.

Nature of Mathematics

Mathematics is one way of trying to understand, interpret, and describe our world. There are a number of components that define the nature of mathematics, and these are woven throughout this document. These components include change, constancy, number sense, relationships, patterns, spatial sense, and uncertainty.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics. Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, … can be described as

- skip counting by 2s, starting from 4
- an arithmetic sequence, with first term 4 and a common difference of 2
- a linear function with a discrete domain

(Steen 1990, 184).

Constancy

Different aspects of constancy are described by the terms **stability**, **conservation**, **equilibrium**, **steady state**, and **symmetry** (AAAS–Benchmarks 1993, 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems. Examples of constancy include the following:

- The area of a rectangular region is the same regardless of the methods used to determine the solution.
- The sum of the interior angles of any triangle is 180°.
- The theoretical probability of flipping a coin and getting heads is 0.5.

Number Sense

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (British Columbia Ministry of Education 2000, 146). A true sense of number goes well beyond the skills of simply counting, memorizing facts, and the situational rote use of algorithms. Number sense develops when students connect numbers to real-life experiences and use benchmarks and referents. This results in students who are computationally fluent, flexible with numbers, and have intuition about numbers. The evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections.

Relationships

Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects, and concepts. The search for possible relationships involves the collection and analysis of data, and describing relationships visually, symbolically, orally, or in written form.

Patterns

Mathematics is about recognizing, describing, and working with numerical and non-numerical patterns. Patterns exist in all strands, and it is important that connections are made among strands. Working with patterns enables students to make connections within and beyond mathematics. These skills contribute to students' interaction with an understanding of their environment. Patterns may be represented in concrete, visual, or symbolic form. Students should develop fluency in moving from one representation to another. Students must learn to recognize, extend, create, and use mathematical patterns. Patterns allow students to make predictions and justify their reasoning when solving problems. Learning to work with patterns in the early grades helps develop students' algebraic thinking, which is foundational for working with more abstract mathematics in higher grades.

Spatial Sense

Spatial sense involves visualization, mental imagery, and spatial reasoning. These skills are central to the understanding of mathematics. Spatial sense enables students to reason and interpret among and between 3-D and 2-D representations and identify relationships to mathematical strands. Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 3-D objects and 2-D shapes. Spatial sense offers a way to interpret and reflect on the physical environment and its 3-D or 2-D representations. Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to make predictions about the results of changing these dimensions. For example,

- knowing the dimensions of an object enables students to communicate about the object and create representations
- the volume of a rectangular solid can be calculated from given dimensions
- doubling the length of the side of a square increases the area by a factor of four

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty. Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty. The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation. Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

Curriculum Document Format

This guide presents the mathematics curriculum so that a teacher may readily view the scope of the outcomes that students are expected to achieve during that year. Teachers are encouraged, however, to examine what comes before and what follows after, to better understand how students' learning at a particular grade level is part of a bigger picture of concept and skill development.

The order of presentation in no way assumes or prescribes a preferred order of presentation in the classroom, but simply lays out the specific curriculum outcomes in relation to the overarching general curriculum outcomes (GCOs).

The footer of the document shows the name of the course, and the strand name is presented in the header. When a specific curriculum outcome (SCO) is introduced, it is followed by the mathematical processes and performance indicators for that outcome. A scope and sequence is then provided, which relates the SCO to previous and next grade SCOs. Also for each SCO, there is background information, assessment strategies, suggested instructional strategies, suggested models and manipulatives, mathematical language, and a section for resources and notes. For each section, the guiding questions should be used to help with unit and lesson preparation.

SCO

Mathematical Processes
[C] Communication **[PS]** Problem Solving **[CN]** Connections
[ME] Mental Mathematics and Estimation
[T] Technology **[V]** Visualization **[R]** Reasoning

Performance Indicators

Describes observable indicators of whether students have achieved the specific outcome.

Scope and Sequence

Previous grade or course SCOs	Current grade SCO	Following grade or course SCOs

Background

Describes the “big ideas” to be learned and how they relate to work in previous grade and work in subsequent courses.

Additional Information

A reference to Appendix A, which contains further elaborations for the performance indicators.

Assessment, Teaching, and Learning

Assessment Strategies

Guiding Questions
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Sample tasks that can be used to determine students’ prior knowledge.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Some suggestions for specific activities and questions that can be used for both instruction and assessment

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcome and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Suggested strategies for planning daily lessons.

SUGGESTED LEARNING TASKS

Suggestions for general approaches and strategies suggested for teaching this outcome.

Guiding Questions
- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

SUGGESTED MODELS AND MANIPULATIVES

MATHEMATICAL LANGUAGE

Teacher and student mathematical language associated with the respective outcome.

Resources/Notes

Contexts for Learning and Teaching

Beliefs about Students and Mathematics Learning

“Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.” (National Council of Teachers of Mathematics 2000, 20).

The Nova Scotia mathematics curriculum is based upon several key assumptions or beliefs about mathematics learning that have grown out of research and practice. These beliefs include the following:

- Mathematics learning is an active and constructive process.
- Learning is most effective when standards of expectation are made clear with ongoing assessment and feedback.
- Learners are individuals who bring a wide range of prior knowledge and experiences and who learn via various styles and at different rates.
- Learning is most likely to occur when placed in meaningful contexts and in an environment that supports exploration, risk taking, and critical thinking and that nurtures positive attitudes and sustained effort.

Students are curious, active learners with individual interests, abilities, and needs. They come to classrooms with varying knowledge, life experiences, and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Students develop a variety of mathematical ideas before they enter school. Children make sense of their environment through observations and interactions at home and in the community. Mathematics learning is embedded in everyday activities, such as playing, reading, storytelling, and helping around the home. Such activities can contribute to the development of number and spatial sense in children. Curiosity about mathematics is fostered when children are engaged in activities such as comparing quantities, searching for patterns, sorting objects, ordering objects, creating designs, building with blocks, and talking about these activities. Positive early experiences in mathematics are as critical to child development as are early literacy experiences.

Students learn by attaching meaning to what they do and need to construct their own meaning of mathematics. This meaning is best constructed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of models and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with and translating through a variety of materials, tools, and contexts when constructing meaning about new mathematical ideas. Meaningful discussions can provide essential links among concrete, pictorial, contextual, and symbolic representations of mathematics.

The learning environment should value and respect all students’ experiences and ways of thinking, so that learners are comfortable taking intellectual risks, asking questions, and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must realize that it is acceptable to solve problems in different ways and that solutions may vary.

Goals for Mathematics Education

The main goals of mathematics education are to prepare students to
- use mathematics confidently to solve problems
- communicate and reason mathematically
- appreciate and value mathematics
- make connections between mathematics and its applications
- become mathematically literate adults, using mathematics to contribute to society

Students who have met these goals will
- gain understanding and appreciation of the contributions of mathematics as a science, a philosophy, and an art
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical tasks and projects
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity about mathematics and situations involving mathematics

Opportunities for Success

A positive attitude has a profound effect on learning. Environments that create a sense of belonging, encourage risk taking, and provide opportunities for student success help develop and maintain positive attitudes and self-confidence. Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, participate willingly in classroom activities, persist in challenging situations, and engage in reflective practices.

To experience success, students must be taught to set achievable goals or assess their progress as they work toward these goals. Striving toward success and becoming autonomous and responsible learners are ongoing, reflective processes that involve revisiting the setting and assessing of personal goals.

Engaging All Learners

"No matter how engagement is defined or which dimension is considered, research confirms this truism of education: *The more engaged you are, the more you will learn.*" (Hume 2011, 6)

Student engagement is at the core of learning. Engagement in learning occurs when students are provided with opportunities to become more invested in their learning. This is critical for teachers to take into account when planning and implementing instruction. Effective instruction engages, embraces, and supports all learners through a range of learning experiences that are both age and developmentally appropriate.

This curriculum is designed to provide learning opportunities that are equitable, accessible, and inclusive of the many facets of diversity represented in today's classrooms. When teachers know their students as individual learners and as individual people, their students are more likely to be motivated to learn, persist in challenging situations, and apply reflective practices.

SUPPORTIVE LEARNING ENVIRONMENTS

A supportive and positive learning environment has a profound effect on students' learning. Students need to feel physically, socially, emotionally, and culturally safe in order to take risks with their learning. In classrooms where students feel a sense of belonging, see their teachers' passion for learning and teaching, are encouraged to actively participate, and are challenged appropriately, they are more likely to be successful.

Teachers recognize that not all students progress at the same pace nor are they equally positioned in terms of their prior knowledge of particular concepts, skills, and learning outcomes. Teachers are able to create more equitable access to learning when

- instruction and assessment are flexible and offer multiple means of representation
- students have options to engage in learning through multiple ways
- students can express their knowledge, skills, and understanding in multiple ways

(Hall, Meyer, and Rose 2012)

In a supportive learning environment, teachers plan learning experiences that support *each* student's ability to achieve curriculum outcomes. Teachers use a variety of effective instructional approaches that help students to succeed, such as

- providing a range of learning opportunities that build on individual strengths and prior knowledge
- providing all students with equitable access to appropriate learning strategies, resources, and technology
- involving students in the creation of criteria for assessment and evaluation
- engaging and challenging students through inquiry-based practices
- verbalizing their own thinking to model comprehension strategies and new learning
- balancing individual, small-group, and whole-class learning experiences
- scaffolding instruction and assignments as needed and giving frequent and meaningful descriptive feedback throughout the learning process
- integrating "blended learning" opportunities by including an online environment that extends learning beyond the physical classroom
- encouraging students to take time and to persevere, when appropriate, in order to achieve a particular learning outcome

MULTIPLE WAYS OF LEARNING

"Advances in neuroscience and education research over the past 40 years have reshaped our understanding of the learning brain. One of the clearest and most important revelations stemming from brain research is that there is no such thing as a 'regular student.'" (Hall, Meyer, and Rose 2012, 2) Teachers who know their students well are aware of students' individual learning differences and use this understanding to inform instruction and assessment decisions.

The ways in which students make sense of and demonstrate learning vary widely. Individual students tend to have a natural inclination toward one or a few learning styles. Teachers are often able to detect learning strengths and styles through observation and through conversation with students. Teachers can also get a sense of learning styles through an awareness of students' personal interests and talents. Instruction and assessment practices that are designed to account for multiple learning styles create greater opportunities for all students to succeed.

While multiple learning styles are addressed in the classroom, the three most commonly identified are:

- auditory (such as listening to teacher-modelled think-aloud strategies or participating in peer discussion)
- kinesthetic (such as examining artifacts or problem-solving using tools or manipulatives)
- visual (such as reading print and visual texts or viewing video clips)

For additional information, refer to *Frames of Mind: The Theory of Multiple Intelligences* (Gardner 2007) and *How to Differentiate Instruction in Mixed-Ability Classrooms* (Tomlinson 2001).

A Gender-Inclusive Curriculum and Classroom

It is important that the curriculum and classroom climate respect the experiences and values of all students and that learning resources and instructional practices are not gender-biased. Teachers promote gender equity and inclusion in their classrooms when they

- articulate equally high expectations for all students
- provide equal opportunity for input and response from all students
- model gender-fair language, inclusive practices, and respectful listening in their interactions with students
- identify and openly address societal biases with respect to gender and sexual identity

Valuing Diversity: Teaching with Cultural Proficiency

"Instruction that is embedded in socially meaningful contexts, and tasks that are meaningful and relevant to the lives of students, will engage students in high-level problem-solving and reasoning and enhance students' engagement (Frankenstein 1995; Gutstein 2003; Ladson-Billings 1997; Tate 1995)." (Herzig 2005)

Teachers appreciate that students have diverse life and cultural experiences and that individual students bring different prior knowledge to their learning. Teachers can build upon their knowledge of their students as individuals, value their prior experiences, and respond by using a variety of culturally proficient instruction and assessment practices in order to make learning more engaging, relevant, and accessible for all students. For additional information, refer to *Racial Equity Policy* (Nova Scotia Department of Education 2002) and *Racial Equity / Cultural Proficiency Framework* (Nova Scotia Department of Education 2011).

Students with Language, Communication, and Learning Challenges

Today's classrooms include students who have diverse language backgrounds, abilities, levels of development, and learning challenges. By observing and interacting with students and by conversing with students and/or their families, teachers gain deeper insights into the student as a learner. Teachers can use this awareness to identify and respond to areas where students may need additional support to achieve their learning goals. For students who are experiencing difficulties, it is important that teachers distinguish between those students for whom curriculum content is challenging and those for whom language-based factors are at the root of apparent academic difficulties. Students who are learning English as an additional language may require individual support, particularly in language-based subject areas, while they become more proficient in their English language skills. Teachers understand that many students who appear to be disengaged may be experiencing difficult life or family circumstances, mental health challenges, or low self-esteem, resulting in a loss of confidence that affects their engagement in learning. A caring, supportive teacher demonstrates belief in the students' abilities to learn and uses the students' strengths to create small successes that help nurture engagement in learning and provide a sense of hope.

STUDENTS WHO DEMONSTRATE EXCEPTIONAL TALENTS AND GIFTEDNESS

Modern conceptions of giftedness recognize diversity, multiple forms of giftedness, and inclusivity. Some talents are easily observable in the classroom because they are already well developed and students have opportunities to express them in the curricular and extracurricular activities commonly offered in schools. Other talents only develop if students are exposed to many and various domains and hands-on experiences. Twenty-first century learning supports the thinking that most students are more engaged when learning activities are problem-centred, inquiry-based, and open-ended. Talented and gifted students usually thrive when such learning activities are present. Learning experiences may be enriched by offering a range of activities and resources that require increased cognitive demand and higher-level thinking with different degrees of complexity and abstraction. Teachers can provide further challenges and enhance learning by adjusting the pace of instruction and the breadth and depth of concepts being explored. For additional information, refer to *Gifted Education and Talent Development* (Nova Scotia Department of Education 2010).

Connections across the Curriculum

The teacher should take advantage of the various opportunities available to integrate mathematics and other subjects. This integration not only serves to show students how mathematics is used in daily life, but it helps strengthen the students' understanding of mathematical concepts and provides them with opportunities to practise mathematical skills. There are many possibilities for integrating mathematics in health education, literacy, music, physical education, science, social studies, and visual arts.

Number (N)

GCO: Students will be expected to demonstrate number sense.

SCO N01 Students will be expected to demonstrate an understanding of place value for numbers greater than one million and less than one-thousandth. [C, CN, R, T]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N01.01 Explain how the pattern of the place-value system (e.g., the repetition of ones, tens, and hundreds) makes it possible to read and write numerals for numbers of any magnitude.
N01.02 Describe the pattern of adjacent place positions moving from right to left and from left to right.
N01.03 Represent a given numeral using a place-value chart.
N01.04 Explain the meaning of each digit in a given numeral.
N01.05 Read a given numeral in several ways.
N01.06 Record, in standard form, numbers expressed orally, concretely, pictorially, or symbolically as expressions, in decimal notation, and in expanded notation, using proper spacing without commas.
N01.07 Express a given numeral in expanded notation and/or in decimal notation.
N01.08 Represent a given number using expressions.
N01.09 Represent a given number in a variety of ways, and explain how they are equivalent.
N01.10 Read and write given numerals in words.
N01.11 Compare and order numbers in a variety of ways.
N01.12 Establish personal referents for large numbers.
N01.13 Provide examples of where large whole numbers and small decimal numbers are used.

Scope and Sequence

Mathematics 5	**Mathematics 6**	**Mathematics 7**
N01 Students will be expected to represent, partition, and compare whole numbers to 1 000 000. **N08** Students will be expected to describe and represent decimals (tenths, hundredths, and thousandths) concretely, pictorially, and symbolically.	**N01** Students will be expected to demonstrate an understanding of place value for numbers greater than one million and less than one-thousandth.	**N02** Students will be expected to demonstrate an understanding of the addition, subtraction, multiplication, and division of decimals to solve problems (for more than one-digit divisors or more than two-digit multipliers, the use of technology is expected)

Background

Students are expected to develop awareness of the patterns and the relationships between and among numbers greater than one million and less than one-thousandth. It is not expected that students will perform calculations with these numbers.

Students will extend their knowledge of numbers by discovering patterns that go beyond the millions to the billions, trillions, etc., and beyond thousandths to millionths. Students will discover that the place-value system follows a pattern where

- each position represents ten times as much as the position to its right
- each position represents one-tenth as much as the position to its left
- positions are grouped in threes for purposes of reading numbers
- spaces (not commas) are used to show the positions when writing numbers, with the exception of four-digit numbers where no spaces are used (e.g., 5640)

All students should be aware that numbers extend to the left up to infinity. Students also need to know that the place-value system extends to the right as well and that there are numbers smaller than 0.001 such as ten-thousandths, hundred-thousandths, and millionths.

Students should have many opportunities to

- read numbers greater than one million and less than one-thousandth; for example, 2 456 870 346 is read as two billion four hundred fifty-six million eight hundred seventy thousand three hundred forty-six, while 2456.7564 is read as two thousand four hundred fifty-six and seven thousand five hundred sixty-four ten-thousandths (The word **and** is used for decimal numbers only.)
- read numbers in several ways; for example 6732.14 could be read as six thousand seven hundred thirty-two and fourteen hundredths or as sixty-seven hundred thirty-two and fourteen hundredths
- record numbers; for example, students would write twelve million one hundred thousand in **standard form** (12 100 000), in **decimal notation** (12.1 million), and in expanded notation (10 000 000 + 2 000 000 + 100 000) (**Note:** Scientific notation is introduced in later grades.)
- establish **personal referents** to develop a sense of larger numbers; for example, 500 is the capacity of the local arena, 10 000 is the population of the town, or a class collection contains a million small objects

Through these experiences, students will develop flexibility in identifying and representing numbers greater than 1 000 000 and less than 0.001. It is also important for students to gain an understanding of the relative size (magnitude) of numbers through real-life contexts that are personally meaningful (e.g., computer memory size, professional athletes' salaries, Internet search responses, populations, or the microscopic world).

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Assessing Prior Knowledge

Tasks such as the following could be used to determine students' prior knowledge.

- Invite students to place counters on a place-value chart to represent a number stated orally. The digital form can be written once the chart is filled in, and the number can be read back.
- Present a riddle to the class such as, I have 25 hundredths and 4 tenths. What am I? Have students use a model of their choice to represent the solution to the riddle.

Whole-Class/Group/Individual Assessment Tasks

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Invite students to explain at least three things they know about a number with 10 digits.
- Ask students to describe when 1 000 000 000 of something might be a large amount and when it might be a small amount?
- Ask students to express 0.0674 in at least three different ways.
- Ask students to write the standard form of numbers that include decimals and/or whole numbers.
  - two million and thirty-seven thousandths
  - two and thirty-seven ten-thousandths
  - one billion and two thousandths
  - sixteen and forty-seven hundred-thousandths
- Ask students to describe how the bold-faced digits in the following two numbers are the same and how they are different.

  5**4**6 397 305 3**4**8 167 903 927

  Extend the task to decimals:

  0.00**7**0 0.000**7**
- Have students write a report on what they have learned about decimals and what questions they may now have concerning the topic.
- Provide students with thousandths and ten-thousandths grids. Ask them to shade in the grids, one at a time, to show the following decimals:
  - 0.004
  - 0.203
  - 0.023
  - 1.799
  - 0.0001
  - 0.0010
  - 1.0150

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Ask students to find various representations for multi-digit and decimal numbers in newspapers and magazines. Encourage discussion on the need for accuracy in reporting these numbers and the appropriate use of rounded numbers.
- Write decimals using place-value language and expanded notation to help explain equivalence of decimals.
  0.2 means 2 tenths
  0.20 means 2 tenths + 0 hundredths
  0.200 means 2 tenths + 0 hundredths + 0 thousandths
- Renaming decimals can also help explain equivalence of decimals.
  0.2 = 0.200
  0.20 = 0.200
  0.200 = 0.200
- Ensure that proper vocabulary is used when reading all numbers. Provide opportunities for students to read decimals in context. Saying decimals correctly will help students make the connection between decimals and fractions 5.0072 should be read as five and seventy-two ten-thousandths not as five point zero zero seven two.
- Include contexts that lend themselves to using large numbers, such as astronomical data and demographic data. Contexts that lend themselves to decimal thousandths include sports data and metric measurements. An interesting task involving decimals might require students to complete a chart such as, in 0.1 years, I could ...; in 0.01 years, I could ...; in 0.001 years, I could ...

SUGGESTED LEARNING TASKS

- Create a class book that includes real-world examples of very large numbers and very small decimals (e.g., population of Mexico City; length of an ant's antenna in centimetres).
- Present a metre stick or a strip of cash register tape as a number line from zero to one billion. Ask students where half a billion, one hundred million, ten million, one million, etc., would be on this number line.
- Prepare and shuffle five sets of number cards (0–9 for each set). Ask students to select nine cards and have them arrange the cards to make the greatest possible or the least possible whole number. Invite students to read each of the numbers. Consider extending the task by asking students questions, such as:
  - How many different whole numbers could be made using the nine digits selected?
  - How many $1000 bills would one get if the greatest and least numbers displayed represented money amounts? This could be extended to explore the number of tens, hundreds, etc., in the number.
- Ask students to determine the value of a whole number between 2.03 million and 2.35 million.
- Ask students to find a value between 0.0001 and 0.00016.
- Present this library information to students: Metropolitan Toronto Library has 3 068 078 books; Bibliothèque de Montreal has 2 911 764 books; North York Public Library has 2 431 655 books. Ask students to rewrite the numbers in a format such as ☐.☐ million or ☐.☐☐ million books. Then ask them to make comparison statements about the number of books held in these libraries.
- Construct a cubic metre, and explore the quantity of centimetre cubes that will fit in the large cube. Extend this to include a discussion of the size of the block required to represent ten million, one hundred million, one billion, and ten billion.
- Provide students with a list of numbers written both in words and in standard numeral form. Ask them to compare these numbers by placing them in a place-value chart.
  - 26.0043
  - 0.13
  - seventy ten-thousandths
  - four and fourteen ten-thousandths
- Ask students to use the following numbers to answer the questions below:
  8.0254   2.086   0.83   24.9181
  - In which number does 8 represent a value of 8 hundredths?
  - In which number does 2 represent a value of 2 tens?
  - In which number does 0 represent the value of 0 ones?
- Ask students to write the number 23.0876 in words.
- Invite students to roll a number cube five times. Using the numbers rolled, create a decimal number with a value between 1.0001 and 9.9999.
- Using numeral cards with digits 0–9 on them, ask students to create decimal numbers that are in a desired range. For example, using five different number cards, create a number that can be found between 1.0009 and 1.5001. Ask students to write a journal entry explaining their thinking.
- Ask students to use the number 619 723 766 to answer the following questions.
  - What is the value of the 9?
  - What is the value of the 3?
  - Choose a place value and show how it is ten times greater than the place value to its immediate right.

- Write a number, such as 32 765 345, on the board. Ask students how many millions are in the number. How many thousands? Ten thousands? Ask them to justify their thinking.
- Pose the following problem to students: Joe said 3 450 000 is greater than 27 450 000 because 3 is greater than 2. Ask students to decide whether Joe is correct and to explain their thinking using words, numbers, and/or pictures.
- Using several number lines, incorrectly place one or two numbers on each, and ask students to work in small groups to explain why the numbers are not correctly placed. Invite students to share their responses with the group. Ask them to justify their thinking.
- Ask students to generate a number with between 7 and 10 digits. Invite them to find classmates with numbers that are similar (place value). Once they have found a group they belong to, have them order their numbers from least to greatest. Then have the class order all of the numbers from least to greatest. Invite each student to read his/her number. (This task can be done in silence so students have to examine the other numbers carefully.) This task can be done using decimal numbers as well.

SUGGESTED MODELS AND MANIPULATIVES

- base-ten blocks
- Cuisenaire rods
- decimal squares
- metre sticks
- number lines
- thousandths grid

MATHEMATICAL LANGUAGE

Teacher	Student
▪ decimal notation	▪ decimal notation
▪ expressions	▪ expressions
▪ magnitude	
▪ multiplicative nature, expanded notation	▪ expanded notation
▪ period	
▪ place value	▪ place value

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students, K–8* (Small 2008), pp. 229, 143
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 283, 199
- *Teaching Student-Centered Mathematics, Grades 5–8* (Van de Walle and Lovin 2006), pp. 109–112

Notes

SCO N02 Students will be expected to solve problems involving whole numbers and decimal numbers. [ME, PS, T]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicator s to determine whether students have achieved the corresponding specific curriculum outcome.

N02.01 Determine whether technology, mental mathematics, or paper-and-pencil calculation is appropriate to solve a given problem and explain why.
N02.02 Identify which operation is necessary to solve a given problem and solve it.
N02.03 Determine the reasonableness of an answer.
N02.04 Estimate the solution and solve a given problem using an appropriate method (technology, mental mathematics, or paper-and-pencil calculation).
N02.05 Create problems involving large numbers and decimal numbers.
N02.06 Use technology, mental mathematics, or paper-and-pencil calculation to solve problems involving the addition, subtraction, multiplication, and division of whole numbers.
N02.07 Use technology, mental mathematics, or paper-and-pencil calculation to solve problems involving the addition and subtraction of decimal numbers

Scope and Sequence

Mathematics 5	Mathematics 6	Mathematics 7
N02 Students will be expected to use estimation strategies, including front-end, front-end adjusted, rounding, and compatible numbers, in problem-solving contexts.	**N02** Students will be expected to solve problems involving whole numbers and decimal numbers.	**N02** Students will be expected to demonstrate an understanding of the addition, subtraction, multiplication, and division of decimals to solve problems (for more than one-digit divisors or more than two-digit multipliers, the use of technology is expected).

Background

Students continue to use the four operations to solve problems with large whole numbers and should use addition and subtraction to solve problems with decimal numbers (limited to thousandths). Multiplication and division involving decimal numbers will be introduced in SCO N08. Students will also be expected to create problems for others to solve. Problem-solving situations should be embedded in a meaningful context as often as possible.

Provide students with opportunities to solve unfamiliar types of problems and encourage students to persevere. Problem solving requires students to build on existing knowledge and develop their own strategies. Encourage students to demonstrate persistence as they work through a challenging problem. It is understandable for students to not immediately know how to go about solving a particularly challenging problem. Solving these challenging problems with minimal assistance will help empower students. It is important that students communicate their thinking processes about the problem and discuss their strategies with other students.

In previous years, students have worked with a variety of problem-solving strategies. These strategies will prove useful to students in approaching a wide variety of problems. In order for students to successfully use problem-solving strategies, the strategies should be explicitly discussed with students, preferably after a student successfully uses it. There is a value in naming the strategies for students so they can discuss them and use them again. As students work through solving problems and begin to learn new strategies to solve these problems, you may consider posting these strategies in the classroom. A mathematics wall or bulletin board designated to problem solving could be displayed for students. Students can select from the strategy list when they are facing a new or challenging problem and when they are not sure how to proceed. Some possible problem strategies are as follows:

- “Act it out.
- Use a model.
- Draw a picture.
- Guess and test.
- Look for a pattern.
- Use an open sentence.
- Make a chart/table or graph.
- Solve a simpler problem.
- Consider all possibilities.
- Consider extreme cases.
- Make an organized list.
- Work backwards.
- Use logical reasoning .
- Change your point of view.”

(Small 2008, p. 41–42)

Technology, such as calculators and computers, are often useful tools and time-saving devices when solving problems with large numbers. However, students need to determine when the use of these tools is appropriate and when mental mathematics or other paper-and-pencil strategies are more appropriate. For example, when presented with a problem such as, $12 000 000 is won in the lottery and is shared with three winners. How much does each person receive?, students are expected to solve the problem mentally, rather than using paper and pencil or a computer or calculator. Students need to consider the context of the question and the numbers involved when deciding which approach—mental mathematics, paper-and-pencil, or calculator—is most appropriate. Mental mathematics is a potential approach for solving calculations before technology is used.

When students determine that technology is the most appropriate approach to use to solve a given problem, they should estimate answers to determine a reasonable answer before using technology. Students should not assume an answer determined with technology is automatically correct. Having students determine the reasonableness of an answer and explaining their thinking is a powerful way to assess understanding and learning.

There are many interesting sources of large numbers, both on the Internet and in reference books. Possible examples that could be discussed are world populations, quantity of data electronic devices can hold (e.g., gigabyte or terabytes), salaries, and astronomy.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to add 6785 + 1834. Explain how they know their answer is reasonable using estimates in their explanation.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to look at the area of countries in the world and draw conclusions and comparisons with the area of Canada. Students can present their findings in the form of a project using technology, graphs, and/or illustrations.
- Provide students with information regarding the populations of some of the capital cities of Canada.

City	Population in 2011
Fredericton	56 224
Charlottetown	59 325
Halifax	403 188
St. John's	192 326
Toronto	5 741 419
Winnipeg	753 555
Edmonton	1 176 307
Ottawa	1 239 140

Ask students to solve problems using the information such as,

(a) If we combined the population of all of the cities in the chart, except Toronto, would it be more or less than the population of Toronto?

(b) Approximately how many more people live in Toronto than Ottawa?

(c) Marc added the population for the capitals of the four Atlantic provinces and said the answer is more than the population of Winnipeg. Is this reasonable? Explain.

- Ask students to choose two types of careers in entertainment (e.g., professional athletes, actors, singers). Have them research the top five salaries in each career. Invite them to generate questions for others to solve, and include an answer key. This can be presented in the form of a project.
- Ask students to describe a situation in which it would be more appropriate to use a calculator than paper-and-pencil calculations to solve a problem. Ask them to explain why they would use a calculator rather than pencil and paper to get the answer.
- Provide students with problems involving large numbers, and have them estimate the solution and solve the problems. Invite students to explain their approach and the operations that were used.
- Give students a virtual amount of money to spend (i.e., spend a million dollars) and have them research from the Internet, catalogues, etc., items they would buy to spend the money. Students could create a poster or other display to communicate their spending choices.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Invite students to research populations of cities and/or provinces in Canada and cities and/or countries of the world. Using this information, students can estimate differences, compare populations, and draw conclusions about Canada compared to the world.
- Investigate the concept of billions. Although these are rarely found in students' experiences, numbers of this magnitude relate to national debt, personal fortunes, populations, or pieces of trivia (e.g., "How long is a billion millimetres?").
- Invite students to use information from the above sources to create problems for their classmates to solve. Students are asked to estimate, then use technology to check answers.

- Include print resources such as the *Canadian Global Almanac* (Global Press 2005), *Guinness World Records 2013* (Guinness World Book Limited 2014), and a world atlas. Use children's literature to provide a context for large numbers to create and solve problems.
- Conduct Internet searches for data relating to any topic of student interest, such as sports, money, planetary distances, or populations. Students can also explore the number of responses ("hits") they get when searching for information on the Internet.

SUGGESTED LEARNING TASKS

- Provide students with appropriate data and ask them to determine how much farther away Jupiter is from Earth than from Mars.
- Invite students to create a variety of "outlandish" problems involving lengths. For example,
  - How many toothbrushes are required to make a line that is 2 km long?
  - How many pennies must be lined up to make a kilometre?
- Ask students to create problems based on information provided, such as the following:

Population 2009	
World	6 790 062 216
China	1 338 612 968
United States	307 212 123
Japan	127 078 679
Germany	82 329 758
Canada	33 487 208

- Provide students with sports data such as the batting averages of some baseball players. (Batting averages are represented using decimal numbers.) Ask them to calculate the spread between the player with the highest batting average and the player with the lowest batting average. Invite students to create problems using the averages on the list.
- The following chart shows the populations of five countries from greatest to least:

Country	**Population in 2012**
India	1 205 073 612
United States	313 847 465
Indonesia	248 645 008
Pakistan	190 291 129
Japan	127 368 088

- Ask students the following questions:
  - The population of Russia is 141 862 011. If the population of Russia is added to the chart, between what two countries would it be listed? Explain how you know.
  - If we combined the populations of Pakistan and Japan, would the result be more or less than the population of the US?
  - Is there any one country that has approximately double the population as another? Explain.
  - If all the populations of the listed countries were combined, would we have more than or less than two billion people? How do you know? Students should show how they arrived at the answer.

- Invite students to conduct research to find situations where decimals are added and subtracted in everyday life and present their findings in a video, as an oral report, or as a written report.
- Ask students what they would rather have for their allowance: one penny a day that doubles each day or $1000 a month? Explain their thinking, and encourage them to use a table to show their solution.
- Tell students, Amy has 500 daytime minutes on her cell phone each month. She is using, on average, 37 minutes a day. Ask students how many days she will have before her minutes are used? Ask them to suggest the number of minutes Amy should use each day to ensure she will not run out of daytime minutes each month.
- Tell students John's class sold 104 magazine subscriptions and Yvonne's class sold 108. The profit on each subscription was $11. One student estimated the total profit was $230. Ask students if this estimate is reasonable. Explain how you know.
- Provide sample problems, such as the following, and ask students to solve them. Have them share with their classmates how they found their answer:
  - Mr. Ron collects recyclables. He counted 32 full bags of bottles. Mr. Ron knows he has more than 400 recycled bottles. Ask students to come up with some possible amounts of bottles that could be in each bag if there were the same number of bottles in each bag.
  - Mr. Yen's greenhouse has rows of flowers. There are 72 flowers in each row with 1080 flowers in total. Ask students to find the number of rows in Mr. Ben's greenhouse.
- Ask students to think about one million dimes, and answer the following questions:
  - How many loonies would that be?
  - How many $100 bills would it take to equal the one million dimes?
  - How high would the stack of dimes be if you piled one on top of the other?
  - What would the mass of one million dimes be?
  - Estimate how long it would take to count one million dimes.
  - What could you buy with one million dimes?
- Ask students to write about a situation in which estimated numbers were used.
- Pose the following: Janell has $500 to buy eight games. Each game costs $37. Janell wonders if she has enough money to buy them all. Invite students to help Janell by explaining how they know if she really does have enough money. Ask them to explain how estimation can help to determine the solution to the problem?
- Invite students to make a list of three or four situations in which they would use a calculator to solve a problem. Ask them to explain why they would use a calculator rather than pencil and paper to get the answer.
- Tell students that Gilles and Tom each rounded numbers to 2.4 million. Ask them if this means they both started with the same exact number? Ask them to explain their thinking using pictures, numbers, and words.
- Ask students to research the distance of each planet from the sun. Record these distances in decimal millions. Ask students to create problems that could be solved using this information and have their partner solve them (e.g., order the planets from least to greatest according to their distance from sun). Rename the numbers and record them in a place-value chart.

SUGGESTED MODELS AND MANIPULATIVES

- calculators
- computers

MATHEMATICAL LANGUAGE

Teacher	Student
▪ decimals ▪ estimate ▪ mental mathematics ▪ paper and pencil ▪ reasonableness ▪ technology	▪ decimals ▪ estimate ▪ mental mathematics ▪ paper and pencil ▪ reasonableness ▪ computers, calculators

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students, K–8* (Small 2008), pp. 47–50, 57, 236–239
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 106–109, 116, 290–293
- *Teaching Student-Centered Mathematics, Grades 5–8* (Van de Walle and Lovin 2006), pp. 124–126
- *Canadian Global Almanac* (Global Press 2005)
- *Guinness World Records 2013* (Guinness World Book Limited 2014)

Notes

SCO N03 Students will be expected to demonstrate an understanding of factors and multiples by ▪ determining multiples and factors of numbers less than 100 ▪ identifying prime and composite numbers ▪ solving problems using multiples and factors [PS, R, V]
[C] Communication **[PS]** Problem Solving **[CN]** Connections **[ME]** Mental Mathematics and Estimation **[T]** Technology **[V]** Visualization **[R]** Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N03.01 Identify multiples for a given number and explain the strategy used to identify them.
N03.02 Determine all the whole number factors of a given number using arrays.
N03.03 Identify the factors for a given number and explain the strategy used (e.g., concrete or visual representations, repeated division by prime numbers, or factor trees).
N03.04 Provide an example of a prime number, and explain why it is a prime number.
N03.05 Provide an example of a composite number, and explain why it is a composite number.
N03.06 Sort a given set of numbers as prime and composite.
N03.07 Solve a given problem involving factors or multiples.
N03.08 Explain why 0 and 1 are neither prime nor composite.

Scope and Sequence

Mathematics 5	Mathematics 6	Mathematics 7
N03 Students will be expected to describe and apply mental mathematics strategies and number properties to recall, with fluency, answers for basic multiplication facts to 81 and related division facts.	**N03** Students will be expected to demonstrate an understanding of factors and multiples by ▪ determining multiples and factors of numbers less than 100 ▪ identifying prime and composite numbers ▪ solving problems using multiples and factors	**N01** Students will be expected to determine and explain why a number is divisible by 2, 3, 4, 5, 6, 8, 9, or 10, and why a number cannot be divided by 0.

Background

The focus of the mathematics curriculum is problem solving. Students need opportunities to do tasks that enhance problem solving and reasoning that includes factors, multiples, and prime and composite numbers.

Multiples of a whole number are the products of that number and any other whole number. To find the first four multiples of 3, multiply 3 by 1, 2, 3, and 4 to get the multiples 3, 6, 9, 12. Multiples of a number can also be found by skip counting by that number.

Factors are numbers that are multiplied to get a product (3 and 4 are factors of 12). If a number can be expressed as a product of two or more whole numbers, then the whole numbers are called factors of that number. To help students understand the meanings for the terms **factor** and **multiple**, students could explore these concepts and write their own definitions (e.g., factor × factor = multiple).

The whole number factors for a number can be found by dividing that number by smaller whole numbers and looking to see if there is a remainder of zero. At this point, students will also recognize that

- the whole number factors of a number are never greater than the number
- the greatest whole number factor is always the number itself; the least whole number factor is 1
- the second whole number factor is always half, or less than half, the number (unless the number is prime)
- the multiple of a number always has that number as a factor

"Students will discover that some numbers have many factors, some have a few, and some have only 1 or 2. For example:

Factors of 24	**Factors of 6**	**Factors of 97**
(8 factors) 1 24 2 12 3 8 4 6	(4 factors) 1 6 2 3	(2 factors) 1 97

As shown in the chart above, factors come in pairs, although some numbers have an odd number of different factors (square numbers), and the number 1 has only 1 factor. For example:

Factors of 16	**Factors of 1**
(5 factors) 1 16 2 8 4 4	(1 factor) 1 1

Organized lists [as shown in the charts above] are a way of determining factors in a systematic fashion, beginning with 1 and the number itself, and then 2 or the next possible factor and its factor partner." (Small 2008, 150–151)

A prime number is defined as a number that has only two factors: 1 and itself. For example, 29 only has factors of 1 and 29 and is, therefore, a prime number. Students should recognize that the concept of prime numbers applies only to whole numbers.

A composite number is a number with more than two factors and includes all non-prime numbers other than 1 and 0. For example, 9 is a composite numbers because it has factors of 1, 3, and 9.

It is important for students to understand that 0 and 1 are not classified as prime or composite numbers. The number 1 has only one factor (itself). Zero is not prime because it has an infinite number of divisors, and it is not composite because it cannot be written as a product of two factors that does not include 0.

Although students should have strategies for determining whether or not a number is prime, it is not essential for them to be able to quickly recognize prime numbers. However, it is expected that students will be able to readily identify even numbers (other than 2) as non-primes (composites) as even numbers will have a minimum of three factors: 1, 2, and the number itself.

Students should be encouraged to accurately use language such as multiple, factor, prime, and composite. As well, encourage students to explore numbers and become familiar with their composition.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask, If you buy muffins in boxes of 6, how many muffins are in 7 boxes? How would the number of muffins change if you bought 9 boxes? If you needed 36 muffins for a party, how many boxes would you buy?

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to express 36 as the product of two factors in as many ways as possible.
- Invite small groups of students to find the number less than 50 (or 100) that has the most factors. Ensure students can explain their process and justify their answer.
- Have students show all the factors of 48 by drawing or colouring arrays on square grid paper.
- Invite students to solve problems involving factors and multiples, such as the following:
  - Mr. Reeves has 24 students in his class. How many different size groups of students can he make so that all groups are the same size?
- Ask students if it is possible to list all of the multiples of 12. Ask them to explain their reasoning.
- Ask students to list all of the factors of 8 and the first ten multiples of 8.
- Have students explain, without dividing, why 2 cannot be a factor of 47.
- Ask students to identify a number with five factors.
- Ask students to find three pairs of prime numbers that differ by two (e.g., 5 and 7).

- Ask, Why is it easy to know that certain large numbers (e.g., 4 283 495) are not prime, even without factoring them?
- Tell students that the numbers 2 and 3 are consecutive numbers, both of which are prime numbers. Ask, Why can there be no other examples of consecutive prime numbers?
- Invite students to use technology to help them determine the prime numbers up to 100. Ask them to prepare a report describing as many features of their list as they can.
- Encourage students to draw diagrams (such as rectangles or factor trees) to show why a given number is or is not prime (e.g., 10, 17, 27).
- Ask students, Is it possible for an even number, other than 2, to be prime? Explain.

Follow-up on Assessment

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

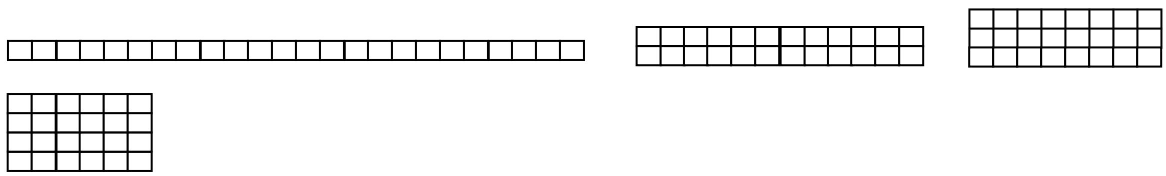
Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

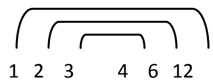
Choosing Instructional Strategies

Consider the following strategies when planning daily lessons.

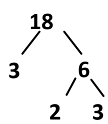
- Ask students to determine the factors of a number by arranging square tiles into as many different arrays (rectangles) as possible. Record the unit length and width of each rectangle. For example, if 24 tiles were used, the rectangles would be 1 × 24, 2 × 12, 3 × 8, and 4 × 6. These are the factor pairs for 24. Have students record their rectangles/factor partners on grid paper. Students should discover that some numbers only have one rectangle. This is an effective approach to introducing prime numbers. Grid paper can also be used to explore this concept.



- Ask students to investigate other numbers to find their factor pairs. Students may use organized lists to determine factors (i.e., begin with 1 and the number itself, then 2 or the next possible factor and its factor partner, etc.)



- Invite students to factor odd composite numbers (e.g., 33, 39). Students sometimes mistake these for prime as they do not readily see how they are factored.
- Invite students to skip count on number lines or other models such as same colour Cuisenaire rods or equal lengths of connected base-ten unit cubes to find multiples of a number.
- Invite students to use the constant function on their calculators to explore multiples of a number. They may also use calculators to systematically test for factors of a number: ÷ 1, ÷ 2, ÷ 3, ÷ 4, etc.
- Explore other strategies, such as factor trees, to determine prime and composite numbers.



SUGGESTED LEARNING TASKS

- Explore the sieve of Eratosthenes to identify the prime numbers to 100. Ask students to discuss any patterns they notice.
- Ask students to express even numbers greater than 2 in terms of sums of prime numbers. (Sample answers may include 4 = 2 + 2, 6 = 3 + 3, 8 = 3 + 5, ..., 48 = 43 + 5, 50 = 47 + 3, ...). Explore this idea further by asking if every even number greater than 2 can be written as the sum of two prime numbers (Goldbach's conjecture).
- Ask students to name numbers with a given quantity of factors (e.g., numbers with 6 factors: 12, 18, 20, etc.).
- Present students with a set of numbers that each have several factors (e.g., 12, 18, 24, 30, or 36). With counters and linking cubes, ask students to attempt to find a way to separate them into equal sets. With arrays, students build rectangles that have the given number of squares. For each arrangement, a multiplication equation should be written. These equations would represent the factors of that number. (Van de Walle and Lovin 2006b, 63).
- Ask students to find a number that has exactly four factors and another one that has five factors. Ask them to explain any patterns they see.
- Invite students to draw one or more rectangles to show that 8 is a factor of 16 and 24.
- Tell students that they are going to look for multiplication expressions and the corresponding rectangular array for several numbers. Their task includes trying to find all the multiplication expressions and rectangular arrays for each number. Have square tiles available that students can use them to explore possible arrays. Once they have created an array, they should draw it on grid paper. Ask students to write a multiplication sentence for each array. Students should group together all arrays with the same number of squares. After identifying the multiplication expression and the rectangular arrays, students should look for patterns in the factors and rectangular arrays. For example, students may look at which numbers have the least number of arrays and, therefore, the least number of factors. Which numbers have arrays that form a square? Which numbers have a factor of 2? What can you say about the factors for even numbers? Do even numbers always have 2

as a factor? What do you notice about the factors of odd numbers? Encourage students to think about why different patterns occur.

- Ask students to solve the following problems using words, pictures, and numbers:
  - Jill has an ice cream sandwich that measures 10 cm × 10 cm. She wants to cut the ice cream into squares. What possible sizes could the squares be? How many squares of each size would be cut?
  - Harry's dad had 36 Halloween treats that he wanted to share evenly among treat bags. What are the different possibilities of the number of bags that he could fill?
  - Juan bought some $10 computer games. Damian bought some $15 computer games. They each spent less than $200, but they both spent the same amount. How much could they have spent?
  - Tell students you are thinking about a number that is a multiple of 2 and 6. Ask them to identify some possible numbers that you could be thinking about.
  - The cafeteria is having a promotion. Every second student receives free milk and every sixth student receives an orange. If 60 students are served at the cafeteria that day, which students received free milk? An orange? Both?
  - Tell students that Olivia was in a class of 24 people. Her teacher told the class to line up and that every second student would receive a pencil and every sixth student would receive an eraser. If Olivia wanted both a pencil and an eraser, ask students which position in the line Olivia should be in.
  - There are 84 students in four grades and they are arranged into teams with the same number on each team. How many teams are there and how many students might there be on each team? How many possible solutions can you find to this problem?
- Tell students you were trying to figure out the multiples of 8 and this is the list you came up with: 0, 8, 16, 23, 32 and 40. Ask, is the list complete? Is it correct? Ask them to explain their thinking.
- Have students choose a number from 2–10. Ask them to list at least five multiples of that number. Ask them to explore any patterns they see in the multiples and to discuss why this may happen.
- Tell students that a number has exactly four prime factors. Ask them to think about what this number could be and to explain their thinking.
- Ask students to draw two different factor trees for 56 and for 32. Ask them to explain why it is possible to draw two different factor trees for each number.
- Ask students if they can name a composite number for which only one factor tree can be drawn.
- Invite students to determine how many factor trees they can draw for the number 13. Ask them to explain their thinking.

SUGGESTED MODELS AND MANIPULATIVES

- base-ten blocks
- coloured tiles
- Cuisenaire rods
- geo-boards
- grid paper
- hundred charts
- linking cubes
- metre stick
- number lines

MATHEMATICAL LANGUAGE

Teacher	Student
▪ composite numbers ▪ factor trees ▪ factors ▪ greatest factor, least factor, second factor ▪ multiples ▪ prime numbers ▪ products ▪ skip counting ▪ square numbers	▪ composite numbers ▪ factor trees ▪ factors ▪ greatest factor, least factor, second factor ▪ multiples ▪ prime numbers ▪ products ▪ skip counting

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students, K–8* (Small 2008), pp. 149–152, 154–155
- Making Mathematics Meaningful to Canadian Students, K–8, Second Edition (Small 2013), pp. 204–207, 209–210

Notes

SCO N04 Students will be expected to relate improper fractions to mixed numbers and mixed numbers to improper fractions. [CN, ME, R, V]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N04.01 Demonstrate, using models, that a given improper fraction represents a number greater than 1.
N04.02 Express improper fractions as mixed numbers.
N04.03 Express mixed numbers as improper fractions.
N04.04 Place a given set of fractions, including mixed numbers and improper fractions, on a number line, and explain strategies used to determine position.
N04.05 Represent a given improper fraction using concrete, pictorial, and symbolic forms.
N04.06 Represent a given mixed number using concrete, pictorial, and symbolic forms.

Scope and Sequence

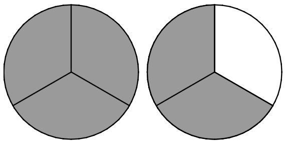
Mathematics 5	Mathematics 6	Mathematics 7
N07 Students will be expected to demonstrate an understanding of fractions by using concrete, pictorial, and symbolic representations to ▪ create sets of equivalent fractions ▪ compare and order fractions with like and unlike denominators	**N04** Students will be expected to relate improper fractions to mixed numbers and mixed numbers to improper fractions.	**N05** Students will be expected to demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially, and symbolically (limited to positive sums and differences).

Background

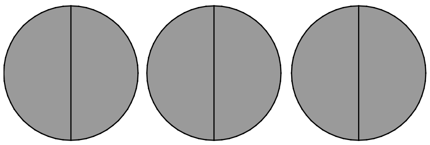
Improper fractions and mixed numbers will be new to students in Mathematics 6. This work provides students with opportunities to build on their prior understanding of equivalent fractions and their ability to compare proper fractions with like and unlike denominators. In Mathematics 6, students now work on fractions that are greater than 1 and relate these fractions to mixed numbers.

Students need to see and use a variety of models and pictures such as pattern blocks, fraction pieces, and number lines when studying fractions. This includes linear, area, and set models. In Mathematics 6, students extend their understanding of fractions to learn that an improper fraction represents a fraction greater than one.

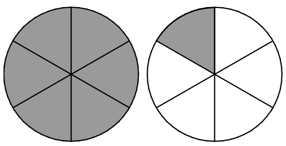
Through the use of models, students should discover that fractions with the numerator greater than their denominator are greater than one (e.g., $\frac{5}{3}$, $\frac{6}{2}$, $\frac{7}{6}$).



$= \frac{5}{3}$

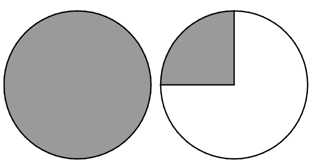


$= \frac{6}{2}$



$= \frac{7}{6}$

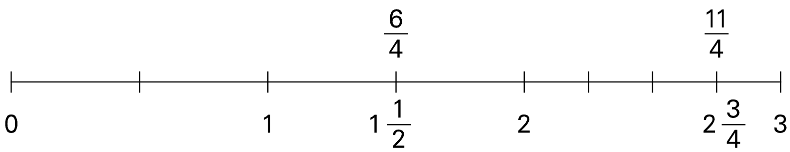
It is important for students to understand that an improper fraction, such as $\frac{5}{4}$ can also be expressed as a mixed number that is composed of a whole number and a proper fraction (e.g., $1\frac{1}{4}$).



$= 1\frac{1}{4}$

Students should fluently move between the mixed number and improper fraction formats of a number. Rather than only applying a rule to move from one format to the other, students should be encouraged to focus on the meaning. For example, since $\frac{14}{3}$ is 14 thirds and it takes 3 thirds to make one whole or 1, then 12 thirds would equal 4 wholes, so $\frac{14}{3}$ represents 4 wholes and 2 thirds of another one whole or $4\frac{2}{3}$. Often it is easier for students to grasp the magnitude of mixed numbers rather than improper fractions. For example, a student may know that $4\frac{1}{3}$ is a bit more than 4, may not have a good sense of the size of $\frac{13}{3}$.

Students should be able to place mixed numbers and improper fractions on a number line easily when they have benchmarks to use, such as closer to zero, closer to one-half, closer to one, closer to one and one-half, or closer to 2. Having these benchmarks helps students visualize the placement and order of these fractions. The concept of equivalent fractions that students learned in Mathematics 5 will also be helpful in developing additional benchmarks.



It is important that students have an opportunity to explore, through a problem-solving context and the use of a variety of models, that fractions are connected to multiplication and division. Students may discover that dividing the numerator by the denominator is a procedure that can be used to change an improper fraction to a mixed number. It would, however, be inappropriate just to tell students to divide the denominator into the numerator to change an improper fraction to a mixed number before they develop the conceptual understanding for this.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Assessing Prior Knowledge

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to place the following fractions on a number line: $\frac{1}{2}, \frac{9}{10}, \frac{4}{5}, \frac{1}{5}$ and explain the strategy they used to determine the location of each fraction.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to model $\frac{7}{4}$ to show that it is greater than one whole.
- Ask students to model an improper fraction and explain how they know it is an improper fraction.
- Invite students to explain how they know that $\frac{5}{4}$ must be more than one whole.
- Ask students, If 14 people at a party each want $\frac{1}{3}$ of a pizza, how many pizzas would be needed?
- Ask students to use coloured tiles to show why $3\frac{1}{3} = \frac{10}{3}$. Observe whether or not they make wholes of 3 (or 6 or 9 ...) squares.
- Provide students with several mixed numbers and improper fractions that are equivalent (e.g., $2\frac{3}{4} = \frac{11}{4}$). Ask them to show if the numbers are equal and to explain their thinking concretely, pictorially, and symbolically.
- Have students write as many improper fractions as they can with the numbers 3, 6, 7, and 8. Have them represent one of the improper fractions using a model or a picture.
- Invite students to explain a situation when it would be a good idea to express an improper fraction as a mixed number.
- Write and model a mixed number, with the same denominator, that is greater than $\frac{3}{3}$, but less than $\frac{6}{3}$.
- Provide students with several mixed numbers and improper fractions.
  For example, $2\frac{1}{3}$, $\frac{7}{4}$, $\frac{5}{3}$, $2\frac{3}{4}$, $1\frac{4}{5}$.
  Have students place the numbers on an open number line to demonstrate their relative magnitude.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

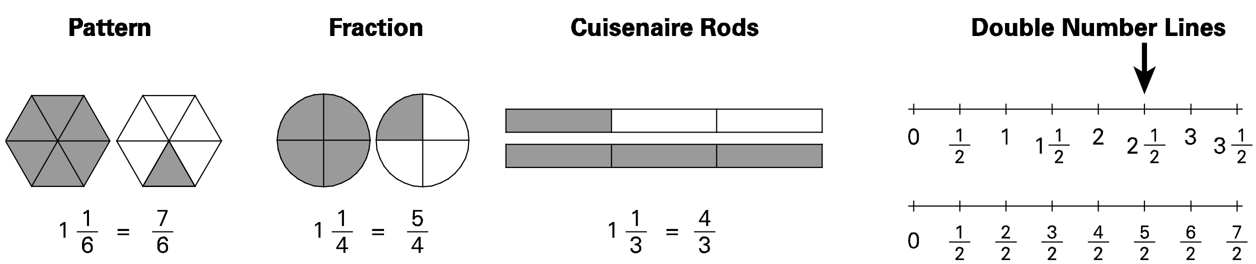
Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

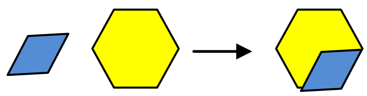
- Explore improper fractions and mixed numbers in a variety of ways and use a variety of different models. Some examples are as follows:



- Using pattern blocks, have students build and count fractional parts and continue beyond a whole: $\frac{1}{3}, \frac{2}{3}, \frac{3}{3}, \frac{4}{3}, \frac{5}{3}$, etc. Ask them to show another way to represent the improper fractions (e.g., $\frac{5}{3} = 1\frac{2}{3}$). Gradually transition to doing this task without the pattern blocks (or other models).
- Provide students with frequent opportunities to use number lines (including double number lines) to explore the placement of mixed numbers and improper fractions. Ensure students are able to explain their strategy, focusing on the use of benchmarks.
- Ask students to visualize fractions based on their experiences with various models. They should be able to draw a variety of representations for the same fraction (e.g., $\frac{7}{3}$).

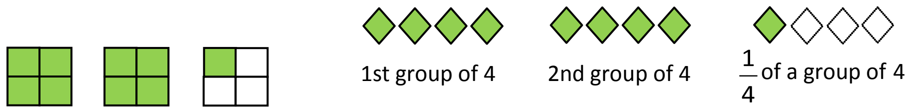
SUGGESTED LEARNING TASKS

- Ask students to model mixed numbers and improper fractions in various ways (e.g., $1\frac{3}{4} = \frac{7}{4}$).
- Ask students to determine what fraction the blue rhombus represents if the hexagon represents one whole. Using pattern blocks, have students identify another name for $\frac{14}{3}$.



- Invite students to represent the following improper fractions using rectangles.
  - $\frac{5}{4}$
  - $\frac{3}{2}$
- Ask students to solve problems such as,
  - Jamir has 15 quarters in his pocket. How many whole dollars does he have?
  - At a party, a tray of strawberries was placed on the table. Each strawberry on the tray had been cut in half. If Daniel ate 9 of the strawberry halves, how do you know that he ate between 4 and 5 whole strawberries? Invite students to use models, pictures, and numbers to show their thinking.
  - Tell students that a tea biscuit recipe calls for $\frac{4}{3}$ cups of flour. Mr. Gordon is not sure what this means. Ask students to help Mr. Gordon by explaining to him what $\frac{4}{3}$ means and to tell him how much flour he needs using a mixed number.
  - Tell students that it takes four 250-mL cups to fill a 1-L bottle. Ask them to determine how many 1-L bottles they could fill if they had ten 250-mL cups. Ask them to represent their solution as a mixed number and as an improper fraction.
  - Tell students it takes $\frac{1}{3}$ of an hour to bake one batch of cookies. Ask students to explain how long it would take if they had 5 batches of cookies to bake. Ask them to represent their answer using a mixed number and an improper fraction.
  - Four friends were at a party. Horatio stated he ate $\frac{5}{3}$ of pizza while Amy stated she ate $\frac{5}{4}$ of pizza. Larry said that Amy ate more pizza than Horatio. Ask students to determine whether Larry is correct? Invite students to explain their thinking using pictures, numbers, and words.
- Create a set of equivalent mixed number and improper fraction cards and distribute a card to each student. Invite students to find their equivalent partner. Have students line up by pairs in ascending order (a temporary number line on the floor might be helpful for students). This task should be done after students have had an opportunity to develop their understanding with models.

- Ask students to model $\frac{9}{4}$ and tell how many groups of 4 are in 9. For example,



- Tell students you modelled the improper fraction $\frac{8}{6}$ using pattern blocks. Ask them how they know you used 8 blocks. Ask them to determine which pattern block was used to model $\frac{8}{6}$ and to explain how they know.
- Write the following statement on the board, "All improper fractions must be greater than one whole." Ask students to determine, using models and pictures, whether this statement is true or false. Have students justify their thinking about the statement.
- Invite students to work in pairs. Each student can use any type of manipulative available to model an improper fraction. Ask students to pass their model to their partner. Each partner must determine if the model represents an improper fraction. Each partner must explain his or her thinking using a picture and the symbolic form.
- Using linking cubes, show students a model of one whole. For example, use five same-coloured linking cubes to represent one whole. Invite students to use this whole to explore different ways to create an improper fraction that would come between 1 and 2.
- Provide index cards, and ask students to create cards that name an improper fraction and that represent the improper fraction (e.g., students draw five rhombuses to represent $\frac{5}{3}$ when one hexagon represents one whole, and then create the corresponding card with the symbolic form of $\frac{5}{3}$). Students can then combine their completed cards and play a matching game in which they have to match the picture of the improper fraction with the symbolic form of that fraction.
- Ask students to think of a mixed number that is a little less than $\frac{9}{5}$. Invite them to show how they know and to explain their thinking.
- Ask students to model and then draw a picture to show that $\frac{5}{2} = 2\frac{1}{2}$.
- Invite students to make a plan to teach their parents about improper fractions and mixed numbers. Have students use models, pictures, numbers, and words to show their parents how to express a mixed number as an improper fraction. Students should be encouraged to use technology to create their presentations.
- Encourage students to work in pairs. Each pair of students will use a deck of cards containing the numerals 1–9. Students will shuffle the numeral cards and will deal four cards to each player. Students use any two of the four numeral cards in their hand to create the greatest possible improper fraction. In turn, each player will reveal their improper fraction, determining which partner has the greatest fraction. Students may have to convert these improper fractions to mixed numbers to help them compare the numbers. The player with the greatest improper fraction will score one point. The first player with five points wins.

- Using the same set of numeral cards described above, invite students to work in pairs. Shuffle the cards and deal three cards to each partner. Students use the cards to create the greatest possible mixed number. In turn, each player will reveal their mixed number. The player with the greatest mixed number will score one point. The first player with five points wins.
- Ask students to think of possible values for when $\frac{13}{?}$ is an improper fraction that is between 2 and 3. Ask students to determine if there is more than one answer and explain how they know.
- Ask students to choose a mixed number, and tell them to keep it a secret from their classmates. Set up centres around the room at which students model their number in one centre, draw it in another, and then represent it using an improper fraction in another. After all students have had an opportunity to represent their number at each of the three centres, bring the class together. Ask students to match all the models to the corresponding pictures and numbers.
- Provide students with a number line marked with 0, 1, 2, 3, 4, and 5. Invite students to place the following improper numbers and mixed numbers on the number line: $\frac{3}{2}$, $3\frac{1}{2}$, $\frac{5}{4}$, $4\frac{1}{5}$, $1\frac{2}{5}$. Ask students to choose one of the numbers and to explain its placement.
- Ask students to explain how they would *immediately* know that $2\frac{2}{5}$ is greater than $1\frac{7}{8}$?
- Ask students to choose two improper fractions or two mixed numbers, and compare and order them. Invite students to explain to a friend how they know they have ordered their numbers correctly.
- Have students write two improper fractions and two mixed numbers that are between 4 and 5 and explain their thinking.
- Tell students that the answer to a problem is $2\frac{1}{3}$. Ask them what the question might be?
- Ask students to use any manipulative to model or represent $\frac{5}{3}$. Discuss with students their choice of models, and lead the discussion to help them see or link the $\frac{5}{3}$ to $1\frac{2}{3}$. Invite students to explain some of their personal strategies that help them to understand that $\frac{5}{3}$ is the same as 1 and $\frac{2}{3}$.

Suggested Models and Manipulatives

- coloured tiles
- Cuisenaire rods
- double number lines
- egg cartons
- fraction circles
- fraction pieces
- number lines
- pattern blocks

MATHEMATICAL LANGUAGE

Teacher	Student
▪ benchmarks ▪ denominator ▪ format ▪ improper fractions ▪ linear, area and set models ▪ meaning ▪ mixed numbers ▪ numerator ▪ proper fractions	▪ benchmarks ▪ denominator ▪ improper fractions ▪ linear, area, and set models ▪ meaning ▪ mixed numbers ▪ numerator ▪ proper fractions

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students, K–8* (Small 2008), p. 202
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), p. 256
- *Teaching Student-Centered Mathematics, Grades 5–8* (Van de Walle and Lovin 2006), p. 69

Notes

SCO N05 Students will be expected to demonstrate an understanding of ratio, concretely, pictorially, and symbolically. [C, CN, PS, R, V]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N05.01 Represent a given ratio concretely and pictorially.
N05.02 Write a ratio from a given concrete or pictorial representation.
N05.03 Express a given ratio in multiple forms, such as "three to five," 3:5, 3 to 5, or $\frac{3}{5}$.
N05.04 Identify and describe ratios from real-life contexts and record them symbolically.
N05.05 Explain the part-whole and part-part ratios of a set (e.g., For a group of three girls and five boys, explain the ratios 3:5, 3:8, and 5:8.).
N05.06 Solve a given problem involving ratio.
N05.07 Verify that two ratios are or are not equivalent using concrete materials.

Scope and Sequence

Mathematics 5	Mathematics 6	Mathematics 7
N07 Students will be expected to demonstrate an understanding of fractions by using concrete, pictorial and symbolic representations to ▪ create sets of equivalent fractions ▪ compare and order fractions with like and unlike denominators	**N05** Students will be expected to demonstrate an understanding of ratio, concretely, pictorially, and symbolically.	**N03** Students will be expected to solve problems involving percents from 1% to 100% (limited to whole numbers). **SP04** Students will be expected to express probabilities as ratios, fractions and percents.

Background

Understanding ratio is critical to the development of proportional reasoning. Have students compare and determine the equivalence of two ratios based on concrete models, rather than relying on rules. Students need to consider ratios in a variety of contexts.

Ratios and fractions are both **comparisons**. A ratio is a way to represent the comparison of two numbers or quantities. Ratios can be used to compare part-to-part or part-to-whole. Ratios may be expressed in words, and in number form with a colon between the two numbers. For example, the ratio of two boys compared to five girls can be expressed as two to five, 2 to 5, or 2:5. If the ratio was expressed as five to two, 5 to 2, or 5:2, it would be the comparison of 5 girls to 2 boys. It is, therefore, possible to express the comparison of two numbers or quantities in more than one way.

When working with ratios, the items and the order in which they are being compared must always be stated. For example,

- The ratio of faces to hearts is 4:1.
- The ratio of hearts to faces is 1:4.
- The ratio of faces to all is 4:5.
- The ratio of all to hearts is 5:1.

All fractions are ratios. A fraction is a part-to-whole comparison. If $\frac{3}{5}$ of a rectangle is shaded, the ratio of shaded parts to the whole is 3:5. However, not all ratios are fractions. As noted above, a ratio can be a part-to-part comparison. In the case of the rectangle with three of the five parts shaded, the ratio of shaded parts to unshaded parts (a part-to-part comparison) is 3:2. This is not a fraction. This could be confusing for students because a ratio can also be written using fractional form. In the example, this ratio (3:2) could also be expressed as $\frac{3}{2}$. It is recommended that ratios written in fractional form be read using ratio language, such as as three is to two rather than as three-halves.

Ratios may be generated in geometric, numerical, and measurement situations. Some examples follow.

Geometric situations
- The ratio of the number of sides in a hexagon to the number of sides in a square (6:4).
- The ratio of the number of vertices to the number of edges in a rectangular prism (8:12).
- The ratio of the number of vertices in a hexagon to the number of sides (6:6).

Numerical situations
- The ratio comparing the value of a quarter in cents to the value of a dime in cents (25:10).
- The ratio comparing the number of cups of water to the number of cups of rice when cooking (2:1).
- The ratio comparing the number of cans of frozen juice concentrate to the number of cans of water required when mixing it (1:3).

Measurement situations
- The ratio comparing the perimeter of a square to the side length of a square (4:1).
- The ratio describing the size of scale models or the scale on a map (1:15).

Equivalent ratios can be explored and modelled with concrete materials. This can be accomplished by engaging students with problems within a real-world context. Encouraging students to write and solve their own problems will help them construct and consolidate their understanding of equivalent ratios.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to select a proper fraction. Ask them to write two equivalent fractions for the fraction they have chosen.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to select up to 20 tiles of different colours so that pairs of colours show the following ratios: 4 to 5, 2:1, $\frac{1}{5}$.
- Give students the following information and ask them to write and read ratio comparisons. Have students explain their ratios. Students should be able to express their ratios as fractions, words, and numbers.

  4 cats 3 goldfish 2 hamsters
- Ask students to make a drawing that shows a ratio situation (e.g., for every one pencil, there are three pieces of paper). Ask them to write ratios to describe what the picture shows and describe the different ratios it represents.
- Tell students that the ratio of boys to the total number of students in the class is 13:28. Ask how many girls are in the class?
- Ask students what would the ratio of legs to heads would be in a group of bears. Of people? Of spiders?
- Ask students to explain why they might describe the ratio below as 4:1 (all to girls) or as 1:4 (girls to all). Are there other ratios that can be used to describe what is given? (B = boy; G = girl)

  B B B G
- Ask students to solve problems such as the following: It is evening and Joy-Lynn is out in the yard with her brother. Joy-Lynn is one metre tall and casts a shadow that is three metres long. If Joy-Lynn stands on her brother's shoulders, which are 1.5 m above the ground, how long a shadow will she and her brother cast?

Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

Choosing Instructional Strategies

Consider the following strategies when planning daily lessons.

- Use counters, other simple models, or the students themselves to introduce the concept of ratio as a comparison between two numbers.

  For example, in a group of three boys and two girls:
  - 3:2 tells the ratio of boys to girls (part-to-part)
  - 3:5 tells the ratio of boys to the total group (part-to-whole)
  - 2:5 tells the ratio of girls to the total group (part-to-whole)
  - 2:3 tells the ratio of girls to boys (part-to-part)

  Students should read “3:2” as “3 to 2” or “3 ___ for every 2 ___.”
- Explore ratios that occur in everyday situations (e.g., the ratio of water to concentrate to make orange juice is 3:1 or “3 to 1”).
- Ask students to use colour tiles, pattern blocks, linking chains, or other models to represent ratio comparisons.
- Encourage students to write and solve each other’s ratio problems.

SUGGESTED LEARNING TASKS

- Provide students with a recipe for lemonade: 4 cups of water, 1 cup of lemon juice, 1 cup of sugar. Have students write about the various ratios that can be drawn from this data. Ask which juice will be stronger flavoured: 3 cans of water for 1 can of concentrate or 4 cans of water for 2 cans of concentrate? Ask students to identify which ratios are also fractions (part-to-whole comparisons).
- Invite students to poll their classmates to determine what pets they have (or other topics such as eye colour, shoe size, hair colour, etc.). Ask them to write part-to-part and part-to-whole ratio comparisons. Require students to write their ratios in words, using a colon, and in fraction form (for part-to-whole only).
- Ask students to model two situations that can be described by the ratio 3:4. Specify that the situations must involve a different total number of items.
- Ask the student to find the following body ratios, comparing results with others.
  - wrist size:ankle size
  - wrist size:neck size
  - head height:full height
- Ask students to show a given ratio pictorially or concretely (e.g., to show 4:5 [part-to-part], one possible solution would be

  ○ ○ ○ ○ ● ● ● ● ●

  Invite students to write three other possible ratios for the model.
- Ask students to write a number of ratios that relate to sport or other real-world situations. For example, compare the number of players on the ice in hockey compared to the number on a soccer field.
- Provide students with a problem such as the following:
  - Mrs. Gupta bought six apples and some oranges at the store. The ratio of apples to oranges was 3:2. Altogether, how many apples and oranges did Mrs. Gupta buy? Explain your answer pictorially, symbolically, and concretely.
- Tell students that the Sharma family has a mother, father, two daughters, and one son. The part-to-part ratio of males to females is 2:3. The part-to-whole ratio of males to whole family is 2:5. Ask students to represent these ratios using counters.
- Invite students to draw pictures of their families. Ask students to write a part-to-part ratio and a part-to-whole ratio to describe their picture (e.g., number of arms to legs and number of children to people). Allow time for students to share their pictures and ratios with the class. Next, ask students to switch pictures with a partner. Give each student a strip of paper and ask them to write a word problem involving equivalent ratios to go with their classmate's picture. Display the pictures and word problems around the classroom. Allow students time to solve all the problems by having students do a gallery walk in pairs.
- Present the following diagram to students:

  x x x o

  x x x o

  x x x o

  Ask students to write equivalent ratios demonstrated through this diagram and to explain their thinking.

- For each of the following ratios, 4:6, 10:30, 3:5, and 4:5, ask students to find an equivalent ratio in which one of the terms is 20.
- Tell students that in a class of 30 students, there are 20 girls. Ask them to explain why the ratio of boys to girls is 1:2.
- Ask students to create a picture representing various groups of items and write two equivalent ratios that can be found in the picture. Ask them to explain their thinking.

SUGGESTED MODELS AND MANIPULATIVES

- colour tiles
- counters
- linking cubes
- pattern blocks

MATHEMATICAL LANGUAGE

Teacher	Student
▪ comparison	▪ comparison
▪ equivalent	▪ equivalent
▪ equivalent fractions	▪ equivalent fractions
▪ geometric, numerical, and measurement	
▪ multiple forms	▪ multiple forms
▪ part-to-part and part-to-whole	▪ part-to-part and part-to-whole
▪ proportional reasoning	
▪ ratio	▪ ratio

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students, K–8* (Small 2008), pp. 250–251, 253–255
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 304–305, 308–310
- *Teaching Student-Centered Mathematics, Grades 5–8* (Van de Walle and Lovin 2006), pp. 155, 156–157, 159, 161

Notes

SCO N06 Students will be expected to demonstrate an understanding of percent (limited to whole numbers) concretely, pictorially, and symbolically. [C, CN, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N06.01 Explain that "percent" means "out of 100."
N06.02 Explain that percent is a ratio out of 100.
N06.03 Represent a given percent concretely and pictorially.
N06.04 Record the percent displayed in a given concrete or pictorial representation.
N06.05 Express a given percent as a fraction and a decimal.
N06.06 Identify and describe percent from real-life contexts, and record them symbolically.
N06.07 Solve a given percent problem involving benchmarks of 25%, 50%, 75%, and 100%.

Scope and Sequence

Mathematics 5	Mathematics 6	Mathematics 7
N09 Students will be expected to relate decimals to fractions and fractions to decimals (to thousandths).	**N06** Students will be expected to demonstrate an understanding of percent (limited to whole numbers) concretely, pictorially, and symbolically.	**N03** Students will be expected to solve problems involving percents from 1% to 100% (limited to whole numbers). **SP04** Students will be expected to express probabilities as ratios, fractions and percents.

Background

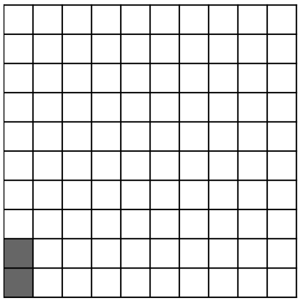
When students have made the connection between fractions and decimals, percent may be introduced. Base-ten blocks and hundredths grids provide models to help students make the connection between fractions, decimals, and percent.

Percent is a part-to-whole ratio that compares a number to 100. "Percent" means "out of 100" or "per 100." Students should understand that percent, on its own, does not represent a specific quantity. For example, 90% might represent 9 out of 10, 18 out of 20, 45 out of 50, and 90 out of 100. This is the first year for students to explore this concept.

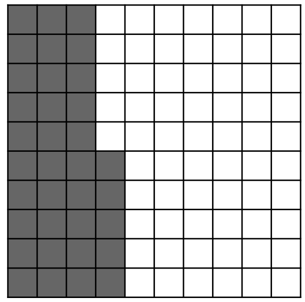
Percent can always be written as a decimal or vice versa. For example, 26% means 26 out of 100, which is the same as 0.26. Both mean 26 hundredths or $\frac{26}{100}$. Students should use this reasoning and be able to express a percent as a fraction and as a decimal. This is more meaningful than using an arbitrary rule such as, to change a decimal to a percent, move the decimal two places to the right.

Students should recognize

- situations in which percent is commonly used
- diagrams showing parts of a set, whole, or measure that represent various percentages (e.g., 2%, 35%)



2 %



35 %

- the relationship between the percent and corresponding decimals and ratios (e.g., 48%, 0.48, 48:100)
- the percent equivalents for common fractions and ratios such as $\frac{1}{4}$ = 25%, $\frac{1}{2}$ = 50%, and $\frac{3}{4}$ = 75%
- percent on its own does not represent a specific quantity; instead, it represents a special ratio (e.g., 50% might represent 50 out of 100 or 20 out of 40).

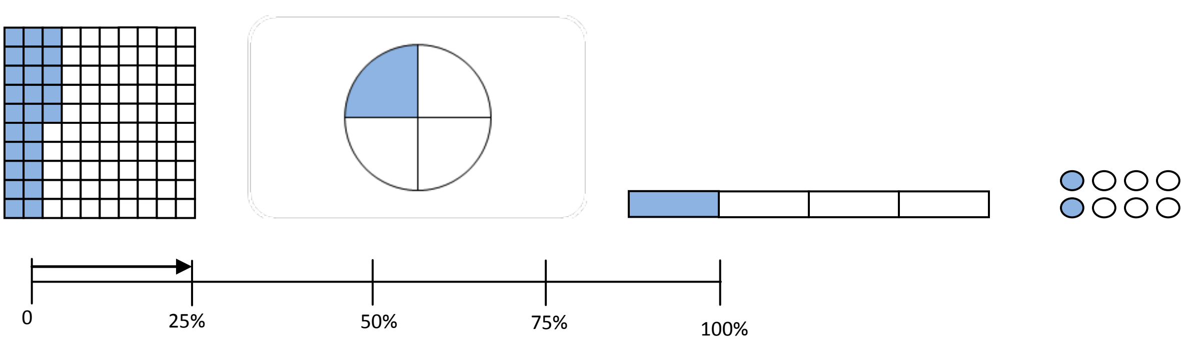
In Mathematics 6, students work with percents between 1 and 100.

Number sense for percent should be developed through the use of these basic **benchmarks**:

- 100% is one whole
- 50% is one-half
- 25% is one-fourth or one-quarter
- 75% is three-fourths or three-quarters

Students are not expected to solve problems that involve calculating percent such as 15%, 35%, 33%, 67%, etc. They are, however, expected to solve problems using the benchmarks of 25% , 50%, 75%, and 100%.

It is important for students to use a variety of representations of percent to help deepen their understanding. For example, 25% can be represented in a variety of ways as shown below.



Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Assessing Prior Knowledge

Tasks such as the following could be used to determine students' prior knowledge.

- Give student partners six cards with different base-ten block pictures on them. Ask them to order the numbers represented and read the decimals to one another. Explain that the flat represents 1 for this task.

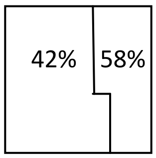
Whole-Class/Group/Individual Assessment Tasks

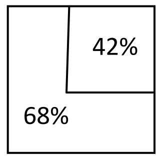
Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students,
  (a) Which is the least? The greatest? Explain your answer.
  $\frac{1}{20}$ 20% 0.02
  (b) Which one doesn't belong? Explain your choice.
  $\frac{3}{4}$ 0.75 0.34 75%
- Ask students what percent of a metre stick is 37 cm?
- Invite students to examine a set of objects and describe different ratio and percent equivalents.
- Ask students to name percents that indicate
  - almost one whole (all) of something
  - very little of something
  - a little less than one-half of something
- Tell students that 60 new floor tiles are being installed in a room. The tiles used must be the following colours: 25% must be blue; 5:10 must be red; 0.20 must be green; the rest are yellow. How many would there be of each colour? Invite students to draw and colour a picture on grid paper to

show what the room tiles may look like and explain how they decided how many of each colour needed to be used.

- Ask students to describe a situation when 45% can be greater than 90%.
- Ask students what is incorrect about each of the following diagrams and have them justify their answers.

(a) 

(b) 

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

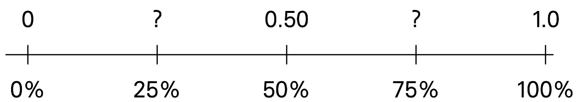
Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

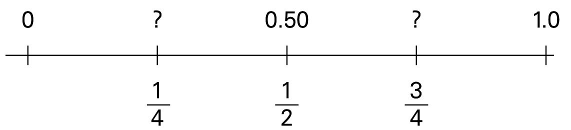
CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Provide students with many opportunities to work with partially shaded hundredths grids, determining the decimal, fraction, ratio, and percent that is shaded.
- Make charts, including symbolic representations, for fractions, decimals, and percents that are equal.
- Use virtual manipulatives that are available on the Internet and interactive whiteboard software.

- Invite students to predict percentages, give their prediction strategies, and then check their predictions. For example, ask them to estimate the percentage of
  - each colour of counter, if a total of 100 blue, red, and green counters are shown on an overhead for 10 seconds
  - a hundred grid that is shaded in to make a picture
  - red counters when 50 two-coloured counters are shaken and spilled
- Use a double number line as a useful tool to model and solve simple percent equivalencies and problems. Extend this to include fraction equivalencies as well.





SUGGESTED LEARNING TASKS

- Ask students to draw a design on a hundredths grid and describe the percent that is shaded.
- Invite students to create a pencil crayon quilt made of patches of various colours. They can describe the approximate or exact percentages of the total quilt that is each colour.
- Tell students that Jane is covering her floor with tiles. The cost of covering the whole floor is $84. How much will she have spent on tiles when 25% of her floor is covered? Use a number line to help model.

- Invite students to collect examples of situations from newspapers, flyers, or magazines in which percent is used, and invite them to make a collage for a class display.
- Ask students to estimate the percentage of time students spend each day doing certain activities (e.g., attending school, physical activity, eating, sleeping).
- Provide students with different size pieces of scrap paper, and ask students to tear off about 60% from their piece. Ask them to explain their thinking to a partner. Repeat with other percentages.
- Ask students to estimate the percentage of pages in a magazine that have advertisements on them.
- Tell students that approximately 50% of all people in Canada over 18 years of age vote when it is time to elect a new prime minister. If 50% of your class voted, how many people would that equal? How about in your grade? How about in your school? How about in your community? Was this percentage easy or difficult to work with and why? What would happen if the percentage was 75% or 25%? Would you use the same strategy or a different strategy to find your answer?
- Changing to newer, more energy-efficient light bulbs can save up to 50% on your electric bill. If a person's electric bill was $30 before changing light bulbs, what would the bill be with the newer light bulbs? Talk to your family about your electric bill. How much could you save? Or how much are you

already saving? Make a list of additional ways your family could both conserve energy and save money.

- Ask students to compare 20% and 0.02 on a hundredths grid. Ask students to tell which is greater? Ask them to explain their answer.
- Ask students to name percents that indicate
  - almost one whole of something
  - very little of something
  - a little less than one-half of something

  Ask students to explain their thinking.
- Ask students to estimate the percentage of red that is shown on the Canadian flag. Have them explain their thinking.
- Ask students to shade hundredths grids to show a particular percent, such as 20% or 60%. Ask them to identify what percent is left unshaded.
- Encourage students to use the Internet or print resources to find out such things as
  - What percent of Earth is water?
  - What percent of the rainforests are in danger?
  - What percent of animals are endangered?
- Have students create a collage showing how percents are used in daily life.
- Invite students to draw a design on a hundredths grid and describe the percentage of the grid covered by each colour in their design. Ask questions such as, How many more squares would you have to cover to fill in the grid?
- Encourage students to use the Internet, a geography book, or other print resources to locate flags of various countries. Invite students to choose the flags of three different countries and determine the percent of a flag represented by a particular colour? Ask students to determine the fraction of the flag represented by that colour? Ask students to determine the ratio of the colour to the whole flag?
- Ask students to explain using models, pictures, or words why a decimal can be represented as a percent.
- Ask students to choose a fraction and a percent that are not equivalent. Ask them to use pictures, numbers, and words to explain which is greater.
- Present students with a problem such as, The school has raised $800 to buy new sports equipment. 50% of the money will be spent on volleyball equipment. 25% will be spent on basketball equipment. The remaining money will be used to purchase new scooters. How much money was spent on each item? Ask students to use a number line and benchmarks to help them solve the problem.

SUGGESTED MODELS AND MANIPULATIVES

- double number lines
- hundredths circles
- hundredths grids

MATHEMATICAL LANGUAGE

Teacher	Student
▪ benchmarks ▪ fraction and decimal ▪ percent ▪ percentage ▪ quantity ▪ ratio	▪ benchmarks ▪ fraction and decimal ▪ percent ▪ percentage ▪ quantity ▪ ratio

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students, K–8* (Small 2008), pp. 257–260
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 313–315
- *Teaching Student-Centered Mathematics, Grades 5–8* (Van de Walle and Lovin 2006), pp. 118–120

Notes

SCO N07 Students will be expected to demonstrate an understanding of integers contextually, concretely, pictorially, and symbolically.
[C, CN, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N07.01 Extend a given number line by adding numbers less than 0 and explain the pattern on each side of 0.
N07.02 Place given integers on a number line and explain how integers are ordered.
N07.03 Describe contexts in which integers are used (e.g., on a thermometer).
N07.04 Compare two integers; represent their relationship using the symbols <, >, and =; and verify using a number line.
N07.05 Order given integers in ascending or descending order.

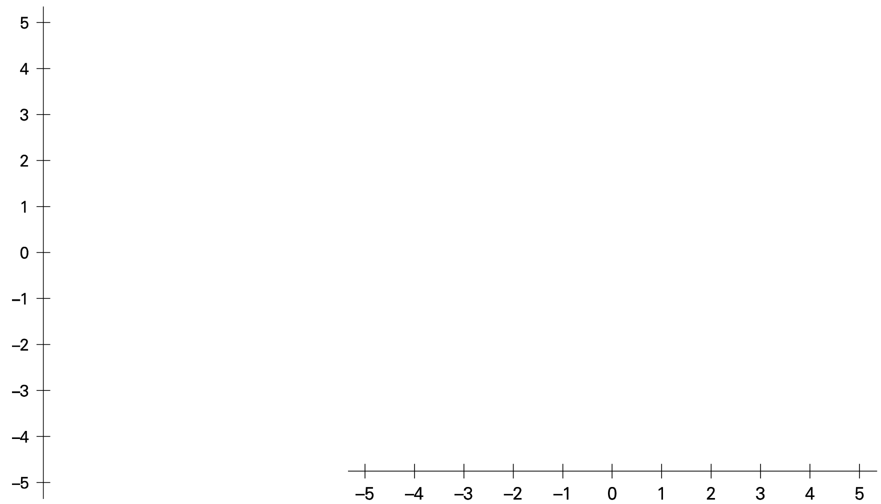
Scope and Sequence

Mathematics 5	Mathematics 6	Mathematics 7
—	**N07** Students will be expected to demonstrate an understanding of integers contextually, concretely, pictorially, and symbolically.	**N06** Students will be expected to demonstrate an understanding of addition and subtraction of integers, concretely, pictorially, and symbolically.

Background

Students are familiar with numbers greater than or equal to 0. This outcome introduces them to integers. Students will explore the set of **integers**, which includes positive and negative whole numbers and zero. “The set of integers, therefore, consists of the positive whole numbers, the opposites of the whole numbers, or negative numbers, and 0, which is neither positive nor negative.” (Van de Walle and Lovin 2006c, 139)

Negative integers may have been part of the day-to-day life of students through experiences such as temperatures below zero and thermometers. To build on this informal understanding, it is beneficial to start with a vertical number line that resembles a thermometer, but also include horizontal models.



Useful contexts when considering negative integers include
- temperatures
- golf scores above and below par
- money situations involving owing money (debits or debts) and earning money (credits)
- distance above and below sea level
- sports scores (goals for and goals against)

When working with number lines, students will recognize that
- 0 is neither positive nor negative
- each negative integer is the mirror image of a positive integer with respect to the 0 mark and is, therefore, the same distance from 0
- “Any negative integer is always less than any positive integer.
- A positive integer closer to 0 is always less than a positive integer farther away from 0.
- A negative integer closer to 0 is always greater than a negative integer farther away from 0.”

(Small 2008, 269, and 2013, 325)

Students will read –5 as negative 5, not as minus 5, to minimize confusion with the operation of subtraction. It is also important for students to recognize that positive integers do not always show the “+” symbol. If no symbol is shown, the integer is positive (0 is neither positive nor negative).

Students need to intuitively make sense of signed quantities by using real examples. These may include sports scores, elevation, and temperatures. Students need to use concrete models such as two-coloured counters and walk-on number lines to model integers. From this work, students need to intuitively recognize that the sum of an integer and its mirror image totals 0. This pair of integers are called opposites. For example, positive three (+3) and negative three (–3) are opposites. They are equidistant from 0 on a number line. Their sum is 0. They are mirror images of each other with respect to 0.

In prior grades, students compared numbers using the vocabulary of **greater than** and **less than** and were introduced to the > and < symbols in Mathematics 3. In Mathematics 6, students will represent comparisons of integers using these symbols.

Addition and subtraction situations involving integers is addressed in Mathematics 7, so students are not expected to perform these operations in Mathematics 6.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Provide students with a blank number line. Have them place positive and negative integers on the line and explain their thinking.

  –5 3 0 –2 2 –1 6

- Ask students, How many negative integers are greater than –7?
- Tell students that a number is 12 jumps away from its opposite on a number line. Ask, What is the number?
- Ask students to explain why –4 and +4 are closer to each other than –5 and +5.
- Ask students to design a simple game for which positive and negative points might be awarded. Have students play and keep track of their total scores.
- Ask students to flip over two playing cards (red cards could represent negative integers and black cards could represent positive integers). Record the comparison symbolically with numbers and the symbols > and <.
- Ask students to explain whether it is true that
  (a) a negative number further from 0 is less than a negative number that is close to 0
  (b) a negative number is always less than a positive number
  (c) a positive number is always greater than a negative number

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

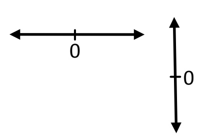
Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Provide students with open number lines in different orientations to explore the placement of integers.



- Explore examples of situations where negative integers are used in various media.
- Ask students to divide a sheet of paper into three parts with the headings Negative, Positive, and Zero. As situations arise throughout the teaching of this outcome, have students record the situation under the headings that best describes it. For example, rise in temperature (positive), spending money (negative), freezing point (zero).
- Give each student a card with an integer recorded on it. Ensure that the set of cards includes pairs of integers, such as +7, –7, and a card with 0. Invite the student holding the “0” card to stand at the front of the classroom in the middle of the room. Invite the remaining students to create a human number line by placing themselves in order according to the card they were given.
- Use a thermometer (vertical number line) to compare integers and record the comparison symbolically ($-8 < 5$; $6 > -7$; $4 < 9$; $-3 > -4$).

SUGGESTED LEARNING TASKS

- Invite 10 students to come to the front of the class. Place an integer on a sticky note on their backs. The students must, without talking, rearrange themselves in ascending order by moving each other.
- Ask students to place a variety of integers at the appropriate places on a number line.
- Ask students to play a game using a deck of cards. The red cards will be used for negative integers and the black cards will be used as positive integers. Each student flips over one card. The student holding the card with the highest value, wins both cards. Play continues until one student has captured all the cards.
- Invite students to choose ten cities and research the temperature for a specific date. Students will then enter the data into a table from warmest to coldest temperatures. Students may use a vertical number line to facilitate this task.
- Ask students to write an integer for each of the following situations:
  - A person walks up eight flights of stairs
  - An elevator goes down seven floors
  - The temperature falls by seven degrees
  - Josh deposits $110 dollars in the bank
  - The peak of the mountain is 1123 m above sea level
- Ask students to investigate **opposite integers** by plotting points such as +5 and –5 on a number line. Ask students to answer questions such as, What do you notice about these integers? Why do you think number pairs such as –5 and +5 are called opposites?
- Ask students to determine whether it is true that any positive number is greater than any negative number. Ask them to use a number line to explain their thinking.
- Provide students with incomplete number lines. Ask them to fill in any missing numbers.
- Ask students to research cities that are below sea level, at approximately sea level, and some that are above sea level. Using a chart, ask them to list these cities from lowest elevation to highest.
- Students may wish to research pro-golf players databases in which they compile data on scores. Explain that golf scores are reported in positive and negative numbers, where positive numbers show how many shots above par were needed to sink the ball. A negative number would be how many shots under par were needed to sink the ball. For example, on a hole that was par 5, it is suggested that it would take five shots to sink the ball. If a player took three shots, he/she would score –2 for that hole. If it took him six shots to sink the ball, his score would be +1. Ask students to rank players according to their scores.
- Tell students that there are 10 numbers between a set of integers. Ask them to identify some possible numbers that could be contained in this set and explain their thinking?
- Create a number line on the board and incorrectly place a negative number on the positive side of the number line. Ask students to decide if this number line is correct and ask them to justify their thinking.
- Invite students to illustrate or discuss a situation in which they have encountered something that could be represented with a negative number.
- Tell students that Josef and John are standing on a number line. Josef is six spaces away from John. Josef is standing on a negative number and John is on a positive number. Ask students to determine some possible numbers Josef and John may be standing on. Ask students to explain the strategies used to solve this problem.
- Provide students with a blank number line. Give them positive and negative integers to place on the line with them choosing their end points and benchmarks.

- Ask students to design a game for which positive and negative points may be awarded. Ask students to play and keep track of their score.
- A number is 12 jumps away from its opposite on a number line. Ask students what the number could be and how they know.
- Ask students to choose two negative integers. Ask them to compare these numbers by describing a context in which they could be used (temperature) and use this context to compare using the less than / greater than symbols.

Suggested Models and Manipulatives

- playing cards
- thermometer
- two-coloured counters
- vertical and horizontal number lines

Mathematical Language

Teacher	Student
▪ "+" and "–" symbols ▪ ascending, descending ▪ greater than, >, less than, <, equal, = ▪ integers ▪ order, compare ▪ zero, negative numbers, positive numbers	▪ "+" and "–" symbols ▪ ascending, descending ▪ greater than, >, less than, <, equal, = ▪ integers ▪ order, compare ▪ zero, negative numbers, positive numbers

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students, K–8* (Small 2008), pp. 267–269
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 323–325
- *Teaching Student-Centered Mathematics, Grades 5–8* (Van de Walle and Lovin 2006), pp. 138–139

Notes

SCO N08 Students will be expected to demonstrate an understanding of multiplication and division of decimals (one-digit whole number multipliers and one-digit natural number divisors). [C, CN, ME, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N08.01 Model the multiplication and division of decimals using concrete and visual representations.
N08.02 Predict products and quotients of decimals using estimation strategies.
N08.03 Place the decimal point in a product using front-end estimation (e.g., For 15.205 × 4, think 15 × 4, so the product is greater than 60.).
N08.04 Place the decimal point in a quotient using front-end estimation (e.g., For $25.83 ÷ 4, think $24 ÷ 4, so the quotient is greater than $6.).
N08.05 Use estimation to correct errors of decimal point placement in a given product or quotient without using paper and pencil.
N08.06 Create and solve story problems that involve multiplication and division of decimals using multipliers from 0 to 9 and divisors from 1 to 9.
N08.07 Solve a given problem, using a personal strategy, and record the process symbolically.

Scope and Sequence

Mathematics 5	Mathematics 6	Mathematics 7
N05 Students will be expected to demonstrate, with and without concrete materials, an understanding of multiplication (two-digit by two-digit) to solve problems. **N06** Students will be expected to demonstrate, with and without concrete materials, an understanding of division (three-digit by one-digit), and interpret remainders to solve problems.	**N08** Students will be expected to demonstrate an understanding of multiplication and division of decimals (one-digit whole number multipliers and one-digit natural number divisors).	**N02** Students will be expected to demonstrate an understanding of the addition, subtraction, multiplication and division of decimals to solve problems (for more than one-digit divisors or two-digit multipliers, the use of technology is expected).

Background

Students will have had experience multiplying and dividing whole numbers in previous grades. The emphasis in Mathematics 6 will continue to be on the understanding of these two operations rather than the mastery of one traditional algorithm. As students extend their learning to multiplying decimal numbers by one-digit whole number multipliers and dividing decimal numbers and one-digit natural divisors, the use of **estimation** is essential to help students ensure the **reasonableness** of their answer. “When estimating, thinking focuses on the meaning of the numbers and the operations and not on counting decimal places.” (Van de Walle and Lovin 2006c, 124–125)

Students must place the decimal point in products and quotients using estimation skills, and not rely on a rule for simply counting decimal places because counting decimal places does not promote an understanding of place value or number sense. The important concept students need to understand is that the place value of the digit in the product or quotient will change according to the placement of the decimal. In considering an example such as $1.255 \times 2 = 2.51$, students will see that counting the decimal places will not help them verify if this answer is correct.

When considering the multiplication of decimals, students should recognize that, for example, 0.8 of something will be almost that amount, but not quite, and 2.4 multiplied by an amount will be double the amount plus almost another half of it. It is important for students to understand that estimation is a useful skill in their lives and regular emphasis on real-life contexts should be provided. Ongoing practise in computational estimation is a key to developing understanding of number and number operations and increasing mental process skills. Although rounding has often been the only estimation strategy taught, there are other strategies (many of which provide a more accurate answer) that should be part of a student's repertoire. Please refer to the background for performance indicator N08.02 for a description of the estimation strategies expected.

A connection should be made between multiplication and division. Multiplication can be used to estimate **quotients**. For example, to estimate a quotient for 74.3 divided by 8, students may think $8 \times 9 = 72$ and $8 \times 10 = 80$. Students would then explain that they know the quotient will be between 9 and 10.

Ensure proper vocabulary when reading multiplication and division problems. This will assist students in making the connection between their knowledge of whole number multiplication and division, and multiplication and division involving decimal numbers. For example, 4×6 is similar to 4×0.6; 4 groups of 6 ones is 24 ones or 24; 4 groups of 6 tenths is 24 tenths or 2.4.

Students should have frequent opportunities to solve and create word problems for the purpose of answering real-life questions of personal interest. These opportunities provide students with a chance to practise their computational skills and clarify their mathematical thinking.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Assessing Prior Knowledge

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to use a model to show how to find the total money collected for photos if 43 students each bring in $23.
- Tell students that at the T-Shirt Shop you can buy T-shirts in packages of eight. One package costs $130. At Big Deals, a T-shirt costs $18. Does Big Deals have the better price? How do you know? Have students record and explain their process.

Whole-Class/Group/Individual Assessment Tasks

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Tell students that you have multiplied a decimal by a whole number and the estimated product is 5.5. What might the two numbers be?
- Ask students to draw or build a model to illustrate 3 × $2.80 and show the answer. Ask students to create a story problem based on the multiplication sentence and share it with a partner to solve.
- Provide a student with a supermarket checkout slip and tell them that it represents a family's weekly groceries. Have students estimate the total amount spent per day or per month by that family.
- Ask students for an estimate of the total cost of eight pens at $0.79 each. Ask what estimating strategy they used and if there is another way to easily estimate the answer.
- Tell students that the class wants to buy six books that cost $11.85 each. How much money will they need? What is the best choice for the correct answer?

  (a) 7.11　　(b) 71.10　　(c) 711.0
- Ask students to estimate the mass of each egg in kilograms, if they know that the total mass of a half dozen eggs is 0.226 kg.
- Ask students to place the decimal in each product. Ask them to explain how estimation helped them correctly place the decimal point in the product.

  (a) 4 × 2.459 = 9836　　(b) 24.35 × 8 = 1948
- Ask students to identify which of the following is the best estimate for 13.7 × 9 and explain why.
  (a) 13.0 × 9　　(b) 14.0 × 9　　(c) 15.0 × 9　　(d) 14.0 × 10　　(e) 10.0 × 9

Follow-up on Assessment

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

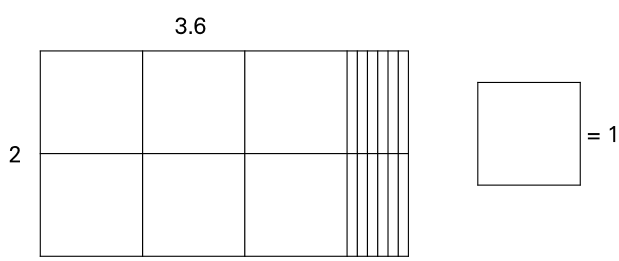
Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Ensure students use proper vocabulary related to multiplication (factors, product) and division (divisor, dividend, quotient) that they have learned in previous grades.
- Ask students to look for benchmark decimals that are easy to multiply and divide. For example, ask students why someone might estimate 516 × 0.48 by taking half of 500.
- Provide opportunities for students to create and solve missing factor and missing divisor/dividend problems, involving decimals, to support the connection between multiplication and division.
- Use the “area model” both concretely with base-ten blocks and pictorially to represent multiplication and division before moving to the symbolic. For example, 2 × 3.6 could be modelled as shown:



- Additional examples of the array model can be found in the Mathematics 5 curriculum document.

SUGGESTED LEARNING TASKS

- Provide students with a number sentence that has decimals missing or misplaced in either the answer or the question. For example, 2.34 × 6 = 1404 has a decimal is missing in the product. Invite students to determine where the decimal should be placed using estimation strategies such as “front-end.”

- Ask students to estimate each of the following and tell which of their estimates is closer and how they know.
  - Three videos games at $24.30/game **or** five teen magazines at $8.89/magazine
  - Nine glasses of fruit smoothies at $2.59/glass **or** four veggie pitas at $4.69/pita
- Tell students that it takes about 9 g of cookie dough to make one cookie. Renee checks the label on the package and finds she has 145.6 g of dough. About how many cookies can she make?
- Ask students to measure side lengths of objects in the classroom to the nearest tenth of a centimetre or hundredth of a metre and then estimate the area of those objects (e.g., side lengths of their desks, their textbooks, or the top of tables).
- Ask students to solve problems that involve dividing the price for a pizza. For example, four people sharing a pizza for $14.56. Change the amount of people and the price of the pizza for more problems.
- Tell students that the cashier told Samantha that her total for 3 kg of grapes at $3.39/kg was $11.97. How did Samantha use estimation to know that the cashier had made a mistake?
- Provide real-world problems involving multiplication and division of decimals where the multiplier/divisor is a one-digit whole number. For example, Jean works at the library for $10.65/hour. Saturday he worked eight hours. What were his earnings? Tom works at a local restaurant. He worked for 7 hours on Sunday and made $74.55. How much does he make in one hour?
- Ask students to figure out how much they need to pay, if they went to the restaurant with three friends and the bill came to $26.88. Students should assume that each person pays their equal share.
- Give students a choice of several multiplication expressions involving decimals multiplied by a single-digit whole number. Ask them to choose one or two sentences and estimate the solution. Next, ask students to explain how they estimated the product(s) and to justify their thinking.
- Ask students to use the information given to determine the better buy.

  Apple juice: 2 L for $1.99 or 4 L for $3.89

  Oranges: four for $0.99 or six for $1.59

  Bananas: 3 kg for $1.89 or 5 kg for $3.19

  Ask students to explain how they know.
- Tell students a decimal number was rounded to 3. Ask them what that number could be.
- Give students a grocery store flyer and ask them to select any item for purchase. Ask them if they were going to purchase six of the same item, approximately how much would it cost. They will need to be able to explain the strategy for estimation. Extend this to a specific dollar amount (e.g., $100) and ask students to select the number of this item that they could purchase with this amount of money without going over.
- Ask students to draw or build a model to illustrate 4 × $1.36 and show the answer. Ask students to create a story problem based on the multiplication sentence and share it with a partner to solve.
- Give students a grocery flyer and ask them to choose three of their favourite items. Ask them to determine how many of each item they could buy with $90.
- Ask students to determine how much more five cans of juice cost at $1.29 each than six cans at $0.99 each.
- Present the following: Milk at school costs $0.55. If there are eight students in your class who each order one milk per day, how much would it cost each day for milk in your class? How much would it cost for one week?

- Pose the following to students: If one school basketball jersey costs \$18.49, approximately how much would it cost to buy nine jerseys. Show how you got your answer pictorially and explain your thinking.
- Tell students to look at the following number sentence where the decimal point has been left out. Ask them to estimate the product to determine the placement of the decimal. Ask them to explain how estimation helped them correctly place the decimal point in the product.
  - 3 × 16.17 = 4851
  - 15.97 × 3 = 4791
  - 4.326 × 7 = 30282
- Ask students to explain why the product of 0.6 and 3 will have a digit in the tenths place. Use words, pictures, and numbers to explain your thinking.
- Ask students to explain whether or not John is right when he says that the answer to 4 × 4.5 is 0.18. Ask them to use pictures, numbers, and words to explain their thinking.
- Give students several examples of multiplication sentences with the answers given. Place the decimal point in an incorrect spot and ask students to explain why the decimal place does not go there and to explain where it should go and why.

  4.35 × 6 = 2.615

  6.487 × 2 = 129.74
- Ask students if they were to estimate the answer for \$21.57 × 5, would it be greater or less than one hundred? Ask them to explain their thinking.
- Present problems such as the following and ask students to solve them and to explain their thinking.
  - Jamal wanted to pay his three friends \$10.15 each for helping him paint his shed. Ask students to estimate the total amount of money that Jamal will have to pay to his friends.
  - Fred calculated that 315.2 × 2 = 63.04. How do you know his answer is incorrect? What is the correct answer? Show your work.
  - A person's hair grows an average of 0.83 cm a month. About how long would a child's hair grow in nine months if they never had a haircut? Explain your thinking.
  - Mr. Brown took his family of eight to a local restaurant. A meal for each person cost \$9.59. Estimate what Mr. Brown's bill will be before taxes. Then, calculate the actual cost of the meal before taxes.
  - Susie had 25.55 metres of string. She needed to hang five balloons from the gym ceiling. How much string did she use for each balloon if she hung each one equally?
  - A group of seven students ordered pizza and the total cost, including tax, was \$51.45. Ask students, How much would each student have to pay if they shared the cost equally?
- Ask students to use base-ten blocks or decimal squares to solve the following:
  - 4.8 × 2
  - 8.12 × 3
  - 6.3 ÷ 2
  - 12.5 ÷ 5
- Ask students to find the numbers that, when multiplied, give the products shown:

  ______ . ______ × ______ = 16.4
- Ask students to think of a situation where it is more practical to estimate the quotient involving a decimal dividend rather than finding the actual answer. Ask them to explain their thinking.
- Ask students to think of a situation where front-end estimation would not be the best estimation strategy to use when solving a division problem involving decimals.
- Ask students to write a story problem using the following division sentence 96.6 ÷ 7.

SUGGESTED MODELS AND MANIPULATIVES

- area models
- base-ten blocks
- calculator
- grid paper
- metre sticks
- money
- number lines
- open array
- place-value chart
- ten-frames (combined to make 100)

MATHEMATICAL LANGUAGE

Teacher	Student
▪ decimal point ▪ decimals ▪ division ▪ divisors ▪ estimation, reasonableness ▪ multiplication ▪ multipliers ▪ product ▪ quotient	▪ decimal point ▪ decimals ▪ division ▪ divisors ▪ estimation, reasonableness ▪ multiplication ▪ multipliers ▪ product ▪ quotient

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students, K–8* (Small 2008), pp. 238–240
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 292–294
- *Teaching Student-Centered Mathematics, Grades 5–8* (Van de Walle and Lovin 2006), pp. 126–128

Notes

SCO N09 Students will be expected to explain and apply the order of operations, excluding exponents, with and without technology (limited to whole numbers). [CN, ME, PS, T]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N09.01 Demonstrate and explain, with examples, why there is a need to have a standardized order of operations.

N09.02 Apply the order of operations to solve multi-step problems with or without technology (e.g., computer, calculator).

Scope and Sequence

Mathematics 5	Mathematics 6	Mathematics 7
—	**N09** Students will be expected to explain and apply the order of operations, excluding exponents, with and without technology (limited to whole numbers).	**N02** Students will be expected to demonstrate an understanding of the addition, subtraction, multiplication and division of decimals to solve problems (for more than one-digit divisors or two-digit multipliers, the use of technology is expected).

Background

Now that students have developed all four basic operations with whole numbers, it is important to provide them with situations in which they can recognize the need for the order of operations. Students should recognize that the convention for **order of operations** is necessary in order to maintain consistency of results in calculations. The purpose of the order of operations is to ensure that the same answer is reached regardless of who performs the calculations.

Context provides students with a reason for the convention for order of operations. Students may be introduced to order of operations by solving a question such as, What is the total cost of theatre tickets for a family with two parents and three children if children's tickets cost $8 each and adult tickets cost $14 each? After solving the problem, ask students to write an expression to explain how they solved the problem. When students write an expression, such as, $3 \times \$8 + 2 \times \$14$, ask if this solution makes sense if it is solved as

$3 \times \$8 = \$24$
$\$24 + 2 = \$26$
$\$26 \times 14 = \$364$

This should help them relate to why the operations are not performed in the order they appear.

When more than one operation appears in an expression or equation, the operations must be performed in the following order:

- operations in parentheses first
- divide or multiply from left to right, whichever operation comes first
- add or subtract from left to right, whichever operation comes first

Students should be taught that **parentheses** ( ) are sometimes referred to as **brackets** [ ]. Some calculators have parentheses that can be entered during calculations, and the use of this function could be used by students. It is important that students recognize that most calculators will not use the order of operations when doing calculations. Students need to enter the digits on calculators following the correct order of operations.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to solve an order of operations expression and then describe what could have happened if the order of operations steps were not followed. For example, students might describe what an incorrect solution could be.
- Tell students that as a result of some faulty keys, the operation signs in these problems did not print. Use the information that is supplied to help determine which operations were used.

  (a) (7 □ 2) □ 12 = 2 (b) (12 □ 4) □ 4 = 7
- Tell students that because the shift key on the keyboard did not work, none of the parentheses appeared in the following equations. If the student has the right answers to both problems, identify where the parentheses must have been.

  (a) 4 + 6 × 8 – 3 = 77 (b) 26 – 4 × 4 – 2 = 18
- Ask students to use their calculators to answer the following question: Chris found the attendance reports for hockey games at the stadium to be 3419 and 4108. If tickets were sold for $12 each, and

expenses for the stadium were $258,712, what was the profit for the two games? Have students write out the equation to demonstrate their understanding of order of operations.

- Ask students to place parentheses in the following equation to determine the various possible solutions.

  4 + 5 × 6 – 2 =

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Invite students to work in groups to answer the equation, 8 – 2 × 4 + 10 ÷ 2, then share their answers. Discuss why some found different answers and the need for rules so we all get the same answer. This could be extended by asking students where parentheses could be placed to get the largest or smallest possible answer.
- Ask students to create word problems from given expressions such as 4 × 10 + 8 × 3.
- Provide students with a variety of equations that do not have parentheses and explore the possible solutions depending on where parentheses are placed.
- Apply the rules for order of operations by modelling a variety of problem solutions. Students can check to determine whether their calculator follows the rules for order of operations. Depending on the type, calculators may yield different results. Students must be aware that most calculators will not use the order of operations to calculate equations automatically.

SUGGESTED LEARNING TASKS

- Ask students to write a number sentence for the following problem: What is the total cost for theatre tickets for a family with two parents and three children if children's tickets cost $9 and adult tickets cost $12? When students write a number sentence such as, 3 × $9 + 2 × $12, ask if this solution makes sense:

  3 × $9 = $27

  $27 + 2 = $29

  $29 × $12 = $348
- Ask students to write number sentences for the following problems and solve them using the order of operations. Consider solving the number sentences for (a) and (b) by ignoring the order of operations. Would the solution make sense in terms of the problem? Discuss.
  (a) Ms. Janes bought the following for her project: five sheets of pressboard at $9 a sheet, 20 planks at $3 each, and two litres of paint at $10. What was the total cost?
  (b) Three times the sum of $35 and $49 represents the total amount of Jin's sales on April 29. When his expenses, which total $75, were subtracted, what was his profit?
- Tell students that Lauren had to answer the following skill-testing questions to win the contest prize.

  (a) 234 × 3 – 512 ÷ (2 × 4)   (b) 18 + 8 × 7 – 118 ÷ 4

  Ask students to determine the winning answers.

  Lauren was told that the correct answer for "b" is 16, but Lauren disagreed. Ask students to explain what the contest organizers might have done incorrectly in solving the question that caused them to get 16 for the answer.
- Ask students to place parentheses in the following equation to explore how many different solutions are possible.   10 + 2 × 8 – 6 ÷ 2 = ?
- Ask students to find some skill-testing questions that can be found on contests. Ask students to answer the question and compare the answers when following the order of operations and the answers when you do not follow them. Discuss the importance, in terms of the contest, of following these rules.
- Ask students to explain why it is necessary to know the order of operations to compute 4 × 7 – 3 × 6. Ask them to compare the solution of the previous problem with the solution of 4 × (7 – 3) × 6. Ask whether the solutions are the same or different and why.
- Provide students with a set of numbers and a target solution as shown below. Invite students to explore and discover where they can place operational symbols and parentheses to achieve the target solution. For example:
  - 3, 6, 3, 4. Solution = 11   Possible answer: 3 + (6 ÷ 3) × 4
  - 3, 6, 3, 4. Solution = 108   Possible answer: (3 + 6) × (3 × 4)
  - 3, 6, 3, 4. Solution = 6   Possible answer: (3 × 6) – (3 × 4)
- Tell students that Molly was doing her mathematics homework when all of a sudden her pet mouse came along and began chewing her paper. When Molly looked at the paper, she noticed all the operation symbols were missing. Ask students to help Molly put these symbols and numbers back to make the statements true. Parentheses should be included where necessary.
  - 12 ? 8 ? 3 ? 2 = 26
  - 8 ? 6 ? 4 ? 2 = 10

SUGGESTED MODELS AND MANIPULATIVES

- calculators
- computers
- two-coloured counters

MATHEMATICAL LANGUAGE

Teacher	Student
▪ calculators ▪ consistency ▪ convention ▪ multi-step ▪ operations ▪ parentheses ▪ standardized order	▪ calculators ▪ always the same ▪ multi-step ▪ operations ▪ parentheses ▪ order

Resources/Notes

Print

- *Teaching Student-Centered Mathematics, Grades 5–8* (Van de Walle and Lovin 2006), p. 132

Notes

Patterns and Relations (PR)

GCO: Students will be expected to use patterns to describe the world and solve problems.

GCO: Students will be expected to represent algebraic expressions in multiple ways.

SCO PR01 Students will be expected to demonstrate an understanding of the relationships within tables of values to solve problems. [C, CN, ME, PS, R, V]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

PR01.01 Generate values in one column of a table of values, given values in the other column, and a pattern rule.
PR01.02 State, using mathematical language, the relationship in a given table of values.
PR01.03 Create a concrete or pictorial representation of the relationship shown in a table of values.
PR01.04 Predict the value of an unknown term using the relationship in a table of values, and verify the prediction.
PR01.05 Formulate a rule to describe the relationship between two columns of numbers in a table of values.
PR01.06 Identify missing terms in a given table of values.
PR01.07 Identify errors in a given table of values.
PR01.08 Describe the pattern within each column of a given table of values.
PR01.09 Create a table of values to record and reveal a pattern to solve a given problem.

Scope and Sequence

Mathematics 5	Mathematics 6	Mathematics 7
PR01 Students will be expected to determine the pattern rule to make predictions about subsequent terms (elements).	**PR01** Students will be expected to demonstrate an understanding of the relationships within tables of values to solve problems.	**PR01** Students will be expected to demonstrate an understanding of oral and written patterns and their equivalent linear relations. **PR02** Students will be expected to create a table of values from a linear relation, graph the table of values, and analyze the graph to draw conclusions and solve problems.

Background

A solid foundation in analyzing and understanding patterns is fundamental to student success and progress in mathematics. In previous grades, students created, extended, and described repeating, increasing, and decreasing patterns. Students learned that a pattern rule describes how each and every term of the pattern is generated and includes the starting point. For example, the pattern rule below could be described as, start at three, add three each time.

xxx	xxx	xxx	xxx
	xxx	xxx	xxx
		xxx	xxx
			xxx
3	6	9	12

Describing how a pattern changes from one term to the next term describes a recursive relationship. For example, in the pattern above, students would have observed that the number of Xs increases by three each time. This is recursive thinking and tells how to find the value of a term given the value of the preceding term.

In Mathematics 5, students continued to describe the recursive relationship in a pattern (i.e., that is how a pattern changed from one term to the next term). However, students were also expected to predict terms in a pattern without using recursive thinking. In order to do this, students began to use tables and charts to display patterns, see relationships, and develop functional thinking. For most students, these tables and charts made it easier to see the patterns from one term to the next within a column of the table or chart (recursive thinking) and they supported students in discovering a rule or relationship (functional thinking) that connected the term number with the term value (between the columns of the table or chart). In the example above and in the chart below, students in Mathematics 5 were expected to see that they can determine the number of Xs (term value) by multiplying the term number by 3. This is functional thinking focused on the relationship between the term and the term value.

Term #	1	2	3	4	5	6	?	…	20
Number of Xs	3	6	9	12	?	?	?	…	?

Students in Mathematics 5 also learned that when using functional thinking they could determine a rule described by a **mathematical expression**. For example, in the pattern above, the rule could be described as $3 \times n$ or $3n$. Therefore the 20th term would be 3×20 which is 60.

In Mathematics 6, students will continue to use tables to help them determine the relationship between the term number and the term value (functional thinking). They will continue to describe the relationship or pattern rule through the use of algebraic expressions. The focus this year is to generate a table of values from an algebraic expression and to derive the expression from a given table of values. Through their understanding of representing patterns concretely, extending patterns, finding missing values, and creating algebraic expressions, students will use their knowledge of patterns to solve problems.

At this grade level, the patterns should include one or more operations, and the pattern rule should be complete enough so that the rule could be used to find missing or subsequent terms in the pattern.

Similarly, given a pattern rule, students will create the pattern from the rule. For example, given the pattern rule, multiply each term by three and subtract one, students should produce the following:

$(1 \times 3) - 1 = 2$
$(2 \times 3) - 1 = 5$
$(3 \times 3) - 1 = 8$
$(4 \times 3) - 1 = 11$

From this work, students will develop the sequence 2, 5, 8, 11, … and the table of values shown below.

Term Number	1	2	3	4
Term Value	2	5	8	11

Finally, students will describe the pattern rule as $(3 \times n) - 1$ or $3n - 1$.

In Mathematics 6, students also learn to make predictions for any term in a given pattern. When students recognize the functional relationship, they will be able to predict the 5th, 10th, or even the 20th term value without recording all the term values in between. Input/output or function machines and tables of values are good models to use to help students focus on functional thinking. The number pattern 1, 3, 5, 7, 9, ... can be shown as follows:

Input	Output
1	1
2	3
3	5
4	7
5	9

When any two of the three components (input, output, pattern rule) are known, the third component can be determined. Data from the input/output machine can then be translated into a table of values with the inputs in one column (the term number) and the outputs (the term value) in another column. Students who are able to use functional reasoning will determine that the rule for this pattern is $2n - 1$. The table of values and pattern rule should be used to predict missing terms.

Term number (n)	1	2	3	4	5
Term value ($2n - 1$)	1	3	5	7	9

Students will use tables to organize and graph the information that a pattern provides. Each pair of numbers (term number, term value) forms an ordered pair that can then be graphed on a coordinate grid. Students will derive a pattern rule and create a table of values for a given linear relationship and create a graph from a table of values. This concept is connected to outcomes PR02, SP01, and SP03.

Students will describe verbally (oral and written), make connections, and move freely among all the representations. Problems can be created and solved using the different representations.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Invite students to fill in missing terms from number sequences and identify the pattern rules.
  4, ___, 12, ___, 20, ...
  18, 16, 14, ___, ___, ...
  2.4, 2.7, ___, ___, 3.6, ...

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

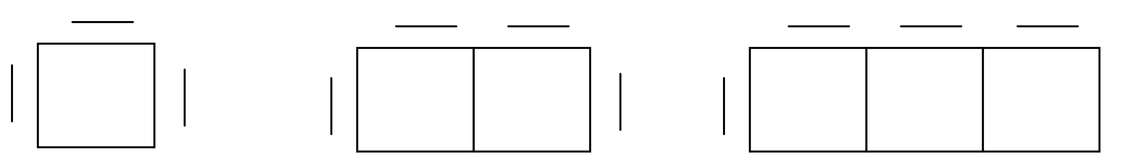
Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to fill in the missing values in a given table.

Term number	1	2	3	?
Term value	4	8	?	16

- Ask students to refer to the following table to answer these questions:

Number of tables	Number of chairs
1	4
2	6
3	8
4	10
5	12



(a) What is the pattern rule for the number of chairs you would need for the tables? Explain your thinking.

(b) Use this rule to predict the number of chairs for 10 tables.

(c) Create a graph to show the values in the table (PR02).

- Present students with tables containing an error in the right column and ask students to identify the value that does not fit the pattern. Ask students to explain why the value in question is incorrect. Ask them to justify their choice.
- Ask students to identify the pattern within each column of the given table:

Number of tables	Number of chairs
1	7
2	12
3	17
4	22
5	27

- Present students with the following problem:

  Sheila works in a computer repair shop. She gets paid \$75 a day plus \$5 for every computer she fixes.

  (a) Create a table to display the total amount of money Sheila could make in a day for any number of computers she might fix.

  (b) Write a pattern rule that you could use to find the total amount of money Sheila could make in a day for any number of computers she might fix.

  (c) Use your rule to determine how much money Sheila would make if she fixed 12 computers in one day.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?

- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Ask students to identify the relationship, rule, and state the value for the 3rd and 12th terms for a given table.
- Present students with a correct pattern rule and a table containing incorrect values. Have students become "Data Detectives" to find and correct the errors.

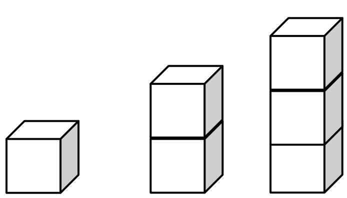
SUGGESTED LEARNING TASKS

- Invite students to create the following pattern with counters, develop a table of values to display the information, write the relationship, and then graph it. Have students predict the value of unknown terms, such as the 10th or 15th term.

- Ask students to fill in the blanks in the table below, describe the relationship for each, and write the rule.

Side Length (cm)	1	2	3	4	5	6	?
Perimeter (cm)	6	12	18	?	30	?	48

- Ask students to describe a real-world situation that depicts a pattern. For example, a taxi ride costs \$2.50 to start and then \$0.40 for each kilometre. How much does it cost to travel 1 km? 2 km? 3 km? Invite students to record the pattern, create a table of values, and graph the relationship. Ask them to determine the total cost of a 15-km trip.
- Tell students that a statue is shaped like a tower and is made of a single column of cubes (see diagram below). A painter has been hired to paint all the cube faces that are visible. This would include the side and top faces as well. The bottom face of each cube is not visible. Students may build the model using linking cubes or blocks. Ask students to create a table of values to record the number of faces that need to be painted on towers 1, 2, 3, 4, and 5 blocks high. Ask students to find the number of faces that would have to be painted for a tower containing 10 blocks. 20 blocks.



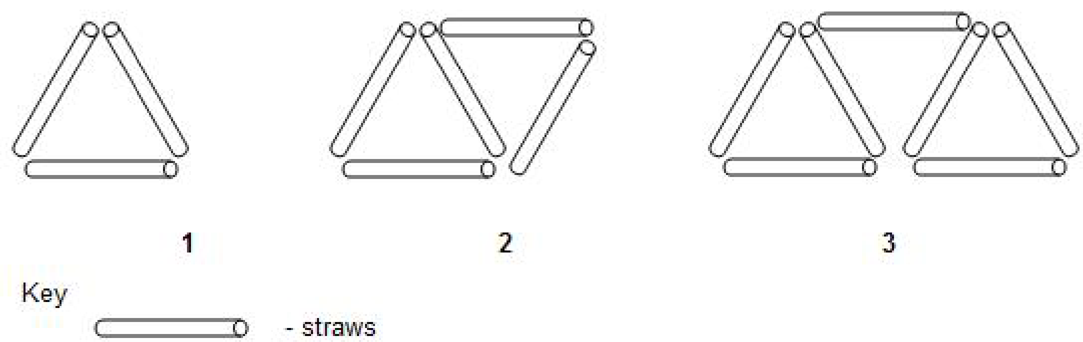
- Tell students that a train has eight wheels and pulls cars, each having four wheels. The table of values below shows the number of wheels on a train with different numbers of cars being pulled. Invite students to draw or make a model of the train with each number of cars being pulled.

Number of cars	0	1	2	3	4
Number of wheels	8	12	16	20	24

- Present students with the following situation: You are going to play paintball with your friends. It costs $20 for admission and an additional $5 for every round of paintball. This relationship can be represented by the expression $5b + 20$. Use this pattern rule to complete the table of values below.

Number of rounds	1	2	3	4	5
Total cost	$25	?	?	?	?

- Ask students to create a table of values to represent the pattern below.
  (a) Write a pattern rule to describe the change within each column.
  (b) Predict the number of straws for diagram 10.



- Ask students to find the missing values in the table below, based on the patterns observed. Ask, How many sandwiches would each person get? Following this pattern, predict how many sandwiches would be needed if 60 people attended the picnic. How many could attend if 90 sandwiches were provided?

Number of people	3	6	?	12	15	?
Number of sandwiches	6	?	18	24	?	36

- Provide students with the tables below. For each table, ask students to determine a pattern rule that can be used to describe the relationship between all the input (term number) and output (term value) combinations. Remind students that the pattern rule has to apply to all the pairs of term numbers and term values in a table, not just the first pair.

Input	Output
1	2
2	3
3	4
4	5
5	6

Input	Output
1	5
2	9
3	13
4	17

- Present problems, such as those below, for students to solve.
  (a) Ted is having a pot-luck dinner. He has prepared four dishes of food for the dinner and has asked all his invited guests to each bring two dishes. The number of dishes at the party depends on the number of guests that come.
    (i) Write a pattern rule that could be used to determine the number of dishes that will be at the dinner for any number of guests that might attend.
    (ii) Use this pattern rule to complete the table of values below.

Number of guests	0	1	2	3	4
Number of dishes	?	?	?	?	?

  (b) Jill works in a store for a wage of $9 per hour. Help Jill complete the following table to show her total earnings after each hour worked in a day. Some values were omitted on each side of the table. Find the missing values:

Hours worked	Total earnings
2	?
?	$36
6	?
?	$72

  Ask students to explain how they derived each omitted value using the pattern rule.

  If Jill wants to buy two pairs of jeans that cost $46 each, how many hours does she need to work in order to buy the jeans?

SUGGESTED MODELS AND MANIPULATIVES

- linking cubes
- pattern blocks
- square tiles
- toothpicks

MATHEMATICAL LANGUAGE

Teacher	Student
▪ increase, decrease ▪ pattern rule ▪ predict ▪ relationships ▪ table ▪ term ▪ term number ▪ term value ▪ unknown term ▪ values ▪ verify	▪ increase, decrease ▪ pattern rule ▪ predict ▪ relationships ▪ table ▪ term ▪ term number ▪ term value ▪ unknown term ▪ values ▪ verify

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students, K–8* (Small 2008), pp. 573–576
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 612–615

Notes

SCO PR02 Students will be expected to represent and describe patterns and relationships, using graphs and tables. [C, CN, PS, R]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

PR02.01 Translate a pattern to a table of values, and graph the table of values (limited to linear graphs with discrete elements).
PR02.02 Create a table of values from a given pattern or a given graph.
PR02.03 Describe, using everyday language, orally or in writing, the relationship shown on a graph.

Scope and Sequence

Mathematics 5	Mathematics 6	Mathematics 7
PR01 Students will be expected to determine the pattern rule to make predictions about subsequent terms.	**PR02** Students will be expected to represent and describe patterns and relationships, using graphs and tables.	**PR01** Students will be expected to demonstrate an understanding of oral and written patterns and their equivalent linear relations. **PR02** Students will be expected to create a table of values from a linear relation, graph the table of values, and analyze the graph to draw conclusions and solve problems.

Background

Mathematics is often referred to as the study of patterns, as they permeate every mathematical concept and are found in everyday contexts. The various representations of patterns including physical models, **table of values**, **algebraic expressions**, and graphs provide valuable tools in making generalizations of mathematical relationships.

Patterns include **repeating** patterns and increasing patterns. An example of a repeating pattern is 1, 2, 2, 1, 2, 2, 1, 2, ...). An example of an increasing pattern is 7, 14, 21, 28, ... Increasing patterns include **arithmetic** (adding or subtracting the same number each time) and **geometric** (multiplying or dividing the same number each time) situations. Some patterns may be a combination of the two. Patterns can be represented using concrete materials and pictures. Students should be able to describe these patterns using words (for example, three times a number, add five) and symbols ($3k + 5$).

Students need to make a connection between the information on a graph and the table of values. They need practice transferring the information on a graph to a table of values and vice versa.

Certain relationships, such as the number of pets owned by a student or the number of siblings, produce discrete data or points. Discrete data has finite values and, usually, is data that can be counted. The data between the points have no meaning or value. As a result, the points on the linear graph should not be connected, and no inferences can be made about values between two data points.

The outcome should be done in conjunction with outcome PR01.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Show students a table that demonstrates the relationship between the number of students going to a movie and the total cost of the tickets. Ask students to describe the relationship between the students and the cost of the tickets using a mathematical expression. Then, ask students to use the pattern to determine the number of students at the movie if the total cost of the tickets is $98.

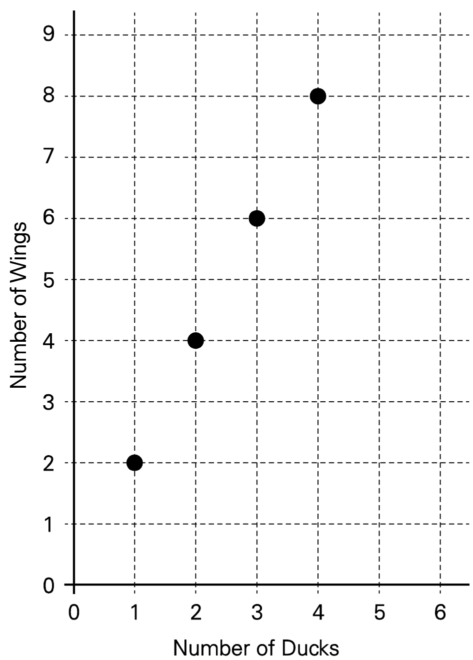
Students	1	2	3	4	?
Cost of tickets	$7	$14	$21	$28	$98

- Give students linking cubes and ask them to create an odd number pattern, starting with one cube, then adding two each time with one to the bottom right and one on the top (L shape). The L shape will increase each time. Ask students to create a table of values for the pattern and then graph it.

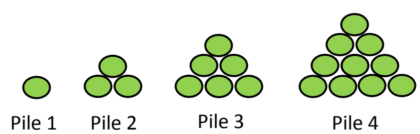
WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to create the table of values from a graph, such as the one below, and describe the pattern rule in words and symbols.



- Provide a visual pattern such as the one below. Ask students to create and graph its table of values and describe the relationship. How many shapes would be needed to make the eighth pile?



  Ask students to create a graph to display the relationship between the number of tricycles and the numbers of wheels. They should represent this data in a table of values as well.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

Choosing Instructional Strategies

Consider the following strategies when planning daily lessons.

- Provide students with linear graphs with discrete data to analyze and have them create corresponding tables of values. Have them describe the relationship shown in the graph using words and symbols.
- Present students with tables and graphs and ask them to identify the patterns represented in each. The graphs and tables should be simple so that the pattern can be readily seen by students.

Suggested Learning Tasks

- Tell students that Mary walked 3 km each day for seven days. Ask students to create a table of values for this data, describe the pattern, and make a graph.
- Invite students to create a concrete and pictorial display of a table of values showing the balance in a bank account or the height of a plant as it grows. Ask students to graph the information.
- Present students with an increasing pattern made from linking cubes. Ask them to make a table of values for the pattern. Then, ask them to create a graph for the same pattern. Ask students to describe the relationship represented by the pattern, the table of values, and the graph.

Suggested Models and Manipulatives

- graph paper
- linking cubes
- pattern blocks
- square tiles
- toothpicks

MATHEMATICAL LANGUAGE

Teacher	Student
▪ algebraic expression ▪ discrete data ▪ geometric ▪ increasing ▪ linear graph ▪ linear relationship ▪ repeating ▪ table of values ▪ translate	 ▪ geometric ▪ increasing pattern ▪ linear graph ▪ repeating pattern ▪ table of values ▪ translate

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students, K–8* (Small 2008), p. 589–590
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 628–629
- *Teaching Student-Centered Mathematics, Grades 5–8* (Van de Walle and Lovin 2006), pp. 270–271

Notes

SCO PR03 Students will be expected to represent generalizations arising from number relationships using equations with letter variables. [C, CN, PS, R, V]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

PR03.01 Write and explain the formula for finding the perimeter of any regular polygon.
PR03.02 Write and explain the formula for finding the area of any given rectangle.
PR03.03 Develop and justify equations using letter variables that illustrate the commutative property of addition and multiplication (e.g., $a + b = b + a$ or $a \times b = b \times a$).
PR03.04 Describe the relationship in a given table using a mathematical expression.
PR03.05 Represent a pattern rule using a simple mathematical expression, such as $4d$ or $2n + 1$.

Scope and Sequence

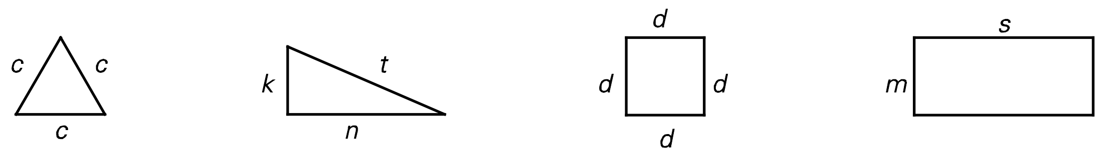
Mathematics 5	Mathematics 6	Mathematics 7
PR02 Students will be expected to solve problems involving single-variable, one-step equations with whole number coefficients and whole number solutions.	**PR03** Students will be expected to represent generalizations arising from number relationships using equations with letter variables.	**PR01** Students will be expected to demonstrate an understanding of oral and written patterns and their equivalent linear relations. **PR02** Students will be expected to create a table of values from a linear relation, graph the table of values, and analyze the graph to draw conclusions and solve problems. **PR04** Students will be expected to explain the difference between an expression and an equation. **PR05** Students will be expected to evaluate an expression given the value of the variable(s).

Background

Depending on how a variable is used, it can have a number of interpretations. In Mathematics 4 and Mathematics 5, students used a variable to represent a single value. In this outcome, the variable is being used to generalize a pattern. For example, variables are used in equations, such as $A = l \times w$, or expressions that are true for all numbers such as $a \times b = b \times a$.

Mathematical patterns and number relationships occur in all areas of mathematics and can be generalized using **algebraic equations**. Previously, students have learned to build and model repeating and increasing patterns, and then develop tables and charts to record them. Tables and charts are graphic organizers that allow students to see mathematical relationships. The next step is to be able to describe these patterns and relationships using an **expression**. An expression may include letters to represent the variables' elements, numbers, and operations (+, –, ×, ÷). It is important that students are able to create and generalize pattern rules to present mathematical situations.

One opportunity for students to make generalizations arising from number relationships is when they explore perimeter and area in outcome M03. One of the goals of this outcome is to make a connection between the concepts to create generalized formulas using variables.



Perimeter $= c + c + c$
$= 3c$

Perimeter $= k + t + n$

Perimeter $= d + d + d + d$
$= 4d$

Perimeter $= m + s + m + s$
$= 2m + 2s$

Area $= m \times s$
$= ms$

Another example of a number relationship generalization is the **commutative property**. Earlier experiences with number combinations have led students to see that addition and multiplication are commutative: changing the order of the **addends** or **factors** does not change the answer. Using **variables** to represent the idea that order does not matter is a good way to describe the property (e.g., $a + b = b + a$ or $a \times b = b \times a$).

We do not write the multiplication sign when a letter variable is used (e.g., instead of writing $3 \times n$, write $3n$). Many students misinterpret $3n$ to mean a number in the thirties, so it is very important that this convention be made clear. Word expressions and word problems should be used in this outcome to reinforce mathematical expressions (e.g., "4 times the number of apples" could be expressed as "4*a*" where *a* represents the number of apples).

Students should also have opportunities to develop mathematical relationships and expressions from the patterns found in tables such as those investigated in outcomes PR01 and PR02.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to draw a diagram of a balance scale and solve single-variable, one-step equations such as the following:
  $18 + n = 31$ $81 = 9p$ $8k = 56$ $m \div 6 = 7$

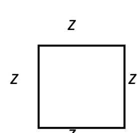
WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

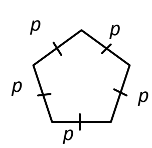
Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Invite students to create an expression for the following patterns:
  - a number doubles
  - a number triples and 2 is added every time
  - 5 is subtracted from a number
  - 2 is added to a number and the sum is doubled
- Provide students with a number of equations such as $27 + 15 = n + 27$. Observe whether the students misinterpret the meaning of the variable, of the meaning of the equal sign, or the commutative property by answering 42. Include multiplication equations as well (see also PR04).
- Invite students to explain how the following two expressions are the same and different. Explain using models, pictures, and words.
  $m \times n$ $n \times m$
- Provide the following table and ask students to generalize the relationship with an expression.

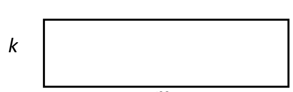
Side length (cm)	1	2	3	4	5
Perimeter (cm)	4	8	12	16	20

- Invite students to write and explain the formula for finding the perimeter of any regular polygon (equilateral triangle, square, regular pentagon, regular hexagon, etc.) using variables.





- Ask students to write and explain the formula for finding the area of any given rectangle using variables.



Follow-up on Assessment

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

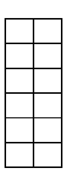
Choosing Instructional Strategies

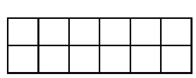
Consider the following strategies when planning daily lessons.

- Ask students to examine the perimeter of regular polygons with various side lengths. They could record the data for a regular hexagon as shown in the table below.

Side length (n) (cm)	1	2	3	4	5	6
Perimeter (P) (cm)	6	12	18	24	30	36

  The next step is to have students generalize the pattern they have found for the perimeter of regular hexagons, stating the pattern rule as an algebraic equation: P(regular hexagon) = 6*n*
- Other types of generalizations can be developed through measurement and pattern tables as students explore the perimeters of other regular polygons and areas of rectangles in M03.
- Reinforce the concept that multiplication is commutative by having students build an array model of a multiplication fact using linking cubes or tiles. Have them turn the model to show the factors in a different order, illustrating the concept that the same product results. The final step is to have students replace the factors with variables.



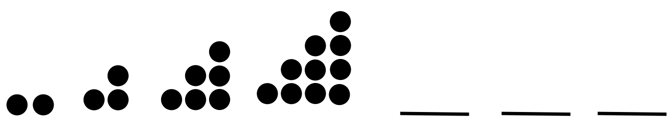


$a \times b = b \times a$

$6 \times 2 = 2 \times 6$

Suggested Learning Tasks

- Play Guess My Rule. Describe a number pattern and invite students to create the mathematical expression that matches the number pattern. For example, “I double every time” or “Divide me in half and add three every time.”
- Provide a table of values and have students generalize the pattern rule and record it as an algebraic equation.
- Provide students with pictures or models of the first three steps of an increasing pattern. Invite students to extend the pattern for several more steps, record the pattern in a table, and look for the relationship. Ask them to write the relationship as an expression and use the expression to predict entries at any step.



Term Number	Term Value
1	2
2	3
3	6
4	10

Suggested Models and Manipulatives

- 2-D shapes
- colour tile
- linking cubes
- pattern blocks

Mathematical Language

Teacher	Student
▪ addends, factors ▪ addition, multiplication ▪ commutative property ▪ equations, variables ▪ formula, area, perimeter ▪ mathematical expression ▪ pattern rule ▪ regular polygon	▪ addends, factors ▪ addition, multiplication ▪ commutative property ▪ equations, variables ▪ formula, area, perimeter ▪ mathematical expression ▪ pattern rule ▪ regular polygon

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students, K–8* (Small 2008), p. 380, 397–398, 583
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 427, 443–444, 621
- *Teaching Student-Centered Mathematics, Grades 5–8* (Van de Walle and Lovin 2006), pp. 253–254, 277

Notes

SCO PR04 Students will be expected to demonstrate and explain the meaning of preservation of equality concretely, pictorially, and symbolically. [C, CN, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

PR04.01 Model the preservation of equality for addition using concrete materials, such as a balance, or using pictorial representations, and orally explain the process.
PR04.02 Model the preservation of equality for subtraction using concrete materials, such as a balance, or using pictorial representations, and orally explain the process.
PR04.03 Model the preservation of equality for multiplication using concrete materials, such as a balance, or using pictorial representations, and orally explain the process.
PR04.04 Model the preservation of equality for division using concrete materials, such as a balance, or using pictorial representations, and orally explain the process.
PR04.05 Write equivalent forms of a given equation by applying the preservation of equality and verify using concrete materials (e.g., $3b = 12$ is the same as $3b + 5 = 12 + 5$ or $2r = 7$ is the same as $3(2r) = 3(7)$).

Scope and Sequence

Mathematics 5	Mathematics 6	Mathematics 7
PR02 Students will be expected to solve problems involving single-variable, one-step equations with whole number coefficients and whole number solutions.	**PR04** Students will be expected to demonstrate and explain the meaning of preservation of equality concretely, pictorially, and symbolically.	**PR03** Students will be expected to demonstrate an understanding of preservation of equality by ▪ modelling preservation of equality, concretely, pictorially, and symbolically ▪ applying preservation of equality to solve equations

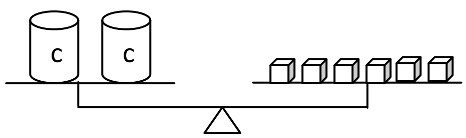
Background

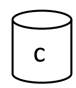
Students developed an understanding of equality and inequality in previous grades. The focus here is to model concretely, pictorially, and symbolically and explain the preservation of equality, before learning to solve equations. Concrete models such as counters and balances and pictorial representations should be used to demonstrate how equations work like balance scales or see-saws. Most students intuitively understand the concept of a balance scale. A balance scale will remain balanced if equal amounts are added to or subtracted from both sides of the scale (additive reasoning). They also need to understand multiplicative reasoning. Both sides of the scale can be multiplied or divided by the same factor. Therefore, when all four operations—addition, subtraction, multiplication, and division—are applied to an equation, the concept of balancing is preserved. Students need to use balances, bags and cubes, and pictures to investigate and model equivalence to prepare students informally for symbolic representation.

Students began exploring the concept of equality in Mathematics 2 and began solving equations in a basic form in Mathematics 3. A misconception for some students may be that the equal sign indicates an answer. They will need further practice and reinforcement in Mathematics 6 to view the equal sign as a symbol of **equivalence** and balance, and represents a relationship, not an operation.

Through the use of balance scales and concrete representations of equations, students will understand that the equal sign means that the quantity on the left of the equal sign is the same as the quantity on the right. When these quantities balance, there is **equality**. When there is an imbalance, there is **inequality**. The work in Mathematics 6 extends this concept so that students discover that any change to one side must be matched with an equivalent change to the other side in order to maintain the balance (preserve the equality). For example, if four is added to one side of the equation, four must be added to the other side in order to preserve the equality.

When using variables, or representing variables using concrete objects such as paper bags or boxes, students need to be directly taught that if the same variable or object is used repeatedly in the same equation, then there is only one possible value for that variable or unknown. For the example below, $c + c = 6$ or $2c = 6$ (c must represent the same number).





c must represent 3 units

represents 1 unit

In Mathematics 3 and 4, **variables** are represented using a variety of symbols, such as circles and triangles. In Mathematics 5, students were introduced to using letters as variables. However, students may have the misconception that $7w + 22 = 109$ and $7n + 22 = 109$ will have different solutions because the letter representing the variable has changed. Also they may see letters as objects rather than numerical values. Conventions of notations using variables may also produce misunderstandings. For example, $j \times z$ is written as jz, but 3×5 cannot be written as "35" and $2g$, where $g = 4$, means 2 times 4, not 24.

Students will explore equivalent forms of a given equation by applying the **preservation of equality** and verify using concrete materials on a balance. They should draw and record the original equation and then add the same amount to both sides. Students should observe that no matter how much they add, the scale will remain balanced as long as they add the same amount on each side. This will help students observe how the equality of the two sides of the equation is preserved. This type of investigation should be repeated to explore subtracting the same amount from both sides, multiplying both sides by the same factor (e.g., double each amount), or dividing both sides by the same divisor.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

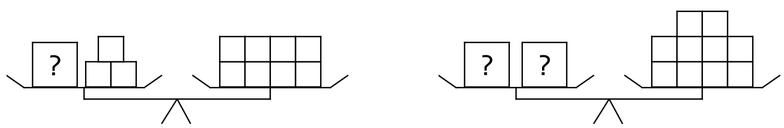
Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to write equations to describe the balance representations, such as the following:



WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to model the following equations using balance scales and various materials.

  Examples:

  $12 + 2s = 18$

  $17 = 5b - 3$

  $3p = 18 \div 2$
- Ask students to determine whether the following forms of pairs of equations are equivalent.

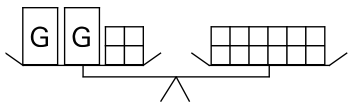
  $4t = 8$ and $4t + 2 = 10$

  $8k = 40$ and $2k = 10$

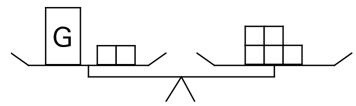
  $12 = j + 7$ and $15 = j + 10$

  $9 = 3s$ and $18 = 9s$
- Invite students to model and write two other equations that are equivalent to $4b = 12$. Ask them to explain how they know the equations are equivalent.
- Ask students to determine if $2g + 3 = 7$ and $3g + 4 = 8$ are equivalent forms of equations. Ask them to explain using models.

- Ask students to write an equation that represents each model:

(a) 

$2G + 4 = 10$

(b) 

$G + 2 = 5$

□ = 1 unit

Ask them to explain what *G* represents in each equation.

After students have recorded the equations, ask,

  - Are the forms of the equations for these two scales equivalent? How do you know?
  - Model and record what will happen if you add two cubes to each side of the balance in (a). Draw the results. Repeat for subtracting two from each side of (a).
  - Model, draw, and record what happens if you multiply both sides of (b) by 3.
  - Model, draw, and record what happens if you multiply both sides of (a) by 3.
- Ask students to write an equation to represent the following situations:
  - Bethany is 3 years older than Toby. Toby is 21 years old. Write and model an equation to represent the problem. Write an equivalent equation to represent the problem that preserves equality.
  - There are 11 muffins on a tray. There were 24 at the start. Some have been eaten. How many muffins are missing from the tray? Write and model an equation to represent the problem. Write an equivalent equation to represent the problem that preserves equality.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

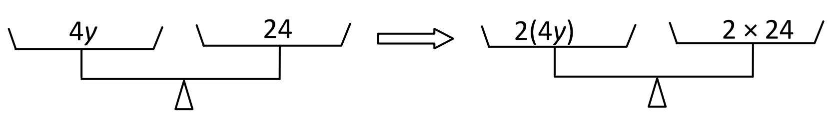
CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

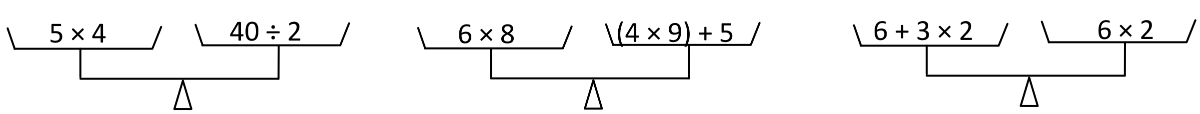
- Use a balance to model equations with bags to represent variables (unknown amounts) and linking cubes or blocks to represent numbers.
- Build known quantities on balance scales to model equality and also model how changes to one side must be matched with equivalent changes on the other. For example, model three cubes plus five cubes on one side and eight cubes on the other. Have students record the equation. Then model adding four to both sides, subtracting two from both sides, doubling both sides, halving both sides, etc. Have students record the equations.
- Model concrete examples of equations that include a variable, such as $3 + x = 10$. Model and record the preservation of equality when 5 is added to each side (e.g., $3 + x + 5 = 10 + 5$). Also explore the preservation of equality using subtraction, multiplication, and division on both sides of the equation.
- Explore the preservation of equality for multiplication by determining whether each side of the equation was multiplied by the same amount. For example, $2r + 3 = 11$ and $6r + 9 = 33$ would be equivalent because all terms in the first equation were multiplied by 3 (tripled). Use a balance to verify.
- Use websites, such as the National Library of Virtual Manipulatives (Utah State University 2014), to provide opportunities to further explore this outcome.

SUGGESTED LEARNING TASKS

- Extend the task of the Tilt or Balance game (Van de Walle and Lovin 2006c, 279) to include adding and subtracting variables.
- Provide illustrations of pan balances that show equal expressions. Ask students to draw and record the shown equation, then draw and record the results when adding the same amount to both sides, subtracting the same amount from both sides, multiplying both sides by the same factor, and dividing both sides by the same divisor.



- Provide a variety of illustrations of pan balances with expressions on each side. Ask students to determine if they balance and to explain their thinking.



- Ask students to draw or model (using a two pan-balance or number line) each set of equations below. Ask them to explain whether each pair of equations are equivalent or not.
  - $n + 2 = 6$ and $n + 3 = 7$
  - $2m + 1 = 9$ and $2m + 2 = 8$
  - $5p + 3 = 18$ and $4p + 3 = 18$
  - $4y = 20$ and $8y = 40$
  - $3k = 12$ and $9k = 2$
  - $2n + 2 = 6$ and $2n + 4 = 6$

- Provide students with cards on which are written the following equations:
  - $15 - s = 9$
  - $11 = 22 - s$
  - $17 - s = 11$
  - $4s = 24$
  - $3s = 12$
  - $4s + 2 = 26$
  - $3s - 4 = 8$
  - $21 - s = 14$
  - $14 - s = 8$
  - $18 = 16 + s$
  - $7 + s = 15$
  - $15 = 13 + s$
  - $9 + s = 17$
  - $0 = 11 - s$

  Lay out the equation cards (face up) so students can see what is on the cards. Ask students to match the equation card with its corresponding equivalent equation. When all equation cards are matched, ask students to choose one equation and create a word problem. Ask students to trade word problems with each other and solve one belonging to a classmate.

Suggested Models and Manipulatives

- balance scales
- base-ten blocks
- colour tiles
- geometric solids
- linking cubes
- number lines
- objects to represent the variables such as pattern blocks

Mathematical Language

Teacher	**Student**
▪ equality ▪ equivalence ▪ inequality ▪ preservation ▪ relationship ▪ variables	▪ equality ▪ equivalence ▪ inequality ▪ preservation ▪ relationship ▪ variables

Resources/Notes

Internet

- National Library of Virtual Manipulatives (Utah State University 2014)
  http://nlvm.usu.edu/en/nav/vlibrary.html

Print

- *Making Mathematics Meaningful to Canadian Students, K–8* (Small 2008), p. 587–588
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 626–628
- *Teaching Student-Centered Mathematics, Grades 5–8* (Van de Walle and Lovin 2006), pp. 279–281

Notes

Measurement (M)

GCO: Students will be expected to use direct and indirect measure to solve problems.

SCO M01 Students will be expected to an understanding of angles by ▪ identifying examples of angles in the environment ▪ classifying angles according to their measure ▪ estimating the measure of angles using 45°, 90°, and 180° as reference angles ▪ determining angle measures in degrees ▪ drawing and labelling angles when the measure is specified [C, CN, ME, V]
[C] Communication **[PS]** Problem Solving **[CN]** Connections **[ME]** Mental Mathematics and Estimation **[T]** Technology **[V]** Visualization **[R]** Reasoning

Performance Indicators

M01.01 Identify examples of angles found in the environment.
M01.02 Classify a given set of angles according to their measure (e.g., acute, right, obtuse, straight, reflex).
M01.03 Sketch 45°, 90°, and 180° angles without the use of a protractor, and describe the relationship among them.
M01.04 Estimate the measure of an angle using 45°, 90°, and 180° as reference angles.
M01.05 Measure, using a protractor, given angles in various orientations.
M01.06 Draw and label a specified angle in various orientations using a protractor.
M01.07 Describe the measure of an angle as the measure of rotation of one of its sides.
M01.08 Describe the measure of angles as the measure of an interior angle of a polygon.

Scope and Sequence

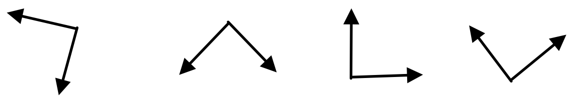
Mathematics 5	Mathematics 6	Mathematics 7
G05 Students will be expected to identify right angles.	**M01** Students will be expected to demonstrate an understanding of angles by ▪ identifying examples of angles in the environment ▪ classifying angles according to their measure ▪ estimating the measure of angles using 45°, 90°, and 180° as reference angles ▪ determining angle measures in degrees ▪ drawing and labelling angles when the measure is specified	**M01** Students will be expected to demonstrate an understanding of circles by ▪ describing the relationships among radius, diameter, and circumference ▪ relating circumference to pi ▪ determining the sum of the central angles ▪ constructing circles with a given radius or diameter ▪ solving problems involving the radii, diameters, and circumferences

Background

Students have been previously introduced to the idea of angles during their study of polygons, but in Mathematics 6 the properties of angles are explored in greater depth. Frequently, angles are defined as the intersection of two **rays** (**arms**) at a common point called a **vertex**. It is more useful, however, for students to conceptualize an angle as a **turn** (rotation) and the measure of the angle as the amount of turn (rotation). “To build an understanding of the connection between angles and turns, students benefit from opportunities to model angles by physically turning arms made from cardboard strips or commercial Geostrips ... fastened together at one end ...” (Small 2008, 457)

It is important for students to understand that

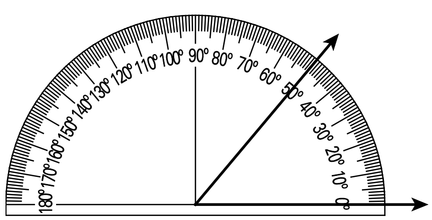
- a larger angle corresponds to a greater turn from the starting position
- the length of the rays (arms) of the angle does not affect the turn amount and, therefore, does not affect angle size
- the orientation of an angle does not affect its measurement or classification



It is also important that students learn the different types of angles and be able to **classify** them as

- **acute** (more than 0° and less than 90°)
- **right** (exactly 90°)
- **obtuse** (more than 90° and less than 180°)
- **straight** (exactly 180°)
- **reflex** (more than 180° and less than 360°)

Prior to measuring with a protractor, students should begin by making their own instruments. Using different size wedges to develop the notion of angle measurement is very important. Moving from an informal angular unit of measure to degrees will help students understand the concept of angular measurement. Students should learn how to use a **protractor** to measure angles accurately. When drawing or measuring angles, students need to be reminded that the centre point of the protractor needs to be lined up with the vertex of the angle, and the initial arm of the protractor must line up exactly with one ray of the angle.



Students typically use protractors with double scales and will need to learn how to determine which set of numbers to use in a given situation. This is best accomplished by first having the student estimate the size of the angle with known benchmark angles such as 45°, 90°, and 180° and then decide which reading makes the most sense. For example, the angle shown above is obviously an acute angle and, therefore, its measure is 50°, not 130°.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

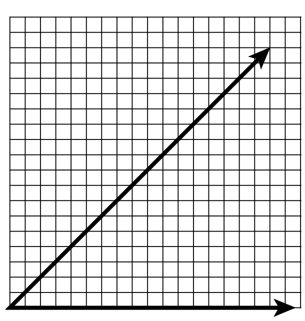
Tasks such as the following could be used to determine students' prior knowledge.

- Provide students with drawings of right angles, greater than right angles, and less than right angles in different positions. Invite students to label each angle as either right, greater than right, or less than right.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

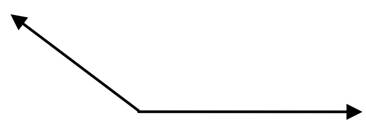
Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to combine two or more pattern blocks to make examples of acute, right, straight, and obtuse angles. Have them record by tracing each one on paper.
- Tell students that the hands of a clock are forming a given angle (such as 45°). Ask what time might be shown.
- Show students the diagram below and ask why it is easy to tell that it is 45°.



- Show students an angle of, for example, 135° and tell them that someone said that it was 45°. Ask them to explain how they think such an error could be made.
- Provide students with various angles and have them measure each with a protractor.
- Ask students to draw angles with specified measures using a protractor.
- Ask students how a 90° angle could be used to construct a 45° angle?

- Invite students to identify angles in objects in the classroom and name the types of angles on the shapes. Have students estimate the sizes of the angles.
- Ask students to identify the angles in various 2-D polygons and on the faces of 3-D shapes and name the types of angles on the shapes. Have students estimate and then measure the sizes of the angles.
- Tell students that Iain measured the angle below and said it measured 50°. Ask students to explain Iain's error?



FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

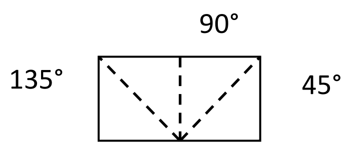
Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

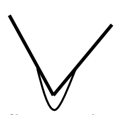
- Invite students to identify angles in a variety of real-life contexts (e.g., angles formed by the two hands of a clock, by the intersection of two roads, and by the blades of scissors or hedge clippers).
- Show students angles (with arms of different lengths) in various positions and of different sizes. Ask them to estimate the measure of each (e.g., almost 45°, 90°, 180°, etc.) of the angles.
- Ask students to stand with their arms closed together pointing out in the same direction to one side. This shows (0°). Then have them raise one arm up until it points directly up (90°), then continue rotating their arm until their arms are out straight to make a straight angle (180°).
- Use children's literature to explore protractors and the different types of angles.

- Invite students to create their own non-standard protractors. Provide students with pieces of translucent paper (tracing paper or waxed paper). Have them fold the paper in half, forming a right angle or square corner. Explain that angles are measured in degrees and that a right angle is 90 degrees. Ask them to fold once again and determine and name the new angles created by the folds. Discuss the measurement of these folds and how they can assist with estimation of angle sizes.



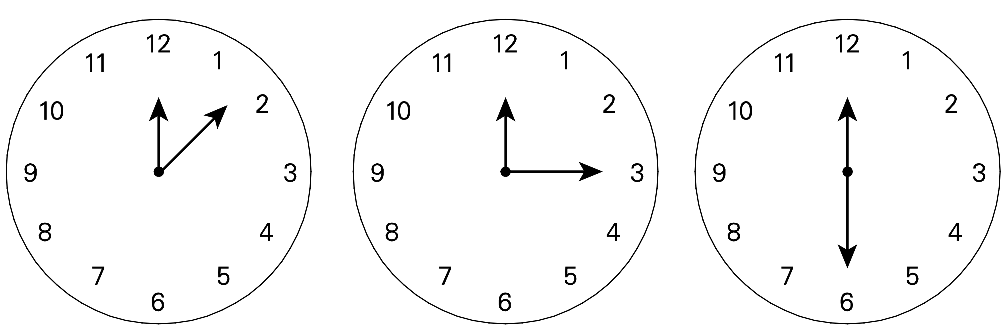
SUGGESTED LEARNING TASKS

- Invite students to investigate angles in various shapes using the corner of a piece of paper as a reference for right angle. Does it fit the angle of the shape or is the angle greater/less than the corner of the paper?
- Invite students to make various angles with pipe cleaners or geo-strips (e.g., almost a right angle, about 45°, a right angle, a straight angle, a reflex angle).

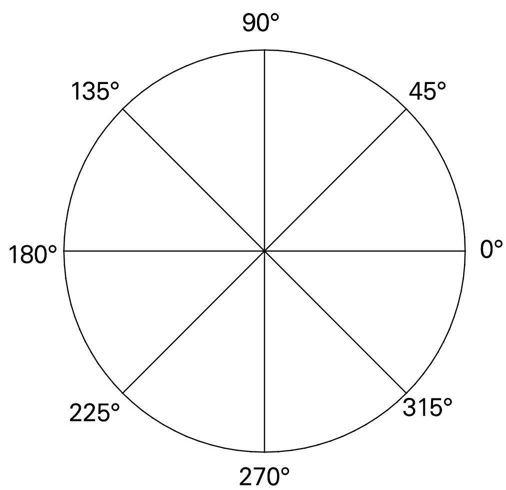


reflex angle

- Ask students to explore the angles in the six different pattern blocks. Which blocks have only acute angles? Only obtuse angles? Both acute and obtuse angles? Only right angles?
- Ask students to measure the angles found in various letters of the alphabet.
- Ask students to explain where acute, right, obtuse, straight, and reflex angles could be identified in the classroom.
- Ask students to identify angles in their surrounding environment. They may do this over the course of a class or a whole day. Ask them to keep a log of where and on what object the angle was identified. Students should create a sketch of the object, highlighting the identified angle in a different colour. They should include a brief description of the angle measure relative to the quarter-turn (right angle), half-turn (straight angle) and three-quarter turn benchmarks. Using the log of angles that they previously identified from their surroundings, ask students to label each of the angles they recorded as acute, right, obtuse, straight, or reflex and to estimate their measure in degrees.
- Ask students to identify the type of angle formed by the hands of a clock at various times of the day. Identify the type of angle created by each time in the clocks shown below.



- Invite students to construct their own 360° protractor using a circular piece of paper. Through this process, 45°, 90°, and 180° benchmarks may be established.
  - Students must first be aware that a circle (full turn) has a measure of 360°.
  - Fold the circle in half. If a full circle measures 360°, a half of a circle must measure 180°.
  - Fold the half circle in half again. This represents a quarter of the whole circle. If half of the circle measures 180°, a quarter must measure 90°.
  - Fold the quarter circle in half again. This represents an eighth of the whole circle. If a quarter of the circle measures 90°, an eighth must measure 45°.
  - Once the circle is unfolded it will be evident that there are eight sets of 45° angles in the whole. Choose a fold line as 0°; label each fold line in the counter-clockwise direction as a consecutive multiple of 45°, as shown below.



  Students now have a guide by which they can estimate the measure of any angle.

- Ask students to sketch each of the following angles without using a protractor. They should label each angle correctly and use an arc to indicate the direction or rotation.
  (i) $\angle ABC$ is 135°
  (ii) $\angle DOG$ is 275°
  (iii) $\angle LMN$ is 88°
  (iv) $\angle ZYX$ is 190°
  (v) $\angle PRQ$ is 100°
  (v) $\angle GEF$ is 290°

  Ask students to explain which benchmarks they found helpful in sketching each angle and how they used these to produce their sketch. Focus on the reasoning students used to arrive at the sketches they produced.

- Ask students to create an original flag. Give students a list of angle measures that must be incorporated in their flag design. Students can create their own flag design that incorporates the specified angle measures. A colour or line type key can be used to indicate the specified angle measures (e.g., Create a flag design that contains at least one 45°, one 120°, and one 155° angle).

  This task can be extended by having students measure any additional angles that may have been formed in their flag creation.

- Ask students to use a straightedge and protractor to draw the following angles:

  (a) 54° (c) 75°

  (b) 135° (d) 156°

  Ask them to label each of the constructed angles with the correct name and its specified measure.
- Ask students to create pieces of art that incorporate a number of specified angle measures. For example, Draw a scene that contains a 45°, 125°, 270°, and a 285° angle. Use a protractor to construct these angles. Label each of the constructed angles using an arc and the specified measure. If students do not want to write the angle measures on their artwork, they may wish to use a colour key. This would involve colouring the arms of each specified angle a different colour and using a key/legend to indicate its measure.
- Ask students to pose so that an assigned angle is displayed somewhere on their bodies.

SUGGESTED MODELS AND MANIPULATIVES

- 2-D and 3-D shapes
- clock hands
- fraction circles
- geo-boards
- geo-strips
- pattern blocks
- Power Polygons
- protractors

MATHEMATICAL LANGUAGE

Teacher	Student
▪ acute, obtuse, right, straight, and reflex angles ▪ classifying ▪ measure, protractor ▪ reference angles ▪ degrees ▪ orientation ▪ turn ▪ rotation ▪ polygon ▪ rays ▪ vertex	▪ acute, obtuse, right, straight, and reflex angles ▪ classifying ▪ measure, protractor ▪ reference angles ▪ degrees ▪ orientation ▪ turn ▪ rotation ▪ polygon ▪ rays ▪ vertex

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students, K–8* (Small 2008), pp. 348–349, 455–466
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 398–399, 501–513
- *Teaching Student-Centered Mathematics, Grades 5–8* (Van de Walle and Lovin 2006), pp. 190–191, 237–239

Notes

SCO M02 Students will be expected to demonstrate that the sum of interior angles is 180° in a triangle and 360° in a quadrilateral. [C, R]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

M02.01 Explain, using models, that the sum of the interior angles of a triangle is the same for all triangles.

M02.02 Explain, using models, that the sum of the interior angles of a quadrilateral is the same for all quadrilaterals.

Scope and Sequence

Mathematics 5	Mathematics 6	Mathematics 7
G02 Students will be expected to name, identify, and sort quadrilaterals, including rectangles, squares, trapezoids, parallelograms, and rhombi, according to their attributes.	**M02** Students will be expected to demonstrate that the sum of interior angles is 180° in a triangle and 360° in a quadrilateral.	**M01** Students will be expected to demonstrate an understanding of circles by ▪ describing the relationships among radius, diameter, and circumference ▪ relating circumference to pi ▪ determining the sum of the central angles ▪ constructing circles with a given radius or diameter ▪ solving problems involving the radii, diameters, and circumferences

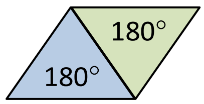
Background

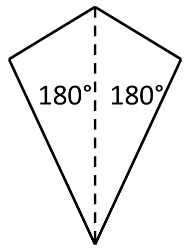
In previous grades, students have investigated some of the attributes of polygons, such as side lengths and vertices. Students will build on these experiences in Mathematics 6 as angles and other properties are investigated in greater depth. It is recommended that M01 and G01 be taught prior to this outcome, so that students are familiar with the measurement of angles, the different types of triangles, and the vocabulary to name and describe them.

Through explorations, students should discover that the angles of a triangle add to 180°. This can be done using paper models and/or dynamic geometry software such as The Geometer's Sketchpad (Key Curriculum 2013) or SMART Notebook (SMART Technologies 2013). Different types of triangles (acute-angled, isosceles, obtuse-angled, equilateral, etc.) need to be used so that students discover that this property applies to all types of triangles.

Once students have an understanding of this property, it would be beneficial to have students measure the interior angles of the triangles using a protractor and find the sum. Students may notice that in some cases their sum does not total 180° exactly, but is very close. It is important that students recognize the potential for human error in measurement.

Exploration of the angle properties of triangles should be extended to **quadrilaterals** by concretely investigating the relationship between triangles and quadrilaterals. Students should discover that two triangles can be combined to create a quadrilateral and conclude that the sum of the angles of a quadrilateral is 360° (180° + 180°).





Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

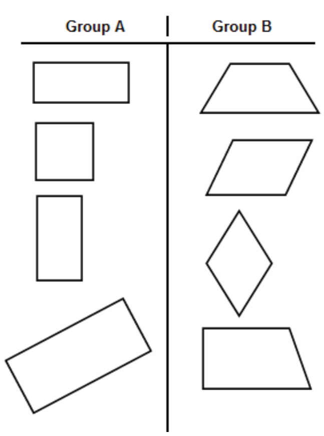
Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

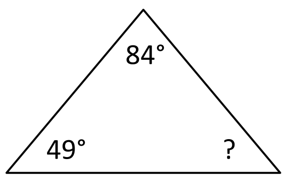
- Provide students with a pre-sorted set of quadrilaterals and ask them to identify the sorting rule.



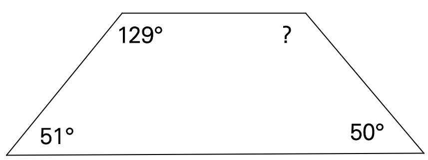
WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students whether a triangle can have more than one obtuse angle. Why or why not? Explain using numbers, pictures, and/or words.
- Ask students whether a triangle can have two right angles. Why or why not? Explain using numbers, pictures, and/or words.
- Ask students to explain how knowing that the sum of the angles in a triangle equals 180° helps them to know the sum of the angles in a quadrilateral. Have students explain their thinking using numbers, pictures, and/or words.
- Invite students to solve to find the measure of the third angle of a triangle when the measures of the other two angles are given.



- Invite students to solve to find the measure of the fourth angle of a quadrilateral when the measures of the other three angles are given.



- Give students the measure of one angle in a triangle. Ask them to derive three other pairs of possible angle measures for the remaining two angles. For example, a triangle has a 45° angle. What are three possible sets of measures for the remaining two unknown angles?
  - Three possible angle sets are as follows:
    - > 45° (given) + 45° + 90° = 180°
    - > 45° (given) + 20° + 115° = 180°
    - > 45° (given) + 30° + 105° = 180°

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

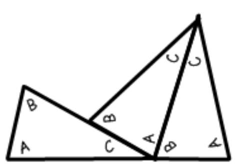
Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

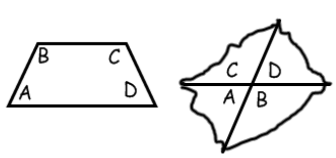
CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

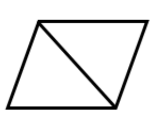
- Invite students to draw a triangle of any type and label the angles *A*, *B*, *C*. Students should cut out the triangle and then tear off the three angles and place the three vertices together to form a 180° angle. Have students measure and record the three angles and then find their sum.
- Invite students to cut out three congruent triangles with angles *A*, *B*, and *C* labelled. Ask students to rotate the triangles so the three different vertices meet at one point to form a 180° angle. Use graphic software or interactive whiteboards and repeat this task.



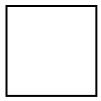
- Cut out a quadrilateral and label the four vertices. Have students tear off the four corners and join the vertices together. Highlight the 360° sum.



- Ask students to draw and cut out a quadrilateral once the sum of the angles of a triangle has been explored and determined. Have them determine that a quadrilateral can be made up of two triangles and the sum of the angles of those two triangles equals 360°.



- Explore how the characteristics of a square are helpful for students in remembering the fact that every quadrilateral has a sum of angles equal to 360°.



SUGGESTED LEARNING TASKS

- Invite students to each draw a variety of different triangles. Have them measure, record, and add the angles of each one. Have them discuss their findings until they reach the conclusion that the sum of the angles of *any* triangle is 180°. Repeat this task using a variety of quadrilaterals.
- Provide a variety of triangles with the measures of two angles shown. Ask students to find the measure of the third angle using their understanding of the sum of the angles of a triangle (without a protractor).
- Ask students to predict the interior angle of an equilateral triangle, and then check by measuring with a protractor.
- Provide students with a variety of quadrilaterals with the measures of three of the angles given. Students must find the measure of the fourth angle without a protractor.
- Organize stations within the classroom. Each station will have a quadrilateral or triangle cut-out. All interior angles would be labelled on the cut-out except for one angle, which has been torn off each shape. Assign students in small groups or pairs to a station. Ask them to find the missing angle measure for their given quadrilateral or triangle. Groups will rotate from station to station until they have found all the missing angle measures.

SUGGESTED MODELS AND MANIPULATIVES

- attribute blocks
- pattern blocks
- Power Polygons
- protractors
- tangrams

MATHEMATICAL LANGUAGE

Teacher	Student
▪ acute angle ▪ acute triangle ▪ equilateral triangle ▪ interior angles ▪ isosceles triangle ▪ obtuse angle ▪ obtuse triangle ▪ quadrilaterals ▪ right angle ▪ right triangle ▪ scalene triangle ▪ sum ▪ triangles	▪ acute angle ▪ acute triangle ▪ equilateral triangle ▪ interior angles ▪ isosceles triangle ▪ obtuse angle ▪ obtuse triangle ▪ quadrilaterals ▪ right angle ▪ right triangle ▪ scalene triangle ▪ sum ▪ triangles

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students, K–8* (Small 2008), pp. 295–296
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 351–352

Software

- The Geometer's Sketchpad (Key Curriculum 2013)
- SMART Notebook (SMART Technologies 2013)

Notes

SCO M03 Students will be expected to develop and apply a formula for determining the ▪ perimeter of polygons ▪ area of rectangles ▪ volume of right rectangular prisms [C, CN, PS, R, V]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

M03.01 Explain, using models, how the perimeter of any polygon can be determined.
M03.02 Generalize a rule (formula) for determining the perimeter of polygons.
M03.03 Explain, using models, how the area of any rectangle can be determined.
M03.04 Generalize a rule (formula) for determining the area of rectangles.
M03.05 Explain, using models, how the volume of any rectangular prism can be determined.
M03.06 Generalize a rule (formula) for determining the volume of rectangular prisms.
M03.07 Solve a given problem involving the perimeter of polygons, the area of rectangles, and/or the volume of right rectangular prisms.

Scope and Sequence

Mathematics 5	Mathematics 6	Mathematics 7
M03 Students will be expected to demonstrate an understanding of volume by ▪ selecting and justifying referents for cubic centimeter (cm^3) or cubic metre (m^3) units ▪ estimating volume, using referents for cubic centimeter (cm^3) or cubic metre (m^3) ▪ measuring and recording volume (cm^3or m^3) ▪ constructing rectangular prisms for a given volume	**M03** Students will be expected to develop and apply a formula for determining the ▪ perimeter of polygons ▪ area of rectangles ▪ volume of right rectangular prisms	**M02** Students will be expected to develop and apply a formula for determining the area of triangles, parallelograms, and circles.

Background

The basic concepts of perimeter, area, and volume have been introduced and explored in previous grades. Students have estimated and worked with both non-standard and standard units. Students need to have many opportunities to experiment with developing their own formulas for calculating perimeter, area, and volume. Developing their own formula to see how they connect and interrelate is much more important than just memorizing a formula. The emphasis for Mathematics 6 is to have students discover the most efficient strategies for finding these measures. These explorations should eventually elicit from students the traditional **formulas** for **perimeter of polygons**, **area of rectangles**, and **volume of right rectangular prisms**. This outcome is closely connected to PR03 where students use letter variables to express a formula.

As a result of prior experiences, students should conceptualize perimeter as the total distance around a closed object or figure. They might observe that, for certain regular polygons, the perimeter is particularly easy to compute.

- Equilateral triangle: The perimeter is three times the side length.
- Square: The perimeter is four times the side length.
- Rectangle: The perimeter is double the sum of the length and the width.

Students will be familiar with the concept of area from Mathematics 4, where they determined the area of rectangles using standard units. “From earlier work with multiplication and the array meaning or model of multiplication, students will know that, to determine the total number of squares, you multiply the number of rows of squares by the number of squares in each row.” (Small 2008, 397–398) Students need to have many opportunities to experiment with the relationships among length, width, and area to develop their own formulas for area of rectangles (remind students that a square is a special type of rectangle).

Volume has been studied in Mathematics 5. Students should recognize volume as the amount of space taken up by a 3-D object or the amount of cubic units required to build and fill the object.

Students should also recognize that each of the three dimensions of the prism affects the volume of the object. Development of the concept of using the **area of the base** as part of the formula for volume of a right rectangular prism will be helpful for work in later grades as volume of other 3-D objects is explored.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Assessing Prior Knowledge

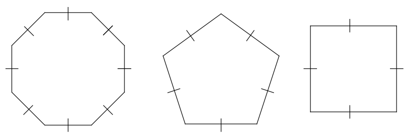
Tasks such as the following could be used to determine students’ prior knowledge.

- Ask students to identify a 3-D object that could be measured in cubic centimetres and a 3-D object that would be measured in cubic metres and explain.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Tell students that the perimeter of a triangle is 15 cm. Have them describe and draw the possible side lengths. (**Note:** If outcome G01 has been completed, the type of triangle can be specified—scalene, isosceles, etc.)
- Ask students to explain how a formula can be used to determine the perimeter of the following regular polygons.



- Provide students with area problems to solve such as the following:
  - A teen mowed two lawns. One lawn was 10 m × 12 m, and the other was 15 m × 10 m. The teen charges $3 for each 10 m^2. How much did she charge to mow the two lawns?
  - Zack needs to mail a present to his cousin. The box is 24 cm long, 15 cm wide, and 5 cm high. The shipper charges $0.75 for each 1 cm^3 and $3 for the total mass. How much will it cost to ship the package?
- Provide students with the dimensions of a real-world container that is a rectangular prism (e.g., a carton or a cereal box). Ask students to find the perimeter and area of each face. Students should also determine the volume for the prism. Ask students to determine the possible dimensions if the object needed to hold twice as much.
- Explain, using numbers, pictures, and/or words, why a rectangular prism that is 5 cm × 3 cm, with a height of 4 cm must have a volume of 60 cm^3.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

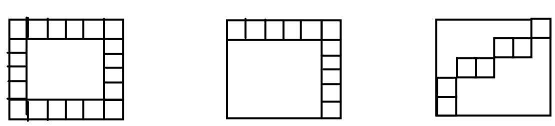
CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Provide pictures of many regular polygons, with the measure of one side provided for each. Have students explore to find the most efficient method for finding the perimeters of each. Lead students to discover that "length of the side + length of the side + length of the side + length of the side ..." is inefficient when multiplication can be used instead. Repeat the task with rectangles and parallelograms.
- Provide students with graphics of many rectangles, including squares, in which the square units are shown and the length and width measures are given. Ask students to find the most efficient way to find the areas of each. Begin with small areas, such as 2 cm × 3 cm, and help students relate these rectangles to the array model for multiplication.
- Invite students to create many different rectangles, including squares, on grid paper. Have them find and record the length, width, and area for each (by counting the squares, if necessary). They should record their findings in chart form so they can look for relationships in the table among the length, width, and area for each. Lead students to develop the formula, length × width (Small 2008, 398).
- Invite students to build a variety of right rectangular prisms. On a chart, have them record the length and width of the base and the height, as well as the volume. Have students look for relationships among these measures and lead students toward developing the formula.

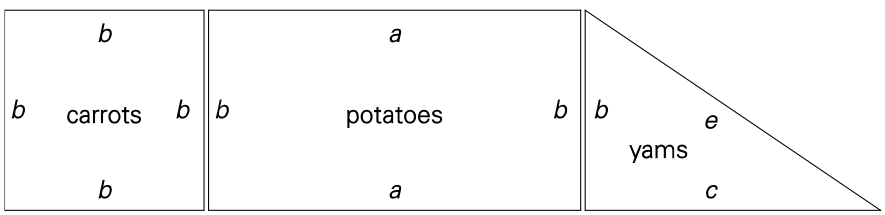
SUGGESTED LEARNING TASKS

- Provide students with a variety of rectangles with incomplete grids. Have them apply the formula to determine their areas.



- Provide students with regular polygons to explore to find patterns between side lengths and to create a rule (formula) for each to calculate the perimeter.
- Present students with rectangular prisms constructed out of linking cubes. Have them calculate the volume. Determine if students use multiplication rather than counting cubes.
- Provide students with linking cubes and ask them to build rectangular prisms of different sizes. In each case, ask them to record the various side lengths and volumes in a table. Ask them to describe the rule (formula) for calculating the volume of any rectangular prism.

- Arrange students into small groups or pairs. Provide each group with a different number of cubic unit manipulatives. These could be cubic centimetre blocks, linking cubes, or both, depending on what is available. Invite each group to construct the largest right rectangular prism that they can, using the blocks they were given. Have each group determine the volume of their own prism. Students can then leave their prism at their desk as a station. Groups can then rotate around the room finding the volume of the prisms constructed by other groups. Once the task is complete, ask students to share their findings with those of other groups to compare their methods and verify their results.
- Provide students with pattern blocks and other manipulatives of varying shape and size. Ask them to identify which dimensions (sides) on the pattern blocks are of equal length. Trace these patterns on paper. Assign the equal sides a common variable to represent the unknown length that these sides share. Ask students to write an expression to represent the perimeter of each pattern block.
- Provide students with a variety of polygons (squares, rectangle, triangles, parallelograms, etc.) cut out from card stock or construction paper and having common side lengths labelled with a variable. Each polygon represents a different shaped field on a farm. Each different variable indicates the length of each side. Ask students to work in groups to arrange the polygons to create various combinations of different shaped farms.



Next, explain to students that the farmer wants to build a fence around the farm. Ask students to develop a formula to determine the perimeter of the farm (e.g., The formula for the above farm would be $P = 3b + 2a + c + e$.).

Once students have found the formula for the perimeter of the farm, assign each variable a number value and ask students to use their formula to find the perimeter. For example, if $a = 5$ metres, $b = 2$ metres, $c = 3$ metres, $e = 4$ metres, the perimeter of the farm above would be as follows:

P = 3(2 metres) + 2(5 metres) + 3 metres + 4 metres

P = 6 metres + 10 metres + 3 metres + 4 metres

P = 23 metres

SUGGESTED MODELS AND MANIPULATIVES

- 1-cm cubes
- base-ten blocks
- grid paper
- linking cubes
- rulers

MATHEMATICAL LANGUAGE

Teacher	Student
▪ area ▪ area of base ▪ base ▪ formula ▪ height ▪ perimeter ▪ polygon ▪ rectangular prisms ▪ volume	▪ area ▪ area of base ▪ base ▪ formula ▪ height ▪ perimeter ▪ polygon ▪ rectangular prisms ▪ volume

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students, K–8* (Small 2008), pp. 380–381, 397–398, 425–426
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 427–427, 442–444, 471–472
- *Teaching Student-Centered Mathematics, Grades 5–8* (Van de Walle and Lovin 2006), pp. 241–242, 253–254, 257–258

Notes

Geometry (G)

GCO: Students will be expected to describe the characteristics of 3-D objects and 2-D shapes and analyze the relationships among them.

GCO: Students will be expected to describe and analyze position and motion of objects and shapes.

SCO G01 Students will be expected to construct and compare triangles, including scalene, isosceles, equilateral, right, obtuse, or acute in different orientations.
[C, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

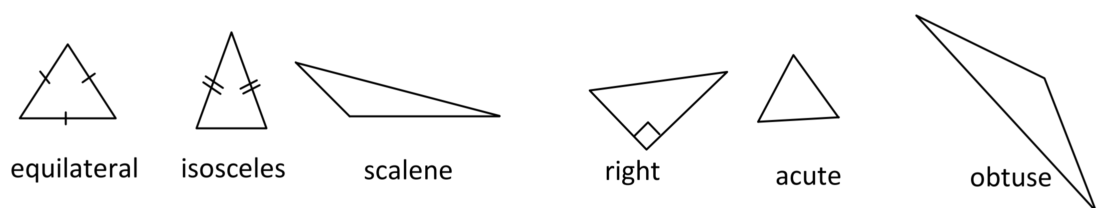
G01.01 Sort a given set of triangles according to the length of the sides.
G01.02 Sort a given set of triangles according to the measures of the interior angles.
G01.03 Identify the characteristics of a given set of triangles according to their sides and/or their interior angles.
G01.04 Sort a given set of triangles and explain the sorting rule.
G01.05 Draw a specified triangle.
G01.06 Replicate a given triangle in a different orientation and show that the two are congruent.

Scope and Sequence

Mathematics 5	Mathematics 6	Mathematics 7
G01 Students will be expected to describe and provide examples of edges and faces of 3-D objects, and sides of 2-D shapes that are parallel, intersecting, perpendicular, vertical, and horizontal.	**G01** Students will be expected to construct and compare triangles, including scalene, isosceles, equilateral, right, obtuse, or acute in different orientations.	**G01** Students will be expected to perform geometric constructions, including ▪ perpendicular line segments ▪ parallel line segments ▪ perpendicular bisectors ▪ angle bisectors

Background

Triangles are three-sided polygons. They can be sorted either by the length of their sides (**equilateral**, **isosceles**, **scalene**) or by the measure of their angles (**right**, **acute**, **obtuse**).



Equilateral triangles have all three side lengths equal. Isosceles triangles have two side lengths equal. Scalene triangles have three different side lengths; none of the side lengths are equal. Students should explore why there are only three possible classifications by side length.

In a right triangle, the measure of one angle is 90°. In an acute triangle, the measure of each of the angles is less than 90°. In an obtuse triangle, the measure of one angle is greater than 90°. Students should explore the reasons for the three different types of triangles in the set of classifications by angle size. For example, a triangle cannot have more than one obtuse angle (greater than 90°) as the angles in a triangle add up to 180°.

Once these two sets of classification (sides and angles) have been studied, teachers should extend students' knowledge to explore how a triangle may fall into two categories at the same time (e.g., a right scalene triangle, an obtuse isosceles triangle).

Students have used the term **congruent** and have had experience comparing and matching 2-D shapes based on attributes. They should now learn the meaning of the markings on the sides of polygons as shown on the triangles on the previous page. To show congruence on a diagram, hatch marks are used. Students must recognize that sides with the same length are indicated using the same hatch mark or variable.

It is important to provide students with frequent opportunities to explore and create different types of triangles. Students should recognize that being given three side lengths, or two angles and one side length, or two side lengths and one angle will result in a unique triangle. Students should construct triangles using a protractor, ruler, geo-strips, geo-board, or technology. Drawing freehand will not produce the required 2-D shape.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Assessing Prior Knowledge

Tasks such as the following could be used to determine students' prior knowledge.

- Provide students with several different 2-D shapes and have them sort and justify their sorting scheme. Observe whether students use correct geometric terminology in their descriptions.

Whole-Class/Group/Individual Assessment Tasks

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Provide students with a set of triangles (be sure to include a variety of different types and different orientations). Have them sort the triangles first according to the length of the sides (equilateral, isosceles, scalene) and explain their sorting rule. Repeat the task having students sort the triangles according to the measures of the angles (right, acute, obtuse) and explain their sorting rule.

- Invite students to draw the following or other examples of triangles that can be classified more than one way (e.g., a scalene right triangle; an isosceles, acute-angle triangle).
- Ask students to construct specific triangles on their geo-boards and record them on dot paper (e.g., an acute triangle that has one side using five pins, a right triangle that is also isosceles, an obtuse triangle that has one side using five pins).
- Provide students with a geo-board and dot paper. Have them create and draw two different
  - scalene triangles
  - isosceles triangles
  - right triangles
  - equilateral triangles
  - acute triangles
  - obtuse triangles
- Ask students to draw various triangle types with specific properties, such as
  - an obtuse triangle with an angle of 130°
  - a triangle with 3 cm and 4 cm sides that form a right angle
  - an equilateral triangle with 10 cm sides
  - an obtuse triangle with a 110° angle and one 5 cm side
- Tell students that one side of a triangle is 20 cm. What might the lengths of the other two sides be for each of the followings kinds of triangles?
  - isosceles
  - scalene
  - equilateral

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?

- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

Choosing Instructional Strategies

Consider the following strategies when planning daily lessons.

- Provide a variety of triangles and ask students to compare and measure the angles so that they are able to discover these patterns: (a) all angles in equilateral triangles are equal; (b) two angles in isosceles triangles are equal; and (c) all angles in scalene triangles are different.
- Ask students to test for congruency by placing one shape on top of another to see if the outlines match exactly.
- Give students cards with examples of right, acute, and obtuse triangles on them. Ask them to sort them into three groups by the nature of their angles and share how they were sorted. Attach the names for these classifications to the students' groups.
- Use Venn diagrams or Carroll diagrams to help organize sorted triangles.

Suggested Learning Tasks

- Prepare pictures on cards or cut-outs of several examples of different kinds of triangles. Ask the students to sort them into three groups and provide their sorting area. Often, they will sort triangles by how their sides look, without knowing the actual names. If so, this will lead to a focus on measuring and comparing the sides, and noting common properties to which the names equilateral, isosceles, and scalene can be attached. (If not, the teacher may sort them, ask students to determine the sorting rule, and do other explorations.)
- Identify everyday examples of each type of triangle—yield sign, bridges, the ends of a Toblerone bar, other support items, ladder against a wall. Students should also examine familiar materials in the classroom, such as pattern blocks and tangrams.
- Provide students with geo-strips of varying lengths. Invite them to investigate the different triangles they can make using three geo-strips at a time and complete a table with their results. This task could be varied by using toothpicks or straws.

Geo-strip Sized Used	Type of Triangle

- Provide students with paper cut-outs of various types of triangles. Have them trace the triangles to explore how many different orientations of the same triangle they can find.
- Invite students to draw a triangle on tracing paper and classify it. Have them fold the paper in order to trace the shape several different ways to create congruent triangles in other orientations.

Suggested Models and Manipulatives

- geo-strips
- grid paper
- straws or pipecleaners
- tangrams

MATHEMATICAL LANGUAGE

Teacher	Student
▪ acute angle ▪ acute triangle ▪ congruent ▪ equilateral ▪ interior angles ▪ isosceles ▪ obtuse angle ▪ obtuse triangle ▪ orientation ▪ protractor ▪ right angle ▪ right triangle ▪ scalene ▪ side length ▪ triangles	▪ acute angle ▪ acute triangle ▪ congruent ▪ equilateral ▪ interior angles ▪ isosceles ▪ obtuse angle ▪ obtuse triangle ▪ orientation ▪ protractor ▪ right angle ▪ right triangle ▪ scalene ▪ side length ▪ triangles

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students, K–8* (Small 2008), pp. 294–295
- Mak*ing Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 350–351
- *Teaching Student-Centered Mathematics, Grades 5–8* (Van de Walle and Lovin 2006), pp. 193–194, 197

Notes

SCO G02 Students will be expected to describe and compare the sides and angles of regular and irregular polygons. [C, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

G02.01 Sort a given set of 2-D shapes into polygons and non-polygons and explain the sorting rule.
G02.02 Demonstrate congruence (sides to sides and angles to angles) in a regular polygon by superimposing.
G02.03 Demonstrate congruence (sides to sides and angles to angles) in a regular polygon by measuring.
G02.04 Demonstrate that the sides of a regular polygon are the same length and that the angles of a regular polygon are the same measure.
G02.05 Sort a given set of polygons as regular or irregular and justify the sorting.
G02.06 Identify and describe regular and irregular polygons in the environment.

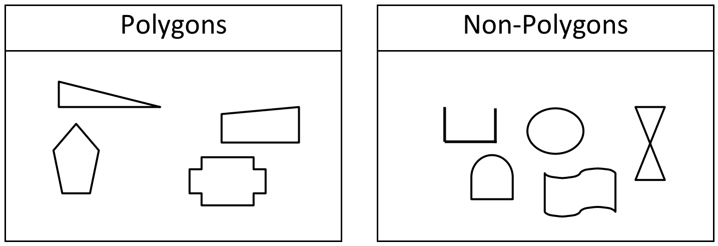
Scope and Sequence

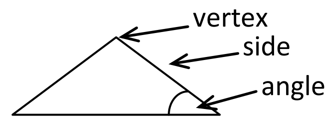
Mathematics 5	Mathematics 6	Mathematics 7
G01 Students will be expected to describe and provide examples of edges and faces of 3-D objects, and sides of 2-D shapes that are parallel, intersecting, perpendicular, vertical, and horizontal. **G02** Students will be expected to name, identify, and sort quadrilaterals, including rectangles, squares, trapezoids, parallelograms, and rhombi according to their attributes.	**G02** Students will be expected to describe and compare the sides and angles of regular and irregular polygons.	**G01** Students will be expected to perform geometric constructions, including ▪ perpendicular line segments ▪ parallel line segments ▪ perpendicular bisectors ▪ angle bisectors

Background

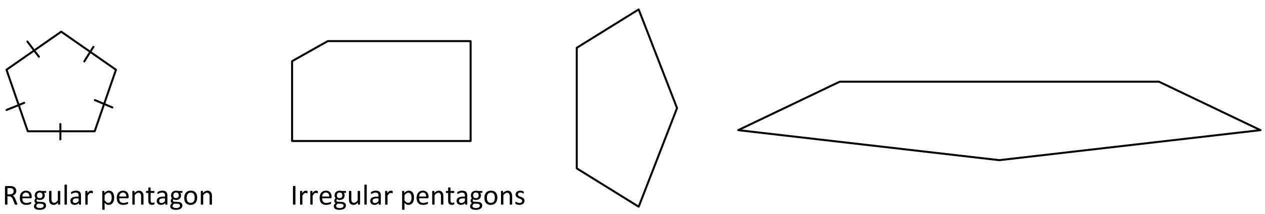
In previous grades, students learned the names of common polygons. The focus in Mathematics 6 is to explore all of the **properties** of **sides** and **angles** of shapes in the classification process. Teachers need to provide students with a variety of sorting activities of 2-D shapes and questions to help guide their investigations.

Polygons are closed 2-D shapes with three or more straight sides. The sides meet only at the **vertices**. A key property of polygons is that the number of sides is always equal to the number of vertices. Shapes that are missing one or more of these attributes are considered **non-polygons**. It is important that students focus on these attributes using proper vocabulary to determine whether the shape is a polygon. A common misconception is to think that triangles and quadrilaterals are not polygons since they have other names.





In Mathematics 6, students will extend their knowledge to include both regular and irregular polygons. **Regular polygons** have all sides and angles equal (e.g., pattern blocks: equilateral triangles, squares, hexagons). **Irregular polygons** do not have all sides or angles that are the same size. Students should be given opportunities to explore both regular and irregular polygons in their environment. Using the attributes of polygons, students should be able to sort into regular or irregular polygons.



Regular pentagon Irregular pentagons

It is also important for students to investigate the concept of congruence by superimposing the shapes (direct comparison by laying one shape on top of the other) and by measuring the sides and angles.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students’ prior knowledge.

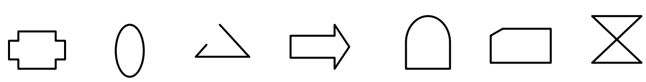
- Provide students with several different 2-D shapes and have them sort and justify their sorting scheme. Observe whether students use correct geometric terminology in their descriptions.

- Ask students to draw quadrilaterals that satisfy a given set of attributes. Be sure to include in the drawings the lengths of the sides and whether or not the opposite sides are parallel. Once they have the quadrilateral drawn, they should be able to identify the shape. For example,
  - a 2-D shape with four straight sides of equal length and four right angles
  - a 2-D shape with four straight sides and four right-angles; one pair of sides is longer than the other
  - a 2-D shape with four straight sides; one pair of sides is parallel with one side longer than the other.

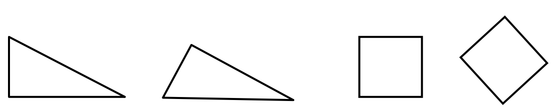
WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

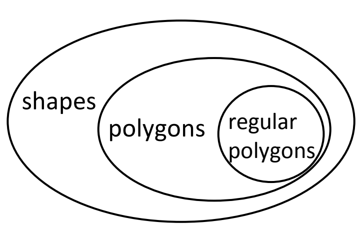
- Provide a set of polygons (paper or other models). Have students determine which are congruent.
- Ask students to draw a polygon and a non-polygon, and explain why one is a polygon and the other is not.
- Provide students with several different polygons (regular and irregular) on cards to sort and have them justify their sorting rule.
- Provide students with cards on which several different shapes (polygons and non-polygons) have been drawn. Ask them to sort the cards and have them justify their sorting rule.



- Invite students to describe the characteristics of a regular polygon and how they would prove a shape is a regular polygon.
- Provide students with dot paper or a geo-board (11 × 11 pin) and have them draw or create two triangles or squares in different orientations. Explain how they know they are congruent.



- Ask students to draw congruent polygons that satisfy a given set of attributes. Students should be able to prove the shapes are congruent by measuring.
- Provide two congruent irregular polygons. Have students prove congruency by measuring and labelling the sides and angles.
- Invite students to sort a set of shapes using a Venn diagram like the one below.



FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

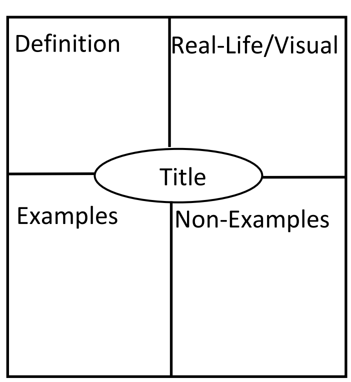
Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Provide students with a template for the Frayer Model and have them fill in the sections, individually or as a group, to consolidate their understanding of the properties of polygons and non-polygons.

  This task may be repeated to distinguish the attributes of regular and irregular polygons.

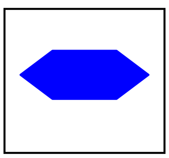


- Invite students to prepare property lists with the headings Sides and Angles. Using a collection of regular and irregular polygons (models or pictures on cards), have students describe the shapes using language such as, all sides equal, two angles the same, opposite sides equal, no sides equal, etc. Then have the students sort the polygons into regular or irregular polygons. Use a Venn diagram or a Carroll diagram to record similarities and differences.

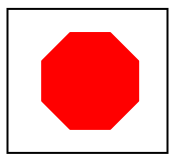
- Provide students with a list of attributes and have them draw a polygon that has the set of attributes. Have students share and compare their polygon with the class.
- Display models or copies of regular polygons on the board. Place a smaller version of the regular polygon on the overhead projector. Have a student move the projector until the image matches with the one taped on the board. This will help to prove the congruence of their angles, regardless of their side lengths. Interactive white boards can also be an effective tool to show congruency of angles of regular polygons.
- Ask students to draw a polygon and a non-polygon, and explain why one is a polygon and the other is not.
- Ask students to agree or disagree with the statement below and to explain their thinking—Because all the angles of a rectangle measure 90°, the angles are congruent. That means that rectangles are regular polygons.

SUGGESTED LEARNING TASKS

- Invite students to work in pairs to prepare a concentration card game with pictures of regular and irregular polygons and their corresponding names.



irregular
hexagon



regular
octagon

- Ask students to trace a regular polygon (e.g., yellow pattern block). Invite students to rotate their shape to prove the congruency of sides and angles. The congruency should be double-checked by measuring the polygon’s angles and sides with a protractor and ruler.
- Invite students to go on a “Polygon and Irregular Polygon” or a “Polygon and Non-Polygon” Scavenger Hunt. Have them sort the polygons they found and explain their rules for sorting.
- Provide several copies of a non-regular polygon that has been rotated and reflected a number of different ways. Have the students cut out one shape and lay it over the others to prove congruency. This can be done using paper drawings or on the computer.
- Invite students to create various types of regular and irregular polygons on geo-boards. Have them also create sets of congruent polygons on their geo-boards in different orientations. Record on grid paper.
- Provide students with pictures of several regular polygons and ask them to measure the angles with a protractor and side lengths with a ruler. Ask them to label them with appropriate hatch marks.
- Provide students with triangular dot paper. Ask them if they can draw a regular pentagon. Once they conclude it is not possible, discuss with them why this is the case.
- Ask students to tell you about the characteristics of a regular polygon. Then ask them which characteristic they prefer to use to check whether a polygon is regular or irregular. Ask why they prefer using this characteristic.
- Provide students with a set of regular polygons. Include several sets of congruent and similar polygons. Ask students to identify pairs of congruent polygons and to explain how they found them.
- Provide students with triangular dot paper. Ask them to draw a regular hexagon. Ask them to show that all sides and all angles are congruent by measuring or superimposing.

- Provide students with a copy of two congruent regular polygons of different orientations. Label the vertices of both polygons. On one, indicate the side lengths and angle measures, but not on the other. Ask students to write the measure of each angle in the second polygon without using a protractor, and the measure of each side length without using a ruler.
- Ask students to answer a question such as, What does it mean when we say two regular polygons are congruent? Use words and pictures to explain your understanding.

SUGGESTED MODELS AND MANIPULATIVES

- attributes
- geo-boards
- geo-strips
- pattern blocks
- Power Polygons

MATHEMATICAL LANGUAGE

Teacher	Student
▪ congruence ▪ polygons ▪ properties ▪ regular, irregular ▪ sides, angles ▪ vertices	▪ congruence ▪ polygons ▪ properties ▪ regular, irregular ▪ sides, angles ▪ vertices

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students, K–8* (Small 2008), pp. 292–296, 316–318
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 348–352, 371–372
- *Teaching Student-Centered Mathematics, Grades 5–8* (Van de Walle and Lovin 2006), pp. 193–194

Notes

SCO G03 Students will be expected to perform a combination of translation(s), rotation(s), and/or reflection(s) on a single 2-D shape, with and without technology, and draw and describe the image. [C, CN, PS, T, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

G03.01 Demonstrate that a 2-D shape and its transformation image are congruent.
G03.02 Model a given set of successive translations, successive rotations, or successive reflections of a 2-D shape.
G03.03 Model a given combination of two different types of transformations of a 2-D shape.
G03.04 Draw and describe a 2-D shape and its image, given a combination of transformations.
G03.05 Describe the transformations performed on a 2-D shape to produce a given image.
G03.06 Model a given set of successive transformations (translation, rotation, or reflection) of a 2-D shape.
G03.07 Perform and record one or more transformations of a 2-D shape that will result in a given image.

Scope and Sequence

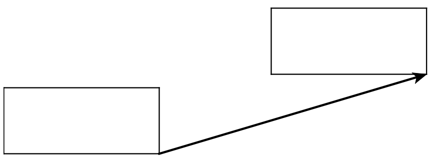
Mathematics 5	Mathematics 6	Mathematics 7
G03 Students will be expected to perform a single transformation (translation, rotation, or reflection) of a 2-D shape (with and without technology) and draw and describe the image. **G04** Students will be expected to identify and describe a single transformation, including a translation, rotation, and reflection of 2-D shapes.	**G03** Students will be expected to perform a combination of translation(s), rotation(s), and/or reflection(s) on a single 2-D shape, with and without technology, and draw and describe the image.	**G03** Students will be expected to perform and describe transformations (translations, rotations, or reflections) of a 2-D shape in all four quadrants of a Cartesian plane (limited to integral number vertices).

Background

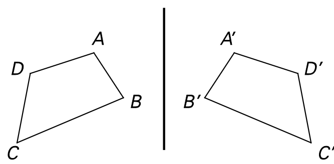
In Mathematics 5, students learned that there are three transformations that change the location of a shape in space, and/or its orientation, but not its size or shape. The three types of transformations are **translations**, **reflections**, and **rotations**. These transformations result in images that are **congruent** with the original object.

- **Translations** move a shape left, right, up, down, or diagonally without changing its orientation. A real-life example of a translation may be a chess piece moving on a chessboard.
- **Reflections** can be thought of as the result of picking up a shape and flipping it over. The reflected image is the mirror image of the original. A real-life example of a reflection may be a pair of shoes.
- **Rotations** move a shape around a **turn centre**. A real-life example of a rotation may be clock hands.

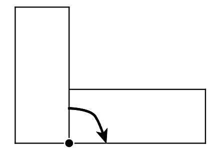
Example of a translation (slide)



Example of a reflection (flip)



Example of a rotation (turn)



In Mathematics 5, students explored translations, rotations, and reflections. They learned to describe translations accurately, using language such as right 2, up 4. By drawing connecting lines between vertices in the pre-image to corresponding vertices in the image, students saw that these lines are all congruent and that each point has been moved in the same distance. Similarly, these lines are all parallel and, thus, each point has been moved in the same direction. Students focused on rotations of one-quarter, one-half, three-quarter, and full turns both clockwise and counter-clockwise with their vertices as centres. To describe reflections (flips), students used language such as, reflected up or reflected to the left. Students drew or traced shapes, and using a Mira, drew mirror lines and the reflected images. Students concluded that the original shape and its image are of opposite orientation. Students joined corresponding vertices with line segments and examined the angles made by the mirror lines with these segments. They concluded that the mirror lines are perpendicular to all segments joining corresponding image points. Students also measured the distance from corresponding vertices to the mirror line. They concluded that corresponding points are equidistant from the mirror lines. In short, the mirror line is the perpendicular-bisector of all segments joining corresponding points. For each transformation, students observed that the image and the pre-image are congruent. Students labelled the original shapes with *A*, *B*, *C*, *D,* ..., and the corresponding vertices of the image with prime notation, *A', B', C', D'*. Once students created images and pre-images, they described how they were the same and how they are different. In discussing the transformations, students considered if the image

- had side lengths congruent to those of the pre-image
- had angle measures congruent to those of the pre-image
- was congruent to the pre-image
- had the same orientation as the pre-image
- had corresponding sides parallel

In Mathematics 6, students are expected to perform a combination of successive transformations with 2-D shapes. This could involve a single type of transformation repeated (such as a translation) or more than one type of transformation (e.g., reflections and translations or a translation and a rotation). Students will need to be able to describe and model the transformations. It is important for students to recognize that some transformations can be described in more than one way.

Students also must be able to create their own designs (SCO G04) using a combination of successive transformations. It is also expected that students be able to analyze existing designs and describe the transformations used to create that design.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Assessing Prior Knowledge

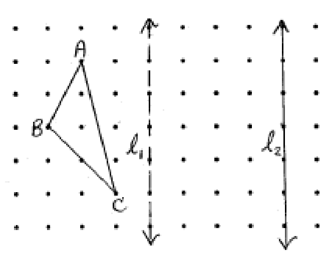
Tasks such as the following could be used to determine students' prior knowledge.

- Provide a 2-D shape and have students show a rotation, reflection, or translation of that shape on grid paper.
- Explain, using words and pictures, how you know if a figure and its image show a translation, reflection, or rotation.

Whole-Class/Group/Individual Assessment Tasks

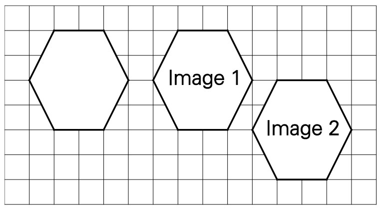
Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to prove that a 2-D shape and its transformation image are congruent.
- Have students locate the image of Δ*ABC* after a reflection across line 1 followed by a reflection across line 2. Ask them what single transformation of Δ*ABC* would have the same result.



- Ask students to determine which transformations were performed on a given shape.

- Present students with three pictures on grid paper of two congruent shapes after two transformations were performed on them. Ask students to predict what two transformations were performed. Could this have been done in more than one way? Could this have been done by a single transformation?



FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Ask students to use shapes from the pattern blocks, tangrams, logic blocks, and other sources to predict and confirm the results of successive rotations, translations, or reflections, or combinations of transformations.
- Give students pictures of shapes and their images showing successive rotations, translations, or reflections, or combinations of transformations. Have them predict what the relationships are and then confirm, using tracing paper or a Mira.
- Ask students to discuss their predictions prior to performing a given transformation to a shape.
- Invite students to investigate such questions as,
  - If a shape undergoes two translations, does it matter in which order they take place?

- Does a reflection followed by a translation produce the same result as the translation followed by the reflection?

SUGGESTED LEARNING TASKS

- Provide pattern blocks, and have students practice successive transformations and draw them on grid paper.
- Invite students to choose a pattern block, perform several transformations of their choice, draw the transformations on grid paper, and have a partner describe the transformations that were performed.
- Ask students to respond in their journal to the following prompts:
  - Explain using words and pictures if a translation can ever look like a reflection.
  - Explain using words and pictures how you know if a figure and its image show a reflection, translation, or rotation.
- Place three geo-boards side by side. Have one student make a scalene triangle on the first geo-board. Ask another student to construct on the second geo-board, the image of this triangle if the right side of the first geo-board is used as a mirror line. Ask another student to construct on the third geo-board, the image of the triangle on the second geo-board under a 90 degree counter-clockwise rotation. Repeat this task using other shapes and transformations.
- Use technology to demonstrate transformations. This could include websites (e.g., “The National Library of Virtual Manipulatives”), The Geometer’s Sketchpad (Key Curriculum 2013) software, and SMART Notebook (SMART Technologies 2013) software.
- Present students with two congruent shapes on grid paper. The first shape represents the pre-image and the second shape represents the image resulting from two transformations. Ask students to write in their journals:

  (a) What two transformations do you predict were performed? Explain your reasoning.

  (b) Draw the missing image.

  (c) Could this have been done more than one way?

  (d) Could this have been done by a single transformation?

SUGGESTED MODELS AND MANIPULATIVES

- geo-boards
- grid paper
- Miras
- pattern blocks
- pentominoes
- tangrams
- tracing paper

MATHEMATICAL LANGUAGE

Teacher	Student
▪ 2-D ▪ combination ▪ congruent ▪ successive ▪ transformations ▪ translations, rotations, reflections	▪ 2-D ▪ combination ▪ congruent ▪ successive ▪ transformations ▪ translations, rotations, reflections

Resources/Notes

Print

- *Teaching Student-Centered Mathematics, Grades 5–8* (Van de Walle and Lovin 2006), p. 215

Software

- The Geometer's Sketchpad (Key Curriculum 2013)
- SMART Notebook (SMART Technologies 2013)

Notes

SCO G04 Students will be expected to perform a combination of successive transformations of 2-D shapes to create a design and identify and describe the transformations. [C, CN, T, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

G04.01 Analyze a given design created by transforming one or more 2-D shapes, and identify the original shape and the transformations used to create the design.
G04.02 Create a design using one or more 2-D shapes and describe the transformations used.
G04.03 Describe why a shape may or may not tessellate.
G04.04 Create a tessellation and describe how tessellations are used in the real world.

Scope and Sequence

Mathematics 5	Mathematics 6	Mathematics 7
G03 Students will be expected to perform a single transformation (translation, rotation, or reflection) of a 2-D shape (with and without technology) and draw and describe the image. **G04** Students will be expected to identify and describe a single transformation, including a translation, rotation, and reflection of 2-D shapes.	**G04** Students will be expected to perform a combination of successive transformations of 2-D shapes to create a design and identify and describe the transformations.	**G03** Students will be expected to perform and describe transformations (translations, rotations, or reflections) of a 2-D shape in all four quadrants of a Cartesian plane (limited to integral number vertices).

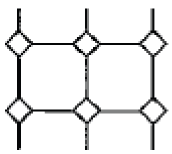
Background

In Mathematics 5, students learned that there are three transformations that change the location of an shape in space, and/or its orientation, but not its size or shape. The three types of transformations are **translations**, **reflections**, and **rotations**. These transformations result in images that are **congruent** with the original object.

- **Translations** move a shape left, right, up, down or diagonally without changing its orientation. A real-life example of a translation may be a chess piece moving on a chessboard.
- **Reflections** can be thought of as the result of picking up a shape and flipping it over. The reflected image is the mirror image of the original. A real-life example of a reflection may be a pair of shoes.
- **Rotations** move a shape around a **turn centre**. A real-life example of a rotation may be clock hands.

In Mathematics 6, students are expected to perform a combination of successive transformations with 2-D shapes. This could involve a single type of transformation repeated or more than one type of transformation (e.g., reflections and translations). Students will need to be able to describe and model the transformations. It is important for students to recognize that some transformations can be described in more than one way.

It is expected that students will be able to analyze existing designs and recognize and describe the transformations used to create patterns found in materials such as wallpaper, fabrics, and borders. Students are also expected to use transformations to create their own patterns and designs using a combination of successive transformations. These patterns will include tessellations. A 2-D figure is said to tessellate if an arrangement of replications of it can cover a surface without gaps or overlapping. For example, if a number of triangles in the pattern blocks were used, you would be able to use them to cover a surface; therefore, this triangle is said to tessellate. Investigations should include some shapes like pentagons and octagons that will not tessellate. The octagon is the shape often used in flooring and tiles with squares used to fill the gaps because octagonal tiles will not tessellate.



Using dot paper allows students to construct tessellations. This provides opportunity to develop spatial sense as well as creativity. Students may explore how transformations are used in real-world applications such as company logos, music, wallpaper design, or art.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to draw a shape, translate it, and then describe and explain the direction and the magnitude of the translation.
- Provide diagrams of rotations and ask, "Which picture shows a quarter turn? Half turn? Three-quarter turn." Invite students to identify the turn centre of the rotation.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Provide students with a 2-D shape and have them follow directions of successive transformations or a combination of transformations.
- Ask students to explain the transformations shown in a pattern, such as fabric, wallpaper, or other designs.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Use wallpaper or fabric as sources of designs that use transformational geometry. Students can look at the designs to find evidence of translations, reflections, and rotations. Have them record the transformations they observe. Many wallpaper and fabric designs incorporate multiple transformations, and some include interesting tessellations.
- Explore examples of transformations in artists' work such as M.C. Escher (2014) (www.mcescher.com).

Suggested Learning Tasks

- Ask students to choose a 2-D shape and create their own design using a combination of successive transformations. Have students record their transformations so the design can be reproduced.
- Use a pentomino to perform a combination of transformations, then sketch the pattern on grid paper.

Suggested Models and Manipulatives

- geo-boards
- grid paper
- Miras
- pattern blocks
- pentominoes
- tangrams
- tracing paper

Mathematical Language

Teacher	Student
▪ 2-D ▪ analyze ▪ design ▪ successive ▪ tessellation ▪ transformations ▪ translations, rotations, reflections	▪ 2-D ▪ analyze ▪ design ▪ successive ▪ tessellation ▪ transformations ▪ translations, rotations, reflections

Resources/Notes

Internet

- M.C. Escher (The M.C. Escher Company 2014)
  www.mcescher.com

Print

- *Teaching Student-Centered Mathematics, Grades 5–8* (Van de Walle and Lovin 2006), pp. 213–214

Notes

SCO G05 Students will be expected to identify and plot points in the first quadrant of a Cartesian plane using whole number ordered pair. [C, CN, V]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

G05.01 Label the axes of the first quadrant of a Cartesian plane and identify the origin.
G05.02 Plot a point in the first quadrant of a Cartesian plane given its ordered pair.
G05.03 Match points in the first quadrant of a Cartesian plane with their corresponding ordered pair.
G05.04 Plot points in the first quadrant of a Cartesian plane with intervals of 1, 2, 5, or 10 on its axes, given whole number ordered pairs.
G05.05 Draw shapes or designs in the first quadrant of a Cartesian plane, using given ordered pairs.
G05.06 Determine the distance between points along horizontal and vertical lines in the first quadrant of a Cartesian plane.
G05.07 Draw shapes or designs in the first quadrant of a Cartesian plane, and identify the points used to produce them.

Scope and Sequence

Mathematics 5	Mathematics 6	Mathematics 7
—	**G05** Students will be expected to identify and plot points in the first quadrant of a Cartesian plane using whole number ordered pair.	**G02** Students will be expected to identify and plot points in the four quadrants of a Cartesian plane using integral ordered pairs.

Background

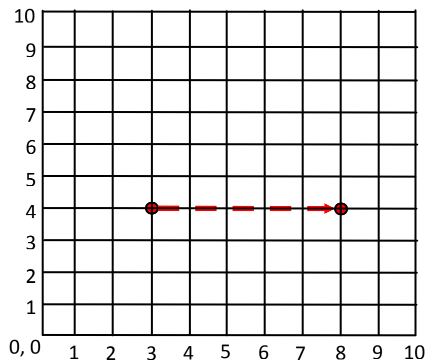
Students have experienced vertical and horizontal number lines in their work with integers. Students will have started to develop an understanding of a coordinate system through graphing activities. They may start by graphing simple relationships, such as the relationship between bicycles and the number of wheels or dogs and the number of paws.

Students need to be able to label the axes of the first **quadrant** of the **Cartesian plane**. They should know that the horizontal axis is commonly called the ***x*-axis** and the vertical is commonly called the ***y*-axis**. Students should extend their knowledge of graphing to determine the location of a point on a Cartesian plane (limited to the first quadrant) using **coordinates**. Coordinates are written as an **ordered pair** and are written in parentheses with a comma to separate the two numbers (4,1). Students need to develop an understanding of ordered pairs as this will be a prerequisite for creating line graphs. Have a discussion about where in everyday life and in what professions we use grids (e.g., GPS Systems, shipping lanes, mapping). This will give students some frame of reference for using grids.

The first number in an ordered pair shows the distance from the **origin (0,0)**, along the horizontal or *x*-axis (how far to move to the right). The second number shows the distance from the horizontal axis along a vertical or *y*-axis (how far to move up). Together these numbers are the ordered pair. For example, if we move 3 right and 4 up, the resulting ordered pair is (3, 4). Students need to know to always start at the origin (point where the two axes meet).

Students need to be able to determine the distance between points on a grid as well. In Mathematics 6, students will be expected to figure out distance between two points either horizontally or vertically on the same line. A real-life application of this concept is determining distances between places on a map using the grid lines or a scale.

The distance between point (3, 4) and point (8, 4) is 5 units.



Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Give students a street map with grid lines on it. Have them determine the distance between locations on this map.
- Tell students that a town map is drawn on a grid. The fire station is at (40,30). There are four fire hydrants, each 20 units from the station in a straight horizontal or vertical line. Have them draw and label axes on the grid, explaining what scale they used, plot the fire station, list the ordered pairs where the hydrants would be found, and plot the points.
- Ask students to explain how to use ordered pairs to describe and locate points on a grid.

- Ask students to predict the shape that was created by plotting points and joining them with straight lines for the following coordinates: (3,0), (4,0), (5,2), (4,5), (3,4), (2,2). Have students then create the shape.
- Tell students that two objects were placed at (0,4) and (3,7) on a grid. Invite them to describe where the second object was placed in relation to the first. Then plot the points and check.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

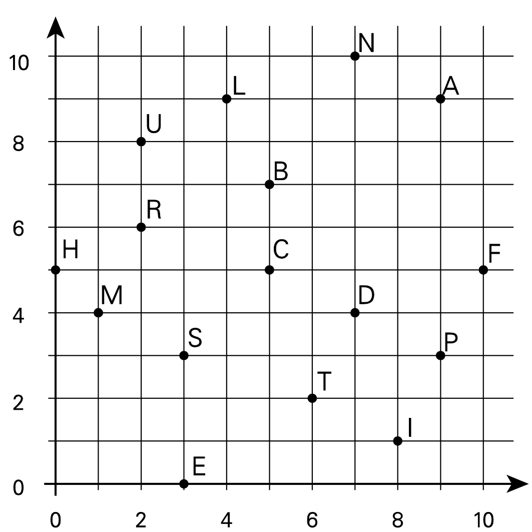
- Display a coordinate grid when you introduce these concepts. Be sure to label the axes correctly. A common error students make when labelling the axes is putting the number between the grid lines.
- Draw and label a coordinate grid on the board. Invite students to explore how they could use two numbers to describe a point on the grid. Introduce terminology such as ordered pair and the origin (0,0). Also, have students use the words **right** and **up** (in this order) as they move along the grid.
- Select points on a grid and have students decide what two numbers name these points. If students say the numbers (1,3) as “three, one” then simply remind them that the first number tells the horizontal distance and the second number tells the vertical distance.

SUGGESTED LEARNING TASKS

- Ask students to plot 10 points in the first quadrant of the Cartesian plane for which the difference between the first and second coordinate is 3.
- Give the coordinates for three vertices of a square (1,2), (1,7), (6,2). Ask students to find the last point and label the coordinates.

- Give students a grid with five points on it and ask them match these to five ordered pairs listed below it.
- Ask students to plot points on grids with different scales (e.g., intervals of ones, twos, fives, tens).
- Create "join-the-dots" pictures on a coordinate grid to reinforce locating coordinates. After they draw their pictures on a grid, they list the coordinates in order of connection. The list of coordinates can be given to other students who then use them to recreate the picture.
- Play a game, such as Battleshape, on the first quadrant of the Cartesian plane. Each player will need two copies of the coordinate grid; one on which to mark his/her shapes and one to keep track of the ordered pairs he/she is asking and whether or not it was a hit or miss.
- Use masking tape to create an *x*- and *y*-axis on the classroom floor. Number the *x*- and *y*-axis with the numbers to 10. Prepare two sets of numeral cards, 0 through 10. Place one set in a bag labelled "*x*-axis" and the other set in a bag labelled "*y*-axis." Pull out a numeral card from each bag to form a coordinate point. (Replace it for the next round when you are done.) Create a spinner divided into four equal sections. Name each section as either right hand, left hand, right leg, or left leg. Spin the spinner to find out which hand or foot must remain on that point. You could divide the class into two teams to help with the large scale of the grid and allow each team to work together to touch each of their coordinate points. If a student "falls" or moves from his coordinate point, the other team wins a point. Students could plot each team's coordinates in alternating colours on a Cartesian plane.
- Ask students to practise locating points along the *x*-axis and the *y*-axis. One of the coordinates of the ordered pair will be zero. They need to make the connection that if one coordinate is 0, the point will need to be plotted on one of the axes. Present students with a grid showing points labelled on the *x*- and *y*-axis and ask students to provide the coordinates for the labelled points.
- Give students a blank grid on which the axes have been labelled. Ask students to randomly place 10 points anywhere on the grid. Call out points at random, and if students have that point on their grid, they mark an X through that point. The first student to have all of their points marked with the X is the winner.
- Give students a grid that has points already labelled with letters as shown below. Ask students to find the letter on the grid represented by each ordered pair. Record the letters, in order, to figure out the message.

  (1,4) (9,9) (6,2) (0,5) (8,1) (3,3) (10,5) (2,8) (7,10)



- Ask students to explain, using words, numbers, and/or pictures, how to use ordered pairs to describe and locate points on a grid.
- Give students a coordinate grid to plot the points listed below and ask them to join them in order. The last point should be joined to the first point. Ask students to describe to the class the figure they have drawn.
  A(2,2) *B*(5,3) *C*(8,2) *D*(7,5) *E*(9,8) *F*(6,7) *G*(5,10) *H*(4,7) *I*(1,8) *J*(3,5)
- Give students a coordinate grid with axes labelled from 0 to 10. Ask students to plot each pair of points on the grid, join the points with a line segment, and find the length of each line segment.

  (a) (4,2) and (7,2)

  (b) (5,7) and (10,7)

  Students can then work with a partner to share pairs of points and find the distance between them.

  Give students a grid with the *x*- and *y*-axes labelled and with various shapes drawn on it. Ask students to name the coordinates of each shape. **Extension:** Draw partial closed figures and ask students to complete the figure and label its coordinates.

SUGGESTED MODELS AND MANIPULATIVES

- grid paper
- maps
- ruler

MATHEMATICAL LANGUAGE

Teacher	Student
▪ axes ▪ Cartesian plane ▪ corresponding ▪ horizontal, vertical, move right, move up ▪ intervals ▪ ordered pairs ▪ origin ▪ plot points ▪ quadrant	▪ axes ▪ Cartesian plane ▪ corresponding ▪ horizontal, vertical, move right, move up ▪ intervals ▪ ordered pairs ▪ origin ▪ plot points ▪ quadrant

Resources/Notes

Print

- *Teaching Student-Centered Mathematics, Grades 5–8* (Van de Walle and Lovin 2006), pp. 215–216

Notes

SCO G06 Students will be expected to perform and describe single transformations of a 2-D shape in the first quadrant of a Cartesian plane (limited to whole number vertices). [C, CN, PS, T, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

G06.01 Identify the coordinates of the vertices of a given 2-D shape (limited to the first quadrant of a Cartesian plane).
G06.02 Perform a transformation on a given 2-D shape, and identify the coordinates of the vertices of the image (limited to the first quadrant).
G06.03 Describe the positional change of the vertices of a given 2-D shape to the corresponding vertices of its image as a result of a transformation (limited to first quadrant).

Scope and Sequence

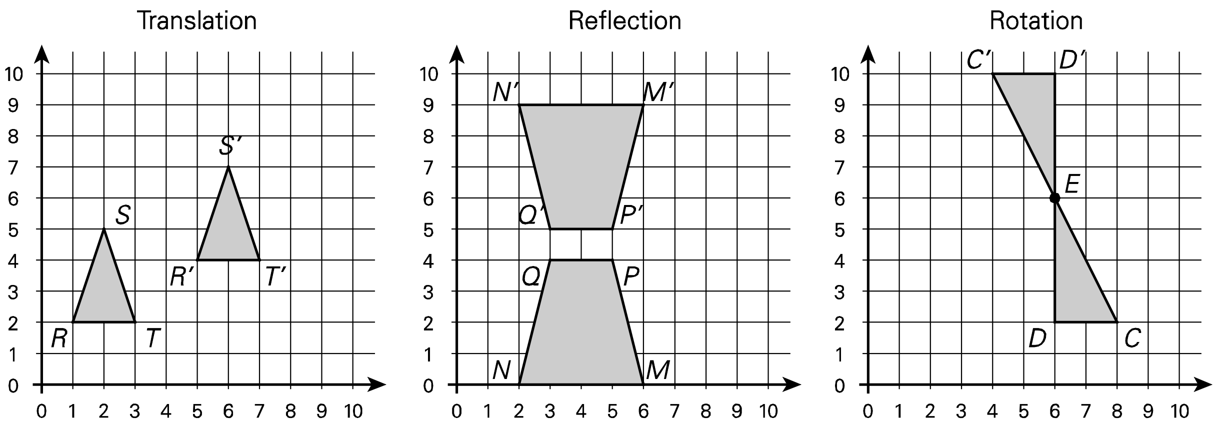
Mathematics 5	Mathematics 6	Mathematics 7
G03 Students will be expected to perform a single transformation (translation, rotation, or reflection) of a 2-D shape (with and without technology) and draw and describe the image. **G04** Students will be expected to identify and describe a single transformation, including a translation, rotation, and reflection of 2-D shapes.	**G06** Students will be expected to perform and describe single transformations of a 2-D shape in the first quadrant of a Cartesian plane (limited to whole number vertices).	**G03** Students will be expected to perform and describe transformations (translations, rotations, or reflections) of a 2-D shape in all four quadrants of a Cartesian plane (limited to integral number vertices).

Background

In Mathematics 5, students learned there are three transformations that change the location of an object in space, or the direction in which it faces, but not its size or shape. These are **translations**, **reflections**, and **rotations**. These transformations are further explored in Mathematics 6 in outcomes G03 and G04. Students will also require knowledge of plotting coordinates on a Cartesian plane as described in G05.

Students are expected to identify and perform these three types of transformations on a Cartesian plane, identify the coordinates of the **image** (*A', B', C', D'*; read as *A* prime, *B* prime, *C* prime, and *D* prime) and describe the change (e.g., when the pre-image below was translated, each *x*-coordinate increased by 4 because the shape was translated 4 units to the right).

Examples of each of these are shown below.



For this outcome, students are only expected to perform a single transformation in the first quadrant.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students’ prior knowledge.

- Invite students to explain the differences and similarities among the three different transformations.
- Provide students with diagrams of different transformations and ask them to label each diagram with the type of transformation the diagram shows.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Provide students with diagrams of different transformations and have them label each diagram with the type of transformation the diagram shows, including the coordinates of the vertices of both the pre-image and the image.
- Ask students to draw a shape (pre-image), translate it, and then describe the positional change of the vertices of the image.
- Ask students to describe how the translation rule can help them identify the coordinates of the vertices of the image.
- Provide a 2-D shape (pre-image) and have students perform a rotation (on a vertex), reflection, or translation on grid paper of that shape (pre-image), label and identify the coordinates of the vertices of both the pre-image and the image and describe the positional change.
- Ask students to explain the differences and similarities among the three different transformations with regard to the Cartesian plane and the coordinates of each pre-image and its image.
- Explain, using words and pictures, how you know if a pre-image and its image show a reflection, translation, or rotation.
- Provide the coordinates for a shape (pre-image) and its transformation. Have students plot and draw both the pre-image and the image and describe the transformation that has occurred.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Provide students with many opportunities to translate a given 2-D shape on a Cartesian plane on grid paper, identify the coordinates of the vertices, and describe the positional change of the vertices.

- Provide students with many opportunities to rotate a given 2-D shape on a Cartesian plane on grid paper, identify the coordinates of the vertices, and describe the positional change of the vertices. Students may trace the original shape on paper (wax or tracing) and use the point of their pencil pressed down on the point of rotation to help them rotate the shape.
- Provide students with many opportunities to reflect a given 2-D shape on a Cartesian plane on grid paper, identify the coordinates of the vertices, and describe the positional change of the vertices. Students may use Miras on the given line of reflection. Include opportunities where the line of reflection is horizontal, vertical, and diagonal.
- Ask students to discuss their predictions prior to performing a given transformation to a shape.
- Explore this concept in other curricular areas such as art and physical education.
- Provide shapes cut from cardstock that have vertices that fit on 1-cm grid paper. Have students practise performing, drawing, and recording various transformations.
- Integrate technology, for example, the Illuminations website (NCTM 2013) in work on this outcome.

SUGGESTED LEARNING TASKS

- Ask students to describe the direction as well as the size/magnitude of a given translation.
- Ask students to determine which transformation was performed on a given shape and identify the coordinates of the vertices of the shape.
- Provide pattern blocks and have students practise each transformation, draw them on a Cartesian plane on grid paper, and identify the coordinates of the vertices of the image.
- Invite students to choose a pattern block, perform a transformation of their choice, draw the transformation on a Cartesian plane on grid paper, and have a partner describe the transformation that was performed, including the coordinates of the original vertices and the new image's vertices.
- Ask students to complete
  - a rotation, given the direction of the turn (clockwise or counter-clockwise), the degree or fraction of the turn (e.g., 90°, three-quarter) and identify the coordinates of the vertices of the shape
  - a translation, given the direction and size/magnitude of the movement
  - a reflection, given the line of reflection and the distance from the line of reflection, limited to remaining in the first quadrant
- Ask students to create a shape on the geo-board, perform a transformation of their choice, and describe the transformation that was performed and the coordinates of the resulting image. Then repeat this on a grid (limited to the first quadrant).
- Respond in their journal to the following prompts:
  - Explain, using words and pictures, if a translation can ever look like a reflection.
  - Explain, using words and pictures, how you know if a figure and its image shows a reflection, translation, or rotation.

SUGGESTED MODELS AND MANIPULATIVES

- geo-boards
- grid paper
- Miras
- pattern blocks

Mathematical Language

Teacher	Student
▪ Cartesian plane ▪ coordinates ▪ image ▪ positional change ▪ transformation ▪ vertices	▪ Cartesian plane ▪ coordinates ▪ image ▪ positional change ▪ transformation ▪ vertices

Resources/Notes

Internet

- *Illuminations: Resources for Teaching Math* (National Council of Teachers of Mathematics 2013) (http://illuminations.nctm.org)

Print

- *Teaching Student-Centered Mathematics, Grades 5–8* (Van de Walle and Lovin 2006), pp. 217–219

Notes

Statistics and Probability (SP)

GCO: Students will be expected to collect, display, and analyze data to solve problems.

GCO: Students will be expected to use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

SCO SP01 Students will be expected to create, label, and interpret line graphs to draw conclusions. [C, CN, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

SP01.01 Determine the common attributes (title, axes, and intervals) of line graphs by comparing a given set of line graphs.
SP01.02 Determine whether a given set of data can be represented by a line graph (continuous data) or a series of points (discrete data) and explain why.
SP01.03 Create a line graph from a given table of values or a set of data.
SP01.04 Interpret a given line graph to draw conclusions.

Scope and Sequence

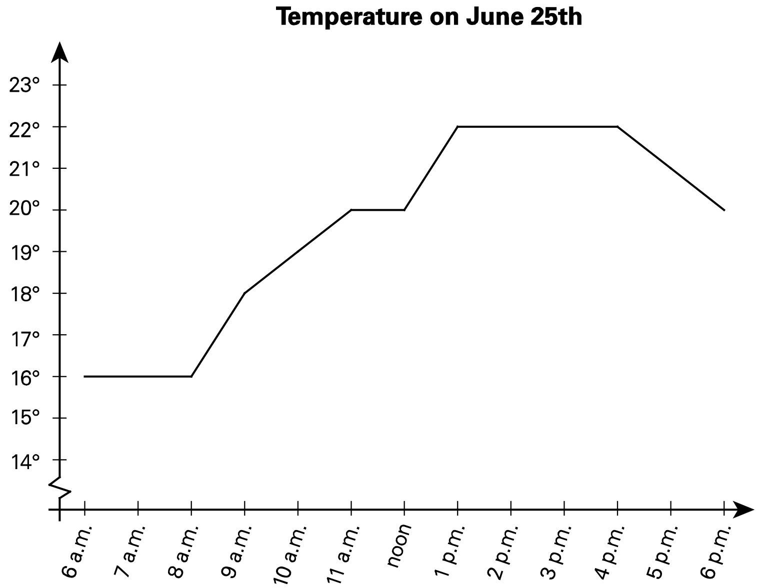
Mathematics 5	Mathematics 6	Mathematics 7
SP02 Students will be expected to construct and interpret double bar graphs to draw conclusions.	**SP01** Students will be expected to create, label, and interpret line graphs to draw conclusions.	**SP03** Students will be expected to construct, label, and interpret circle graphs to solve problems

Background

Students have investigated tables of values and described patterns and relationships using graphs and tables in outcomes PR01 and PR02. This work should be continued as students work with line graphs.

In earlier grades, students collected data, constructed pictographs, bar graphs, and double bar graphs, and read and interpreted those graphs. In Mathematics 6, line graphs are introduced. “A *line graph* is used when there is a numeric value associated with equally spaced points along a continuous number scale.” (Van de Walle 2006c, 323). The points on a line graph are plotted to show relationships between two variables, such as time and temperature. Every point on the line should have a value, but a line graph can also be used to show values between points on the graph. Students should be able to determine the value of the data points. If the data is continuous, the points are then joined to form a line. The distinction between **continuous** and **discrete** data should be emphasized as students investigate line graphs. Continuous data includes an infinite number of values between two points and is shown by joining the data points. Examples of continuous data are temperature change over the course of one day, the cost of gas per litre, plant growth over time, and rainfall in a day. Discrete data has finite values (i.e., data that can be counted, such as the number of pets or the number of siblings), and the data between the points have no value. As a result, the points representing discrete data on the graph should not be connected and no inferences can be made about values between two data points.

The purpose of a line graph is to focus on trends implicit in the data. For example, if students measure the temperature outside every hour during a school day, they could create a graph in which the ordered pairs (hour, temperature) are plotted. By connecting the points with line segments, students see the trend in the temperature. This type of exploration of line graphs links to outcome G05. Ensure that the construction of the line graph and interpretation of the data are not addressed independently. Whenever students construct graphs, the data should be discussed and interpreted.



Line graphs, like bar graphs, should include a title, labelled axes (overall description and specific data categories), and a clear scale. Line graphs are not always straight lines and may be called broken-line graphs.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Assessing Prior Knowledge

Tasks such as the following could be used to determine students' prior knowledge.

- Provide students with a double bar graph and have them identify the title, labels, scale, and legend. Ask them to describe why it is important to include each of these for a double bar graph.

Whole-Class/Group/Individual Assessment Tasks

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Provide students with two line graphs displaying similar data (such as temperature change over time in two different areas) and have students write comparison statements based on the data shown.

- Have students create a line graph based on the following information using appropriate scales, labels, and title.

Number of cups	1	2	3	4
Capacity (mL)	250	500	750	1000

- Ask students to explain (in words or pictures) the difference between continuous and discrete data.
- Provide an example of a line graph and have students create three questions that can be answered from the graph.
- Ask students to explain three situations where it would be appropriate to use a line graph.
- Provide a broken-line graph and have students explain why line graphs are not always linear.
- Provide students with examples of different types of data. Have them determine whether the data is continuous or discrete.
  - The number of students who eat in the cafeteria over a month.
  - The temperature over 48 hours.
  - The attendance at the local movie theatre.
  - Your height over five years.
- Ask students to create a line graph based on the table below. Have them determine about how much rain fell by 5:30 p.m. If the rain continues to fall steadily at the same rate, about how much will fall by 8:00 p.m.?

Time of Day	2:00 p.m.	3:00 p.m.	4:00 p.m.	5:00 p.m.	6:00 p.m.
Total rainfall	3 mm	5 mm	7 mm	9 mm	11 mm

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

Choosing Instructional Strategies

Consider the following strategies when planning daily lessons.

- Ensure students are aware of the parts of line graphs (e.g., titles, labels, scales) using real graphs that are interesting to students.
- Ask students to create line graphs and have them explain and justify the attributes they used when they created their graphs (e.g., scales, labels, title).
- Connect this outcome with prior knowledge of tables of values or sets of data (PR01 and PR02).
- Provide real-world line graphs and ask questions that require students to read and interpret the information found there.
- Have a whole-group discussion about the differences between continuous and discrete data and when to use each type of graph.
- Integrate the use of technology to construct line graphs. It is important that students also have the experience of creating graphs with paper-and-pencil methods.
- Use websites such as Statistics Canada (www.statcan.gc.ca), which include background information, and activities.

Suggested Learning Tasks

- Ask students to collect information about the number of students in the school in grades 1, 2, 3, 4, and 5 and draw a line graph to help show whether there are differences in the number of students in certain grades. Remind students to carefully consider the step size for the vertical scale.
- Ask students to record the changes in temperature over time during the day/week and create an appropriate line graph with a title, labelled axes, and scales.
- Ask students to look up the hockey scores for a favourite team over the course of 10 games and then create a line graph with the ordered pairs (game number, number of goals scored by favourite team). Have them create a second graph with the ordered pairs (game number, goals scored by opposing team) and then compare the two graphs.
- Have a class discussion about the differences between continuous and discrete data and when to use each type of graph.

Suggested Models and Manipulatives

- computer program (spreadsheet or graphing applications)
- graph paper

Mathematical Language

Teacher	Student
▪ continuous and discrete data ▪ create, label, interpret, conclusions ▪ line graphs, broken-line graphs ▪ ordered pairs ▪ table of values, set of data ▪ title, axes, intervals ▪ variables	▪ continuous and discrete data ▪ create, label, interpret, conclusions ▪ line graphs, broken-line graphs ▪ ordered pairs ▪ table of values, set of data ▪ title, axes, intervals ▪ variables

Resources/Notes

Internet

- *Statistics Canada* (Government of Canada 2014)
  www.statcan.gc.ca

Print

- *Making Mathematics Meaningful to Canadian Students, K–8* (Small 2008), p. 505
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), p. 549
- *Teaching Student-Centered Mathematics, Grades 5–8* (Van de Walle and Lovin 2006), pp. 322–323

Notes

SCO SP02 Students will be expected to select, justify, and use appropriate methods of collecting data, including questionnaires, experiments, databases, and electronic media. [C, PS, T]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

SP02.01 Select a method for collecting data to answer a given question, and justify the choice.
SP02.02 Design and administer a questionnaire for collecting data to answer a given question, and record the results.
SP02.03 Answer a given question by performing an experiment, recording the results, and drawing a conclusion.
SP02.04 Explain when it is appropriate to use a database as a source data.
SP02.05 Gather data for a given question by using electronic media, including selecting data from databases.

Scope and Sequence

Mathematics 5	Mathematics 6	Mathematics 7
SP01 Students will be expected to differentiate between first-hand and second-hand data.	**SP02** Students will be expected to select, justify, and use appropriate methods of collecting data, including questionnaires, experiments, databases, and electronic media.	**SP03** Students will be expected to construct, label, and interpret circle graphs to solve problems.

Background

From Mathematics 5, students should recognize that although some data is collected **first-hand** by interviews or observations, much of the data to which they are exposed is **second-hand**. Students should explore, through discussion, how such data might be collected and how reliable they feel it is. For example, if students read that 30% of children in Canada are not physically fit, what might they wonder about the data source? Was a sample used? Were children tested directly or was data collected by asking doctors or teachers? Students should recognize that they must be careful about drawing conclusions from reported data. Becoming familiar with sources for different types of data is valuable to students. There are many different sources of data.

A questionnaire is a collection of survey questions on the same topic. When designing a questionnaire, it is important to formulate good questions. It is helpful to discuss with students the various options in designing their questionnaire (e.g., multiple choice responses or yes/no responses; interviews or independently completed questionnaires).

Data can also be collected by conducting an experiment that is set up to answer a particular question. Electronic media, such as spreadsheets or Internet sites (e.g., Statistics Canada, music databases, Guinness World Records, weather, or sport leagues) are another useful source of data. If information about a large population is needed, there are organized collections of related data called databases, such as those created by Statistics Canada. Sometimes it is not possible to survey every person. In these situations a **sample** of the population is used and the results are then generalized to the entire target group. When this data is analyzed, it is important that students recognize that conclusions drawn from the sample may not be perfectly true for the entire group. A sample should also be carefully selected to avoid potential biases. For example, if someone wanted to determine the favourite type of take-out food in a community, it would not be reliable to only survey patrons of “Pizza King.” This sample of people would likely be biased in favour of pizza. After students have collected their data, have students explore which type of graph(s) would be appropriate to display it.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Assessing Prior Knowledge

Tasks such as the following could be used to determine students’ prior knowledge.

- Invite students to work in groups to generate questions for which the data would be collected first- and second-hand.

Whole-Class/Group/Individual Assessment Tasks

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students why a sample of 5-year-olds might not be the best one to find out what playground equipment an elementary school should have.
- Invite students to describe how they would gather data on the following and justify their method.
  - The three most popular selections out of their school vending machine.
  - The daily high temperature for Halifax for the past three weeks.
  - The number of times “heads” turns up out of 100 flips of a coin.
- Provide two sample survey questions. Ask students which is better. Have students give a reason for their choice.
  (a) How many brothers and sisters do you have? ___

(b) Are you a member of a large family? Yes ___ No ___

- Invite students, in pairs, to design a questionnaire for a given question, administer it, and record the results.
- Ask students to create a graph that shows the growth of the population of New Brunswick over a period of 20 years. What databases could be accessed to find the information?

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Invite students to collect data to solve problems that are relevant to them. Begin by having students choose good survey questions with a limited number of answers (possibly “Other” as one choice). The response options should be discrete and not overlap.
- Ask students to design questionnaires with the audience and situation in mind. Students should be made aware that many factors have the potential to affect the results, including bias and sample size.
- Remind students that data can be first-hand (collected directly by students) or second-hand (collected by others).
- Use websites such as Statistics Canada (www.statcan.gc.ca) as a source of data and additional information on statistics and various data displays.

Suggested Learning Tasks

- Ask students to find the answer to the question, Which hockey player scored the most goals in one season? by using Internet/electronic media databases.
- Invite students to design and conduct experiments to answer a question. For example, an experiment could be on memory where 20 items are viewed for one minute, then covered, and the subject has to name as many as they can.
- Invite students to work in pairs to design a questionnaire for a given question, administer it, and record the results.
- Ask students what sample/data source they would use to answer questions such as the amount of water an average Canadian uses in a day.
- Invite students to design a questionnaire on problems such as, “What nutritious snacks should be placed in our vending machines?” or “How many hours do Grade 6 students spend using the Internet each day?” Have students collect the data and later graph the results (SP03).

Suggested Models and Manipulatives

- computer program (spreadsheet or graphing applications)
- graph paper

Mathematical Language

Teacher	Student
▪ collecting data ▪ design, administer, record, conclusion ▪ first-hand and second-hand data ▪ questionnaires, experiments, databases, electronic media ▪ results ▪ sample ▪ select, gather ▪ select, method	▪ collecting data ▪ design, administer, record, conclusion ▪ first-hand and second-hand data ▪ questionnaires, experiments, databases, electronic media ▪ results ▪ sample ▪ select, gather ▪ select, method

Resources/Notes

Internet

- Statistics Canada (Government of Canada 2014)
  www.statcan.gc.ca

Print

- *Making Mathematics Meaningful to Canadian Students, K–8* (Small 2008), pp. 525–532
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 568–574
- *Teaching Student-Centered Mathematics, Grades 5–8* (Van de Walle and Lovin 2006), pp. 309–310

Notes

SCO SP03 Students will be expected to graph collected data and analyze the graph to solve problems. [C, CN, PS]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

SP03.01 Determine an appropriate type of graph for displaying a set of collected data and justify the choice of graph.

SP03.02 Solve a given problem by graphing data and interpreting the resulting graph.

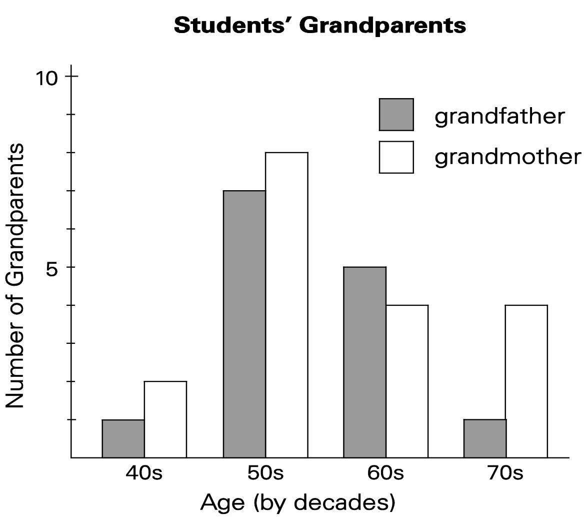
Scope and Sequence

Mathematics 5	Mathematics 6	Mathematics 7
SP02 Students will be expected to construct and interpret double bar graphs to draw conclusions.	**SP03** Students will be expected to graph collected data and analyze the graph to solve problems.	**SP03** Students will be expected to construct, label, and interpret circle graphs to solve problems.

Background

Students should regularly use a variety of graphs to display and organize data. Discuss the different types of graphs that students know and how these are used to display different types of information. By the end of Mathematics 6, students should know how to create and analyze pictographs, line plots, Venn diagrams, Carroll diagrams, bar graphs, double bar graphs, and line graphs. Students will study circle graphs in Mathematics 7.

As described in outcome SP02, data can be collected in surveys, through experiments, or through research. Topics may include areas of mathematics, other curricular areas such as science and social studies, and real-life situations. For example, students might gather information about the ages of their grandparents and display it in various types of graphs.



Grandparents' Ages

Grandparents	
In their 40s: Grandfather Grandmother	 ☺ ☺ ☺
In their 50s: Grandfather Grandmother	 ☺ ☺ ☺ ☺ ☺ ☺ ☺ ☺ ☺ ☺ ☺ ☺ ☺ ☺ ☺
In their 60s: Grandfather Grandmother	 ☺ ☺ ☺ ☺ ☺ ☺ ☺ ☺ ☺
In their 70s: Grandfather Grandmother	 ☺ ☺ ☺ ☺ ☺
☺ = 1 grandparent	

After students have collected their data, they should be able to justify which type of graph(s) would be appropriate to use to display them. Students should recognize that the various types of data displays are not always equally effective or appropriate depending on the type of data. For example, students should recognize that a line graph would not be appropriate for the information displayed in the graphs on the previous page since the data is a count of the number of grandparents in each age range and is, therefore, not continuous. When students create graphs, ensure that they include a title, labels on both axes, and an appropriate scale. Data displays communicate information, so it is important that graphs are accurate, well-organized, and easy to read.

Students need to understand that data is collected to answer questions and solve problems. "When students formulate the questions they want to ask, the data they gather become more and more meaningful. How they organize the data and the techniques for analyzing them have a purpose." (Van de Walle and Lovin 2006c, 309).

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Invite students to draw conclusions from a given double bar graph to answer questions.
  - What information does the graph show?
  - What kinds of data were collected?
  - How many pieces of data were involved?
  - What conclusions can be drawn based on this data?

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to describe the purpose of different types of graphs and give examples of types of data that are appropriate and inappropriate for each (e.g., pictograph, bar graph, line graph).
- Ask students to create a graph that compares two sets of data. Have them explain their choice of graph. Ensure that students include a title, labels on both axes, an appropriate scale, and that their data are well-organized.
- Invite students to answer a given question by performing an experiment or collecting data. Students should record the results, graph the data, and draw conclusions based on the data and graph.
- Provide students with a collection of data and have students graph the information. Consider the student's choice of graph format and the presence of title, labels, appropriate scales, and accurate data representation.
- Provide students with a graph. Ask them to describe what they can interpret from it. Have them graph the same data using a different type of data display.
- Provide students with a graph and have them answer questions that require careful analysis of the data shown.

Follow-up on Assessment

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

Choosing Instructional Strategies

Consider the following strategies when planning daily lessons.

- Examine many real-world pictographs, bar, double bar, and line graphs gathered from newspapers, magazines, and other print media. Discuss why the choice of format is appropriate in each case. Ask students questions that can be answered through careful analysis of the graph.
- Collect data as a class or individually. Have students place the data in a table, and choose an appropriate graph to display them. Ask students to explain their reasoning for their choice of graph.

- Use the Internet as a source of data and possible lesson ideas such as Statistics Canada (www.statcan.gc.ca).
- Provide meaningful questions that students can answer by gathering and graphing data. Examples,
  - If we order T-shirts for our school, what are the most popular sizes we need to get?
  - What were the most frequently observed types of insects during our science investigation?
  - What are the distances our paper airplanes travelled in our "Flight" experiment?
  - What type of fruit was purchased the most at the school canteen or cafeteria?
- Provide students with questions to help them to analyze the data.
  - Which data point is the greatest? Least? Why do you think this is the case?
  - What trend does the data show?
  - What predictions can you make?
  - What questions do you have based on the graph?

SUGGESTED LEARNING TASKS

- Give groups of students examples of different types of graphs. Have them create reasons for when and why we would use this type of graph. Combine ideas and have students present their findings. Students could also create a list of questions relating to the graph that could then be analyzed.
- Use questions related to
  - Favourites: types of music, sports, video games, movies
  - Numbers: amount of money spent on entertainment (movies, etc.), number of pets, hours on computer, number of texts per week
  - Measures: sitting height, arm span, area of foot, time on the bus

  Invite students to collect data, graph, and analyze the results.

  (Van de Walle and Lovin 2006c, 309)
- Present students with "real life" survey questions such as students' satisfaction with cafeteria food, the most popular noon-hour activity, or whether students would like to have a school uniform. Have them collect the data, display it with an appropriate graph, and interpret the results.
- Invite students to explore how data is displayed in other subject areas or in the media. Discuss how the graphs can be analyzed to solve problems.

SUGGESTED MODELS AND MANIPULATIVES

- computer program (spreadsheet or graphing applications)
- grid paper
- prepared graphs from media such as newspapers or magazines

MATHEMATICAL LANGUAGE

Teacher	Student
▪ appropriate graph ▪ create, analyze ▪ display, organize ▪ graph data ▪ interpreting graph ▪ pictographs, line plots, Venn diagrams, Carroll diagrams, bar graphs, double bar graphs, and line graphs	▪ appropriate graph ▪ create, analyze ▪ display, organize ▪ graph data ▪ interpreting graph ▪ pictographs, line plots, Venn diagrams, Carroll diagrams, bar graphs, double bar graphs, and line graphs

Resources/Notes

Internet

- *Statistics Canada* (Government of Canada 2014)
  www.statcan.gc.ca

Print

- *Making Mathematics Meaningful to Canadian Students, K–8* (Small 2008), pp. 493–498
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 538–544
- *Teaching Student-Centered Mathematics, Grades 5–8* (Van de Walle and Lovin 2006), pp. 319–323

Notes

SCO SP04 Students will be expected to demonstrate an understanding of probability by ▪ identifying all possible outcomes of a probability experiment ▪ differentiating between experimental and theoretical probability ▪ determining the theoretical probability of outcomes in a probability experiment ▪ determining the experimental probability of outcomes in a probability experiment ▪ comparing experimental results with the theoretical probability for an experiment [C, ME, PS, T]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

SP04.01 List the possible outcomes of a probability experiment, such as
- tossing a coin
- rolling a die with a given number of sides
- spinning a spinner with a given number of sectors

SP04.02 Determine the theoretical probability of an outcome occurring for a given probability experiment.

SP04.03 Predict the probability of a given outcome occurring for a given probability experiment by using theoretical probability.

SP04.04 Conduct a probability experiment, with or without technology, and compare the experimental results to the theoretical probability.

SP04.05 Explain that as the number of trials in a probability experiment increases, the experimental probability approaches the theoretical probability of a particular outcome.

SP04.06 Distinguish between theoretical probability and experimental probability, and explain the differences.

Scope and Sequence

Mathematics 5	**Mathematics 6**	**Mathematics 7**
SP03 Students will be expected to describe the likelihood of a single outcome occurring, using words such as **impossible**, **possible**, and **certain**. **SP04** Students will be expected to compare the likelihood of two possible outcomes occurring, using words such as **less likely**, **equally likely**, or **more likely**.	**SP04** Students will be expected to demonstrate an understanding of probability by ▪ identifying all possible outcomes of a probability experiment ▪ differentiating between experimental and theoretical probability ▪ determining the theoretical probability of outcomes in a probability experiment ▪ determining the experimental probability of outcomes in a probability experiment ▪ comparing experimental results with the theoretical probability for an experiment	**SP04** Students will be expected to express probabilities as ratios, fractions, and percents.

Background

Students were introduced to the concept of probability in Mathematics 5. Students have conducted probability experiments in Mathematics 5, and it will be helpful to review, with students, the meaning of probability—the chance of an event happening out of all possible outcomes. Revisit the concepts of "more likely," "less likely," "equally likely," "possible," "impossible," or "certain." Give students opportunities to experiment with probability and provide them with meaningful explorations. The materials students use to conduct experiments should be familiar. This becomes important when students are determining all the possible outcomes of an experiment.

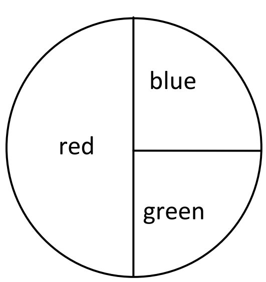
Probability is a measure of how likely an event is to occur. Probability is about the prediction of an event over the long term rather than predictions of individual, isolated events. **Theoretical probability** can sometimes be obtained by carefully considering the possible outcomes and using the rules of probability. For example, in flipping a coin, there are only two possible outcomes, so the probability of flipping a head is, in theory, $\frac{1}{2}$. Often in real-life situations involving probability, it is not possible to determine theoretical probability. We must rely on observation of several **trials** (experiments) and a good estimate, which can often be made through a data collection process. This is called **experimental probability**. As students gather data, they should learn that as the sample size increases, the experimental probability approaches the value of the theoretical probability.

Theoretical probability of an event is the ratio of the number of favourable outcomes in an event to the total number of possible outcomes, when all possible outcomes are equally likely. Simply stated, theoretical probability describes what "should" happen and helps predict the experimental probability.

$$\text{Theoretical probability} = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

If a number cube labelled 1 to 6 is rolled, there are six equally likely possible outcomes (1, 2, 3, 4, 5, 6). In determining the theoretical probability of rolling one of these possible outcomes, for example rolling a 4, one outcome is compared to six possible outcomes, so the theoretical probability of rolling a 4 is $\frac{1}{6}$.

A critical consideration in determining theoretical probability is the likelihood of an outcome. In the case of rolling a number cube, all six outcomes are equally likely since all 6 numbers have an equal chance of being rolled. On the spinner below, however, the three outcomes (red, blue, and green) are not equally likely. The theoretical probability of landing on red is $\frac{1}{2}$ not $\frac{1}{3}$. This can be determined by identifying the fractional part of the spinner that is red.



Experimental probability, or the relative frequency of an event, is the ratio of the number of observed successful occurrences of the event to the total number of **trials**. The greater the number of trials, the closer the experimental probability approaches the theoretical probability. Before conducting experiments, students should predict the probability whenever possible.

$$\text{Experimental probability} = \frac{\text{Number of observed successful occurrences}}{\text{Total number of trials in the experiment}}$$

Students should describe probabilities using fractions.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

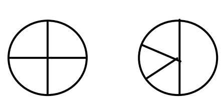
Tasks such as the following could be used to determine students' prior knowledge.

- Invite students to toss a coin twenty-five times and record their results in a chart. Then, ask them to flip the coin another twenty-five times and record the results. Ask them to compare and explain the results.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to create a spinner for which there are four equally likely outcomes and another spinner for which the four outcomes are not equally likely. Have them predict the probability for each spinner's outcomes.



- Provide students with a bag with 10 red cubes and five blue cubes. Ask students to determine the theoretical probability of picking a blue cube.
- Ask students to list the outcomes that result when two dice are rolled and the numbers are subtracted. Conduct the experiment six times and compare theoretical and experimental probabilities. Then conduct the experiment 60 times and compare the results with the first results. Explain what happens when you increase the number of trials in a probability experiment.
- Provide students with a 10-sided die and have them determine the theoretical probability of rolling a prime number (2, 3, 5, 7). Have students roll the die 5 times, 10 times, and 50 times and compare the experimental probability result of each with the theoretical probability. Ask them to explain why it is important to have more than a few trials in a probability experiment.
- Tell students that you rolled a pair of number cubes (with the numbers 1 to 6) 25 times and the sum of the numbers was eight on four of the rolls. What is the theoretical and experimental probability that the sum is eight?
- Invite students to explain how a scientific experiment is like a probability experiment. They should focus on the differences between theory/hypothesis and experimental results.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

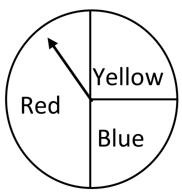
Consider the following strategies when planning daily lessons.

- Introduce simulations—experiments that indirectly model a situation. Students will have had experience directly determining experimental probabilities in Mathematics 5. An example of a simulation is creating a spinner that represents a basketball player who makes their free throws 8 times in 10. The spinner has 0.8 of the face labelled Hit and 0.2 labelled Miss. This can also be simulated with a 10-sided die: the numbers 1 to 8 representing Hits and numbers 9 and 10 representing Misses.

Either model can be used to simulate

- the probability of making exactly three shots in the next five tries
- the probability of missing the first shot, but making the next three in a row
- the probability of missing five shots in a row

- Invite students to explore situations for which outcomes are equally likely. In these cases, they should list the outcomes and count the number of items on the list to determine probabilities. Students must also recognize, however, when outcomes are not equally likely and take this into account. For example, using the spinner shown, students might list the outcomes as “red,” “yellow,” and “blue” and assume that since there are three outcomes, each has a probability of $\frac{1}{3}$. This, however, is not the case. Students might benefit from reconfiguring the spinner to show equally likely outcomes by dividing the red section into two equal pieces. Now the outcomes might be “red 1,” “red 2,” “yellow,” and “blue” and each outcome would now have a probability of $\frac{1}{4}$. Because there are two red sections, the probability of red is, therefore $\frac{2}{4}$.



Suggested Learning Tasks

- Provide pairs of students with 24 linking cubes of different colours and a paper bag. Have them determine the theoretical probability for selecting each colour from the bag. Next, have them conduct the experiment by drawing and replacing one cube at a time for 50 trials. Compare the theoretical and experimental probabilities and discuss.
- Ask students to determine approximately how many boxes of cereal will need to be purchased before a consumer collects each of six possible prizes contained therein. This simulation can be performed by rolling a die, recording the prize number won (based on the roll of the die), continuing until at least one of each number is rolled, repeating the experiment several times, and determining, on average, the number of rolls (purchases) required.
- Invite students to discuss how probability is used in the media. Ask students to find examples of how probability is used to influence people in advertisements, the Internet, newspapers, and magazines.

Suggested Models and Manipulatives

- cards
- coins
- linking cubes
- number cubes
- spinners

MATHEMATICAL LANGUAGE

Teacher	Student
▪ experimental probability ▪ outcomes ▪ probability ▪ ratio ▪ sample size ▪ theoretical probability ▪ trials	▪ experimental probability ▪ outcomes ▪ probability ▪ ratio ▪ sample size ▪ theoretical probability ▪ trials

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students, K–8* (Small 2008), pp. 546–547, 548–552
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 588–589, 590–594
- *Teaching Student-Centered Mathematics, Grades 5–8* (Van de Walle and Lovin 2006), pp. 334–336

Notes

Appendices

Appendix A:
Performance Indicator Background

Number (N)

SCO N01 Students will be expected to demonstrate an understanding of place value for numbers greater than one million and less than one-thousandth. [C, CN, R, T]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

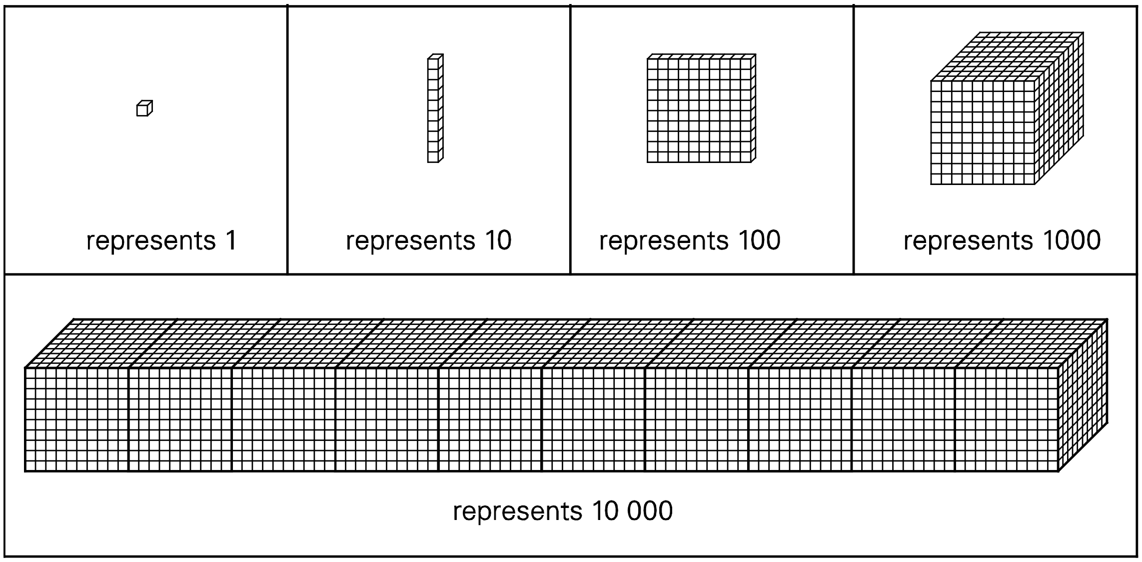
Performance Indicators

N01.01 Explain how the pattern of the place-value system (e.g., the repetition of ones, tens, and hundreds) makes it possible to read and write numerals for numbers of any magnitude.
N01.02 Describe the pattern of adjacent place positions moving from right to left and from left to right.
N01.03 Represent a given numeral using a place-value chart.
N01.04 Explain the meaning of each digit in a given numeral.
N01.05 Read a given numeral in several ways.
N01.06 Record, in standard form, numbers expressed orally, concretely, pictorially, or symbolically as expressions, in decimal notation, and in expanded notation, using proper spacing without commas.
N01.07 Express a given numeral in expanded notation and/or in decimal notation.
N01.08 Represent a given number using expressions.
N01.09 Represent a given number in a variety of ways, and explain how they are equivalent.
N01.10 Read and write given numerals in words.
N01.11 Compare and order numbers in a variety of ways.
N01.12 Establish personal referents for large numbers.
N01.13 Provide examples of where large whole numbers and small decimal numbers are used.

Performance Indicator Background

Whole numbers are discussed first. A discussion of decimal numbers follows.

N01.01 It is important for students to develop a sense of the size of these numbers through concrete and pictorial models. Students should be encouraged to think about how big is one million or one billion. Students can construct a concrete representation of some of these numbers. For example, students could be asked to determine the dimensions of a base-ten rod that would represent 10 000 if the small cube represents 1. They could then use materials, such as rolled newspaper and tape, to build this rod. They could also build the subsequent base-ten flat that would represent 100 000, and extend this to building a base-ten cube to represent 1 000 000. The cube representing 1 000 000 will measure one cubic metre, which will make a nice connection to measurement work. Students could then speculate whether a base-ten flat representing 100 000 000 would fit inside the classroom or whether a base-ten cube representing 1 000 000 000 would fit inside the gymnasium. Such constructions help students to develop a conceptual understanding of large numbers.



Numbers written in standard form are organized and written in groups of three digits. Some authors call each of these groups a **period**. It is not important to highlight the term **period** nor is it intended that students use this terminology. Students should be given opportunities to examine large numbers in real-world contexts that show the pattern of grouping digits in periods. They should be able to explain how this patterning facilitates the reading of numbers. An example, such as 582 582 582, written in words or reading the word name may help students see the pattern more clearly. 582 582 582 is read as five hundred eighty-two **million**, five hundred eighty-two **thousand**, five hundred eighty-two since the digits 582 appear in the millions, in the thousands, and in the ones periods. Within each period, there are hundreds, tens, and ones. Students need to learn the grouping names (ones, thousands, millions, etc.) to facilitate the reading and writing of numbers. They will recognize that each period has a similar pattern of 100, 10, and 1 of the given period unit. The recognition of this pattern will help students to read larger numbers with which they are unfamiliar.

N01.02 Place-value concepts are important in developing number sense. When students examine large numbers, they develop a greater sense of the patterning in the place-value system. This exploration will help students to recognize the regularity of the patterns that are inherent in the place-value system. Students should also be able to explain the relationship between each place-value position and its neighbour positions, namely a group of ten in one position makes a group of one in the position to the left, and a group of one in any position makes a group of ten in the position to the right. Students have used this principle to regroup and trade in previous grades, but they should be able to state that this pattern continues to work regardless of the size of the number. Students should be able to explain that the digits 0–9 are used cyclically and indicate the number of units in any given place. This will help them to develop an understanding of the patterning of the digits in the place-value system. For example, each place value is ten times the value of the unit to its right. This also extends to the idea that the tens place is 10 times the ones, the hundreds is 100 times the ones, the thousands is 1000 times the ones, and so on.

N01.03 As students begin working with a place-value chart to represent large numbers, concentrate on their understanding of the number as a whole, as well as the value of each digit in the number. To simply place digits in the proper space in the place-value chart does not indicate an understanding of the value of that digit. Students need to understand how to transfer the numbers from the place-value chart to standard form with correct spacing, as well as in written form and expanded form.

When students begin exploring the value of numbers and are presented with a number such as 7 324 169, they may be asked to indicate what the 7 represents. Students need to see the 7 means seven million, which is written as 7 000 000. Students should also understand, for example, in the number 345 461, the 5 is in the thousands place, so it represents five thousand, but there are actually 345 thousands in this number.

Students should learn that the position of a digit determines its value. Students should also recognize and work with the idea that the value of a digit varies, depending on its position or place, in a numeral. Use of the place-value chart can support the development of this understanding.

For example, students would record the number 124 987 453 in a place-value chart as

Billions	Hundred Millions	Ten Millions	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones
	1	2	4	9	8	7	4	5	3

Students could also be presented with a numeral and could use counters to model the number in the place value chart. For example, if presented with the numeral 23 124 302, students would record the following:

Billions	Hundred Millions	Ten Millions	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones
		●●	●●●	●	●●	●●●●	●●●		●●

Students could then write this number in expanded form as 20 000 000 + 3 000 000 + 100 000 + 20 000 + 4000 + 300 + 2.

N01.04 Students should be able to read large numbers beyond one million and should also begin to recognize that the name of a number is connected to the number of digits, for example, that 3 000 000 is three million and that 20 000 000 000 is twenty billion. Students should be able to read a number such as 2 371 209 413 and should recognize and work with the idea that the value of a digit varies, depending on its position or place in a numeral. Students should recognize the value represented by each digit in a number, as well as what the number means as a whole. The digit 2 in 42 845 701 represents 2 millions whereas the digit 2 in 3200 represents 2 hundreds. Students should be able to explain the meaning of the digits, including numerals with all digits the same (e.g., for the numeral 222 222 222, the first digit represents 2 hundred millions, the second represents 2 ten millions, the third represents 2 millions, the fourth represents 2 hundred thousands, the fifth digit represents 2 ten thousands, the sixth digit represents 2 thousands, the seventh digit represents 2 hundreds, the eighth digit represents 2 tens, and the ninth digit represents 2 ones).

It is important to spend time developing a good understanding of the meaning and use of the digit 0 in numbers. Students need many experiences using base-ten materials and place-value charts to model numbers with zeros as digits and to make connections with the symbols for numbers with zeros as digits. Teachers should ask students to write the numerals for numbers read to them, such as seventy million three thousand five hundred forty or nine hundred thousand two hundred eight. When a number, such as seven hundred million four thousand five hundred forty-three, is written in its symbolic form using digits, the digit 0 is called a place holder. If the digit 0 was not used, the number would be recorded as 74 543, and you would mistakenly think that the 7 represented 70 000 instead of 700 000 000.

N01.05 and **N01.10** Students should read a given numeral without using the word and. For example, 12 537 421 is read as twelve million five hundred thirty-seven thousand four hundred twenty-one, not as twelve million five hundred thirty-seven thousand four hundred and twenty-one. When reading numbers, the word and is reserved for the decimal.

Students should also have experience reading numbers in several ways. For example, 1 938 147 may be read as one million nine hundred thirty-eight thousand one hundred forty-seven but might also be read as 19 hundred thousands, 38 thousands, one hundred forty-seven; or 193 ten thousands, 8 thousands, 1 hundreds, 4 tens, 7 ones; or 1938 thousands, 14 tens, 7 ones; or 19 hundred thousands, 37 thousands, 11 hundreds, 3 tens, 17 ones.

Students will also need to be able to write the number words for the numbers they encounter and read numbers written in words. The accepted convention for writing number words is as follows

- fifty-six
- three hundred fifty-six
- four thousand three hundred fifty-six
- twenty-six thousand nine hundred fifty-six
- one hundred forty-six thousand nine hundred fifty-six
- one million one hundred forty-six thousand nine hundred fifty-six

When students write numbers in words, they must consider the place value of each digit, thus solidifying the importance of the periods. For example, to write 19 946 219 in words, students must recognize that they start with the largest period, in this case millions, and continue with the successive periods. Students name each period once they say the total number in that period. In 19 946 219, nineteen must be followed with the period name, million, and nine hundred forty-six must be followed with the period name, thousand.

Students must have a deep understanding of numbers and be able to rename numbers in a variety of ways. Students should recognize that 1 000 000 000 is just another expression for

- 10 hundred millions
- 100 ten millions
- 1000 millions
- 10 000 hundred thousands
- 100 000 ten thousands
- 1 000 000 thousands
- 10 000 000 hundreds
- 100 000 000 tens
- 1 000 000 000 ones

N01.06 Students must be able to record numbers heard and read numbers written symbolically. Students should be given many opportunities to record numbers in symbolic form. Numbers written in standard form are organized and written in groups of three digits.

Students are expected to write a given numeral using proper spacing without commas. We do not use the comma because in many countries using the metric system, the comma is used as the decimal point. The accepted convention for four-digit numbers is to not leave a space (e.g., 4567). For numbers with 5 or more digits, leave a small space between each group of three digits starting from the right (e.g., 470 389 006). If too large a space is used, the number may be misinterpreted as two separate numbers.

When provided with a number represented as a model, expression, expanded notation, decimal notation, place-value chart, or words, students need to be able to record the number symbolically in more than one way. For example, if presented with a model or picture of 1 large flat, 2 large rods, 5 large cubes, 2 flats, 3 rods, and 4 small cubes, the number can be recorded in many ways including, 125 234; 100 000 + 20 000 + 5000 + 200 + 30 + 4; or 1 hundred thousands, 2 ten thousands, 5 thousands, 2 hundreds, 3 tens, 4 ones.

As students continue work with large numbers, they sometimes need to rename numbers using decimal notation. They need to understand that the number to the left of the decimal names the whole number and the digits to the right of the decimal names the part of the number. For example, in the number 43 431 509 there are 43 whole millions and 431 thousands. We would write this as 43.4 million. Estimation also plays a role in renaming numbers using decimal notation. Students need to see that a number such as 3 450 000 is about 3.5 million. The focus should be on students' reasoning and estimating of these numbers.

Students should also understand that when we are dealing with extremely large numbers it is very difficult to be accurate. If asked to find the population of Canada, students could use a search engine to see that in the 2008 census, the population was 33 311 389 people. However, it is impossible to calculate the population of the nation at any given time since it is continuously changing. Therefore, giving this population count is only an estimate. When asked about the population of Canada, a reasonable estimate would be 33.3 million people.

N01.07 Expressions may be recorded in the expanded notation (additive expanded form). For example, 814 256 is written in expanded form as 800 000 + 10 000 + 4000 + 200 + 50 + 6.

Expanded form can be demonstrated in either of the following ways:
27 456 721 = 20 000 000 + 7 000 000 + 400 000 + 50 000 + 6000 + 700 + 20 + 1
27 456 721 = (2 × 10 000 000) + (7 × 1 000 000) + (4 × 100 000) + (5 × 10 000) + (6 × 1000) + (7 × 100) + (2 × 10) + (3 × 1).

Students should be exposed to both forms. To foster deep understanding of expanded form, students should be given numbers that include zeros, such as 5 000 302. Also, expanded form should be given in various orders such as (4 × 10 000) + (3 × 100 000) + (2 × 100).

N01.08 Students have had ample opportunities with concrete, pictorial, and verbal representations of base-ten models in previous grades. They will now record the base-ten partitions for large numbers as an expression. For example, 2 793 159 may be expressed as 2 000 000 + 700 000 + 93 159. It is important to model the correct use of the term **expression** to students. An expression names a number. Sometimes an expression is a number such as 2 793 159. Sometimes an expression shows an arithmetic operation, such as 2 793 000 + 159. The number 2 793 159 may also be represented by its partitions, such as 2 000 000 + 700 000 + 93 000 + 159, or 500 000 + 500 000 + 500 000 + 500 000 + 700 000 + 70 000 + 10 000 + 10 000 + 3000 + 159. Numbers can also be represented by a difference expression, such as 3 000 000 – 206 841 or 2 800 000 – 6841. Students should also be provided with opportunities to write the numeral represented by a given expression.

N01.09 Students must have a deep understanding of numbers and be able to represent and rename numbers in a variety of ways. They should be able to translate from one representation to another, for example, from a place-value chart to a numeral or from a base-ten picture to expanded form. Students should be also able to explain why the representations are equivalent. For example, students should be able to represent 129 842 as 129 thousands, 842 ones; as 12 ten thousands, 98 hundreds, 42 ones; or as 12 ten thousands, 9 thousands, 84 tens, 2 ones. Students should be able to explain why each of the representations is equivalent 129 842.

N01.11 Comparing and ordering numbers is fundamental to understanding numbers. Students should investigate meaningful contexts to compare and order two or more numbers, both with and without models. For example, ask them to compare and order populations of communities or capacities of arenas.

Students must understand that when comparing two whole numbers with the same number of digits, the digit with the greatest place value needs to be addressed first. For example, when asked to explain why one number is greater or less than another, students might say that 28 251 424 < 34 367 539 because 28 251 424 is less than 30 million while 34 367 539 is greater than 30 million. When comparing 2 516 056 and 2 615 046, students should begin comparing the millions, and then compare each place value to the right.

Students should not only recognize numbers that are greater or less than a number, but should also be able to name numbers between any two given numbers. For example, if they know the populations of two large cities, they should be able to identify a population that would fall between these two amounts.

Students should be provided with opportunities to examine large numbers and use place-value arguments to explain which number is larger. Students should also be able to place large numbers in approximate positions on a number line given benchmarks. Teachers should use number lines often and provide opportunities for students to construct various number lines. In previous grades, students have had experience working with number lines that begin with numbers other than 0 and have a variety of end numbers with and without hatch marks. Students will continue to work with empty number lines and number lines with hatch marks and benchmarks. There are many real-world contexts that can be used to help students make sense of large numbers (e.g., they can be asked to compare and order populations of various countries of the world or different large cities in Canada).

N01.13 It is important that students view large numbers and small numbers in print and electronic media in order to see examples of large numbers used in the real world. As students begin to work with numbers greater than one million, it can become more challenging to provide meaningful examples to represent these numbers. Various texts, media, and technology may provide students with real-life examples of large numbers and can give students a context in which they can understand what these numbers mean. The Guinness Book of World Records, and other subject texts, such as social studies and science, may also provide examples of large number usage.

DECIMAL NUMBERS

Work at this level should concentrate on having students understand that the place-value system extends to the left of the decimal as it does to the right. It is here that it should be emphasized that the majority of real-life exposure to decimal numbers is unlikely to extend below the thousandths. However, it is important for students to know that the place-value system extends beyond thousandths and that they can use the patterns of the place-value chart to assist them in reading and writing these decimal numbers.

Students can often use the same strategies when reading and writing decimal numbers as those used to read and write whole numbers.

Show students how to write the numbers that are less than one thousandth in words to get them familiar with saying the words. For example, when talking about two ten thousandths (0.0002), students begin writing or saying this as two parts out of ten thousandths, or two in ten thousandths. They then place this number on the place-value chart to show that the 2 is in the ten thousandths place. Therefore, we say that the number is 2 ten thousandths.

Some students may have trouble reading and writing decimal numbers. When writing numbers in the place-value chart, demonstrate to students how the value of the last digit depends on which period it is placed in. In Example A below, we would read the number as thirteen ten thousandths. In Example B, the last digit is in the ten thousandths place, therefore we would read the number as two and four thousand five hundred sixty-seven ten-thousandths.

	Tens	Ones.	Tenths	Hundredths	Thousandths	Ten Thousandths	Hundred Thousandths	Millionths
Example A					1	3		
Example B		2	4	5	6	7		

As students continue comparing decimal numbers, it must be reiterated that not all strategies to compare whole numbers will work for decimal numbers. For example, when comparing 3456 with 345, it is evident to see that 3456 is greater because there are more digits in that whole number. When comparing decimal numbers, however, this strategy may not work. For example, if asked to determine which number, 0.234 or 0.2287, is greater, students may think that 0.2287 is greater because it has more digits. Help students understand that 0.234 is greater because 0.234 is greater than 0.23 and 0.2287 is less than 0.23. It is through this work that student understanding of the place-value system will be seen. The place-value chart, as well as number lines, can be used in the comparison of decimals. For most students, it will be easier to compare decimals with the same number of decimal places. For decimal numbers that do not have the same number of decimal places, students can be shown that they can place a desired number of zeros at the end of the number without changing the value of the number. It will be necessary here to discuss how one tenth, for example, is the same as 10 hundredths, 100 thousandths, and 1000 ten thousandths. Writing these numbers on a place-value chart or under each other to show the relationship would be helpful. For example,

0.1
0.10
0.100
0.1000

Students' understanding of decimal numbers involving ten thousandths is an extension of their understanding of the place-value system. It can be difficult to provide meaningful examples of numbers that extend beyond the thousandths. Using examples and contexts, such as parts per million (ppm) in science to explain the amount of a chemical in a solution, may be one way to help students understand where these numbers could be used.

As students continue to work with decimal numbers beyond thousandths, they can be shown how these very small numbers are related to larger numbers (millions) using the place-value chart.

SCO N02 Students will be expected to solve problems involving whole numbers and decimal numbers. [ME, PS, T]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

N02.01 Determine whether technology, mental mathematics, or paper-and-pencil calculation is appropriate to solve a given problem and explain why.
N02.02 Identify which operation is necessary to solve a given problem and solve it.
N02.03 Determine the reasonableness of an answer.
N02.04 Estimate the solution and solve a given problem using an appropriate method (technology, mental mathematics, or paper-and-pencil calculation).
N02.05 Create problems involving large numbers and decimal numbers.
N02.06 Use technology, mental mathematics, or paper-and-pencil calculation to solve problems involving the addition, subtraction, multiplication, and division of whole numbers.
N02.07 Use technology, mental mathematics, or paper-and-pencil calculation to solve problems involving the addition and subtraction of decimal numbers.

Performance Indicator Background

In previous grades, students were expected to perform paper-and-pencil calculations to determine
- sums and differences involving whole numbers (up to four-digit numbers)
- products involving whole numbers (up to three-digits by a one-digit multiplier; and 2 two-digit numbers)
- quotients involving whole numbers (up to three-digits by a one-digit divisor)

As well, students were expected to perform paper-and-pencil calculations to determine sums and differences of decimal numbers (up to thousandths). In Mathematics 6, students are expected to continue to perform these calculations to solve problems.

Paper-and-pencil calculation of sums, differences, products, and quotients of large multi-digit numbers is not an expectation for Mathematics 6. Specifically, students are not expected to perform paper-and-pencil calculation of
- multi-digit addition and subtraction of whole numbers with five or more digits or decimal numbers beyond thousandths
- multiplication of whole numbers involving three-digit factors
- division of whole numbers involving two-digit divisors

It is, however, expected that students will estimate sums, differences, products, and quotients of questions involving such numbers, and will then use a calculator to determine the actual sum, difference, product, or quotient. It is also expected that students will use mental mathematics to calculate multi-digit sums, differences, products, and quotients when appropriate. For example, students are expected to mentally calculate the sum of 200 000 + 300 000, the difference of 120 000 – 30 000, the product of 73 × 1000, and the quotient of 35 000 ÷ 5.

Expectations for paper-and-pencil calculations for multiplication and division involving decimal numbers are described in outcome N08. Specific expectations for mental mathematics and estimation involving whole numbers and decimal numbers in Mathematics 6 follow.

N02.01 Students need to be aware of when to use technology. There are situations where using a calculator is more appropriate than using mental mathematics or paper-and-pencil strategies. Although it is essential for students to master the basic facts, to become comfortable using mental mathematics strategies, and to calculate sums, differences, products, and quotients using paper-and-pencil methods, calculator use can enhance student learning if used appropriately. Sometimes, it is beneficial to involve students in solving problems that focus on the problem-solving process and not the actual calculations within the problem. Using a calculator can enable a student to focus on the problem solving rather than tedious multi-digit calculations.

Solving problems involving large numbers may seem overwhelming to some students. Breaking the problem down into smaller steps, or using smaller numbers will help students understand how to go about solving the problem. It may also be important to emphasize the necessity of estimating the answer to the problem before or after the solution is found. Calculators can assist students in solving problems involving large numbers. As well, students should be aware that calculator and technology usage is a part of everyday work. Discuss with students the types of jobs that would use calculators and other technology as part of the work environment. Some occupations include bankers, accountants, educators, doctors, nurses, scientists, stock brokers, contractors, and architects.

Students need to be given opportunities to see that there will be times when getting an exact answer is not possible. For example, it may be difficult to know exactly how many juice boxes to buy for the school population on sports day. Estimation would be used to come close to a reasonable number. Other real-life examples could be explored to emphasize the importance of knowing how to estimate, such as knowing how much money would be needed to buy a list of items at the store or how much lumber is needed to build a dog house.

Many times students confuse estimation with guessing and may offer answers or estimates without thinking or giving any thought to the numbers involved. Students must understand that in order to estimate, they must do something with the numbers (rounding, comparing, using referents/benchmarks, using compatible numbers to do mental mathematics computations, etc.) whereas guessing is a random response without using a strategy.

Students entering Mathematics 6 should have quick recall of all basic facts for the four operations. Recall of basic addition facts was expected in Mathematics 2, subtraction facts in Mathematics 3, multiplication facts in Mathematics 4, and division facts in Mathematics 5. As well, students will have a variety of strategies for mental calculations that have been addressed in previous grades. In Mathematics 6, students will apply these strategies to large numbers and to decimal numbers.

MENTAL MATHEMATICS FOR ADDITION

Addition Facts Applied to Multiples of Powers of 10

Knowledge of all single-digit addition facts was an expectation in Mathematics 2. These facts were applied to multiples of powers of 10 in earlier grades. In Mathematics 6, these facts will continue to be applied to multiples of powers of 10 and will be further applied to tenths, hundredths, and thousandths, and extended to very large numbers such as 2 million or 0.8 billion.

The strategies for 88 of the 100 facts involving single-digit addends are as follows:

- Doubles Facts
- Plus-One Facts
- Near-Doubles (1-Aparts) Facts
- Plus-Two Facts

- Plus Zero Facts
- Make-10 Facts

There are a variety of strategies that can be used for the last 12 facts. Further information about the basic addition and subtraction fact-learning strategies can be found in the curriculum documents for Mathematics 2 and 3.

Examples:
- For 40 + 60, think, If 10 from the 60 is given to the 40, the question becomes 50 + 50 or 100.
- For 4000 + 5000, think, 4000 and 4000 is 8000, so 1000 more is 9000; or think 4 and 5 is 9, but these are thousands, so the answer is 9000.
- For 0.07 + 0.05, think, If one-hundredth from 0.07 is moved to 0.05, the question becomes 0.06 + 0.06, or 0.12; or think, 7-hundredths plus 5-hundredths is 12-hundredths which is 12-hundredths (0.12).

Sample Tasks:
- 90 + 60
- 80 increased by 30
- 600 girls and 600 boys. How many children?
- $5000 + $9000
- 20 000 + 30 000
- 0.6 + 0.3
- 0.5 kg plus 0.7 kg
- 0.04 m increased by 0.08 m
- The sum of 0.09 and 0.06

Students can apply patterns with addition, as well as other operations.

Input	Output
0	4
1	7
2	10
3	

(Add 4)
(Add 6)
(Add 8)

When presented with an increasing pattern, such as 100, 300, 500, 700, ____ , students might think, Each number is 200 more than the number before, so the next number in the pattern is 900 because 700 + 200 = 900.

When presented with an increasing pattern, such as 10 000, 40 000, 70 000, 100 000, ______, the student might think, Each number is 30 000 more than the number before, so the next number in the pattern is 130 000 because 100 000 + 30 000 = 130 000.

Front-End Addition
This strategy is applied to questions that involve two combinations of non-zero digits, one combination of which may require regrouping. The strategy involves first adding the digits in the highest place-value position, then adding the non-zero digits in another place-value position, and making any needed regrouping. After a review of this strategy applied to whole numbers, it should be extended to large numbers such as millions or billions and to tenths, hundredths, and thousandths.

Examples:

- For 26 + 37, think, 20 plus 30 is 50, 6 plus 7 is 13, and 50 plus 13 is 63.
- For 307 + 206, think, 300 plus 200 is 500, 7 plus 6 is 13, and 500 plus 13 is 513.
- For 3600 + 2500, think, 3 thousand plus 2 thousand is 5 thousand, 6 hundred and 5 hundred is 11 hundred, and 5 thousand and 11 hundred is 6100.
- For 25 000 + 38 000, think, 20 thousand plus 30 thousand is 50 thousand, 5 thousand plus 8 thousand is 13 thousand, and 50 thousand plus 13 thousand is 63 thousand (63 000).
- For 7.3 + 2.6, think, 7 plus 2 is 9 and 3-tenths plus 6-tenths is 9-tenths, so the answer is 9 and 9-tenths (9.9).
- For 5.06 + 3.09, think, 5 and 3 is 8, 6-hundredths and 9-hundredths is 15-hundredths, and 8 and 15-hundredths is 8.15.
- For 5.8 million + 2.5 million, think, 5 and 2 is 7, 8-tenths and 5-tenths is 13-tenths, and 7 and 13-tenths is 8 and 3-tenths million (8.3 million).

Sample Tasks:

- 45 + 36
- 18 kg more than 56 kg
- 102 more than 567
- $660 + $270
- 3400 km and 5800 km
- The sum of 2040 and 6090
- 56 000 females and 47 000 males. What is the total?
- $60,080 increased by $10,090
- 3.5 m and 2.4 m
- 4.3 kg more than 7.8 kg
- 7.5 km increased by 2.9 km
- The sum of $0.12 and $0.09

Quick Addition—No Regrouping

This strategy is actually the front-end strategy applied to questions that involve more than two combinations with no regrouping. The questions are always presented visually, and students quickly record their answers on paper. While it could be argued that this is a pencil-and-paper strategy because answers will always be recorded on paper before answers are read, it is included here as a mental-mathematics strategy because most students will do all the combinations in their heads starting at the front end.

This strategy requires students to holistically examine each question to confirm there will be no regrouping. This habit of holistically examining each question as a first step in determining the most efficient strategy needs to pervade all mental mathematics lessons. It is important to present examples of these addition questions in both horizontal and vertical formats. Students should have applied this strategy to multi-digit numbers up to the end of Mathematics 5, so in Mathematics 6 they should apply it to large numbers and to tenths, hundredths, and thousandths. Most likely, students will add the digits in corresponding place values of the two addends without consciously thinking about the names of the place values. Therefore, in the discussion of the questions, you should encourage students to read the numbers correctly and to use place-value names. This will reinforce place-value concepts at the same time as addition.

Examples:
- For 543 + 256, think and record each resultant digit: 5 and 2 is 7, 4 and 5 is 9, and 3 and 6 is 9, so the answer is 799 (seven hundred ninety-nine); or think, **5**00 and **2**00 is **7**00, **4**0 and **5**0 is **9**0, **3** and **6** is **9** to get 799.
- For 2341 increased by 3415, think and record each resultant digit: 2 and 3 is 5, 3 and 4 is 7, 4 and 1 is 5, and 1 and 5 is 6, so the answer is 5756 (five thousand seven hundred fifty-six); or think, **2**000 and **3**000 is **5**000, **3**00 and **4**00 is **7**00, **4**0 and **1**0 is **5**0, **1** and **5** is **6** to get 5756.
- For $23,451 + $41,426, think and record each resultant digit: 2 and 4 is 6, 3 and 1 is 4, 4 and 4 is 8, 5 and 2 is 7, and 1 and 6 is 7, so the answer is $64,877 (sixty-four thousand eight hundred seventy-seven); or think, **2**0 000 and **4**0 000 is **6**0 000, **3**000 and **1**000 is **4**000, **4**00 and **4**00 is **8**00, **5**0 and **2**0 is **7**0, **1** and **6** is **7** to get $64,877.
- For 34.32 + 23.57, think and record each resultant digit: 3 and 2 is 5, 4 and 3 is 7, 3 and 5 is 8, and 2 and 7 is 9, so the answer is 57.89 (fifty-seven and eighty-nine hundredths); or think, **3**0 and **2**0 is **5**0, **4** and **3** is **7**, **3**-tenths and **5**-tenths is **8**-tenths, **2**-hundredths and **7**-hundredths is **9**-hundredths to get 57.89.

Sample Tasks:
- The sum of 291 and 703
- $\begin{array}{r} 537 \\ \underline{+\ 341} \end{array}$
- There were 333 girls and 144 boys at the concert. What was the total attendance?
- $4532 + $2367
- There are 8107 people in town and 1742 people on the outskirts. What is the total population?
- 372 more than 5116
- $\begin{array}{r} 10\ 357 \\ \underline{+\ 42\ 111} \end{array}$
- 34 680 + 21 318
- The sum of $12,045 and $36,920
- Population of 2.4 billion increased by 3.5 billion.
- 3.5 m and 2.4 m
- The sum of 4.6 and 3.3
- Latrell counted $0.75 in one pocket and $0.14 in the other. How much money does Latrell have in his pockets?
- 5.05 km more than 7.04 km
- 45.5 km + 12.3 km
- 235.6 m increased by 22.2 m
- $\begin{array}{r} \$456.17 \\ \underline{+\ \$502.62} \end{array}$
- 23.08 more than 534.71

Finding Compatibles

This strategy for addition involves looking for pairs of numbers that combine easily to make a sum that is a power of ten that will be easy to work with. In addition to finding compatibles for whole number powers of 10, students should extend this strategy to searching for pairs of decimal numbers that add to 1, 0.1, 0.01, etc. (In some resources, these compatible numbers are referred to as friendly numbers or nice numbers.) You should be sure that students are convinced that the numbers in an addition expression can be combined in any order (the associative property of addition).

Examples:

- For 1 + 7 + 9 + 8 + 3, think, 1 + 9 is 10 and 7 + 3 is 10, so 10 + 10 + 8 is 28.
- For 30 + 75 + 70 + 25, think, 30 + 70 is 100 and 75 + 25 is 100, so 100 + 100 is 200.
- For 300 + 800 + 700 + 600 + 200, think, 300 + 700 is 1000, 800 + 200 is 1000, so 1000 + 1000 + 600 is 2600.
- For 250 + 470 + 750, think, 250 and 750 is 1000, so 1000 and 470 is 1470.
- For 4000 + 5000 + 6000, think, 4000 and 6000 is 10 000, so 10 000 and 5000 is 15 000.
- For 9500 + 2200 + 500, think, 9500 and 500 is 10 000, so 10 000 plus 2200 is 12 200.
- For 0.4 + 0.3 + 0.6, think, 4-tenths and 6-tenths is 1, so 1 and 3-tenths is 1.3.

Sample Tasks:

- 60 + 30 + 40 + 70
- The total of three items costing $75, $95, and $25.
- The sum of 200, 700, 500, 800, and 300.
- The total of three deposits: $50, $460, $950.
- 5000 + 3000 + 5000 + 7000
- $2500 and $3500 and $7500.
- 8000 km + 4000 km + 6000 km + 7000 km + 2000 km
- Total of three items: $1000, $5000, $9000.
- 0.2 + 0.4 + 0.3 + 0.8 + 0.6
- 6-tenths + 9-tenths + 4-tenths + 1 tenth
- The sum of three lengths: 0.09 m, 0.13 m, 0.01 m
- Gina has $0.50 and Joe has $0.75. Zaria has $0.25 more than their total. How much money does Zaria have?

Break-up and Bridge

This strategy involves starting with the first number in its entirety and adding the place values of the second number, one-at-a-time, starting with the largest value. In Mathematics 6, the practice items should include large whole numbers as well as numbers in the tenths, hundredths, and thousandths along with those numbers used in previous grades. Remember that the practice items should only include questions that require two combinations with one regrouping.

Examples:

- For 45 + 36, think, 45 and 30 (from the 36) is 75, and 75 plus 6 (the rest of the 36) is 81.
  In symbols: 45 + 36 = (45 + 30) + 6 = 75 + 6 = 81.
- For 537 + 208, think, 537 and 200 is 737, and 737 plus 8 is 745.
  In symbols: 537 + 208 = (537 + 200) + 8 = 737 + 8 = 745.
- For 5300 + 2800, think, 5300 and 2000 (from the 2800) is 7300 and 7300 plus 800 (the rest of 2800) is 8100.
  In symbols: 5300 + 2800 = (5300 + 2000) + 800 = 7300 + 800 = 8100.
- For 34 000 + 27 000, think, 34 000 plus 20 000 is 54 000, and 54 000 plus 7000 is 61 000.
  In symbols:
  34 000 + 27 000 = (34 000 + 20 000) + 7000 = 54 000 + 7000 = 61 000.
- For two items costing $3.60 and $5.70, think, $3.60 and $5 (from the $5.70) is $8.60, and $8.60 plus $0.70 (the rest of $5.70) is $9.30.
  In symbols: $3.60 + $5.70 = ($3.60 + $5.00) + $0.70 = $8.60 + $0.70 = $9.30.

Sample Tasks:

- 46 + 36
- 17 more than 64
- The sum of $370 and $440
- 365 increased by 109
- 2500 + 3700
- The sum of 16 800 km and 1300 km
- The total of 4070 girls and 3080 boys
- 7009 increased by 2008
- 46 000 + 37 000
- The total of $66,000 and $15,000
- 56 000 increased by 24 000
- 17 000 km more than 28 000 km
- 4.7 m + 3.5 m
- The total of 15.6 km and 10.7 km
- 12.5 kg increased by 5.6 kg
- $1.65 + $2.20
- The sum of 4.06 m and 3.07 m
- 4.56 kg more than 4.40 kg
- 1.234 + 4.503
- 14.056 + 3.743

Compensation

This strategy involves changing one number in the addition question to a nearby multiple of a power of ten, carrying out the addition using that multiple of a power of ten, and adjusting the answer to compensate for the original change. Students should understand that the number is changed to make it more compatible, and that they have to hold in their memories the amount of the change. In the last step, it is helpful if they remind themselves that they added too much so they will have to take away that amount. This strategy is perhaps most effective when one of the addends has an 8 or 9 in its lowest place value, although some students are comfortable using it with a 7 as well. In Mathematics 6, the questions should include numbers in the thousands and tens of thousands, as well as numbers in the tenths, in the hundredths, and in the thousandths along with those numbers used in previous grades.

Examples:

- For 52 + 39, think, 40 is easier to work with than 39. Then 52 plus 40 is 92, but I added 1 too many; so, to compensate, I subtract one from my answer, 92, to get 91.
- For 345 + 198, think, 200 is easier to work with than 198. Then 345 + 200 is 545, but I added 2 too many; so, I subtract 2 from 545 to get 543.
- For 4500 + 1900, think, 2000 is easier to work with than 1900. Then 4500 + 2000 is 6500, but I added 100 too many; so, I subtract 100 from 6500 to get 6400.
- For 34 000 + 9900, think, 10 000 is easier to work with than 9900. Then 34 000 plus 10 000 is 44 000, but I added 100 too many; so, I subtract 100 to get 33 900.
- For 59 000 + 25 000, think, 60 000 plus 25 000 is 85 000, but I added 1000 too many; so, I subtract 1000 to get 84 000.
- For 4.6 + 1.8, think, 4.6 plus 2 is 6.6, but I added 2-tenths too much; so, I subtract 2-tenths from 6.6 to get 6.4 (six and 4-tenths).
- For 0.54 plus 0.29, think, 54-hundredths + 30-hundredths is 84-hundredths, but I added 1-hundredth too much; so, I subtract 1-hundredth from 84-hundredths to compensate to get 83-hundredths, or 0.83.

Sample Tasks:

- 58 + 9
- 49 + 38
- 265 + 399
- $198 more than $465
- 3456 km increased by 999 km
- The sum of 2998 and 3525
- 16 000 + 39 000
- The sum of 28 000 and 65 000
- The total of $38,000 and $9900
- 74 000 km increased by 18 000
- 3.9 m + 2.5 m
- 3.5 km more than 4.8 km
- $0.36 + $0.39
- $2.47 more than $4.99

Make Multiples of Powers of Ten

In previous grades, students would have been introduced to this strategy as Make-10, Make-10s, and Make-100s, and Make-1000s. In Mathematics 6, this strategy should be extended to include Make-10 000, Make-1, and Make-1s.

Like the compensation strategy, this strategy is best applied when one of the addends has an 8 or 9 in its lowest place value, and it makes use of the compatibility of multiples of powers of ten in addition. This strategy, however, involves getting the amount needed to make one addend a multiple of a power of ten from the other addend, thus changing both addends to numbers that are easier to combine. A common error is for students to forget that both addends have changed; this means that more has to be kept in their short-term memories. Therefore, questions used for reinforcement should not involve too many non-zero digits. The compensation and the make-multiples-of-powers-of-ten strategies should be compared so students are clear about how they are alike and how they are different because both strategies are appropriately applied to the same questions.

Examples:

- For 92 + 69, think, If 1 is taken from 92 and given to 69, the question becomes 91 + 70, which is easier to add to get 161.
- For 298 + 345, think, If 2 is taken from 345 and given to 298, the question becomes 300 + 343, which is easier to add to get 643.
- For 650 + 190, think, If 10 is taken from 650 and given to 190, the question becomes 640 + 200, which is easier to add to get 840.
- For 34 000 + 28 000, think, If 2000 is taken from the first addend and given to the second addend, the question becomes 32 000 + 30 000, which is easier to add to get 62 000.
- For 56 700 + 3900, think, If 100 is taken from the first addend and given to the second addend, the question becomes 56 600 + 4000, which is easier to add to get 60 600.
- For 1.3 + 0.9, think, If 1-tenth is taken from the first addend and given to the second addend, the question becomes 1.2 + 1, which is easier to add to get 2.2.
- For 1.4 + 2.9, think, If 1-tenth is taken from the first addend and given to the second addend, the question becomes 1.3 + 3, which is easier to add to get 4.3.
- For 3.98 + 4.24, think, If 2-hundredths is taken from the second addend and given to the first addend, the question becomes 4 + 4.22, which is easier to add to get 8.22.

Sample Tasks:
- 45 + 29
- $298 more than $465
- 6476 increased by 999
- The sum of 18 000 and 46 000
- The total of 78 200 km and 9900 km
- $56,000 increased by $18,000
- 1.9 m + 2.6 m
- 4.5 km more than 5.8 km
- $0.25 + $0.59
- $3.56 more than $2.99

MENTAL MATHEMATICS FOR SUBTRACTION

Some of the following material is a review from Mathematics 5, but it is necessary to include it here to consolidate the understanding of subtracting by tenths, hundredths, and thousandths.

Applying Subtraction Facts to Multiples of Powers of 10
This strategy applies to calculations involving the subtraction of two numbers with the same place values and with only one non-zero digit. In Mathematics 6, the application of this strategy should be extended to numbers with tenths, hundredths, and thousandths. The strategy involves subtracting the single non-zero digits as if they were the single-digit subtraction facts and then attaching the appropriate place-value name and symbols. This strategy should be reviewed and modelled with base-ten blocks. Since this strategy rests on students' knowledge of subtraction facts, the quick recall of which was expected in Mathematics 3, the facts should be reviewed and consolidated. The principal strategy advocated for these facts is the think-addition strategy, although the back-through-10 strategy and the up-through-10 strategy are also helpful when the minuends are greater than 10.

Examples:
- For 80 – 30, think, 8 tens subtract 3 tens is 5 tens, or 50; or think, 8 subtract 3 is 5, but this is 5 tens, so the answer is 50.
- For 1500 – 600, think, 15 hundreds subtract 6 hundreds is 9 hundreds, or 900; or think, 15 subtract 6 is 9, but this is 9 hundreds, so the answer is 900.
- For 6000 – 2000, think, 6 thousands subtract 2 thousands is 4 thousands, or 4000; or think, 6 subtract 2 is 4, but this is 4 thousands, so the answer is 4000.
- For 90 000 – 40 000, think, 9 subtract 4 is 5, but this is tens of thousands, so the answer is 50 000; or think, 90 thousand subtract 40 thousand is 50 thousand or 50 000.
- For 0.8 – 0.5, think, 8-tenths subtract 5-tenths is 3-tenths, or 0.3; or think, 8 subtract 5 is 3, but this is tenths, so the answer is 0.3.
- For 1.4 – 0.7, think, 14-tenths – 7-tenths is 7-tenths, or 0.7; or think, 14 subtract 7 is 7, but this is tenths, so the answer is 0.7.
- For 0.17 – 0.09, think, 17-hundredths subtract 9-hundredths is 8-hundredths, or 0.08; or think, 17 subtract 9 is 8, but this is hundredths, so the answer is 0.08.

Sample Tasks:
- 120 – 70
- $20 less than $90
- 700 kg decreased by 300 kg
- The difference between 1100 km and 400 km
- 6000 minus 1000

- $13,000 less $6000
- 40 000 – 10 000
- 80 000 minus 20 000
- The difference between $90,000 and $50,000
- 120 000 km decreased by 30 000 km
- 0.7 kg – 0.2 kg
- The difference between 1.5 km and 0.6 km
- 0.5 m less than 0.8 m
- 1.6 kg decreased by 0.9 kg
- 0.05 m less than 0.08 m
- 0.16 kg decreased by 0.09 kg

Quick Subtraction
This strategy is actually the front-end strategy applied to subtraction questions that involve no regrouping. If questions only require two subtractions to get an answer, students should be able to do them mentally. However, questions involving three or more subtractions should be presented visually with students quickly recording their answers on paper. While it could be argued that this is a pencil-and-paper strategy for these questions because answers will always be recorded on paper before answers are read, it is included here as a mental mathematics strategy because most students will do all the subtractions in their heads starting at the front end. This strategy requires students to holistically examine each question to confirm there will be no regrouping. This habit of holistically examining each question as a first step in determining the most efficient strategy needs to pervade all mental mathematics lessons. It is important to present examples of these subtraction questions in both horizontal and vertical formats. The numbers should include decimal examples as well as whole number examples.

Most likely, students will subtract the digits in corresponding place values of the minuend and subtrahend without consciously thinking about the names of the place values. Therefore, in the discussion of the questions, you should encourage students to read the numbers correctly and to use place-value names. This will reinforce place-value concepts at the same time as subtraction is reinforced.

Examples:
- For 560 – 120, think, 500 – 100 is 400 and 60 – 20 is 40, so the answer is 440. (Record the answer if required.)
- For 568 – 135, think and record each difference: Subtract 100 from 500, 30 from 60, and 5 from 8 to get 433; or think and record each resultant digit: 5 – 1 = 4, 6 – 3 = 3, 8 – 5 = 3, so the answer is 433 (four hundred thirty-three).
- For 4070 – 3030, think, 4000 – 3000 is 1000 and 70 – 30 is 40, so the answer is 1040. (Record answer if required.)
- For 4568 – 1135, think and record each difference: Subtract 1000 from 4000, 100 from 500, 30 from 60, and 5 from 8 to get 3433; or think and record each resultant digit: 4 – 1 = 3, 5 – 1 = 4, 6 – 3 = 3, 8 – 5 = 3, so the answer is 3433 (three thousand thirty-three).
- For 87 000 – 32 000, think, 80 000 – 30 000 is 50 000 and 7000 – 2000 is 5000 so the answer is 55 000. (Record if required.)
- For 25 786 – 12 125, think and record each subtraction: Subtract 10 000 from 20 000, 2000 from 5000, 100 from 700, 20 from 80, and 5 from 6 to get 13 661; or think and record each difference: 2 – 1 = 1, 5 – 2 = 3, 7 – 1 = 6, 8 – 2 = 6, and 6 – 5 = 1, so the answer is 13 661 (thirteen thousand six hundred sixty-six).

- For 345.84 – 112.42, think and record each subtraction: Subtract 100 from 300, 10 from 40, 2 from 5, 4-tenths from 8-tenths, and 2-hundredths from 4-hundredths to get 233.42; or think and record each digit: 3 – 1 = 2, 4 – 1 = 3, 5 – 2 = 3, 8 – 4 = 4, and 4 – 2 = 2, so the answer is 233.42 (two hundred thirty-three and forty-two hundredths).

Sample Tasks:
- 56 – 21
- 604 – 203
- 590 – 230
- 6700 – 1100
- 4080 – 1020
- 14 000 – 2000
- 38 000 – 1500
- $\begin{array}{r} 537 \\ \underline{-\ 101} \end{array}$
- 304 fewer people than 8605 people
- $3245 less than $7366
- The difference between 1225 km and 3575 km
- Subtract 575 from 3889
- 45 678 – 21 543
- $\begin{array}{r} 83\ 419 \\ \underline{-\ 21\ 417} \end{array}$
- The difference between $96,475 and $5,125
- 75 575 km decreased by 31 235 km
- 213.7 kg – 101.2 kg
- The difference between 456.9 km and 45.6 km
- 45.12 m less than 57.75 m
- $\begin{array}{r} 575.86 \\ \underline{-\ 125.36} \end{array}$

Back through a Multiple of a Power of Ten

This strategy involves subtracting a part of the subtrahend to get to the nearest multiple of a power of ten, and then subtracting the rest of the subtrahend. This strategy is most effective when the subtrahend is relatively small compared to the minuend. In previous grades, students would have been introduced to this strategy as back-through-10 and back-through-10s and 100s. In Mathematics 6, the strategy is extended to going back through larger powers of ten.

Examples:
- For 35 – 8, think, 35 subtract 5 (one part of the 8) is 30, and 30 subtract 3 (the other part of the 8) is 27.
- For 530 – 70, think, 530 subtract 30 (one part of the 70) is 500, and 500 subtract 40 (the other part of the 70) is 460.
- For example: For 8600 – 700, think, 8600 subtract 600 (one part of the 700) is 8000 and 8000 subtract 100 (the rest of the 700) is 7900.
- For 74 000 – 9000, think, 74 000 subtract 4000 (one part of the 9000) is 70 000, and 70 000 subtract 5000 (the rest of the 9000) is 65 000.
- For 4.5 – 0.9, think, 4.5 – 0.5 (one part of 0.9) is 4, and 4 subtract 0.4 (the other part of 0.9) is 3.6.
- For 1.63 – 0.07, think, 1.63 subtract 0.03 (one part of 0.07) is 1.6, and 1.6 subtract 0.04 (the other part of 0.07) is 1.56.

Sample Tasks:
- $\begin{array}{r} 57 \\ \underline{-\ 8} \end{array}$
- 9 fewer people than 92 people
- $40 less than $210
- The difference between 630 km and 80 km
- Subtract 600 from 2300
- 7500 less 700
- 45 000 – 8000
- 83 400 minus 600
- The difference between $42,000 and $7000
- 33 000 km decreased by 5000 km
- 13.2 kg – 0.7 kg
- The difference between 23.5 km and 0.8 km
- 0.06 m less than 1.21 m
- $2.53 – $0.07

Up-through a Multiple of a Power of Ten
This strategy involves determining the difference between the two numbers in two steps starting from the smaller. First, determine the difference between the subtrahend and the next multiple of a power of ten, then determine the difference between that multiple of a power of ten and the minuend, and finally add these two differences to get the total difference. This strategy is particularly effective when the two numbers involved are quite close together, although in making change in money situations, this is the principal strategy that traditionally has been used, regardless of the difference. For example, to get the change from a $20-bill for an item that costs $6.95, you select a nickel to get to $7, a $1 coin and a $2 coin to get to $10, and a $10-bill to get to $20. In Mathematics, this strategy should be applied to decimal numbers.

Examples:
- For 84 – 77, think, It is 3 from 77 to 80 and 4 from 80 to 84, so the total difference is 3 plus 4, or 7.
- For 613 – 594, think, It is 6 from 594 to 600 and 13 from 600 to 613, so the total difference is 6 plus 13, or 19.
- For 2310 – 1800, think, It is 200 from 1800 to 2000 and 310 from 2000 to 2310, so the total difference is 200 plus 310, or 510.
- For 57 000 – 49 000, think, It is 1000 from 49 000 to 50 000 and 7000 from 50 000 to 57 000, so the total difference is 1000 + 7000, or 8000.
- For 12.4 – 11.8, think, It is 2-tenths from 11.8 to 12 and 4-tenths from 12 to 12.4, so the total difference is 2-tenths plus 4-tenths, or 0.6.
- For 6.12 – 5.99, think, It is 1-hundredth from 5.99 to 6.00 and 12-hundredths from 6.00 to 6.12, so the total difference is 1-hundredth plus 12-hundredths, or 0.13.
- For 12.54 – 12.48, think, It is 2-hundredths from 12.48 to 12.5 and 4-hundredths from 12.5 to 12.54, so the total difference is 2-hundredths + 4-hundredths, or 0.06.

Sample Tasks:
- $\begin{array}{r} 57 \\ \underline{-48} \end{array}$
- 92 – 86
- $140 less than $210
- The difference between 630 km and 580 km
- 2400 minus 1700

- 8500 decreased by 7800
- 45 000 – 38 000
- 83 000 less 79 000
- The difference between $42,000 and $35,000
- 35 000 km subtract 26 000 km
- 13.2 kg – 12.7 kg
- The difference between 23.5 km and 22.8 km
- 1.99 m less than 2.21 m
- $2.53 – $2.45

Break-up and Bridge
This strategy involves starting with the minuend in its entirety and subtracting the values in the place values of the subtrahend, one-at-a-time, starting with the largest. If students were modelling subtraction on a number line, they would probably naturally use this strategy.

Examples:
- For 92 – 26, think, Start with 92 and subtract 20 (the tens place of 26) to get 72, and then subtract 6 (the ones place in 26) from 72 to get 66.
- For 745 – 207, think, Start with 745 and subtract 200 (the hundreds place in 207) to get 545, and then subtract 7 (the ones place in 207) from 545 to get 538.
- For 860 – 370, think, Start with 860 and subtract 300 (the hundreds place in 370) to get 560, and then subtract 70 (the tens place in 370) from 560 to get 490. (Likely a Back-through-100s Strategy in the last step.)
- For 8300 – 2400, think, Start with 8300 and subtract 2000 to get 6300, and then subtract 400 from 6300 to get 5900. (Likely a Back-through-1000s Strategy in the last step.)
- For 5750 – 680, think, Start with 5750 and subtract 600 to get 5150, and then subtract 80 from 5150 to get 5070. (Likely a Back-through-100 Strategy in the last step.)
- For 47 000 – 28 000, think, Start with 47 000 and subtract 20 000 to get 27 000, and then subtract 8000 from 27 000 to get 19 000. (Likely a Back-through-10 000s Strategy in the last step.)
- For 24 500 – 2700, think, Start with 24 500 and subtract 2000 to get 22 500, and then subtract 700 from 22 500 to get 21 800. (Likely a Back-through-100s Strategy in the last step.)

Sample Tasks:
- $\begin{array}{r} 74 \\ \underline{-\ 36} \end{array}$
- 53 – 25
- $306 less than $870
- The difference between 640 km and 170 km
- 750 minus 260
- 803 decreased by 306
- 5400 – 1500
- 7100 less 2600
- The difference between $8020 and $3050
- $\begin{array}{r} 6425 \\ \underline{-\ 307} \end{array}$
- 63 000 – 25 000
- The difference between 66 500 km and 18 000 km
- $75,500 – $4900
- 10 600 less than 32 100

Compensation

This strategy for subtraction involves changing the subtrahend to the next multiple of a power of ten, carrying out the subtraction, and then adjusting the answer to compensate for the difference between the original subtrahend and the multiple of a power of ten that was used. Students should understand that the subtrahend is changed to make it more compatible, and that they have to hold in their memories the amount of that change. In the last step, it is helpful if they remind themselves that they subtracted too much, so they will have to add that amount back on. This strategy is most effective when the digit in the lowest non-zero place value is an 8 or a 9. In Mathematics 6, this strategy should be extended to numbers involving decimal tenths, hundredths, and thousandths.

Examples:

- For 36 – 8, think, 36 – 10 = 26, but I subtracted 2 too many so I add 2 to 26 and get 28.
- For 85 – 29, think, 85 – 30 = 55, but I subtracted 1 too many so I add 1 to 55 to get 56.
- For 145 – 99, think, 145 – 100 = 45, but I subtracted 1 too many so I add 1 to 45 to get 46.
- For 750 – 190, think, 750 – 200 = 550, but I subtracted 10 too many so I add 10 to 550 to get 560.
- For 5700 – 997, think, 5700 – 1000 is 4700, but I subtracted 3 too many so I add 3 to 4700 to get 4703.
- For 3600 – 990, think, 3600 – 1000 is 2600, but I subtracted 10 too many so I add 10 to 2600 to get 2610.
- For 24 000 – 995, think, 24 000 – 1000 is 23 000, but I subtracted 5 too many so I add 5 to 23 000 to get 23 005.
- For 56 000 – 980, think, 56 000 – 1000 is 55 000, but I subtracted 20 too many so I add 20 to 55 000 to get 55 020.
- For 47 000 – 19 000, think, 47 000 – 20 000 is 27 000, but I subtracted 1000 too many so I add 1000 to 27 000 to get 28 000.

Sample Tasks:

- $\begin{array}{r} 57 \\ \underline{-\ 29} \end{array}$
- 92 less 38
- $399 less than $875
- The difference between 630 km and 298 km
- 450 minus 190
- 830 decreased by 380
- 5700 – 997
- 4500 less 1990
- The difference between $7500 and $2900
- 6500 km subtract 1980 km
- 23 000 – 1997
- The difference between 33 000 km and 2980 km
- $64,000 – $9900
- Subtract 29 000 from 92 000

Balancing for a Constant Difference

In subtraction questions that require regrouping, this strategy can be used most effectively. By adding the same amount to both numbers in order to get the subtrahend to a multiple of a power of ten any regrouping is eliminated, so the subtraction is much easier to do. This strategy needs to be carefully introduced because students need to be convinced it actually works! They need to understand that by adding the same amount to both numbers, the two new numbers have the same difference as the original two numbers. Examining possible numbers on a metre stick that are a fixed distance apart can help students with the logic of this strategy. (For example, place a highlighter that is more than 10 cm long against a metre stick so that its bottom end is at the 18-cm mark, note where its top end is located, and write the subtraction sentence that gives the length of the highlighter. Repeat by placing the bottom end of the highlighter at the 20-cm mark. Ask, Is the length of the highlighter the same in both number sentences? Which subtraction would be easier to do?)

Students have been introduced to this strategy in earlier grades. In Mathematics 6, they should extend this strategy to numbers involving decimal tenths, hundredths, and thousandths. (Note: Because both numbers change in carrying out this strategy, many students may need to record the changed minuend to keep track, especially for numbers greater than two-digit.) This strategy should be compared to the compensation strategy so students see how it is alike and how it is different.

This strategy can lead to a very effective pencil-and-paper strategy for questions in which the minuends are multiples of powers of ten. These questions traditionally required subtracting with regrouping from one, or more, zeros; however, if 1 is subtracted from both numbers, the questions will require no regrouping. For example, for 4000 – 3467, if 1 is subtracted from both the minuend and the subtrahend, the question becomes 3999 – 3466, which is then much easier to subtract by quick subtraction.

Examples:
- For 87 – 19, think, If 1 is added to both numbers, the question becomes 88 – 20, which is easy to subtract to get 68.
- For 345 – 198, think, If 2 is added to both numbers, the question becomes 347 – 200, which is easy to subtract to get 147.
- For 5600 – 1990, think, If 10 is added to both numbers, the question becomes 5610 – 2000, which is easy to subtract to get 3610.
- For 7800 – 3998, think, If 2 is added to both numbers, the questions becomes 7802 – 4000, which is easy to subtract to get 3802.
- For 45 000 – 19 000, think, If 1000 is added to both numbers, the question becomes 46 000 – 20 000, which is easy to subtract to get 26 000.
- For 67 000 – 29 999, think, If 1 is added to both numbers, the question becomes 67 001 – 30 000, which is easy to subtract to get 37 001.
- For 52 000 – 9800, think, If 200 is added to both numbers, the question becomes 52 200 – 10 000, which is easy to subtract to get 42 200.

Sample Tasks:
- $\begin{array}{r} 77 \\ \underline{-\ 39} \end{array}$
- 53 – 28
- $399 less than $875
- The difference between 640 km and 198 km
- 750 minus 290
- 830 decreased by 380
- 5400 – 997
- 7500 less 2990
- The difference between $8500 and $3900
- $\begin{array}{r} 6500 \\ \underline{-\ 1980} \end{array}$
- 43 000 – 2997
- The difference between 66 000 km and 4980 km

- $75,000 – $9900
- Subtract 38 000 from 92 000

MENTAL MATHEMATICS FOR MULTIPLICATION

Some of the following material is a review from Mathematics 4 and 5, but it is necessary to include it here to consolidate the understanding of multiplying by tenths, hundredths, and thousandths; the related division by tens, hundreds, and thousands; and the reverse of multiplying by tens, hundreds, and thousands.

Quick Multiplication (No Regrouping)
Note: This pencil-and-paper strategy is used when there is no regrouping and the questions are presented visually instead of orally. It is included here as a mental mathematics strategy because students will do all the combinations in their heads starting at the front end.

Examples:
- For 52 × 3, simply record, starting at the front end, 150 + 6 = 156.
- For 423 × 2, simply record, starting at the front end, 800 + 40 + 6 = 846.

Sample Tasks:
- 43 × 2 =
- 2 × 1.42 =
- The perimeter of a square with a side length of 4.2 cm
- 3 groups of 12.3

Multiplying by 10, 100, and 1000
This strategy involves keeping track of how the place values have changed.

Multiplying by 10 increases all of the place values of a number by one place. For 10 × 67, think, the 6 tens will increase to 6 hundreds and the 7 ones will increase to 7 tens; therefore, the answer is 670.

Multiplying by 100 increases all of the place values of a number by two places. For 100 × 86, think, the 8 tens will increase to 8 thousands and the 6 ones will increase to 6 hundreds; therefore, the answer is 8600. It is necessary that students use the correct language when orally answering questions where they multiply by 100. For example, the answer to 100 × 86 should be read as 86 hundred, which is equivalent to 8 thousand 6 hundred.

Multiplying by 1000 increases all the place values of a number by three places. For 1000 × 45, think, the 4 tens will increase to 40 thousands and the 5 ones will increase to 5 thousands; therefore, the answer is 45 000. This could also be thought of as 45 ones becoming 45 thousands instead of dealing with the digits separately. It is necessary that students use the correct language when orally answering questions where they multiply by 1000. For example, the answer to 1000 × 45 should be read as 45 thousand.

Sample Tasks:
- $73 × 1000 =
- 5 m = ___ cm
- 4.5 × 10 =
- 4 tenths times one hundred
- 2.3, 23, 230, ____, ____, ____

Compensation for Multiplication

This strategy for multiplication involves changing one of the factors to a ten, hundred, or thousand; carrying out the multiplication; and then adjusting the answer to compensate for the change that was made. This strategy could be carried out when one of the factors is near ten, one hundred, or one thousand.

Examples:

- For 6 × $4.98, think, 6 times 5 dollars less 6 × 2 cents; therefore, $30 subtract $0.12, which is $29.88.
- For 3.99 × 4, think, 4 × 4 is 16. Then I subtract 4 × 0.01 (0.04), which is 15.96.

Sample Tasks:

- How much will it cost for five CDs if each CD costs $19.98?
- $9.99 × 8 =
- Find the perimeter of a square with each side measuring 4.99 cm each.

Halving and Doubling

This strategy involves halving one factor and doubling the other factor in order to get two new factors that are easier to calculate. While the factors have changed, the product is equivalent, because multiplying by one-half and then by 2 is equivalent to multiplying by 1, which is the multiplicative identity. Halving and doubling is a situation where students may need to record some sub-steps.

Examples:

- For 42 × 50, think, one-half of 42 is 21 and 50 doubled is 100; therefore, 21 × 100 is 2100.
- For 500 × 88, think, double 500 to get 1000 and one-half of 88 is 44; therefore, 1000 × 44 is 44 000.
- For 6 × 2.5, think, one-half of 6 is 3 and double 2.5 is 5; therefore, 3 × 5 is 15.
- For 4.5 × 2, think, double 4.5 to get 9 and one-half of 2 is 1; therefore, 9 × 1 is 9.
- For 140 × 35, think, one-half of 140 is 70 and double 35 is 70; therefore 70 × 70 is 4900.

Sample Tasks:

- 86 × 50 =
- 8 × 2.5 =
- The product of 140 and 5
- Five-tenths of one hundred twenty
- The area of a rectangular garden with dimensions 8 m and 2.5 m
- How many hours in 5 days?

Front-end Multiplication or the Distributive Principle in 10s, 100s, and 1000s

This strategy involves determining the product of the single-digit factor and the digit in the highest place value of the second number, and adding to this product a second sub-product. This strategy is also known as the distributive principle.

Examples:

- For, 62 × 3, think, 3 times 6 tens is 18 tens, or 180, and 3 times 2 is 6, so 180 plus 6 is 186.
- For, 2 × 706, think, 2 times 7 hundreds is 14 hundreds, or 1400, and 2 times 6 is 12, so 1400 plus 12 is 1412.
- For, 5 × 6100, think, 5 times 6 thousands is 30 thousands, or 30 000, and 5 times 100 is 500, so 30 000 plus 500 is 30 500.

Sample Tasks:
- 62 × 4 =
- Four glasses of milk each with 250 mL.
- 4 × 2100 =
- A pair of factors of ____ are 6 and 3100.
- The area of a bathroom tile measuring 75 mm × 8 mm

Finding Compatible Factors
This strategy for multiplication involves looking for pairs of factors whose product is a power of ten and re-associating the factors to make the overall calculation easier. This is possible because of the associative property of multiplication. An example would be 2 × 78 × 500: think, 2 times 500 is 1000, and 1000 times 78 is 78 000.

Examples:
- For 25 × 63 × 4, think, 4 times 25 is 100, and 100 times 63 is 6300.
- For 2 × 78 × 500, think, 2 times 500 is 1000, and 1000 times 78 is 78 000.
- For 5 × 450 × 2, think, 2 times 5 is 10, and 10 times 450 is 4500.

Sometimes this strategy involves factoring one of the factors to get a compatible.
- For 16 × 25, think, 16 has 4 as a factor (4 × 4), so think 4 × (4 × 25) = 4 × 100 = 400.
- For 25 × 28, think, 28 has 4 as a factor (4 × 7), so think (25 × 4) × 7, so 4 × 25 is 100, and 100 times 7 is 700.
- For 68 × 500, think, 68 has 2 as a factor (2 × 34), so 500 times 2 is 1000, and 1000 times 34 is 34 000.

Sample Tasks:
- 4 × 38 × 25 =
- 250 × 16 =
- Find the product of 2, 12, and 50.
- One box contains 50 bags of peppermints. Each bag contains 81 peppermints. How many peppermints are in two boxes?

MENTAL MATHEMATICS FOR DIVISION

Quick Division (No Regrouping)
This pencil-and-paper strategy is used when there is no regrouping and the questions are presented visually instead of orally. It is included here as a mental mathematics strategy because students will do all of the combinations in their heads starting at the front end.

Examples:
- For 640 ÷ 2, simply record, starting at the front end, 300 + 20 = 320.
- For 1290 ÷ 3, simply record, starting at the front end, 400 + 30 = 430.

Sample Tasks:
- 360 ÷ 3 =
- How many groups of 8 are there in 7280?
- The length of a side of a square with a perimeter of 84 cm

Dividing by Ten, Hundred, and Thousand
This strategy involves keeping track of how the place values have changed. Division by a power of ten should be understood to result in a uniform “shrinking” of hundreds, tens, and units that could be

demonstrated and visualized with base-ten blocks. For example, 600 ÷ 20 can be represented by determining how many groups of 2 rods there are in 6 flats.

Dividing by 10 decreases all the place values of a number by one place. For 340 ÷ 10, think, the 3 hundreds will decrease to 3 tens, and the 4 tens will decrease to 4 ones; therefore, the answer is 34.

Dividing by 100 decreases all the place values of a number by two places. For, 7500 ÷ 100, think, the 7 thousands will decrease to 7 tens and the 5 hundreds will decrease to 5 ones; therefore, the answer is 75.

Dividing by 1000 decreases all the place values of a number by three places. For, 63 000 ÷ 1000, think, the 6 ten thousands will decrease to 6 tens and the 3 thousands will decrease to 3 ones; therefore, the answer is 63.

This strategy can also be applied to a task in which the divisor is a multiple of 10 and the dividend is a multiple of the divisor, such as 480 ÷ 60. Since 48 is a multiple of 6, 480 must be a multiple of 60, in fact 480 = 8 × 60 so 480 ÷ 60 = 8.

Sample Tasks:
- 80 ÷ 10 =
- 72 000, 7200, 720, ____, ____, ____
- Twenty-two thousand divided by one hundred
- How many metres are there in forty-six thousand kilometres?

Division Using the Think-Multiplication Strategy
This is a convenient strategy to use when dividing mentally. For example, when dividing 60 by 12, think, What is multiplied by 12 to get 60? This could be used in combination with other strategies. For 920 ÷ 40, think, 20 groups of 40 would be 800, leaving 120, which is 3 more groups of 40 for a total of 23 groups.

Sample Tasks:
- 880 ÷ 40 =
- How many groups of 70 are there in 1470?
- Find the length of a rectangle with area of 240 cm2 and a side width of 40 cm.

Using Division Facts for Tens, Hundreds, and Thousands
This strategy applies to dividends of tens, hundreds, and thousands divided by a single-digit divisor. There would be only one non-zero digit in the quotient. For 60 ÷ 3, think, 6 ÷ 3 is 2 and, therefore, 60 ÷ 3 is 20. For 800 ÷ 4, think 8 ÷ 4 is 2 and therefore, 800 ÷ 4 is 200. For 1000 ÷ 5, think 10 ÷ 5 is 2 and therefore 1000 ÷ 5 = 200.

Sample Tasks:
- 90 ÷ 3 =
- 35 000 ÷ 5 =
- Divide five hundred sixty by eight
- Find the side length of an equilateral triangle with a perimeter of 33 000 mm.

Partitioning the Dividend

This strategy involves partitioning the dividend into two parts, both of which are easily divided by the given divisor. Students should look for a ten, hundred, or thousand that is an easy multiple of the divisor and that is close to, but less than, the given dividend. An example might be 156 ÷ 4. A student can think of 156 as 120 + 36 so 120 ÷ 4 = 30 and 36 ÷ 4 = 9 so 156 ÷ 4 = 39.

Examples:

- For 372 ÷ 6, think, (360 + 12) ÷ 6, so 60 + 2 is 62.
- For 3150 ÷ 5, think, (3000 + 150) ÷ 5, so 600 + 30 is 630.

Sample Tasks:

- 248 ÷ 4 =
- 8280 divided by 9
- The quotient of 344 divided by 8
- One-fifth of a year in days

Compensation for Division

This strategy for division involves increasing the dividend to an easy multiple of ten, hundred, or thousand to get the quotient for that dividend, and then adjusting the quotient to compensate for the increase. 156 ÷ 4 could be solved mentally using this strategy. Students might think 156 is almost 160, 160 is 40 sets of 4, but we need one set less, so the quotient must be 39;
or (160 ÷ 4) – (4 ÷ 4) = 40 – 1 = 39.

Example:

- For 348 ÷ 6, think, 348 is about 360 and 360 ÷ 6 is 60 but that is 12 too much, so each of the 6 groups will need to be reduced by 2, so the quotient is 58.

Sample Tasks:

- 304 ÷ 8 =
- $1393 shared between seven people will give each person $____.
- Three times ____ is 264.

N02.02 and **N02.05** Students should create and solve a variety of story problems involving the four operations for whole numbers, and addition and subtraction of decimal numbers. Students have had experience with a variety of story-problem structures working with whole numbers in previous grades. When they learn how to operate with decimals, they should also be exposed to a variety of story-problem structures for addition and subtraction so that they get a full picture of the various contexts in which decimals are used.

Examples of these various types of problems appear in the tables provided below.

<table>
<tr><th colspan="5">Addition and Subtraction Story Structures</th></tr>
<tr><th colspan="3">Join</th><th>Part-Part-Whole</th><th>Compare</th></tr>
<tr><th>Result Unknown</th><th>Change Unknown</th><th>Start Unknown</th><th>Whole Unknown</th><th>Difference Unknown</th></tr>
<tr><td>Mike earned $72.48 last month selling newspapers. This month he earned $81.15. How much money did he earn in all?

$72.48 + $81.15 = ?</td><td>Last week 2115 kg of blueberries were picked in Oxford. Some more blueberries were picked this week, giving a total of 4236 kg of blueberries picked. How many kilograms of blueberries were picked this week?

2115 + ? = 4236
or
4236 – 2115 = ?</td><td>The grade 4 class is fund-raising for a community centre. A donor just gave them $563, and now they have $4998. How much money did they have before the donation?

? + 563 = 4998
or
4998 – 563 = ?</td><td>There are 317 boys and 248 girls in a school. How many students are in the school?

317+ 248 = ?</td><td>Mary bought 12.78 metres of cloth for her sewing projects. Chantella bought 8.85 metres of cloth for her sewing projects. How many more metres of cloth did Mary buy than Chantella bought?

8.85 + ? = 12.78
or
12.78 – 8.85 = ?</td></tr>
<tr><th>Result Unknown</th><th>Change Unknown</th><th>Start Unknown</th><th>Part Unknown</th><th>Smaller or Larger Unknown</th></tr>
<tr><td>Gavin collected 239 seashells in his bucket. He gave his brother 103 of those seashells. How many seashells does he have left?

239 – 103 = ?</td><td>Kayla had 1.56 kg of sugar. She used some sugar to make cookies and has 0.83 kg of sugar left. How much sugar did she use?

1.56 – ? = 0.83
or
15.6 – 0.83 = ?
or
0.83 + ? = 15.6</td><td>A company wants to donate books to schools. They gave the first school 2356 books. They still have 3517 books to donate. How many books did they have to begin with?

? – 2356 = 3517
or
2356 + 3517= ?</td><td>There were 4735 people at a concert. If 1352 of them were children, how many were adults?

1352 + ? = 4735
or
4735 – 1352 = ?</td><td>Our school collected 4387 bottles for the recycling project. Another school collected 2185 more bottles than our school. How many bottles did the other school collect?

4387 + 2185 = ?</td></tr>
</table>

<table>
<tr><th colspan="3">Multiplication and Division Story Structures</th></tr>
<tr><th>Equal Groups</th><th>Comparison</th><th>Combinations</th></tr>
<tr>
<td>Result Unknown
(Given the number of groups and the size of the group, find the result.)

A bag holds 18 carrots. If you have 15 bags of carrots, how many carrots do you have?

15 × 18 = ?

There are 25 rows of chairs in the library. Each row has 19 chairs in it. How many chairs are in the library?

25 × 19 = ?

A grasshopper jumps 9 cm in a single jump. If the grasshopper jumps 36 times, what distance will it have travelled?

9 × 36 = ?</td>
<td>Result Unknown
(Given the initial amount and the multiplier, find the result.)

Kylie ate 5 apples last week. Her brother ate twice as many apples. How many apples did her brother eat last week?

5 × 2 = ?</td>
<td>Result Unknown
(Given the size of the two sets, find the result.)

Khaled has 3 pairs of pants and 5 shirts. How many different outfits can he make?

3 × 5 = ?</td>
</tr>
<tr>
<td>Size of a Group Unknown
(Given the result and the number of equal groups, find the size of the group.) (partition division)

You have 112 chairs. You need to put them in 8 rows. How many chairs will be in each row?

112 ÷ 8 = ?
or
8 × ? = 112</td>
<td>Multiplier Unknown
(Given the result and the initial amount, find the multiplier.)

A frog jumped 2 metres. A kangaroo jumped 12 metres. How many times farther did the kangaroo jump?

12 ÷ 2 = ?
or
2 × ? = 12</td>
<td rowspan="2">One Set Unknown
(Given the result and one of the sets, find the other set.)

Chika likes to eat yogurt with berries for recess. Chika has 5 different kinds of berries that she adds to her yogurt. If she can make 15 different yogurt with berries snacks, how many different kinds of yogurt does she use to make her snacks?

15 ÷ 5 = ?
or
5 × ? = 15</td>
</tr>
<tr>
<td>Number of Equal Groups Unknown
(Given the result and the size of the set, find the number of groups.) (measurement division)

You have 27 photographs. You want to put 3 photographs on each page of your photo album. How many pages will you fill?

27 ÷ 3 = ?
or
3 × ? = 27</td>
<td>Initial Unknown
(Given the result and the multiplier, find the initial amount.)

Thea collected 45 cans for recycling. That was 5 times as many as cans as Beth collected. How many cans did Beth collect for recycling?

45 ÷ 5 = ?
or
5 × ? = 45</td>
</tr>
</table>

Students should have many opportunities to solve and create word problems for the purpose of answering real-life questions, preferably choosing topics of interest to them. These opportunities provide students with a chance to practise their computational skills and to clarify their mathematical thinking.

Students are expected to solve multi-step story problems involving combinations of the four operations as well as creating their own problems. Requiring students to create their own problems provides opportunities for them to explore the operations in depth. It is a complex skill requiring solid conceptual understandings and must be part of the student's problem-solving experiences. It is important that, among the problems presented or created by students, some lend themselves to mental computation, others require paper-and-pencil computation, and still others call for the use of calculators. Calculators are useful for decimal computations involving complicated numbers, multiple calculations, and problem solving.

N02.03 and **N02.04** The ability to estimate computations is a major goal of any modern computational program. For most people in their everyday lives, an estimate is all that is needed to make decisions, and to be alert to the reasonableness of numerical claims and answers generated by others and with technology. The ability to estimate rests on a strong and flexible command of facts and mental calculation strategies.

It is essential that estimation strategies are used by students before attempting pencil-and-paper or calculator computation to help them determine reasonable answers. Teachers should also model this process of estimating before personally doing any calculations in front of the class, and should constantly remind students to estimate before calculating. Students are expected to check the reasonableness of their answers by estimating and employing mental mathematics strategies. When a student asks whether or not their answer to a question is correct, ask the student if the answer makes sense to them, having them justify their response. Having students determine the reasonableness of an answer and explaining their thinking is a powerful way to assess understanding and learning.

While teaching estimation strategies, it is important to use the language of estimation. Some of the common words and phrases are *about*, *just about*, *between*, *a little more than*, *a little less than*, *close*, *close to*, and *near*.

In Mathematics 6, students continue to apply these previously learned estimation strategies to large whole numbers and numbers involving decimal tenths, hundredths, and thousandths.

COMPUTATIONAL ESTIMATION STRATEGIES

Rounding

Students have used rounding to estimate sums, differences, products, and quotients in previous grades. In Mathematics 6, students will continue to use this strategy for all four operations.

Students should consider rounding the smaller factor up and the larger factor down to give a more accurate estimate. For example, estimating the product of 653 × 45 with a conventional rounding rule would be 700 × 50 = 35 000, which would not be close to the actual product of 29 385. Using the rounding strategy above, the 45 would round to 50 and the 653 would round to 600, giving an estimate of 30 000, much closer to the actual product. (When both numbers would normally round up, the above rule does not hold true.)

To round 763 × 36, round 763 (the larger number) down to 700 and round 36 (the smaller number) up to 40, which equals 700 × 40 = 28 000. This produces a closer estimate than rounding to 800 × 40 = 32 000, when the actual product is 27 468.

An example of rounding division questions with a two-digit divisor and a three-digit dividend is in rounding 789 ÷ 89, round 89 to 90 and think, 90 multiplied by what number would give an answer close to 800 (789 rounded)? Since 9 × 9 = 81, therefore 800 ÷ 90 is about 9.

An example of rounding a division question with a two-digit divisor is as follows:

- For 7843 ÷ 30, think of it as 750 tens ÷ 3 tens to get 250 tens.

Sample Tasks:

- 384 × 68 =
- 7011 × 39 =
- The product of 708 and 49
- Find the total cost for 31 students to pay for a year of university when tuition is $6950 each.
- 87 × 371 =
- 48 rows of 562
- About how many hours in a year?
- 411 360 ÷ 71 =
- 810.3 ÷ 89 =
- The quotient of two hundred thirty-three divided by twenty-nine
- 2689 ÷ 90 =
- Forty divided into 3989
- About how many sixties are there in 3494?

Front-end Addition, Subtraction, and Multiplication

This strategy involves combining only the values in the highest place value to get a “ball park” figure. Such estimates are adequate in many circumstances. Although estimating to tenths and hundredths is included here, it is most important to estimate to the nearest whole number.

Examples:

- To estimate 0. 093 + 4.236, think, 0.1 + 4.2 = 4.3 (to the nearest tenth).
- To estimate 0.491 + 0.321, think, 0.4 + 0.3 = 0.7 (to nearest tenth).
- To estimate 3.871 + 0.124, think, 3 + 0 = 3 (to nearest whole number).
- To estimate 5.711 – 3.421, think, 5.7 – 3.4 = 2.3 (to nearest tenth).
- To estimate 3.871 – 0.901, think, 4 – 1 = 3 (to nearest whole number).
- To estimate 3 125 × 6, think, 3 000 × 6 is 6 groups of 18 thousands, or 18 000.
- To estimate 42 175 × 4, think, 40 000 × 4 is 4 groups of 4 ten thousands or 160 000.
- To estimate 3 × 4.952, think, 3 × 5 or 15.
- To estimate 63.141 × 8, think, 60 × 8 or 480.

Sample Tasks:

- 0.701 + 0.001 =
- 10.673 + 20.241 =
- 0.615 increased by 0.013
- 0.512 – 0.111 =
- The difference of 15.3 and 10.1
- 0.09 less than 0.81
- 15.3 g – 10.1 g =

- 7200 × 3 =
- Find the total of three tanks with 8112 litres of oil in each tank.
- The product of 6 and 41 296
- 5 times 3.171 is about ____.
- Estimate the mass of nine containers of hockey pucks with a mass 7.921 kg each.
- Estimate 202.273 × 8.

Front-end Division

This strategy involves rounding the dividend to a number related to a factor of the divisor and then determining in which place value the first digit of the quotient belongs, to get a “ball park” answer. Such estimates are adequate in many circumstances.

Examples:

- For 425 ÷ 8, round the 425 to 400, because we know that 5 × 8 = 40, we know that the first digit in the quotient is a 5 and it is in the tens place, therefore, the quotient is 50.
- For 799 ÷ 9, round 799 to 810, because we know that 9 × 9 = 81, we know that the first digit in the quotient is a 9 and it is in the tens place, therefore, the quotient is 90.

Sample Tasks:

- 191 ÷ 3
- $276.50 shared equally with nine people
- 389 ÷ 3
- 479 ÷ 4

Adjusted Front End or Front End with Clustering

Here are some practice examples for estimating multiplication of two-digit factors by double- and triple-digit factors. Here students may use paper and pencil to record part of the answer.

Example:

- To estimate 93 × 41, think, 90 × 40 is 40 groups of 9 tens, or 3600; and 3 × 40 is 40 groups of 3, or 120; 3600 plus 120 is 3720.

Sample Tasks:

- 86 × 39 =
- 75 × 26

Doubling for Division

Doubling for division involves rounding and doubling both the dividend and divisor. This does not change the solution but can produce “friendlier” divisors.

Examples:

- 2223 ÷ 5 can be thought of as 4446 ÷ 10, or about 445.
- 1333.97 ÷ 5 can be thought of as 2668 ÷ 10, or about 266.

Sample Tasks:

- 243 ÷ 5 =
- $3212.11 shared among 5 people
- Find the length of one side of a regular pentagon with a perimeter of 235 cm.

N02.06 Communicating mathematical understanding and thinking is important in learning new skills and concepts in mathematics. When students can explain how they know an answer is reasonable, it means students understand the process and the problem in a way that makes sense to them. As students continue to solve problems, it is necessary to work on their communication skills—showing how they know. Using pictures, numbers, and words can help show their understanding and can improve their communication skills.

Consider the following situation:

The Toronto Maple Leafs have 42 regular-season games and sell out all their games each season. Their stadium has 18 800 seats.

If we ask students to calculate how many tickets would be sold in a year, they are simply computing two given numbers. A richer way to ask students to engage in problem solving using large numbers would be to ask the following:

Would the Toronto Maple Leafs sell 1 000 000 tickets in a year? If not, how many years would it take to sell 1 000 000 tickets?

Therefore, it is necessary to engage students in appropriate questions and tasks that allow such a process. Open-ended tasks can allow students an opportunity to effectively communicate their reasoning in multiple ways. These representations strengthen the connections students make among the various strands in the mathematics curriculum.

Working with large numbers may cause some difficulty with computation without the use of technology. It is suggested that students use calculators to work with these large numbers for computational purposes when the focus is not on mental mathematics. Take the opportunity to observe students as they use calculators when computing large numbers. Assess students' understanding as they communicate the reasonableness of the answer they found on the calculator.

SCO N03 Students will be expected to demonstrate an understanding of factors and multiples by ▪ determining multiples and factors of numbers less than 100 ▪ identifying prime and composite numbers ▪ solving problems using multiples and factors [PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

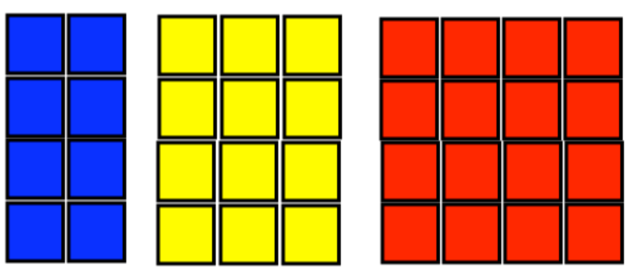
Performance Indicators

N03.01 Identify multiples for a given number and explain the strategy used to identify them.
N03.02 Determine all the whole number factors of a given number using arrays.
N03.03 Identify the factors for a given number and explain the strategy used (e.g., concrete or visual representations, repeated division by prime numbers, or factor trees).
N03.04 Provide an example of a prime number, and explain why it is a prime number.
N03.05 Provide an example of a composite number, and explain why it is a composite number.
N03.06 Sort a given set of numbers as prime and composite.
N03.07 Solve a given problem involving factors or multiples.
N03.08 Explain why 0 and 1 are neither prime nor composite.

Performance Indicator Background

N03.01 Students in Mathematics 6 should be comfortable explaining, in a variety of ways, how to find the factors and multiples for a given number. Students should build models to show both factors and multiples. Students can use a variety of models to explain the meaning of factors and multiples.

Students should build multiples through modelling. For example, the model below shows that 8, 12, and 16 are all multiples of 4 because they can all be organized into four rows.



Explore with students that when we multiply two factors, the product is a multiple of those two factors. For example, since 2 × 5 = 10, 2 and 5 are factors of 10, whereas 10 is a multiple of 2 and 5. Students sometimes do not recognize 0 as a multiple of any number. Small (2008) states, “There are two ways to approach this. One is to observe that, for example, 0 = 0 x 3, so 0 is a multiple of 3. The other is to use patterns. The multiples of 4 are 4 apart, so going down from 4, you get to 0: 24, 20, 16, 12, 8, 4, 0.” (p.155)

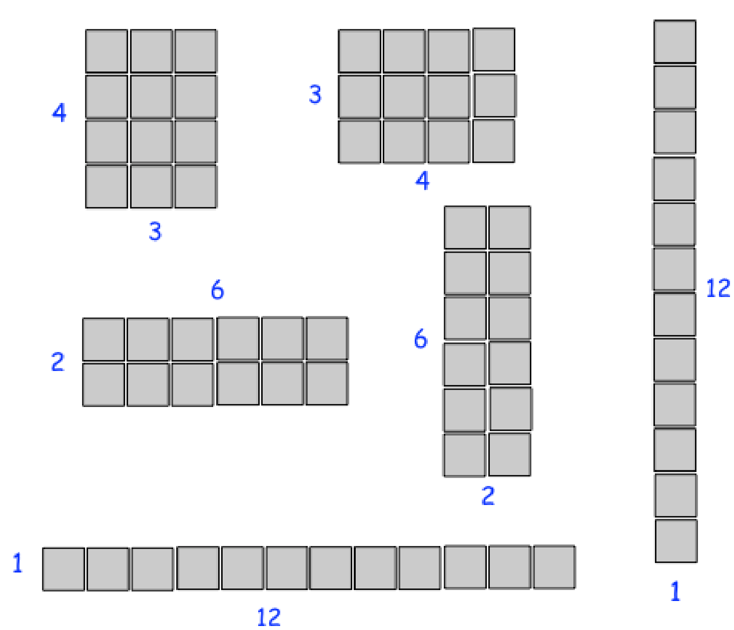
Multiples of a number can be identified using a hundred chart or a number line. Students can start at 0 and then skip count the specified number. For example, when asked to find the multiples of 8, students can start at 0, and shade in every 8th number. The shaded numbers are the multiples of 8. To emphasize 0 as a multiple of every number, it is suggested that a hundred chart that includes 0 be used. Students can also use a number line to skip count by the specified number. Students can also use an organized chart to identify multiples of a number. Here, they can use multiplication of the number by various factors to determine multiples of the number.

Students may use various manipulatives such as linking cubes, counters, buttons, etc., to create equal groups of the specified number to find its multiples. The total amount of items would be the multiple of that number. For example, if students were asked to find the multiples of 5, they would create one group of five and list 5 as a multiple, two groups of 5 would be 10 and 10 can be listed as a multiple of 5. Five groups of 5 would be 25 and 25 would be a multiple. Remind students that 0 is a multiple of every number, and it should be included as a multiple here.

N03.02 and **N03.03** Factors are numbers that when multiplied produce a product. For example, the factors of 12 are 1 and 12, 2 and 6, and 3 and 4. A review of basic multiplication facts may be necessary for students experiencing difficulty. Students may begin working with factors of numbers with which they are comfortable.

When students are introduced to factors, it is a good idea to use various manipulatives such as square tiles or linking cubes to form arrays or area models to identify the factors of a given number. Invite students to use a given number of squares and find the different area models they can make with that amount. The dimensions of the rectangular area models are the factors of the number. For example, to find the factors of 12, give each student 12 tiles or linking cubes and ask them to form a rectangle(s) or an array using only these 12 tiles. Ask students to find other ways to arrange the 12 tiles to make complete rectangles. The diagram below shows the factors of 12.

They could build models like the following to show the factors of 12.



Students should conclude that they can make rectangles with dimensions 1 × 12 (or 12 × 1), 2 × 6 (or 6 × 2), 3 × 4 (or 4 × 3). These numbers would represent the factors of 12. Students sometimes forget to list 1 and the number itself, as factors of a given number. Remind students they are to find all whole number factors.

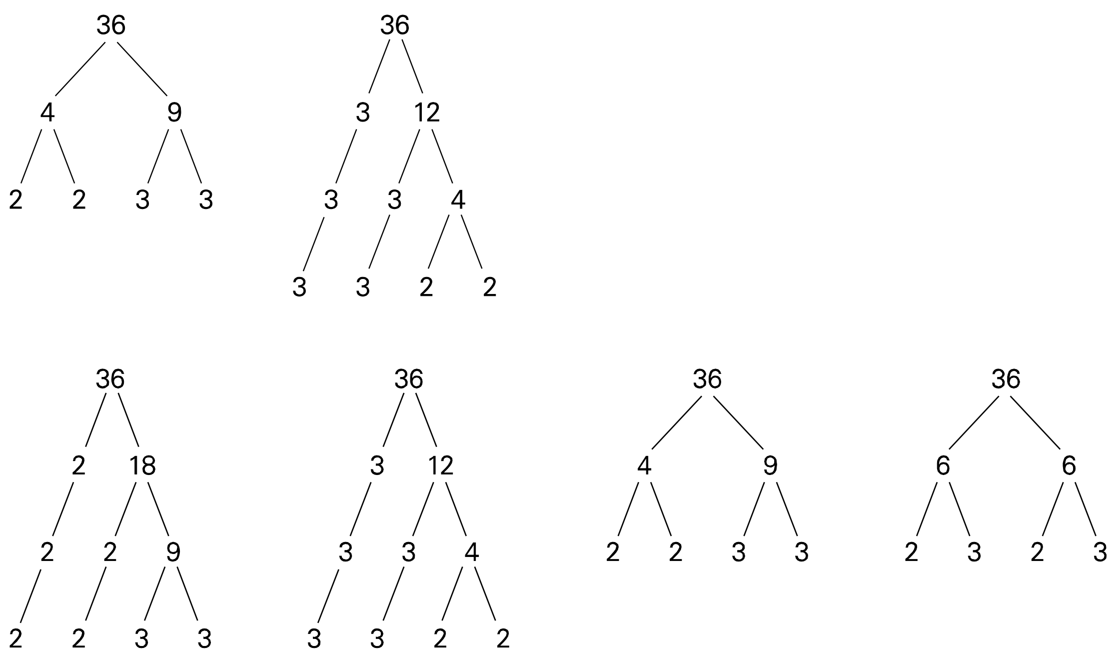
Students examine factors of numbers to begin to develop strategies for determining if a number is a factor of a given number. For example, if the number is even, then 2 is a factor. If the number ends in 0, then 10 is a factor. In Mathematics 7, students will develop divisibility rules and some of this reasoning about factors will lay the foundation for that future work. Students are expected to justify the conjectures they make about factors through referencing models.

When studying factors, students can organize factors to look for patterns. Students can create a table or a list. Students can try to find numbers that have a large list of factors and others that have a short list of factors.

In Mathematics 6, students should understand they can use division to find the factors of a given number. For example, when asked to find all the factors of 12, students can look for different numbers that can divide evenly into 12. These will be the factors of 12. 12 ÷ 1 = 12, so 1 and 12 are factors.
12 ÷ 2 = 6, so 2 and 6 are factors and 12 ÷ 3 = 4; so 3 and 4 are factors.

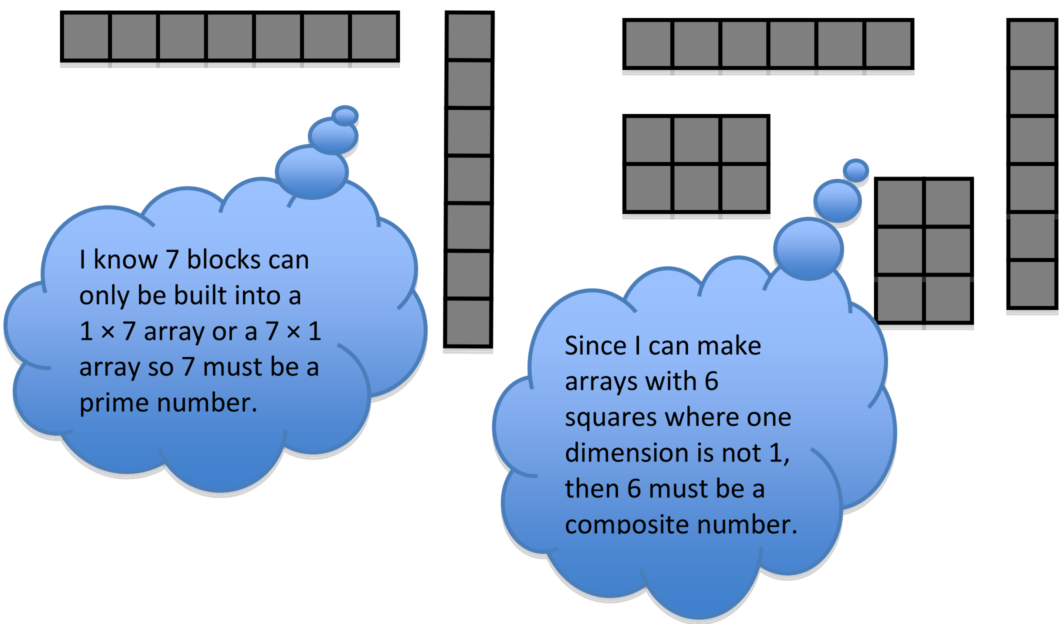
Review with students that they can find the factors of numbers in different ways, such as forming arrays or repeated division.

Another way to find the factors of a number is to use a factor tree (factorization). Ask students to work in groups to factor the number 36. Give students time to employ different strategies to find the prime factors. Have each group present their strategies to the class, looking at similarities and differences in each group's approach and their starting point to factor the number. It is important that students discover that regardless of where they start the factorization process, they will always end with the same prime factors.



N03.04, **N03.05**, **N03.06**, and **N03.08** The work that students have completed on factors will help them as they work on prime and composite numbers. Students see how factors help determine whether a number is prime or composite. Arrays and rectangular arrangements provide students with a visual and concrete representation of how a number can be broken down. Numbers that can only be arranged in one array are prime numbers. Numbers that can be arranged in more than one array are composite. For example, give students several numbers to explore, such as 3, 6, 9, 13, and 16. Ask them to find the factors of the numbers. Ask students if they notice any similarities or differences with the factors of the numbers. Discuss with students the fact that some numbers have only two factors; these are prime numbers. Other numbers have more than two factors; these are composite numbers.

Students should be able to explain the difference between prime and composite numbers using multiple strategies. One strategy they may use is to build rectangular arrays with squares. For a given number, if only a $1 \times n$ or $n \times 1$ array can be built, then the number is prime. If arrays with two dimensions that are each not one can be built, then the number is composite. This provides an opportunity for students to connect their understanding of the dimensions of a rectangle with a given area to the factors of that number.



Another meaningful experience for students in determining prime and composite numbers is to have them use the Sieve of Eratosthenes to determine the prime numbers under 100. To do this, students should be given a hundred chart and asked to use coloured pencils to circle the multiples of numbers. Begin by asking students to cross out the 1. Then skip 2, but circle all the remaining multiples of 2 in one colour. Then skip 3, but circle all the remaining multiples of 3 in a different colour, and so on. Many online applets are available to demonstrate this task. In the end, the composite numbers will be circled and primes will remain uncircled. Students should explain why the uncircled numbers are prime.

~~1~~	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

N03.07 Encourage students to develop their own strategies in determining multiples of numbers. They can use these strategies to help solve various problems involving multiples. For example, ask students to solve problems involving multiples, such as:

- Determine how many packages of wieners (12 wieners in a package) and how many packages of hot dog buns (8 buns in a package) would be needed to fill an order of 72 hot dogs.
- Spiders have 8 legs and ants have 6 legs. There is a container on the table that contains both spiders and ants. The number of spider legs equal the number of ant legs. What are some possible numbers of spiders and ants that would produce this result?

Students continue to work with prime and composite numbers identifying whether a number is prime or composite, factoring a number into its prime factors, and continuing to develop personal strategies to solve problems involving factors. Students need opportunities to apply these skills and ideas in problem solving situations where they are thinking about and reasoning through their work. Students, through working with factoring numbers, are now asked to demonstrate an understanding that composite numbers can be created by multiplying prime numbers together.

After completing the Sieve of Eratosthenes task described on the previous page, ask students to explore the result of multiplying two prime numbers. Students should understand that when they multiply any two prime numbers, the resulting product will always be a composite number. Ask students to explain why this happens. If students work with smaller prime numbers as factors, they will be able to identify on the hundred chart that the resulting product is a composite number.

Posing real-life problems that involve factors will allow students opportunities to see how finding factors of numbers are used outside of the mathematics classroom. These real-world connections help put a value on student work and understanding. Helping students understand the concept of factors can be done through problem solving where a problem can be posed, and once it is solved, the term factor can be introduced.

Consider the following example. Farmer Joe is trying to figure out how to plant a new potato garden. He has a plot of land that can cover a maximum of 100 m^2 and a minimum of 10 m^2. He needs to decide how big to have his garden, but wants some choice on the dimensions of the area. He needs to know which area would give him the most choices for the dimensions, but still have enough room to plant his potatoes. What could you suggest?

Here, students are asked to find areas for Farmer Joe's potato garden that would yield different dimensions, or factors of that area. They should see that numbers like 13, 37, and 59 only have one way to plant the area. Other areas, such as 36, 48, and 54 have many. They should also see that the larger the area, the more potatoes that can be planted.

N03.08 It may be necessary to point out that the number 1 is neither prime nor composite, since it does not fit the definition of a prime or a composite number. The number 1 only has one factor. To be prime, a number must have only two factors—1 and itself. The number 1 cannot be composite because it does not have more than two factors.

0 is another special number. 0 cannot be a prime number because every number is a factor of 0. 0 × 1 does equal 0, but 0 multiplied by anything equals 0. 0 is not a composite number because it cannot be written as a product of two factors, neither of which is itself.

SCO N04 Students will be expected to relate improper fractions to mixed numbers and mixed numbers to improper fractions. [CN, ME, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

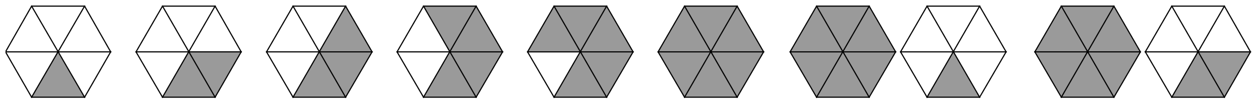
Performance Indicators

N04.01 Demonstrate, using models, that a given improper fraction represents a number greater than 1.
N04.02 Express improper fractions as mixed numbers.
N04.03 Express mixed numbers as improper fractions.
N04.04 Place a given set of fractions, including mixed numbers and improper fractions, on a number line, and explain strategies used to determine position.
N04.05 Represent a given improper fraction using concrete, pictorial, and symbolic forms.
N04.06 Represent a given mixed number using concrete, pictorial, and symbolic forms.

Performance Indicator Background

N04.01 It is important that students have a strong conceptual understanding of improper fractions. Students need to be able to understand and explain that an improper fraction represents more than one whole and that its numerator is greater than its denominator. To create this conceptual understanding, students should model improper fractions using a variety of materials such as fraction circles, fraction squares or rectangles, pattern blocks, geo-boards, number lines, and grid paper. Models help students clarify ideas that are often confused when presented in a purely symbolic mode. Using this approach and encouraging the use of models and pictures, students will begin to develop an understanding of the meaning of improper fractions. For example, ask students to use any manipulative to model $\frac{1}{3}$. Then, ask them to represent $\frac{2}{3}$, and then $\frac{3}{3}$. Ask them to explain why $\frac{3}{3}$ is the same as 1 whole. Next, ask students to determine what fraction they would have if they were given another $\frac{1}{3}$. Lead the discussion to help students see that $\frac{4}{3}$ is greater than 1. Invite students to explain some of their personal strategies that help them to understand that $\frac{4}{3}$ is greater than 1.

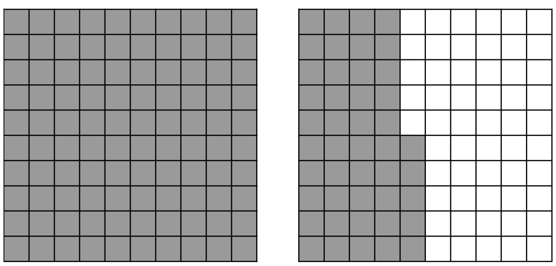
Students will have an understanding of fractional parts, or equal shares that they refer to as thirds, fourths, fifths, tenths, etc. They will recognize that these fractional parts can be counted in the same way as any other set of objects. Fractions greater than one whole can be understood this way. By counting fractional parts, we can help students develop a completely generalized system for naming fractions before they learn about fraction symbolism. For example, if counting sixths, students will say one-sixth, two-sixths, three-sixths, four-sixths, five-sixths, six-sixths or one whole, seven-sixths, eight-sixths. This can be modelled with pattern blocks as illustrated on the following page.



one-sixth	two-sixths	three-sixths	four-sixths	five-sixths	six-sixths	seven-sixths	eight-sixths
$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$	$\frac{6}{6}$	$\frac{7}{6}$	$\frac{8}{6}$

Alternatively, show students manipulatives representing six-fourths. Ask students to tell how many fourths. Ask if the collection is more or less than one whole, or more or less than two wholes. Ask students to explain their thinking. While doing this, prompt students to make informal comparisons. For example, ask them to explore the reasons they would get more than one whole out of six-fourths when they would not get one whole out of four-sixths.

To help students understand that an improper fraction represents a number greater than one, use a hundredth grid (a grid with 100 squares). Presenting the idea that 100 squares in the grid equals one whole (grid), students can explore ways they can represent more than one whole. In the example below, students are asked to shade in 145 blocks and to use their picture to name the improper fraction. Using this type of task will also help students strengthen their understanding of the purpose of the denominator and numerator.



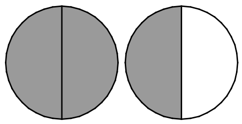
$\frac{100}{100}$ $\frac{45}{100}$

Through exploration of models, students will recognize that some models show fractional parts that make more than one whole, such as $\frac{5}{4}$, $\frac{3}{2}$, and $\frac{6}{5}$.

N04.02 and **N04.03** Students have been modelling, creating, describing, drawing, and naming improper fractions and mixed numbers. Students need to see the connection between improper fractions and mixed numbers as both represent numbers greater than one whole. They also need to recognize that every improper fraction can be expressed as an equivalent mixed number and every mixed number can be expressed as an equivalent improper fraction. To determine equivalent improper fractions and mixed numbers, students must understand that they are different representations of equal value. To develop a conceptual understanding of equivalency, it is important that models be used to generate the different representations of a fraction. Students must understand why a fraction can have another name (e.g., $\frac{7}{6} = \frac{14}{12}$) and yet have the same value. Students should be able to visualize equivalent improper fractions and mixed numbers as the naming of the same region partitioned in different ways as shown on the following page.

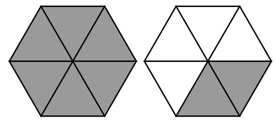
Some manipulative materials that illustrate equivalent improper fractions and mixed numbers include fraction circles, pattern blocks, geo-boards or geo-paper, fraction factory, and egg cartons.

Fraction circles or squares



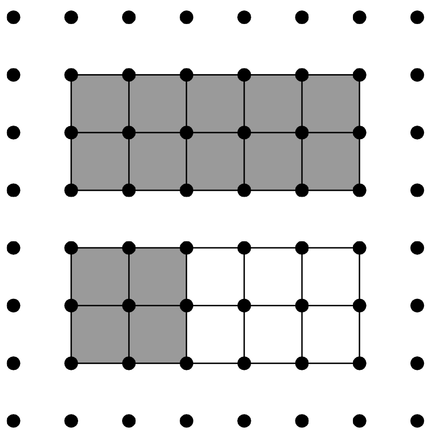
$1\frac{1}{2} = \frac{3}{2}$

Pattern blocks



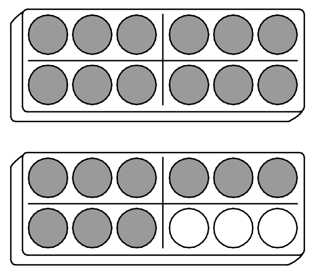
$1\frac{1}{3} = \frac{8}{6} = \frac{4}{3}$

Geo-boards/geo-paper



$1\frac{4}{10} = \frac{14}{10}$

Egg carton

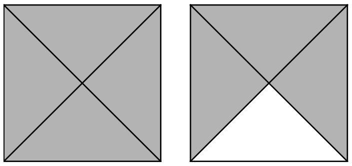


$1\frac{9}{12} = \frac{21}{12}$

Students can use models such as pattern blocks to discover these connections. For example, ask students to model $\frac{7}{2}$ when the yellow hexagon represents one whole. As they build this model, students will see that they have created 3 whole hexagons plus $\frac{1}{2}$ of another hexagon. Ask students to then think about how they could report what they see. Ask them to think of another way to represent this improper fraction.

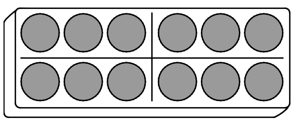
Give students a mixed number such as $3\frac{2}{5}$. Ask students to use models to find an improper fraction that names the same amount. They may use any familiar materials or make drawings, but they must be able to give an explanation for their result. Similarly, ask students to start with a fraction greater than 1, such as $\frac{17}{4}$, and ask them to determine the mixed number and provide a justification for their result. This task can be repeated using other numbers. Students may find it easier if they are able to choose the numbers they work with for this task. (Van de Walle 2006b,140)

The figure below illustrates this idea by showing that the improper fraction $\frac{7}{4}$ and the mixed number $1\frac{3}{4}$ refer to the same fraction of a region, or area; therefore, they are equivalent.

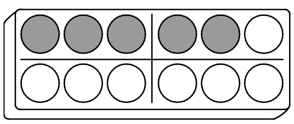


$\frac{7}{4} = 1\frac{3}{4}$

The figure below illustrates another way to show that an improper fraction can be expressed as a mixed number, where both name the same quantity. In this example, it can be seen that there is one full egg carton, or $\frac{12}{12}$ eggs and a part of another carton, namely $\frac{5}{12}$. The resulting mixed number then, would be $1\frac{5}{12}$ or the equivalent improper fraction that would name the amount of the set would be $\frac{17}{12}$.



$\frac{12}{12}$



$\frac{5}{12}$

Through working with these numbers, some students may discover the relationship in multiplying the denominator by the whole number and adding the numerator to get the improper fraction, but it is not the recommended way to introduce or teach the topic. "There is absolutely no reason ever to provide a rule about multiplying the whole number by the bottom number and adding the top number. Nor should students need a rule about dividing the bottom number into the top to convert fractions to mixed numbers." (Van de Walle 2006b, 141) Providing students with ample opportunities to explore these concepts through the use of hands-on activities, using models and pictures, will help students develop an understanding using their own words and in their own way.

N04.04 Students have been working with mixed numbers and improper fractions by representing, modelling, naming, and expressing them in different symbolic forms. As students continue working with these numbers, they can begin thinking about strategies that would enable them to compare and order them. In Mathematics 5, students worked with comparing proper fractions with like and unlike denominators that will now help them compare mixed numbers and allow them to extend their personal strategies to compare improper fractions. When comparing and ordering improper fractions, encourage students to recognize that it may be easier to express the improper fraction as a mixed number where they would compare the whole number first and then look at the proper fraction if needed. For example, when comparing $\frac{6}{4}$ and $\frac{9}{5}$, students could express both as a mixed number namely, $1\frac{2}{4}$ and $1\frac{4}{5}$. Here they could easily see that they are both 1 whole with the first mixed number having an extra $\frac{2}{4}$ or $\frac{1}{2}$ and the second mixed number having an extra $\frac{4}{5}$. Knowing that $\frac{2}{4}$ is the same as $\frac{1}{2}$ and that $\frac{4}{5}$ is greater than $\frac{1}{2}$, students should be able to see that $\frac{9}{5}$ is greater than $\frac{6}{4}$.

Using a number line when solving problems is another strategy that students can use to help show their understanding. As students encounter problems that require them to compare mixed numbers and improper fractions, a logical representation of the understanding would be to place the given numbers on a number line. For example, stretch string across the classroom with various points marked for 0, 1, 2, 3, and 4. You may want to ask students to do this. This string will be used to show students that all proper fractions are between 0 and 1 and all mixed numbers and improper fractions are greater than 1. Students will clip index cards with various proper fractions, improper fractions, and mixed numbers on the string. You may wish to ask students to place various benchmarks and ask them to choose the numbers they want to place. For example, you may wish to ask them to write any mixed number or improper fraction that would come between 1 and 2, or between 3 and 5, that could go on the number line. After each student has had a chance to place some numbers on the number line, have a class discussion to decide if all the placed numbers are in relative positions.

Students should be able to compare fractions having the same denominator, (e.g., $\frac{7}{6} < \frac{11}{6}$ because if a number of equal-sized cakes are each cut into 6 equal pieces, 7 of those pieces are less than 11 of them). This shows that if two fractions have the same denominator, the one with the greater numerator is greater. Students should be able to compare fractions having the same numerator (e.g., $\frac{8}{3} > \frac{8}{5}$ because if 3 people share 8 tea biscuits, they will each get a larger portion than if 5 people share the same 8 tea biscuits). This shows that if two fractions have the same numerator, the one with the greater denominator is less. Students should make these connections as they compare and order fractions. Benchmarks (values used for comparison), such as $\frac{1}{2}$ or 1, can be used to compare fractions (e.g., $\frac{2}{5} < \frac{7}{8}$ because $\frac{2}{5}$ is less than $\frac{1}{2}$, while $\frac{7}{8}$ is more than $\frac{1}{2}$). Number lines can be extended so that students see benchmarks of 1, $1\frac{1}{2}$, 2, $2\frac{1}{2}$, and so on. They can use the benchmarking strategy to compare improper and mixed fractions as well. Students should be encouraged to solve and create problems in context that relate to the comparison of fractions.

Students estimate the size of fractions by grouping them as less than one-half, between one-half and one, between one and one and one-half, etc. For example, $1\frac{2}{5}$ is between 1 and $1\frac{1}{2}$, and therefore, it is estimated to be less than $1\frac{1}{2}$. Benchmarks can be used to identify and create larger or smaller fractions than a given fraction or between two fractions. For example, $\frac{15}{8}$ is greater than the given fraction $\frac{8}{7}$ because $\frac{15}{8}$ is closer to the benchmark of 2 and $\frac{8}{7}$ is closer to the benchmark of 1; and $\frac{6}{5}$ is smaller than the given fraction of $\frac{9}{6}$ because $\frac{6}{5}$ is closer to 1. The fraction $\frac{7}{8}$ is between $\frac{1}{5}$ and $\frac{10}{6}$ because $\frac{1}{5}$ is close to 0, $\frac{7}{8}$ is close to 1, while $\frac{10}{6}$ is close to $1\frac{1}{2}$.

Students should make connections that if two fractions have the same denominator, the size of the numerator will determine if the fraction is larger or smaller (e.g., $\frac{10}{5} > \frac{8}{5}$ because the numerator 10 is greater than 8). If two fractions have the same numerator, then the one with the larger denominator is less (e.g., $\frac{5}{4} < \frac{5}{3}$ because fourths are smaller than thirds).

N04.05 and **N04.06** Provide students with opportunities to use concrete, pictorial, and symbolic forms to represent improper fractions and mixed numbers. This helps students become exposed to these numbers in more than one way, where they are physically working with the number using materials. Drawing a picture to represent the number they are working with helps students solidify the concrete image of the number. The next step in this progression of learning is naming the number using symbols.

Ask students to model a given mixed number, for example 3 and $\frac{2}{6}$ using manipulatives. You may wish to give them a choice of several different mixed numbers depending upon their understanding of mixed numbers. Allow students time to discuss their choice of manipulatives and how their model represents their chosen mixed number. Ask students to then draw a picture to represent this number. (It could be a picture of the model they already used, or the number in a different context.) Again, ask them to explain how their drawings represent the given mixed number. Ask them to then represent this mixed number in symbolic form (using numbers).

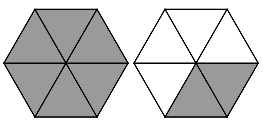
Many types of manipulatives can be used to explore improper fractions and mixed numbers. For example, coloured linking cubes can be used to create different lengths where students can compare the lengths of the joined cubes. Students could create a length of 4 blue cubes and another with 8 yellow cubes. Students could then talk about how the lengths of these two compare. So, blue is $\frac{1}{2}$ the length of yellow. If they then create a length of 6 red cubes, they would say that red is $1\frac{2}{4}$ the length of blue.

Another suggested task in using the linking cubes would be to provide students with 20 of the same coloured linking cubes. Ask students to represent an improper fraction such as $\frac{17}{5}$. They should understand from previous work that $\frac{17}{5}$ means there are 5 parts in one whole and there are 17 parts in all. They should then go on to create towers of 5 linking cubes so they will see that they can create 3 complete towers with 2 cubes left over. This can then be used to help them see that $\frac{17}{5}$ is the same as 3 and $\frac{2}{5}$.

Once students have had ample opportunity building, creating, modelling, drawing, and naming improper fractions, they will be ready to use symbolic forms to represent the fractions with which they are working. Students should be able to easily translate a given improper fraction between various representations such as models, pictures, and then in numbers. Ask students to model an improper fraction and then draw this representation. Ask students to then use numbers to name this fraction.

To help students see the relevance of translating their models of improper fractions to pictures and then to a symbolic form, ask students to prove that $\frac{8}{6}$ is less than $1\frac{1}{2}$. To do this, ask students to use pattern blocks to build $\frac{8}{6}$ and then draw this on paper as a part of their written response to the question.

Students could then go on to show how their picture of the pattern blocks show that $\frac{8}{6}$ is less than $1\frac{1}{2}$.



$$\frac{8}{6} = \frac{6}{6} + \frac{2}{6}$$

$$\frac{6}{6} = 1$$

$$\frac{8}{6} = 1\frac{2}{6}$$

SCO N05 Students will be expected to demonstrate an understanding of ratio, concretely, pictorially, and symbolically. [C, CN, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

N05.01 Represent a given ratio concretely and pictorially.
N05.02 Write a ratio from a given concrete or pictorial representation.
N05.03 Express a given ratio in multiple forms, such as “three to five,” 3:5, 3 to 5, or $\frac{3}{5}$.
N05.04 Identify and describe ratios from real-life contexts and record them symbolically.
N05.05 Explain the part-whole and part-part ratios of a set (e.g., For a group of three girls and five boys, explain the ratios 3:5, 3:8, and 5:8.).
N05.06 Solve a given problem involving ratio.
N05.07 Verify that two ratios are or are not equivalent using concrete materials.

Performance Indicator Background

N05.01 A ratio is a comparison of any two quantities. When investigating the concept of ratio, provide students with various concrete materials to represent these ratios. Use the students themselves, counters, or other simple models such as linking cubes, pattern blocks, or buttons to illustrate the concept of ratio as a comparison between two numbers (or among three or more numbers). These models can help students see the part-to-part and part-to-whole relationships.

Encourage the use of appropriate language (e.g., A ratio such as 3:2 is read as “three to two” or “3 __ for every 2 __”).

Through exploration and making meaningful connections, ratios can be related to everyday situations (e.g., the ratio of water to concentrate to make orange juice is 3:1 or “3 to 1”) or in relation to other topics in mathematics (e.g., students can explore the ratio of the length of one side of a rectangle to the perimeter).

In previous grades, students have often compared two quantities in an additive way. For ratio, students need to understand multiplicative comparison. For example, if you ask students to compare the value of a dime to the value of a penny, they may say that the dime is 9 cents more than the penny. This represents additive thinking but this is not a ratio. Help students view the multiplicative comparison of the value of one dime to the value of one penny as 10 cents to 1 cent or the value of one dime as 10 times the value of one penny. When the ratio of boys to girls is written as 3:2, that comparison is another way of saying that the number of boys is $\frac{3}{2}$ or $1\frac{1}{2}$ times the number of girls or the number of girls is $\frac{2}{3}$ the number of boys. A connection can be made to the relational structure of the place-value system where there is a 1:10 ratio as we move to the right and a 10:1 ratio as we move to the left.

Ratios can be used to make part-to-whole comparisons or part-to-part comparisons. Part-to-whole ratios are fractions because they compare a part with a whole. For example, if a student places 3 red counters and 5 yellow counters on their desk, they can compare the number of red counters to all the counters as 3 to 8, 3:8, or $\frac{3}{8}$. This is a part-to-whole ratio. The students can also compare the number of yellow counters to all the counters as 5 to 8, 5:8, or $\frac{5}{8}$. This is also a part-to-whole ratio. Therefore, students should be reminded that all fractions are ratios. However, not all ratios are fractions. A ratio can also be a part-to-part comparison, for example, students can compare the number of red counters to the number of yellow counters as 3 to 5, 3:5, or $\frac{3}{5}$. This represents a part-to-part ratio. This is particularly problematic because this ratio can be written using fractional form. It is recommended that ratios written with fractional form be read using ratio language (i.e., $\frac{3}{5}$ as three is to five rather than as three fifths). The colon notation and the fraction notation are completely interchangeable; however, the colon notation is more often used with part-to-part comparisons while the fraction notation is favoured with part-to-whole comparison. Fraction notation is more often used when computing with ratios (e.g., determining equivalent ratios).

Students will also encounter ratios that involve the comparison of two quantities that have different units—these ratios are called rates. It is a multiplicative comparison of two quantities with the quantities described in different units. Students will use rates when they shop (e.g., oranges at $1.09/2) when they deal with speed (e.g., 100 km in 2 hours), and when they convert between units of measurement (e.g., 1000 m in 1 km). The mathematics used to talk about rates is the same as the language we used to talk about ratios. Rates sometimes involve a comparison with 1 called unit rates (e.g. $1.25 for 1 avacado, 100 km per 1 hour). In a unit rate, the second term is always 1. Unit rates are very useful when comparing the cost of two or more items to determine the better buy.

N05.02 Ratios can be written symbolically in a number of ways, such as 5:3, 5 to 3, or $\frac{5}{3}$. The 5 is the first term of the ratio and the 3 is the second term of the ratio. Ratios such as 5:3 are read as “five is to three” or “five to three” or “five __ for every three __.”

N05.03 As students continue to work with ratios, provide them with many opportunities to show and understand that ratios can be written in many forms. It is beneficial for students to be able to move easily among different forms when expressing a number.

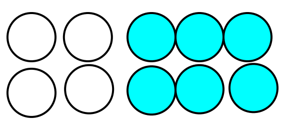
N05.04 A ratio is a multiplicative comparison of two numbers, measures, or quantities of the same type of thing. Ratios should be explored using real-life meaningful contexts (e.g., 3:1 is the ratio of water to concentrate for making orange juice; 3:2 is the ratio of the number of boys to the number of girls in a group of 3 boys and 2 girls). You are using ratio when stating, She is running twice as fast now as last year (2:1).

A scale on a map provides a good context for ratios. Discuss with students the need for this scale or ratio (it is impossible to show the actual size and/or distances on a map). Another example of a ratio in the real world is in mixing gas and oil for chain saws and snowmobiles. The gas:oil ratio might be 50:1. This means that for every 50 L of gas there would be 1 L of oil needed or the amount of gas is 50 times the amount of the oil.

N05.05 To illustrate the difference between part-to-part and part-to-whole ratios of the set, provide small groups of students with a bag containing two different coloured counters. Ask students to compare the counters in as many ways as they can. Invite groups of students to share their findings, and identify which ratios are part-to-part comparisons and which are part-to-whole comparisons.

Discuss part-to-part and part-to-whole ratios with students identifying examples of each type. Once the ratio is given, students could be told to represent it as a part-to-part ratio or it could be left open where students choose which ratio type to use. After creating their representations, encourage students to explain why they chose to represent the ratio the way they did.

N05.07 Students worked with equivalent fractions in Mathematics 5 and should be able to use this concept to help them understand equivalent ratios, as many students will recognize the similarity between equivalent ratios and equivalent fractions. For example, in the diagram below, $\frac{2}{5}$ of the counters in the top row are white, which also illustrates the ratio 2:5. In total $\frac{4}{10}$ of the counters are white or 4:10, so $\frac{2}{5} = \frac{4}{10}$. Therefore, the ratios 2:5 and 4:10 are also equivalent. If 2 of every 5 counters are white, then 4 of every 10 would also be white.



To help students visualize the concept of equivalent ratios, ask them to create a given ratio using two different coloured snap cubes. For example, students can build a model using 3 black cubes and 2 white cubes and describe the ratio as 3:5 (part-to-whole). When looking at the ratio of black to the whole, they would see 3 black to 5 in all or 3:5. Demonstrate for students an equivalent ratio for 3:5 by replicating the original model. We now have an equivalent ratio for 3:5, which is 6:10. By continuing to replicate their original model, they can create additional equivalent ratios.

Using pattern blocks, ask students to explore equivalent ratios by seeing that when the yellow hexagon is one whole, one blue rhombus represents 1:3 or $\frac{1}{3}$ of the hexagon. To create an equivalent ratio, students could use the green triangles to match the same area as one blue rhombus. They will see that it takes two green triangles to create one blue rhombus; therefore, the ratio of triangles to the whole is 2:6, and this clearly illustrates that 1:3 is equivalent to 2:6. Ask students to explore other equivalent ratios using pattern blocks.

Students can use equivalent ratios to make predictions. For example, in a large bag of marbles, the ratio of blue marbles to the total number of marbles is 4:10 (i.e., 4 out of every 10 marbles are blue). Use this to predict the number of blue marbles you would expect in 100 selections.

SCO N06 Students will be expected to demonstrate an understanding of percent (limited to whole numbers) concretely, pictorially, and symbolically. [C, CN, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

N06.01 Explain that "percent" means "out of 100."
N06.02 Explain that percent is a ratio out of 100.
N06.03 Represent a given percent concretely and pictorially.
N06.04 Record the percent displayed in a given concrete or pictorial representation.
N06.05 Express a given percent as a fraction and a decimal.
N06.06 Identify and describe percent from real-life contexts, and record them symbolically.
N06.07 Solve a given percent problem involving benchmarks of 25%, 50%, 75%, and 100%.

Performance Indicator Background

N06.01 Discuss with students that you evaluate their progress in many different ways. One form of assessment is a test and getting the results in the form of a percent. The greatest score you can get is 100%; therefore, to give a percent, it must always be "out of 100." For example, if you get 87% on a test, this means you got 87 marks out of a possible 100 marks ($\frac{87}{100}$). As connections are made to fractions, 100% can be seen as one whole where anything less than that whole is a part or a percent. Ask students to explore and model how 0.87 means the same as 87% or $\frac{87}{100}$ or 87 out of 100. Focus on mathematical language, using 87 hundredths or 87 out of 100, to help students see these connections.

N06.02 Percent is a ratio and, therefore, is another name for a fraction. Percent should be viewed as a part-to-whole ratio that compares a number to one whole that has been divided into 100 equal parts. Students may note the connection to the word "cent" where a cent is $\frac{1}{100}$ of a dollar. Students will not be computing percents where they are procedurally finding the percentage of fractions or ratios at this time and need not work with percents greater than 100, but should recognize

- situations in which percent is commonly used
- diagrams that represent various percents
- the relationship between percents, decimals, and fractions (e.g., 48% = 0.48 = $\frac{48}{100}$)
- that percent is a ratio or a comparison of the percent value to 100 and can be written as __ : 100 and $\frac{?}{100}$
- that finding a percent is the same as finding an equivalent ratio out of 100

Students should understand that determining a percentage is the same as determining an equivalent ratio out of 100. Students should view percent as a special ratio (part-whole) for which one number is compared with one hundred (e.g., 56% represents the ratio 56:100 or the decimal number 0.56 or the fraction $\frac{56}{100}$).

Many problems involving percents will also require students to use their knowledge of equivalent fractions and equivalent ratios. Provide students with an example such as, there were 10 cartons of milk ordered for recess; 7 were chocolate. Therefore, $\frac{7}{10}$ of the cartons of milk were chocolate. This also means $\frac{70}{100}$, 70 out of 100, or 70% of the cartons of milk were chocolate.

N06.03 Students should relate to percent visually. They should be able to readily identify percent from a picture and should recognize that parts of a given picture should always add to give 100%. Provide students with a blank hundredths grid and ask them to use four different colours to shade in the grid. Ask them, for example, to shade 30 blocks red, 20 blocks blue, 45 blocks black, and 5 blocks yellow. Ask students to describe each colour using a fraction, decimal, percent, and a part-to-whole ratio. This task will help students connect these four ways of representing a number.

Number sense for percent should be developed through the use of benchmarks:
- 99% is almost one whole
- 49% is almost one-half
- 10% is not very much
- 1% is close to 0 and is very small in relation to the total

Immediate connections should be made between certain percents and their fraction equivalents, such as 50% and $\frac{1}{2}$, 25% and $\frac{1}{4}$, 75% and $\frac{3}{4}$, and 100% and 1. As well, for other percents students should be encouraged to recognize that percents such as 51% or 49% are close to 50% or $\frac{1}{2}$ and therefore use $\frac{1}{2}$ for estimation purposes.

N06.04 Hundredths grids are excellent resources to use with students to help develop their understanding of percents. Encourage them to also use other concrete representations to facilitate their understanding of percents of objects, as well as equivalences of fractions, decimals, and percents.

Ask students to predict percents, give their prediction strategies, and then check their predictions. For example, ask them to estimate the percent of
- red counters when fifty two-coloured counters are shaken and spilled
- each colour of counter if a total of 100 blue, red, and green counters are shown on an overhead for 10 seconds
- a hundredths grid that is shaded to make a picture

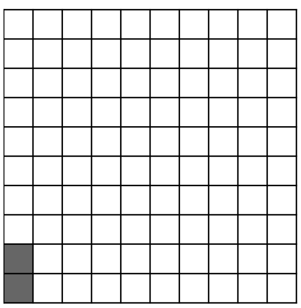
N06.05 Emphasis should be placed on recognizing that percent is not a new concept or idea; it is merely a new notation. For example, $\frac{1}{2}$ of a region, 0.5 of a region, and 50% of a region all represent the same idea. Students need opportunities to explore, illustrate, explain, and express ratios, fractions, and decimals as percents and vice versa. The form that is used depends mainly on the situation; for example, baseball statistics are always presented in decimal number form (a batting average of 0.345), measurements are expressed using fractions and decimals, and weather data are referred to only by percentages (a 30% chance of snow).

Students should be able to relate common percents to fractions such as 25% to $\frac{1}{4}$, 50% to $\frac{1}{2}$, 75% to $\frac{3}{4}$, or 100% to 1. They should not be computing with percents at this time and need not work with percents greater than 100. Throughout the unit students have been making connections using fractions, decimals, percents, and ratios to represent any given number. Working with a hundredths grid is essential to this type of work so students can have a visual representation of the work they are doing.

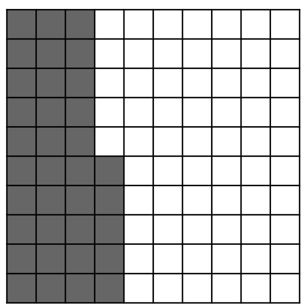
Provide students with a hundredths grid, asking them to shade a percent of the grid. Once the grid is shaded, ask them to create a corresponding card that illustrates the percent shaded on the grid using a decimal, fraction, ratio, and in words.

(These grids and cards will be similar to the materials used in the Decimal Square kit.)

N06.06 Students should explore situations in which percent is commonly used. This could be accomplished by looking for percents in newspapers, magazines, sale flyers, and other advertisements. They should make and interpret diagrams that represent various percentages (e.g., 2% and 35% represented in hundredths grids).



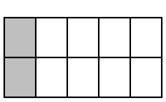
2 %



35 %

Students should recognize and express the relationship between the percent and decimal names of these special ratios (e.g., 48% and 0.48). Using a metre stick as a model for percent is helpful for students to understand that 48 cm is 48% or 0.48 of the stick. Have students explore a variety of geographic or social studies data expressed in terms of percentages (e.g., about 70% of Earth is water, about 68% of Canadian households own microwaves, and over 80% of car passengers wear seat belts). Have students cut sheets of paper and lengths of string to show percents such as 50%, 10%, and 25%. Have students predict percentage results, explain their prediction strategies, and check their predictions.

Students will develop an understanding of the concept of percent as a ratio. In most cases, percent is used as a special ratio to describe a part of a whole rather than one part to another part. For example, in the grid below, the percent of squares shaded should be $\frac{2}{10}$ or $\frac{20}{100}$, or 20% which illustrates a part-to-whole relationship.



N06.07 There are many different applications to the real world where students can use percents and ratios to solve problems. One such example is through the use of scale diagrams; however, students' work and discussion should not be limited to this particular situation. In using scale diagrams as a means of having students solve problems involving ratios and percents, students can look at maps investigating the particular scale used to represent distances and sizes of countries. Based on the scale used, students could calculate distances between identified places.

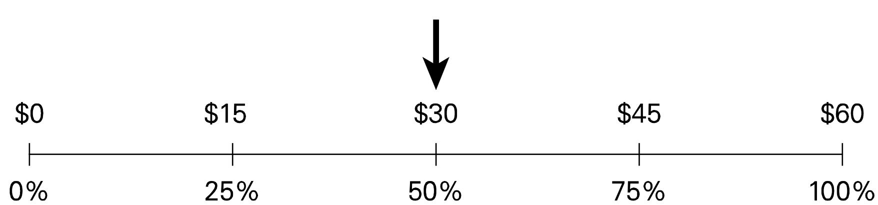
Students may be familiar with model toys, and can readily identify that a model car or motorcycle has a scale of 1:30 to an actual car. Ask them to explore the dimensions of the actual size of the car or motorcycle to discover that the model dimensions are in the numerator and the actual car dimensions are in the denominator. If the model car has a door that is 4 cm in height, students can use their understanding of scales and ratios to determine the height of the actual car door. It is very important to keep numbers simple when representing or comparing various ratios.

Students have been working on making connections between percents, decimals, ratios, and fractions. Through this work, students' understanding of percent will now be extended to determine percents to solve problems. They will now be expected to understand how to estimate and determine a given percent of a number using benchmarks of 25%, 50%, 75%, and 100%. For example, students may be asked to determine 50% of 80 and would think, 50% is one-half, and one-half of 80 is 40.

Number lines are helpful tools when working with percents. Students can see that when they are asked to determine a percent of a given number, the given number is the whole and is represented at the end of the number line. Students would then use their knowledge of benchmarks, using $\frac{1}{2}$ as 50%, $\frac{1}{4}$ as 25% and $\frac{3}{4}$ as 75% to help them estimate and determine the given percentage of the number using the number line. For example, to solve the following problem, a number line can be used as shown.

- Shawn wanted to save $60 for his sister's birthday gift. He thought about it and decided he should have 50% of the money saved by June. How much money would Shawn have saved by June?

  Students need to understand that it if they want to use a number line to help represent a problem, they need to see that, while the end points of their number line begin at 0 and go to 60, for example, that this number line represents 100% of the total amount of money needed. Therefore, the benchmark of one-half or 50% represents $30.



Begin instruction with establishing the benchmark of 50% or one-half. Practice determining 50% of given numbers so students can explore how this is the same as finding the benchmark of one-half on their number line (between 0 and the number in question). When students are comfortable with this idea, lead them to determining 25% and 75% of given numbers. These two percents would represent the $\frac{1}{4}$ and $\frac{3}{4}$ mark on their number line.

SCO N07 Students will be expected to demonstrate an understanding of integers contextually, concretely, pictorially, and symbolically.
[C, CN, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

N07.01 Extend a given number line by adding numbers less than 0 and explain the pattern on each side of 0.
N07.02 Place given integers on a number line and explain how integers are ordered.
N07.03 Describe contexts in which integers are used (e.g., on a thermometer).
N07.04 Compare two integers; represent their relationship using the symbols <, >, and =; and verify using a number line.
N07.05 Order given integers in ascending or descending order.

Performance Indicator Background

N07.01 Negative integers are the opposites of positive integers. Each integer has an opposite that is equidistant from zero. A number line is a useful tool in helping students see the relationship between positive and negative integers. Tape a number line on the floor (use cash register tape or string), whereby the middle or centre of the number line is marked with a 0. Ask students to think about what numbers would go to the left, or below the 0. Ask students to explore these numbers asking them to think about the placement of the numbers that are “less” than 0. Show students that numbers such as +1 and –1 are placed so that they are the same distance away from 0.

N07.02 and **N07.03** Students, almost daily, have experiences that can be modelled with negative numbers. Students may also gain an understanding of negative integers by putting them into meaningful context such as describing floors below the main floor in a building, talking about below sea level, or golf scores that are below par. Negative numbers could also be described in terms of owing money. Discuss some of these situations that can be represented by negative numbers. For example, losing $15 can be represented as –15 or trying to walk up an icy hill but sliding backward three steps can be represented as –3.

Using a number line (painter tape works well), ask students to stand on a negative integer and tell where they would see this number in real life, or to describe a situation that could be represented by this number.

Use play money to represent various situations in which students can see a gain or loss, emphasizing a loss would represent a negative situation. It may be necessary to help students understand that a situation can be represented by a negative number without the result being a negative number. For example, if you have $5 in your pocket and you gave $3 to your friend for his birthday, you would still have $2 left, but it would be a loss of $3.

N07.04 and **N07.05** When comparing and ordering numbers, students should refer to a number line and look at the position of the number on the line in relation to 0 to determine its value. A common misconception students have is that the greater the digit, the greater the value will be. They may see a number like –8 and think it is greater than –1 or +5 just because the digit 8 is greater than the digit 1 or 5. Ask students to refer to the position of the number in relation to 0. Remind students that, when comparing two numbers, any number to the right is always greater than any number to the left. This applies to both positive and negative integers.

When placing integers on a horizontal number line, remind students to first look at the sign to see whether it is a positive or negative number. Then ask them to look at the digit to determine how far away from 0 the number should go. Ensure students understand that all negative numbers are less than 0 and placed to the left of the 0 on the number line.

Students can stand on number lines to help order integers. To illustrate the idea of whether a negative integer is greater than or less than another, ask students to stand on the number in question and ask them to hop to 0 counting how many times they needed to hop to get to 0. Ask them to do the same for the other number and then compare the number of hops it takes to get to 0. For example, when comparing –5 and –8, have a student stand on –5 and jump to 0. Students will see it takes 5 jumps to move from –5 to 0. Next, ask a student to stand on –8 and jump to 0. Students will see it takes 8 jumps to move from –8 to 0. They should understand that because it took more jumps to get to 0 from –8, –8 is farther away from 0. Therefore, –8 would be less than –5. This would be recorded symbolically as $-5 > -8$ or $-8 < -5$.

If students are struggling with this concept, ask them to refer to a thermometer or use a vertical number line to represent the thermometer.

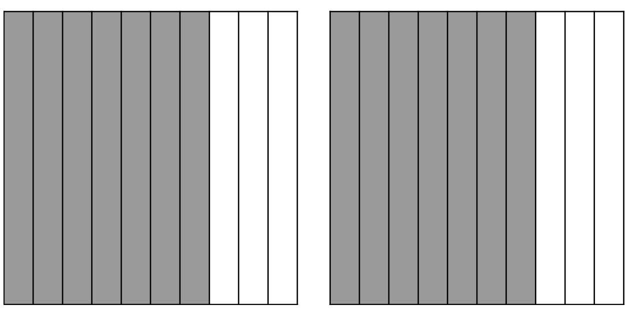
SCO N08 Students will be expected to demonstrate an understanding of multiplication and division of decimals (one-digit whole number multipliers and one-digit natural number divisors). [C, CN, ME, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

N08.01 Model the multiplication and division of decimals using concrete and visual representations.
N08.02 Predict products and quotients of decimals using estimation strategies.
N08.03 Place the decimal point in a product using front-end estimation (e.g., For 15.205 × 4, think 15*m* × 4, so the product is greater than 60.).
N08.04 Place the decimal point in a quotient using front-end estimation (e.g., For $25.83 ÷ 4, think 24 ÷ 4, so the quotient is greater than $6.).
N08.05 Use estimation to correct errors of decimal point placement in a given product or quotient without using paper and pencil.
N08.06 Create and solve story problems that involve multiplication and division of decimals using multipliers from 0 to 9 and divisors from 1 to 9.
N08.07 Solve a given problem, using a personal strategy, and record the process symbolically.

Performance Indicator Background

N08.01 It is important that students draw or build models to show the multiplication sentence involving decimals. For example, to model 2 × 0.7 (two groups of seven-tenths) students would use two decimal squares for 0.7.



Students can see how one whole is created, and how many parts of another whole there are, thus determining that 2 × 0.7 (two groups of seven-tenths) is 1.4.

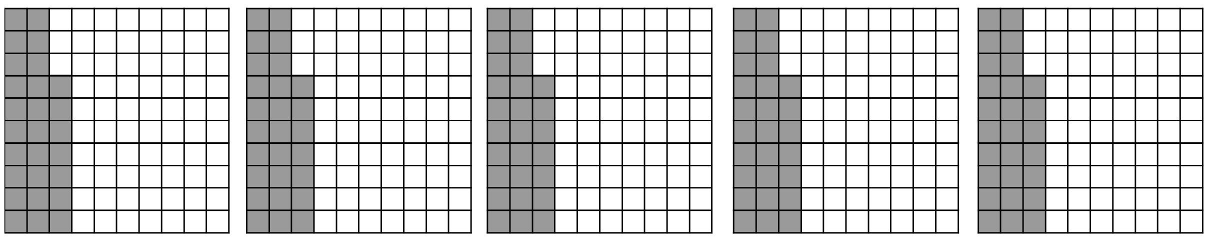
Another way to illustrate this example is to use base-ten blocks. Students can represent 0.7 using seven rods, where a flat represents 1. Students make two groups of 7 rods and see that there is a total of 14 rods. Knowing there are 10 rods in a flat (ten-tenths are equal to 1), students should understand that 14 rods equals one whole and 4 tenths or 1.4.

Students can also use a number line to represent 2 × 0.7. After placing 0.7 on the line, students could move another 0.7 along the line to arrive at 1.4.

It is important for students to use manipulatives such as base-ten materials or decimal squares, as it may help them visualize the concept of multiplication more easily. Also, previous work on multiplying decimals in a money context may enable students to think about what the decimal means in a more meaningful manner. Although students have had prior experience modelling decimals where they would have used different blocks to represent one whole, it may be necessary to review this concept again as some students may struggle with the idea that one whole can be represented in different ways.

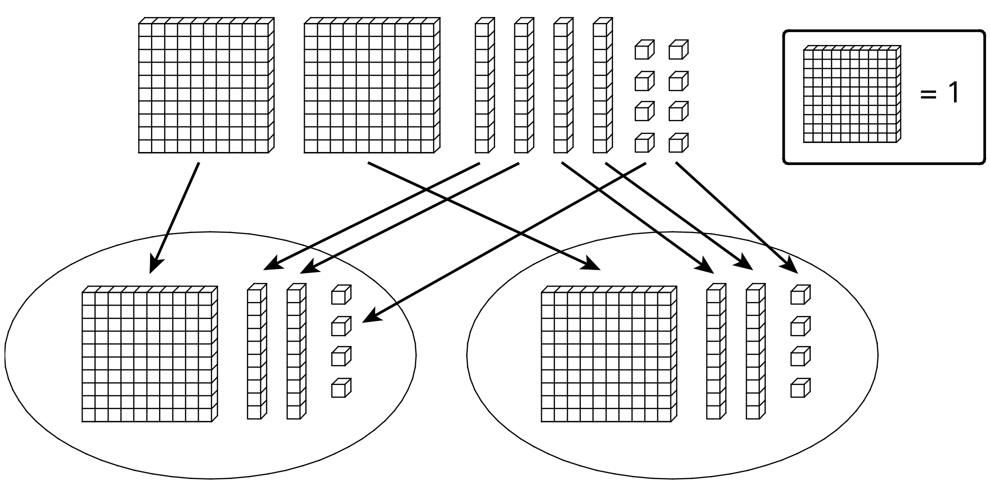
Using decimal squares, ask students to shade a blank hundredths grid (decimal square) to represent 0.27. Ask them how they could now use this square to represent or show 5 × 0.27. Students should learn that if they shaded 5 groups of 0.27 on the grid, they could represent the product of these numbers. Students can then be asked to explain other ways they could obtain the product.

The squares shown here indicate that the shading was done by repeatedly shading 27 parts of each whole square. If the shaded parts are combined, 135 blocks in total are shaded or 1 whole and 35 hundredths.

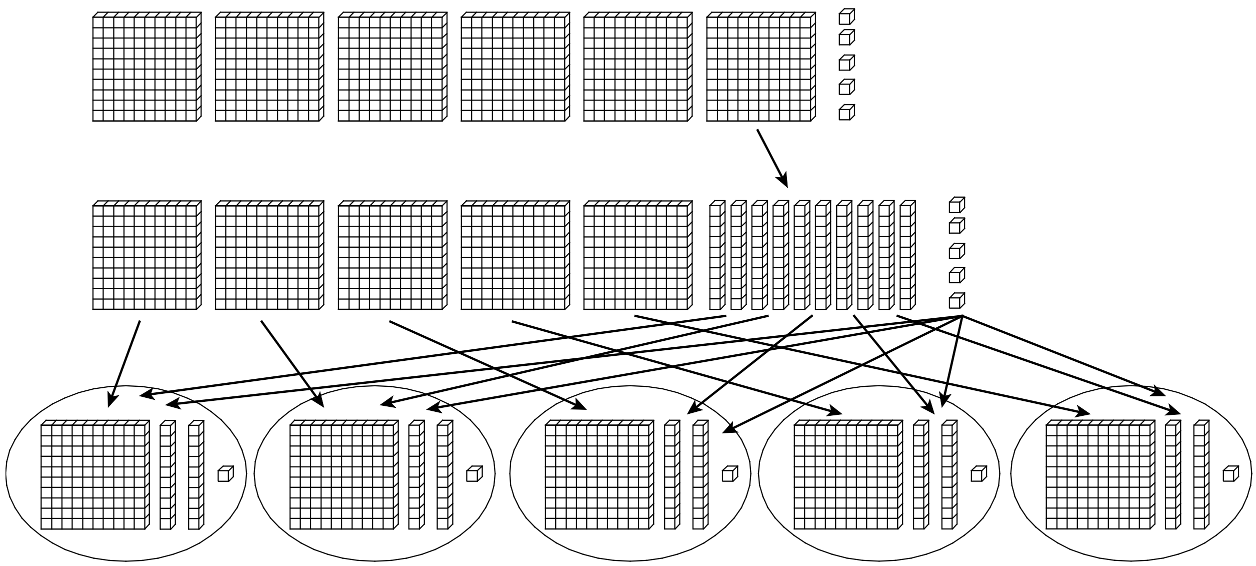


As students proceed through these various models they need to explain their discoveries both orally and in writing. They need to relate the model to an algorithm by explaining each symbolic step in relation to the appropriate part of the model.

Division can be approached in a manner exactly parallel to multiplication. It is very important that students have a lot of opportunities to model the division of decimals using manipulatives and visual representations. Students cannot be expected to solve division problems involving decimals using symbolic notation before completing work with this concept using “hands-on”’ materials. A strategy to introduce division of decimals by whole numbers would be to show students how to break apart a decimal into the desired number of groups. In using base-ten blocks, ask students to model or represent the decimal number to be divided. For example, students may use base-ten blocks to model 2.48 ÷ 2. Ask students to indicate into how many equal groups they need to break the dividend. Following the idea of equal sharing, ask students to share each of the base-ten blocks into the desired number of groups where they would share out all the flats, then the rods and so on.



Students should also solve division problems that require regrouping (e.g., 6.05 ÷ 5, 2.3 ÷ 2, or 3.42 ÷ 3). To model the first problem, students represent 6.05 using 6 flats and 5 small cubes. They will then share 5 of the flats among five groups by placing 1 flat in each group. They will be left with 1 flat, which can be traded for 10 rods. The 10 rods will be shared among the 5 groups. Finally, the five small cubes will be shared among the five groups. It is important that students continue to use manipulatives to model division of a decimal by a whole number when solving problems.



To solve division problems, students may use the think multiplication strategy. Students can look at the number of equal groups and think about the decimal number needed in each group to total the dividend.

For example, to solve 12.33 ÷ 3, students would determine how much was needed in three equal groups to total 12.33. They would think there would be 4 ones in each group since 4 × 3 = 12. They could then consider 0.33 and there would be 0.1 in each group with 0.03 left over. From there, they could see that 0.01 would fit into each group. In total, there would be 4.11 in each group with no remainder. Students beginning this strategy are expected to use base-ten blocks to model their thinking.

Students' attention must be drawn to the fact that when there is a remainder in a division equation involving decimal number dividends, the remainder does not mean a whole number; it means part of a whole number (a decimal). Students should explain the different treatment of remainders and represent the treatment appropriately based on the context. An example, If three people are to share fifty-six dollars, each person would get eighteen dollars and sixty-six cents and there would be two cents left over. In context, the quotient is reported as a hundredths place decimal because it is dollar notation. As a result, the leftover two pennies will not be shared. Another treatment of a remainder is rounding up. For example, How many vans are needed to transport thirty-two children if six children can go in each van? You will need six vans to transport all children as 5.33 vans leaves two children behind. In this context the quotient must be rounded up.

N08.02 Estimation strategies are very important as students work with computing decimals. Students should be encouraged to estimate the product or the quotient before calculating the solution. In doing so, they will be able to determine if their calculated answer is reasonable. Estimation is also an essential tool for students to correctly place the decimal point in the product or quotient.

Students should habitually estimate answers before attempting pencil-and-paper or calculator computations in order to be alert to the reasonableness of answers. Usually, these need only be “ball park” estimates, especially when using a calculator where typical input errors result in place-value mistakes that can be detected from those “ball park” estimates. There are also many instances in life when an estimate is all that is needed. Such estimates should be as close as you can get to the actual answer.

The language of estimation should be used throughout estimation lessons. Some of the common words and phrases related to estimation are *about*, *just about*, *between*, *a little more than*, *a little less than*, *close*, *close to*, and *near*.

It is also important for students to hear and see a variety of contexts for each estimation strategy, so they are able to transfer the use of estimation and strategies to situations found in their daily lives.

Front-End Estimation

This strategy is the simplest of all the estimation strategies. Front-end multiplication involves beginning at the largest place value (e.g., an estimate of 8 × 823.24 would be 8 × 800 or 6400). As such, these front-end estimations will only require the use of the basic facts.

Examples:

- To estimate 5 × 1.437, think, 5 times 1 is 5, so the estimate is 5 and the actual answer will be a little more than 5.
- To estimate 8 × 3.6, think, 8 times 3 is 24, so the estimate is 24 and the actual answer will be a little more than 24.
- To estimate 7 × 8.48, think, 7 times 8 is 56, so the estimate is 56 and the actual answer will be a little more than 56.
- To estimate 63.141 × 8, think, 60 × 8 or 480.
- To estimate 5 × 0.897, think, 5 × 1, or 5.

Front-end estimates are adequate in many circumstances, particularly before using a calculator to perform multi-digit computations. For multiplication, the actual answer will always be more than the front-end estimates since the digits in the other place values are disregarded.

Sample Tasks:

- Estimate 6 × $6.29.
- What is the approximate product of 8 and 2.12?
- Estimate 5 × 4.3.
- What is the approximate area of a 4 cm × 6.5 cm rectangle?
- About how much does Carmelita earn for 7 hours @ $7.45/hour?

Front-End Division

This strategy involves rounding the dividend to a number related to a factor of the divisor and then determining in which place value the first digit of the quotient belongs, to get a “ball park” answer. Such estimates are adequate in many circumstances. To estimate the quotient of 424.53 ÷ 8, round the 425.53 to 400. The first digit is a 5 because 5 × 8 = 40. The digit will be in the tens place. Therefore, the front-end estimate is 5 tens or 50.

Sample Tasks:

- 31.917 ÷ 3 =
- $276.50 shared equally with nine people

While this strategy could be applied to division questions if the divisor is a factor of the highest place value of dividend, division estimation is better done by a rounding strategy.

Rounding

This most common estimation strategy involves rounding one or both numbers to their highest place values so the calculation is more easily done mentally. Rounding numbers to the highest place value enables students to keep track of the rounded numbers and do the calculation in their heads using basic facts; however, rounding to two highest place values would require most students to record the rounded number(s) before performing the calculation mentally. Rounding to one or to two highest place values depends upon how close your estimate needs to be to the actual answer.

There are a number of things to consider when rounding for multiplication and division estimates.

- If one of the factors is a single-digit number, consider rounding the other factor. For example, for 8 × 69.3, rounding 69.3 to 70 and multiplying by 8 is a much closer estimate than rounding 8 to 10 and multiplying 10 by 70.
- If the two factors are two-digit numbers with the ones digits 5 or more, consider rounding the smaller factor up and the larger factor down. For example, for 76 × 3.6, rounding to 70 × 4 produces a closer estimate than rounding to 80 × 4 or to 80 × 3.
- If rounding for a division estimate, look for compatible numbers. For example, for 47.19 ÷ 6, use 48 ÷ 6; for 33.08 ÷ 7.8, use 32 ÷ 8.

Examples:

- To estimate 7 × 6.42, think, 6.42 rounds to 6, so 7 times 6 gives an estimate of 42.
- To estimate 8 × 69.30, think, 69.30 rounds to 70, so 8 times 70 gives an estimate of 560.
- To estimate 3 × 4.952, think, 3 × 5 or 15.

Sample Tasks:

- Estimate 4 × 57.9.
- What is the approximate product of 82.3 and 6?
- Estimate the cost of 5 books @ $17.49 each.
- Estimate 3.2 × 8.
- What is the approximate area of a 3.5 cm × 6 cm rectangle?
- Approximately what is the cost of 9 notebooks at $1.15 each?
- 5 times 3.171 is about _____.
- Estimate the mass of nine containers of hockey pucks with a mass of 7.921 kg each.
- Estimate 202.273 × 8.

The strategy for rounding in division questions with single-digit divisors is to round the dividends to compatibles with the divisors.

Examples:

- To estimate 47.1 ÷ 6, think, Round 47.1 to 48, a compatible with 6 in division, so 48 ÷ 6 gives an estimate of 8.
- To estimate 82.2 ÷ 9, think, Round 82.2 to 81, a compatible with 9 in division, so 81 ÷ 9 gives an estimate of 9.
- To estimate 37.8 ÷ 4, think, Round 37.8 to 36, a compatible with 4 in division, so 36 ÷ 4 gives an estimate of 9.

Some practice items for division:

- Approximately what is $14.50 divided by 8?
- About what is the daily average if 16.35 km was walked in 7 days?
- Estimate 1.16 ÷ 6.

Adjusted Front-End Estimation
This strategy is often used as an alternative to rounding to get closer estimates. It involves getting a front-end estimate and then adjusting that estimate to get a better, or closer, estimate by considering the second-highest place values.

Examples:

- To estimate 4 × 4.26, think, 4 times 4 is 16 and 4 times 0.2 is 0.8, so the estimate is 16 + 0.8, or 16.8.
- To estimate 2.357 × 6, think, 2 times 6 is 12 and 0.3 times 6 is 1.8, so the estimate is 12 + 1.8 or 13.8.
- To estimate 5 × 2.189, think, 5 times 2 is 10 and 5 times 0.1 is 0.5, so the estimate is 10 + 0.5, or 10.5.

Sample Tasks:

- Estimate 3 × $5.67.
- What is the approximate product of 8 and 2.456?
- About what is 5 times 6.237 kg?
- Estimate 4.445 × 7.

N08.03 and **N08.04** Students are expected to place the decimal point in products by methods other than simply counting decimal places in the factors as this does not promote an understanding of place value or number sense. The important concept students need to understand is the place value of the digit in the product will change according to the placement of the decimal. In an example such as 1.255 × 2 = 2.51, students should understand that counting the decimal places will not help them verify if this answer is correct.

Help students to see the decimal number in terms of its value. For example, students should see that 1.62 is a little more than 1 and one-half. When solving a problem that involves multiplying, for example 1.62 by 5, students should see the product would be a little greater than 5, but less than 10 as 1.62 is less than 2. They should also see that the exact product will be a little more than 7.5, as five groups of 1.5 is 7.5.

When making predictions about the product of a decimal number and its whole number multiplier, the product has to be reasonable based on previous knowledge of decimals and place value. When students look at problems involving multiplication, they should be able to come up with a reasonable answer and be able to justify how they came up with that answer. This will not be automatic for some students and practice with manipulatives will be beneficial.

Students need to know about the placement of the decimal points in the product of a multiplication problem. To do this, they could use front-end estimation. For example, students could be presented with the following task:

- There are three CDs that cost $12.69 each. How much money would you need to buy these three CDs? Which is the best choice for the correct answer?
  (a) 380.70
  (b) 3.807
  (c) 38.07

(d) 3807.00

Students think, I round 12.69 to 12 and 12 × 3 = 36. The answer from our choices has to be close to 36.

Patterns can also be used to help students understand the placement of the decimal in the product of two decimal amounts. For example, 420 × 4 = 1680; 42 × 4 = 168; 4.2 × 4 = 16.8.

Decimal placement can also be explored using a calculator, but it is important that students practise applying mental mathematics strategies.

N08.06 Students should be solving, as well as creating, multi-step story problems involving multiplication and division. Requiring students to create their own problems provides opportunities for them to explore the operations in depth. It is a complex skill requiring solid conceptual understandings and must be part of the student's problem-solving experiences. Students should engage with situational contexts that involve equal groups and comparisons. They should also continue to use models such as sets, area models, and number lines in contextual situations. For example, a student might be asked to determine the cost of five objects that cost $3.46 each (equal groups, multiplication) or to compare the length of a killer whale that measures 9.3 m to the length of a cow that is 3.1 m long (comparison, multiplier unknown).

To understand multiplication and division, meaningful context and estimation should always play a role. Estimation should be done before students use paper-and-pencil computations. Estimation focuses on the meaning of the decimal numbers and the operations and not just on counting decimal places. Base-ten blocks can be used to model groups of, or sets of, decimal numbers or area models. Clear connections should be made between the context, the models, and the symbolism. It is important to note that many of the algorithms for multiplication and division of whole numbers can be applied to decimals, often with the same models.

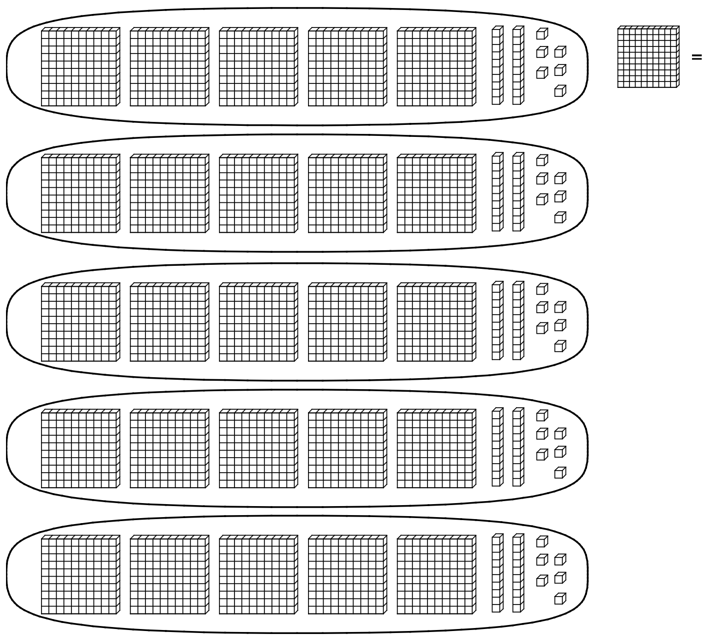
In division, a common context to which the students would relate is the sharing of money or unit pricing. Other possible contexts are sharing metres of ribbons, litres of juice, or kilograms of meat. Explain to students how to round numbers when determining the price of single items (e.g., If you want to buy one can of peas that is priced at 2/99¢, your price per can would be 50¢. If grapefruit are 3/$1, individually they cost 34¢. Students should understand that the "remainder" when they perform the division of a decimal number is different than with whole numbers (e.g., when dividing 3.4 by 3, the remainder "1" at the bottom is really "one-tenth" not "1").

N08.07 Students should continue to use concrete models, such as base-ten blocks, money, and pictures, to make sense of multiplication and division algorithms involving decimals. It is not enough to tell students to multiply or divide, estimate, and decide where to put the decimal point; they need to see why the procedure works. Students should be able to compute products and quotients of decimals using their own personal paper-and-pencil algorithms; however, they should also know when it is appropriate to use mental procedures or calculators. They should also recognize that the algorithms for computing with decimals are directly related to the algorithms for computing with whole numbers. Students require practice estimating products and quotients as well—this should be done before any procedure is undertaken so results can be tested for reasonableness. Encourage them to think first about the product or quotient if there were no decimals; then to take into account the affect on this solution that the decimals play.

As students model the multiplication and division of decimals by whole numbers, they should be recording symbolically and explaining verbally (both oral and written) each step of the progress. For example, when using the area model, students need to relate the model to the algorithm by explaining each symbolic step in relation to the appropriate part of the rectangular model. Estimation can be used in helping to develop their own personal algorithm. Begin with a context problem, estimate first by rounding to whole numbers, do the computation as if it were whole numbers, and then place the decimal using estimation.

Students are encouraged to invent and use personal algorithms to perform multiplication and division calculations. Personal algorithms can be instructive for solving multiplication and division problems as the student develops a better understanding of place value and why these methods work. Personal algorithms can be more conveniently applied in certain situations of multiplication and division. An algorithm that students create is more meaningful to students and to their understanding. These should be developed through the use of concrete materials as well as their understanding of place-value concepts. The more algorithms students know the more it allows them to choose the most efficient one to solve the problem accurately. The application of various algorithms is useful in mental computations. An example of a personal algorithm might be to determine the partial products of each place value, such as:

$$
\begin{array}{rl}
5.26 & \\
\times \quad 5 & \\
\hline
25 & (5 \times 5) \\
1.0 & (5 \times 0.2) \\
0.3 & (5 \times 0.06) \\
\hline
26.3 &
\end{array}
$$



Using the distributive property, students can multiply in parts
(e.g., $6 \times 24.2 = 6 \times (20 + 4 + 0.2) = 120 + 24 + 1.2 = 145.2$).

With the support of models, such as base-ten blocks, students will learn that the process of dividing decimal numbers by whole numbers is identical to that involving the division of whole numbers. For example, to solve 45.2 ÷ 4, a student might think, if a rod represents one, then I know that 45.2 can be represented using 4 flats, 5 rods, and 2 small cubes. I can share 4 flats among 4 groups. Each group gets 1 flat (one flat represents 10). I can share 4 of the rods with 4 groups. Each group gets 1 rod (one rod represents 1). That leaves 1 rod left and I can trade it for 10 small cubes. I have 12 small cubes that can be shared with 4 groups. Each group gets three small cubes (one small cube represents 0.1, so each group gets 0.3).

This may be recorded as

$$
\begin{array}{rl}
4\overline{)45.2} & \\
\underline{-\quad 40} & \quad (10) \\
5 & \\
\underline{-4} & \quad (1) \\
12 & \\
\underline{-12} & \quad (0.3) \\
0 &
\end{array}
$$

Another personal algorithm for division might be to divide the dividend into parts, for example,
96.6 ÷ 6 = (90 + 6 + 0.6) ÷ 6 =
(15 + 1 + 0.1) =
16.1.

Students should understand that the inverse relationship between multiplication and division applies to decimal numbers.

Example:

- 3 × 2.1 = 6.3 has the inverse relationships of 6.3 ÷ 3 = 2.1 and 6.3 ÷ 2.1 = 3.

In division there is a similar relationship where 5.5 ÷ 5 = 1.1 has the inverse relationship of 5 × 1.1 = 5.5 and 1.1 × 5 = 5.5.

SCO N09 Students will be expected to explain and apply the order of operations, excluding exponents, with and without technology (limited to whole numbers). [CN, ME, PS, T]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

N09.01 Demonstrate and explain, with examples, why there is a need to have a standardized order of operations.

N09.02 Apply the order of operations to solve multi-step problems with or without technology (e.g., computer, calculator).

Performance Indicator Background

N09.01 To introduce students to the idea of the order of operations, provide the class with a question similar to 4 + 8 × 2 – 7 and ask them to find the solution. Ask students to share their solutions and discuss if there are other possible solutions for this problem.

Discuss why people may have different answers for this question. Some may add 4 and 8 and then multiply by 2 and subtract 7 to get 14, whereas some others may add 4 to 8 × 2 and then subtract 7 for an answer of 13. Explain that we need to have rules to make sure everyone is getting the same answer. Many times in real life, people are in a situation where there are various operations to calculate in order to solve the problem. To ensure the correct solution is found, these rules must be followed.

To illustrate the necessity of the rule, pose this problem to students: Mac bought 6 pairs of socks for $7 each and a scarf for $4. How much money did Mac spend?

To find the amount of money spent, the equation could be written as 6 × 7 + 4 or 4 + 6 × 7. Explain to students that in order to find the correct answer, we would have to multiply 6 × 7 first, and then add 4 in order for the amount to make sense. If we have 6 pairs of socks and spent $7 on each pair, we would have to multiply these two numbers. It would not make sense to add 4 and 6 and then multiply this number by 7.

N09.02 Students must be aware that most calculators will not use the order of operations to calculate equations automatically. Therefore, they cannot rely on their calculator to solve problems involving multiple operations. Students will need practice entering the digits on the calculator in the order that the operation should be performed.

Patterns and Relations

SCO PR01 Students will be expected to demonstrate an understanding of the relationships within tables of values to solve problems. [C, CN, ME, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

PR01.01 Generate values in one column of a table of values, given values in the other column, and a pattern rule.
PR01.02 State, using mathematical language, the relationship in a given table of values.
PR01.03 Create a concrete or pictorial representation of the relationship shown in a table of values.
PR01.04 Predict the value of an unknown term using the relationship in a table of values, and verify the prediction.
PR01.05 Formulate a rule to describe the relationship between two columns of numbers in a table of values.
PR01.06 Identify missing terms in a given table of values.
PR01.07 Identify errors in a given table of values.
PR01.08 Describe the pattern within each column of a given table of values.
PR01.09 Create a table of values to record and reveal a pattern to solve a given problem.

Performance Indicator Background

PR01.01 The focus here is to identify patterns that increase or decrease between values within and between the columns of a table of values chart and write the pattern rule. It is important that students first identify the value at which the pattern begins and then indicate the amount they increase or decrease from that given value.

In previous grades, students completed tables of values given simple expressions involving one operation. In Mathematics 6, they will be exposed to expressions that involve two operations, most commonly multiplication or division followed by addition or subtraction. For example, Lauren pays $10 to enter an amusement park. Each ride costs Lauren an additional $2. This relationship can be represented by the pattern rule $2n + 10$. Use this pattern rule to complete the table shown here.

Number of Rides	Total Cost
1	12
2	14
3	?
4	?

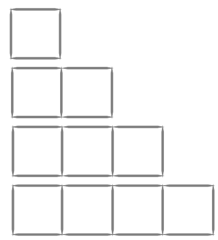
PR01.02 In describing the relationship between the columns of a given table of values, students should be encouraged to use appropriate mathematical language. Invite students to share how they arrived at their conclusions. To create a comfortable environment for students to justify their reasoning, model appropriate language. Brainstorm, with students, other words or phrases that could be used and display these mathematically appropriate words.

Students will identify and explain patterns found in a variety of tables. These patterns can then be used to help students determine an unknown term. Students will be expected to look at patterns functionally. For example, when presented with the table below, students must be able to see that they can determine the value of *A* by adding three to the value of *N*. This is functional thinking.

N	1	2	3	4	5
A	4	5	6	7	8

PR01.03 Give students a table of values and encourage them to visualize the change in each variable, as well as model it pictorially (drawing pictures) or concretely (using manipulatives). Patterns can be modelled using any available manipulatives. For example, students could model the table of values below using toothpicks as shown.

Term #	Term Value
Number of Squares	Number of Toothpicks
1	4
2	7
3	10
4	13



Students will also determine that they can multiply the term number by 3 and then add one ($3n + 1$) to find the term value.

PR01.04 As students explore relationships between numbers in columns of a table of values, they will predict what the missing values might be and determine values for numbers not covered in the table.

PR01.05 In previous grades, students found relationships (pattern rules) within a column and then used that rule to predict terms not in the table of values. This strategy works well when the numbers in the column are in sequence. However, this is not practical when they are asked to find larger terms in the sequence (e.g., find the 50th term in this sequence). Problems such as these will require developing a pattern rule that can be used to determine the value in the second column (term value) based on the corresponding value in the first column (term number).

The objective is that students are able to derive a pattern rule that relates one column of a table of values to the other column.

PR01.06 Students are familiar with using a given pattern rule to complete the right column of a table of values. However, they will now be expected to find missing values in either column of an incomplete table using a given pattern rule. Inverse operations may be addressed here as a possible strategy.

n	*3n*
2	?
4	?
?	18
?	24
10	?

Students will complete the first two missing values by simply applying the pattern rule, $3n$. To find the third and fourth missing values, they will have to work backwards (use the inverse operation). For example, to determine the third missing value, students should think, Three groups of an unknown number gave me 18. If I share 18 into three equal groups, how many will be in each group? (18 divided by 3).

PR01.07 It is important that students are able to identify errors in a given table of values so that they do not extend the pattern incorrectly. They should be able to support their answer. For example, Ravi has a weekly paper route. He gets paid $30 a week. The following table of values shows his earnings over a five-week period. Identify the error in this table.

Number of Weeks	Earnings
1	$30
2	$60
3	$90
4	$100
5	$130

PR01.08 When describing patterns within a given column of a table of values, students sometimes overlook stating the starting value of the pattern. The table below shows changes in the height of a plant over time.

Number of Weeks	Plant Height in cm
1	6
2	10
3	14
4	18
5	22

Most students would describe the change in weeks as "going up by 1" and the change in plant height as "going up by 4." Both of these descriptions do not acknowledge the starting value. A more appropriate description of each would be, "Weeks start at 1 and increase by 1 each time. Height starts at 6 and increases by 4 each time."

PR01.09 Students have previously extended patterns concretely and pictorially. At this point, students must use a pattern rule that relates one column to the other. Creating a table of values to record the pattern is new to Mathematics 6. They may have difficulty determining how each column should be labelled. Students should determine that the values in the second column result from (or are dependent on) the corresponding value in the first column. For instance, in the previous example, plant growth is determined by the number of weeks that have passed.

Students will now be expected to develop an expression and table to solve a given problem. Consider the following example:

- Gloria is going to a community celebration. Admission is $5, and each activity costs $2.
  - (a) Use words to describe how to find the total amount of money Gloria will spend for any number of activities that she may participate in. (Multiply the number of activities by 2 and add 5.)
  - (b) Write an expression to represent the above situation.
  - (c) Use your expression to create a table of values showing how much Gloria will spend if she takes part in 0 to 5 activities.

SCO PR02 Students will be expected to represent and describe patterns and relationships, using graphs and tables. [C, CN, PS, R]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

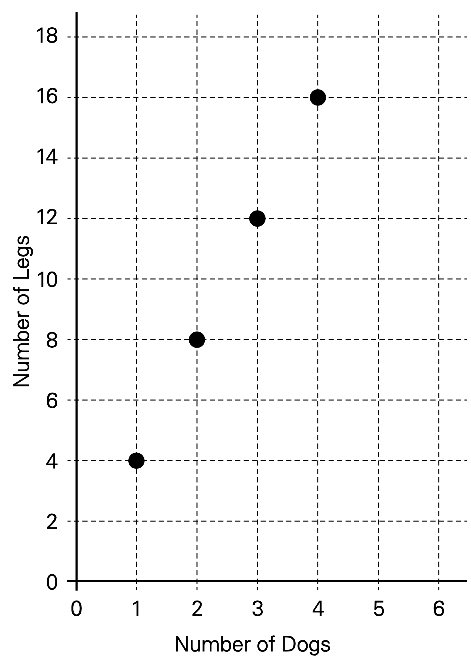
Performance Indicators

PR02.01 Translate a pattern to a table of values, and graph the table of values (limited to linear graphs with discrete elements).
PR02.02 Create a table of values from a given pattern or a given graph.
PR02.03 Describe, using everyday language, orally or in writing, the relationship shown on a graph.

Performance Indicator Background

PR02.01 Students are already familiar with working with tables of values. Now, students will translate a pattern in a table of values by representing that pattern using a linear graph. Linear graphs (see example below) are new for students in Mathematics 6.

Number of Dogs	1	2	3	4
Number of Legs	4	8	12	16



Give students a table of values and ask them to create a graph with the same information. They will label each axis in the same way as the table is labelled. Some simple data that could be used are time and distance information or increasing patterns with linking cubes.

PR02.02 As well as creating graphs from a table of values, students need to be able to create a table of values from a pattern or graph.

Students should be presented with graphs and expected to create tables of values from the graphs. They will need to look for the patterns within the table of values and make the connection between that and the graph.

PR02.03 Students should not only be able to create graphs, but they need to be able to explain what the graph shows. They need to be able to explain what information they get from the graph and what questions can be answered by looking at the graph. They should explain the data and the relationships that are shown. If they are able to explain relationships involving data shown on graphs, they are better able to make connections.

SCO PR03 Students will be expected to represent generalizations arising from number relationships using equations with letter variables.
[C, CN, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

PR03.01 Write and explain the formula for finding the perimeter of any regular polygon.
PR03.02 Write and explain the formula for finding the area of any given rectangle.
PR03.03 Develop and justify equations using letter variables that illustrate the commutative property of addition and multiplication (e.g., $a + b = b + a$ or $a \times b = b \times a$).
PR03.04 Describe the relationship in a given table using a mathematical expression.
PR03.05 Represent a pattern rule using a simple mathematical expression, such as $4d$ or $2n + 1$.

Performance Indicator Background

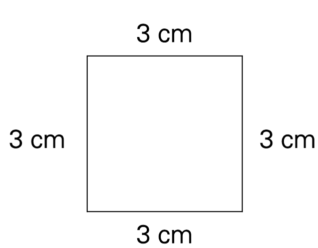
PR03.01 and **PR03.02** Students have learned about perimeter and area in previous grades. At this point, it may be necessary to review area as the amount of flat (two-dimensional) surface within an enclosed shape.

The focus here is on deriving expressions to represent the perimeter of any regular polygon and the area of rectangles, as opposed to actually calculating these values given known measurements.

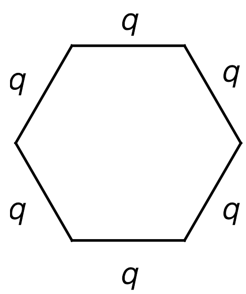
It may be necessary to review the use of square units when measuring area. Students should understand that square units are used to measure the area of 2-D shapes. They will not be expected to understand $n \times n = n^2$ as they will not be dealing with exponents until junior high. However, at this level students may view the exponent of 2 on any squared unit as meaning 2-D. Once students have derived an expression for the area of a rectangle, they will be able to use this formula to find the area of any given rectangle.

Students will develop a number of formulae for measurement in this grade and these formulae depend heavily on an understanding of equality. Students should be able to explain that a formula such as $A = l \times w$ means that the area is equivalent to the measure of the length multiplied by the measure of the width.

In order to develop a formula for finding the perimeter of any regular polygon, provide students with a variety of regular polygons on paper or cardstock. Ask students to identify the dimensions of each of the regular polygons that are congruent. Illustrate that the sum of sides having equal measures can be found through repeated addition and multiplication. For example, in the square (the regular quadrilateral) below, the perimeter may be written as 3 cm + 3 cm + 3 cm + 3 cm or, through multiplication, as 4 × 3cm (four groups of 3 cm) = 12 cm.



In the case where the measure of a side is not specified, students should be to able substitute a variable for the unknown amount and use this to derive their formula. Since the side lengths of a given regular polygon are congruent, the same variable can be used to represent those side lengths. For example, in the regular hexagon below, the side length is unknown so we use the variable *q* to represent it.



Thus, the expression for its perimeter may be written in repeated addition as $q + q + q + q + q + q$ or more conveniently as $6q$.

PR03.03 The commutative property simply shows that the order in which terms are added or multiplied does not affect the final outcome. This can be illustrated to students using real number examples such as,

- $2 + 4 = 6$ and $4 + 2 = 6$
- $5 \times 2 = 10$ and $2 \times 5 = 10$

Students may generalize the commutative property using variables, for example $a + b = b + a$ and $a \times b = b \times a$. “This is a way of saying that no matter what numbers you use to replace *a* and *b*, if you multiply them in one order, you get the same result as multiplying in the other order.” (Small 2008, 583)

PR03.04 Students have worked on writing a word rule to describe the relationship in a given table. Students will now extend this skill and write the pattern that is found in the table using a mathematical expression, or numbers and variables.

PR03.05 Students will take a word rule and use it to generate a mathematical expression using variables. For example, students may be presented with a problem such as, The cost to join minor hockey is $120 per player. Each player must pay an additional fee of $5 for each practice. To represent the total cost for any given player, the word rule would be to multiply the number of practices by $5 and add $120. This can be now written as a mathematical expression, $5p + 120$, where *p* represents any number of practices. Students could then use this expression to generate a table of values showing total costs for various possible numbers of practices.

SCO PR04 Students will be expected to demonstrate and explain the meaning of preservation of equality concretely, pictorially, and symbolically. [C, CN, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

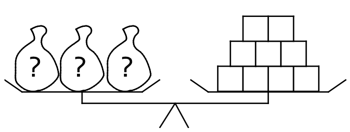
PR04.01 Model the preservation of equality for addition using concrete materials, such as a balance, or using pictorial representations, and orally explain the process.
PR04.02 Model the preservation of equality for subtraction using concrete materials, such as a balance, or using pictorial representations, and orally explain the process.
PR04.03 Model the preservation of equality for multiplication using concrete materials, such as a balance, or using pictorial representations, and orally explain the process.
PR04.04 Model the preservation of equality for division using concrete materials, such as a balance, or using pictorial representations, and orally explain the process.
PR04.05 Write equivalent forms of a given equation by applying the preservation of equality and verify using concrete materials (e.g., $3b = 12$ is the same as $3b + 5 = 12 + 5$ or $2r = 7$ is the same as $3(2r) = 3(7)$).

Performance Indicator Background

PR04.01 and **PR04.02** Students should know the difference between an equation and an expression, but it is still important for them to hear the correct terminology modelled. An equation is a complete number sentence stating that two amounts are the same. Equations must contain an equal sign (e.g., 2 + 3 = 5). A number sentence with a variable is an algebraic equation (e.g., $p + 2 = 3$ reads “Two more than a number is equal to 3.”) This is an algebraic equation because it is stating that an unknown amount plus 2 is the same as 3. An algebraic expression on the other hand is simply a statement without the notion of equivalency. For example, $p + 2$ could be read as, a number increased by two. In this case, the variable p could be any value. The emphasis here is on modelling these equations and showing that if you add or subtract the same amount to/from each side, the equality is preserved.

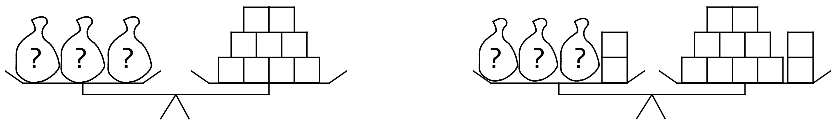
When modelling equations, using the pan balance, bags can be used to represent variables (unknown amounts) and linking cubes or blocks used to represent numbers.

Model a simple equation on a pan balance such as $3n = 9$.



$3n = 9$

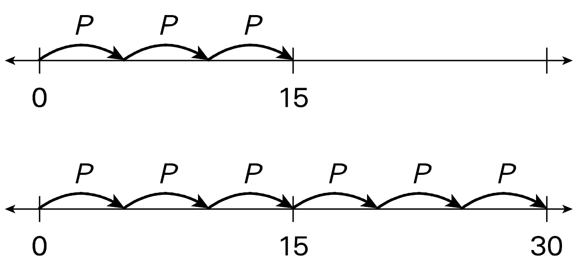
Students can now add a constant amount to each side. No matter how much they add, the scale will remain balanced as long as they add the same amount on each side. This will help students observe how the equality of the two pans (sides of the equation) is preserved.



$3n = 9$ and $3n + 2 = 11$ are equivalent equations.

Once students have had experience creating equivalent equations using the pan balance, additional activities may be modelled using a digital balance from the National Library of Virtual Manipulatives or another online applet, but this should not be a substitute for students using the actual pan balance as a hands-on task.

Equivalent equations can also be modelled on a number line (e.g., Is $3p = 15$ equivalent to $6p = 30$?) Show students a model of $3p = 15$ and $6p = 30$.



These equations are equivalent because the “jumps” are all the same distance.

PR04.03 To ascertain preservation of equality for multiplication, it must be determined whether each side of the equation was multiplied by the same amount. For example, $3n + 2 = 8$ and $6n + 4 = 16$ would both be equivalent equations because all terms on both sides were doubled (×2).

$2n + 3 = 7$ and $6n + 9 = 14$ would not be equivalent because the terms on the left side were tripled ($n \times 3$), but the terms on the right were doubled ($n \times 2$); therefore, equality is not preserved. Use a balance scale to verify.

PR04.05 In order for equations to be equivalent, the same operations have to be performed on each side where the value of the variable does not change. For example, $3n + 1 = 7$ and $3n = 6$ are equivalent equations because 1 is subtracted from each side in the first equation to make the second equation. This is called “preservation of equality.”

Measurement

SCO M01 Students will be expected to demonstrate an understanding of angles by ▪ identifying examples of angles in the environment ▪ classifying angles according to their measure ▪ estimating the measure of angles using 45°, 90°, and 180° as reference angles ▪ determining angle measures in degrees ▪ drawing and labelling angles when the measure is specified [C, CN, ME, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

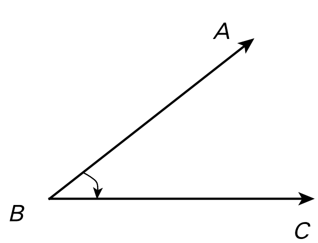
Performance Indicators

M01.01 Identify examples of angles found in the environment.
M01.02 Classify a given set of angles according to their measure (e.g., acute, right, obtuse, straight, reflex).
M01.03 Sketch 45°, 90°, and 180° angles without the use of a protractor, and describe the relationship among them.
M01.04 Estimate the measure of an angle using 45°, 90°, and 180° as reference angles.
M01.05 Measure, using a protractor, given angles in various orientations.
M01.06 Draw and label a specified angle in various orientations using a protractor.
M01.07 Describe the measure of an angle as the measure of rotation of one of its sides.
M01.08 Describe the measure of angles as the measure of an interior angle of a polygon.

Performance Indicator Background

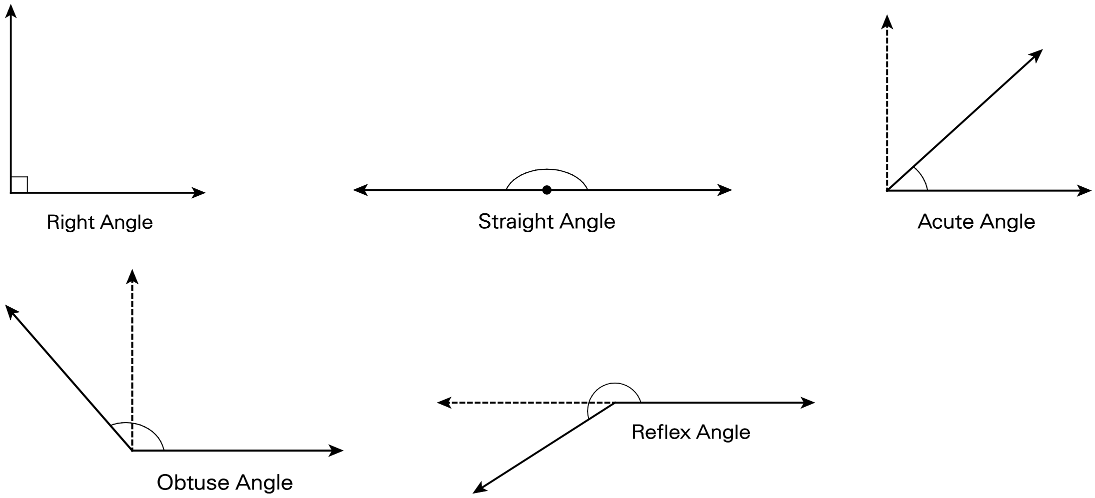
M01.01 Initial exploration of angles will simply involve students identifying two line segments connected at a shared point and thus forming an angle in their surrounding environment. Examples in the classroom may include door and window frames (right angles), adjacent floor tiles (right or straight angles) or hands of a clock (any angle depending upon the given time).

Students need to recognize an angle as a figure formed by two line segments or rays with a common endpoint called a vertex. The line segments or rays are called the arms of the angle. To name an angle, use the angle symbol ($\angle$) followed by the letter representing the vertex. It is often necessary or more efficient to use three letters ensuring that the vertex is the middle letter.



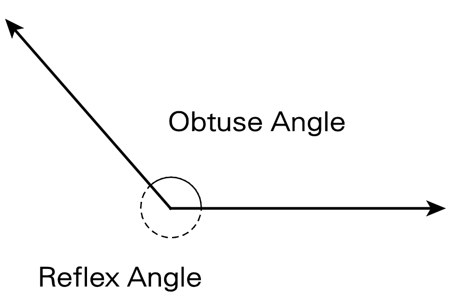
In a diagram as above, angles are usually marked with an arc except for a 90° angle, which has a special notation. The above angle could be named $\angle B$, $\angle ABC$, or $\angle CBA$.

M01.02 As students are familiar with the one-quarter, one-half, and three-quarter turn benchmarks for rotations, and with right angles, it will now be necessary to introduce names of other angles that fall between these benchmarks.



- Right Angle
  - two rays that intersect to form a square corner
  - a right angle has a measure of 90°
  - it is a quarter-turn
- Straight Angle
  - two rays that intersect to form a straight line
  - a straight line, made by two right angles, has a measure of 180°
  - it is a half-turn
- Acute Angle
  - an angle with a measure less than 90° and more than 0°
  - it is less than a quarter-turn
- Obtuse Angle
  - an angle with a measure less than 180° and more than 90°
  - it is more than a quarter-turn but less than a half-turn
- Reflex Angle
  - an angle with a measure less than 360° and more than 180°
  - it is more than a half-turn but less than a full-turn

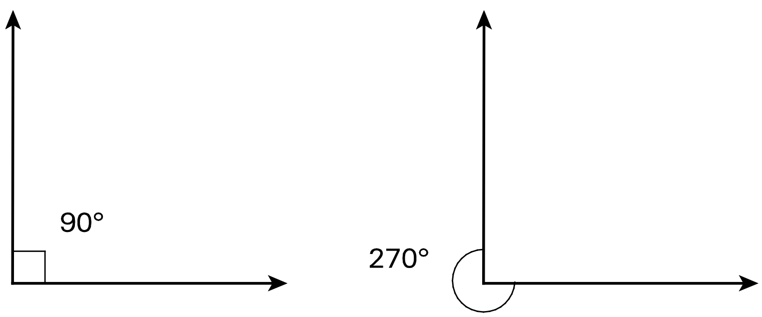
Some students will likely make the extension that an acute or obtuse angle also has a reflex angle on the outside of its arms. It may be appropriate to address this point as a class. Any given angle has a second angle outside of its arms (the remainder of the circular rotation).



M01.03 In addition to estimating the measure of an angle using the 45°, 90°, and 180° benchmarks, students will be sketching these angles without the use of a protractor and describing relationships among them.

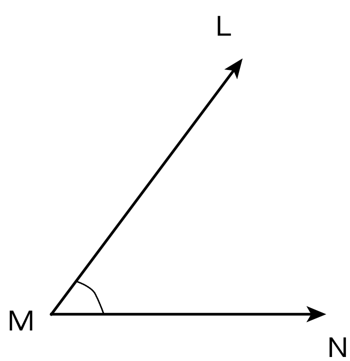
Having estimated angle measures previously, students should be able to visualize each of the 45°, 90°, and 180° benchmarks. They can now use these benchmarks to sketch angles without the use of a protractor.

To avoid confusion as to which angle is being referred to in a particular drawing, a directional arc is drawn between the two arms. Note that a special mark, a small square, is used to indicate a right angle.



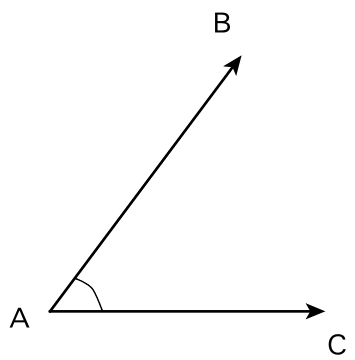
The arc between the arms of the angle above indicates that this drawing is showing a 270° angle and not a 90° angle.

This would also be an appropriate time to introduce the correct method of naming angles. When an angle is presented with three labelled points, the angle would be named using the angle symbol $\angle$ and the three points written with the vertex in the middle.



This angle could be correctly named $\angle LMN$ or $\angle NML$, because in each case the vertex point is in the middle of the name. Emphasize that writing the points in alphabetical order is not a requirement and will not necessarily be correct in all cases, depending on the letter name of the vertex.

This angle could be named $\angle BAC$ or $\angle CAB$. Writing the points in alphabetical order ($\angle ABC$) in this case would be incorrect, as the vertex would not be in the middle.

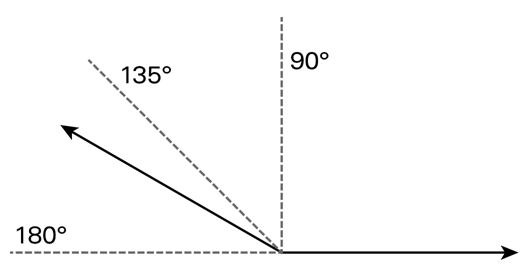


M01.04 The focus in Mathematics 6 is to develop benchmarks for angles of measure and estimate the size of angles based on these benchmarks. Students will become proficient with benchmarks for 45°, 90°, and 180°.

Visualizations of the 45°, 90°, and 180° benchmarks and their combinations can now be used to estimate the measures of other angles.

For example, estimate the measure of the angle below.

Students may visualize or sketch benchmark angles on a separate sheet of paper or directly on the given angle, as shown. The angle in question is between 135° and 180°. Its measure might be estimated at somewhere between 150° to 160°.



Establish the fact that the measure of rotation for a full turn within a circle will always be 360°. This is true regardless of the size (diameter) of the circle. Students will likely have heard "360" as a stunt performed by snow boarders, skateboarders, ice skaters, etc., where the athlete makes a complete spin and ends up facing forward in his/her original position. This may be a good starting point for the introduction of this concept. Students will also likely be familiar with the stunt called a "180," where the athlete performs a half spin and ends up facing opposite the original position. Hence, the term "180" is used because the athlete rotated his/her body in a half circle, and 180° is half of 360°.

Once it has been established that a full turn has a measure of 360° and a half turn measures 180°, the benchmarks of 90° and 45° can be easily introduced. Since a half turn has a measure of 180°, a quarter turn (half of 180°) would thus have a measure of 90° and an eighth of a turn (half of 90°) would measure 45°.

These benchmarks may also be established through student construction of a protractor. Once the 45°, 90°, and 180° benchmarks have been established, students can use these to estimate the measure of other angles.

M01.05 Introduce the concept of the degree. Students will be familiar with the use of metric units such as cm, m, and mm for measuring linear distance. A degree is the unit used to measure rotation within a circle and subsequently can be used to measure angles. Some students will likely attempt to make an association between degrees of a circle and degrees Celsius or Fahrenheit used for measuring temperature. It may be necessary to differentiate between the two. Although the word **degree** is used in both cases, they are units of measurement for two entirely unrelated quantities.

Students often determine the measurement of angles a difficult task because the degree units are so very small coupled with the many lines and the double numbering running clockwise and counter-clockwise on most protractors. Before beginning to use a standard protractor, it is useful to move from the non-standard unit wedges to involving students in making their own protractors. Students are expected to engage in hands-on experiences with angle measurement.

Teachers can provide students with semicircular shapes cut from tracing paper, or construction paper. (The tracing paper would allow students to see the angle vertex and follow the arms in order to measure more easily.) Students fold the semicircle in half, forming a right angle. The teacher explains that angles are measured in degrees and that a right angle is made up of 90 of these degrees. Have this fold named 90°. Ask students to fold once again and determine, and name the size of the new angles generated by the folds. One further fold would provide angles halfway between 0° and 45° and 45° and 90°. Discuss the measurement of these folds with the class and how these can assist with estimation of angle sizes.

Students should be able to estimate angles, within 5–10 degrees of their actual size. To estimate well, students need to have a sense for the sizes of a few angle benchmarks such as 45°, 90°, and 180°. Provide students with geo-strips, pipe cleaners, or straws that can be used to make or show angles. Angles that students are measuring should be drawn in a variety of positions and with arms of varying lengths. Students should share, discuss, and explain their strategies for estimating and measuring angles.

Once students have had practice estimating and measuring the size of given angles using their homemade protractors, they should learn how to use standard protractors to measure angles reasonably accurately. Since these protractors have double scales, students will need to determine which number to use in a given situation. This is best accomplished by having students first estimate the size of the angle and use this estimate to decide which reading makes the most sense. Initially, concentrate on drawing angles between 0° and 90° and then on angles between 90° and 180°. In order to obtain an accurate measure, students should be aware of the importance of positioning the initial arm of the protractor so that it coincides with the first arm of the angle and the centre point on the protractor so it coincides with the vertex of the angle. Again, angles should be drawn in a variety of positions and with arms of varying lengths. Students should share, discuss, and explain their strategies for estimating and measuring angles.

Building upon the benchmarks and estimation skills developed previously, students will now use an actual protractor to measure angles. The initial use of a circular (360°) protractor rather than a semi-circular (180°) protractor is recommended. This will reinforce students' understanding of an angle as a rotation within a full circle.

Students will use a protractor showing all 360 degrees of a circle (full turn) to measure angles to the nearest degree. As this is the first time students have used a protractor, it will be necessary to explain that the degrees on the protractor are only labelled in multiples of ten; however, each unlabelled tick represents 1°, thus allowing them to measure angles between the labelled values.

Not all angles will be oriented with one horizontal arm. This may present a challenge for some students. In any case, the centre point of the protractor must be placed on the vertex of the angle and the zero degree mark of the protractors baseline must be lined up with one of the angle's arms. If this orientation of the angle presents a problem, the page or book may be turned so that one arm becomes horizontal from the students' point of view.

When measuring angles using a protractor, it is important that the reading always begins at zero. Incorrect measures often result from students beginning at the 180° mark. Emphasize to students that they must always line up one arm of the angle with the zero degree and then begin counting up from zero until they reach the next arm. This will avoid incorrect scale readings.

It is also important to recognize that the direction of rotation between angle arms will determine which scale is used on the protractor. Again, lining up the zero degree with the first arm of the angle and counting up to the next arm will avoid any confusion with regard to which scale should be used. Remind students that the arc (hatch mark) between the arms of the angle indicates which angle is being measured.

Estimating angle measures before using the protractor is important in helping students avoid mistaken scale readings. For example, A student might estimate angle $\angle ABC$ to have a measure of a little more than 90°. If they measure the angle and mistakenly read the protractor scale as 88° as opposed to the correct measurement of 92°, they should recognize that this measurement does not agree with their estimation and is probably incorrect.

M01.06 After students have learned how to use a protractor to measure angles; they now will use the protractor to construct angles. Initially, concentrate on having them draw angles between 0° and 90° and then on angles between 120° and 180°. For most students, this is probably their first experience viewing a straight line as an angle with a measure of 180°. In order to produce an accurate drawing, students should be aware of the importance of positioning the 0o on the protractor so that it coincides with the first arm of the angle and the centre point on the protractor so it coincides with the vertex of the angle.

Previously, students have sketched angle approximations using benchmarks. At this point, they will construct angles with a specified angle measure using a protractor and straightedge.

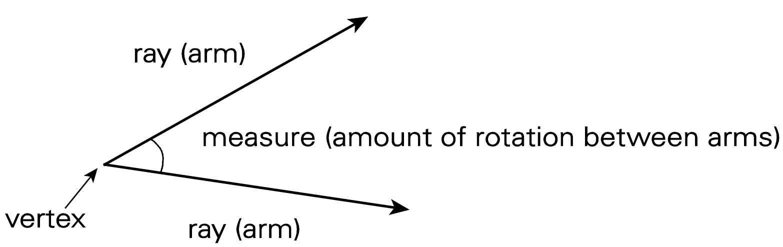
After students have become proficient measuring angles with a protractor, ask them to reverse the process to construct angles with a specified measure using a ruler and protractor. When completed, students should measure their angle using the protractor to ensure that they completed the construction correctly. Discuss the process that was used to construct the angles with the class.

Students may find it easier to use a semicircular (180°) protractor to construct angles to an accurate degree. It is important to note that as with some circular protractors, there is an inner and outer scale, and that to avoid confusion between the two, students must always start measuring from the 0° mark. Ask students to sketch an approximation of the angle using benchmarks before attempting the actual construction. A mental visualization of this approximation may be sufficient for some students at this point.

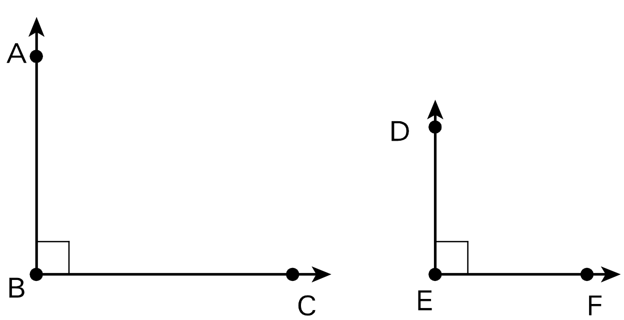
When constructing angles, students should always use a straight edge to construct the initial arm. This arm may be horizontal unless otherwise specified. The centre mark of the protractor would then be placed on the endpoint of the initial arm and the baseline rotated so that the 0° mark is in line with that arm. Keep in mind that the direction of rotation will determine whether the inner or outer scale of the protractor will be used and indicate from which 0° mark (left or right) measurement will begin. Students should then count up from zero to the desired measure, mark that degree, and connect it to the vertex using a straight edge. The angle must be indicated by drawing an arc between the two arms, and any given points should be labelled correctly remembering that the vertex is always found in the middle of the angle name when using the three-letter naming method. Students need many opportunities to practice drawing angles of different measures and orientations in order to attain proficiency.

M01.07 In Mathematics 5, students had some experience with right angles and using fractions of a circle as benchmark angles. Up to this point they have been exposed to quarter, half, three-quarter, and full turns. The measuring of angles in degrees and the subsequent use of a protractor will be new concepts in Mathematics 6. The concept of a circle being comprised of 360° will serve as a convenient starting point for measuring angles in degrees.

It may be necessary to review the concept of an angle as being the amount of rotation between two rays (arms) joined at a shared point (called the vertex). The amount of rotation required to get from one arm of the angle to the other is the angle's measure. The length of the arms does not affect the angle's measure.



For example, if students were asked which of the angles below has the greatest measure, some might respond that angle *ABC* is larger than angle *DEF*. In actual fact, both angles have the same measure (the amount of rotation between the arms is the same).



Some computer programs and on-line applets are available that could allow students to examine the effect of various angle turns. For example, students can rotate an arm to form an angle that is 50° and note that the size of the required turn to complete the straight angles is 130°. This will help them experience first-hand the connection between the two angle measures on their protractors.

SCO M02 Students will be expected to demonstrate that the sum of interior angles is 180° in a triangle and 360° in a quadrilateral.
[C, CN, ME, PS, R, V]

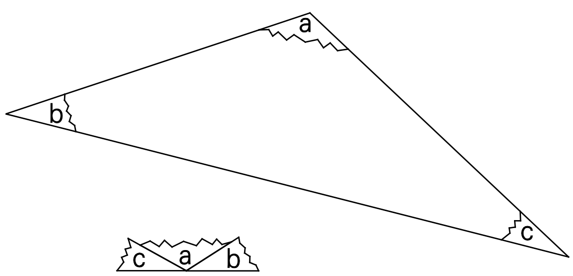
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

M02.01 Explain, using models, that the sum of the interior angles of a triangle is the same for all triangles.
M02.02 Explain, using models, that the sum of the interior angles of a quadrilateral is the same for all quadrilaterals.

Performance Indicator Background

M02.01 To illustrate that the sum of the interior angles of a triangle is the same for all triangles in a visual/tactile manner, create several large triangles of varying shapes and sizes on construction paper or card stock. Ask students to tear the corners off each triangle. Arrange the three corners so that they are all joined at a common vertex to form a straight angle. Students will observe that when the three corners are arranged at a common vertex, the three angles form a half circle (180°).



Tearing off the corners of the triangles and labelling the vertices with letters or symbols will avoid confusion as to which vertices need to be adjacent when aligning the angles. The curved edges also provide a clearer visual for students that the three interior angles of any given triangle form a half circle.

It may be worthwhile to ask students to measure the interior angles of the triangles they used in the above task using a protractor and then find the sum. They may notice that in some cases their sum does not total 180° exactly, but is very close. This would be a good opportunity to discuss sources of human error in measurement.

By using a variety of triangles of various sizes in these explorations, students should conclude that the sum of the interior angles of any triangle will always be 180°.

This concept can also be reinforced using interactive whiteboard technology, computer software, or interactive websites such as Angle Sums found on the National Council of Teachers of Mathematics *Illuminations* site (http://illuminations.nctm.org/Activity.aspx?id=3546). If using interactive whiteboard technology, use the triangle-shape tool to create a triangle of any size or shape. Using the angle measure tool, label the measure of each interior angle. The program will automatically place the correct measurement within the angle. Ask students to drag each vertex of the triangle to a different position on the interactive whiteboard to create a triangle of a new shape and size. The angle measure tool will automatically change the measure of the angle as the student drags the vertex. Students will observe that no matter how they change the shape and size of the triangle, the sum of the interior angles will always be 180°.

Extend this concept to find a missing angle in a given triangle when two other angle measures are known. By subtracting the sum of the two known angles from 180°, the unknown angle measure can be determined.

M02.02 After establishing that the sum of interior angles for any triangle is 180°, students could use this information to explore the sum of the interior angles of any quadrilateral. They must first recognize that any given quadrilateral is composed of two triangles. This can be reinforced using tangram pieces. By combining two tangram triangles to form a quadrilateral, students should deduce that since the sum of the interior angles of a triangle is 180°, then the sum of the interior angles of the quadrilateral would be 360° (180° + 180° = 360°).

Perhaps the most straightforward manner in which to demonstrate this concept is to ask students to draw a diagonal connecting opposite vertices of various quadrilaterals. Students can be provided with these quadrilaterals on construction paper or card stock or they may create their own. Students can then cut the quadrilateral along the diagonal they drew producing two triangles. Having already learned that the sum of the interior angles of a triangle is 180°, they can make the deduction that because any quadrilateral can be deconstructed into two triangles, the sum of the interior angles of any quadrilateral must be 180° + 180° = 360°.

This concept may be approached in a similar fashion as triangles were previously. Provide students with various quadrilaterals. Ask students to tear the corners off each quadrilateral and then arrange them so that their vertices are adjacent. Students will observe that the four angles of any quadrilateral form a full circle (360°).

Interactive whiteboard technology, computer software, or interactive websites may also be used to illustrate this concept. If using interactive whiteboard technology, use the quadrilateral-shape tool to draw a quadrilateral of any shape and size on the interactive whiteboard. Label the measure of the interior angles using the angle measure tool. Ask students to physically change the position of the vertices to create quadrilaterals of different shapes and sizes. The angle measure tool will automatically show the change in each angle, and students will observe that the sum of the four angles of any quadrilateral will always be 360°.

Extend this concept to find unknown angles within quadrilaterals given the measure of any three angles. By subtracting the sum of the three known angles from 360°, the unknown angle measure can be determined.

SCO M03 Students will be expected to develop and apply a formula for determining the ▪ perimeter of polygons ▪ area of rectangles ▪ volume of right rectangular prisms [C, CN, ME, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

M03.01 Explain, using models, how the perimeter of any polygon can be determined.
M03.02 Generalize a rule (formula) for determining the perimeter of polygons.
M03.03 Explain, using models, how the area of any rectangle can be determined.
M03.04 Generalize a rule (formula) for determining the area of rectangles.
M03.05 Explain, using models, how the volume of any rectangular prism can be determined.
M03.06 Generalize a rule (formula) for determining the volume of rectangular prisms.
M03.07 Solve a given problem involving the perimeter of polygons, the area of rectangles, and/or the volume of right rectangular prisms.

Performance Indicator Background

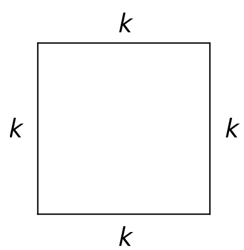
M03.01 Students have been introduced to perimeter in previous grades as the distance around a shape. Perimeter can be illustrated by having a student walk around the edge (perimeter) of the classroom, or trace the edges of their desk or textbook cover with their finger. Although measuring perimeter is often thought of as being different from linear measurement, it is actually only a variation. Students are measuring a linear distance that is not just a straight line. Students began in previous years using indirect measuring (i.e., using a string to measure around a shape, cutting the string, and then measuring the length of the string). Now students will measure directly and add the side lengths. When students investigate the distance around a polygon, they will produce their own formulas for perimeter.

Perimeter can be initially modelled and determined by using drawings of various polygons on centimetre grid paper and using the number of units along the combined sides to find the perimeter. However, modelling of this nature would be restricted to squares, rectangles, or combinations of these as the sides of these polygons will align with the lines on the grid paper.

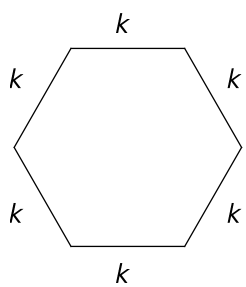
Students will also find perimeter of polygons by measuring the sides of the polygon and finding the sum.

M03.02 The focus here is on deriving a formula that can be used to generalize the process by which the perimeter of any given polygon can be found. In order to generalize a formula for determining the perimeter of a rectangle, students can be presented with a variety of rectangles. Examining the rectangles, students identify which side lengths are congruent. The sum of these congruent sides can be found through repeated addition ($w + w + l + l$) and by multiplication ($2 \times w$ and $2 \times l$) .Thus, the generalized formula for determining the perimeter of the rectangle would be expressed as Perimeter = $2w + 2l$. It may be necessary to discuss with students the convention of writing multiplication involving a variable and a number without using the multiplication symbol. For example, $2 \times w$ would be written as $2w$. This is the form most used in later grades.

In the case of equilateral triangles, squares, or other regular polygons having equal side lengths, there will be no actual length and width dimensions, as all side lengths are congruent. In this case, the same variable may be used to represent each of the congruent side lengths. For example, in the square below the perimeter may be calculated as 3 cm + 3 cm + 3 cm + 3 cm or, through multiplication, 4 × 3 cm (Four groups of 3 cm) = 12 cm. The generalized formula for determining the perimeter of a square could be expressed as $4k$ or $k + k + k + k$.

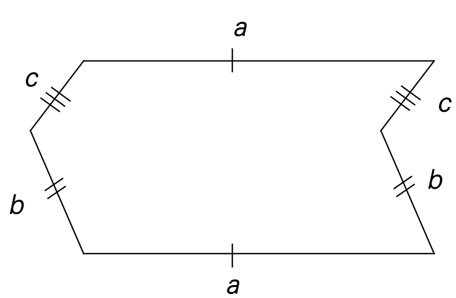


In the case where the measure of a side is not specified, students should be able to substitute a variable for the unknown amount and use this to derive their formula. For example, in the regular hexagon below, the side length is unknown, so we use the variable k to represent it.



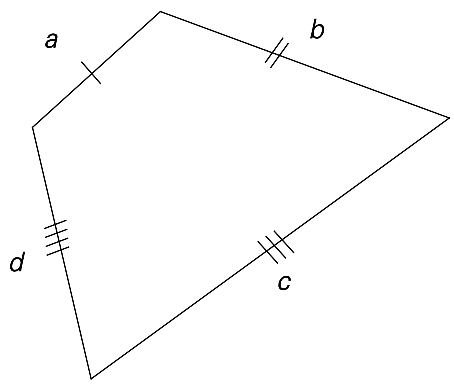
Thus, the expression for the perimeter of regular hexagons may be written in repeated addition as $k + k + k + k + k + k$ or more efficiently as $6k$ because all six side lengths are congruent.

For polygons with two or more different side lengths, addition of the combined congruent dimensions will be required as in the example below.



The expression for the perimeter of the irregular hexagon shown above would be
Perimeter = $2a + 2b + 2c$.

Problems of this nature will not always necessarily involve regular polygons. For example, the quadrilateral below has no side lengths congruent. The expression for the perimeter of this quadrilateral would be written as Perimeter = $a + b + c + d$.



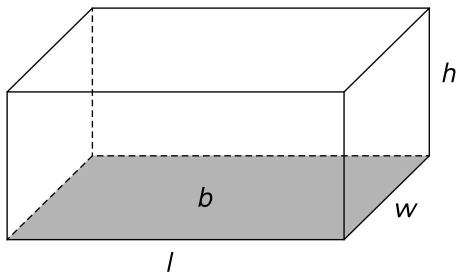
However, as long as students can recognize congruent sides, a formula should be easily derived.

M03.03 and **M03.04** It is important to approach this as an investigation rather than providing a rote formula. Students should be given the opportunity to discover how to find the area of a rectangle independently. Teachers should try to elicit from the students ways to find the areas of squares and rectangles by working with a variety of materials. Teachers should not simply provide the formula—multiplying the length by the width or $A = l \times w$.

Provide students with several pictures of rectangles, all of which have whole number dimensions and drawn on square grid paper. Students should be directed to determine the areas of these rectangles by their own methods. Many will just count the squares. This should be followed by a discussion emphasizing that rectangles can be filled with square centimetres in rows and columns and the connection to arrays and the area model in multiplication. Then provide students with 1 cm × 5 cm grid strips and ask them to use only these strips to find the areas of other rectangles. They should see that these strips could be used to find in each rectangle the number of square centimetres in each row and the number of rows. Discussion again should centre around the idea that if we know the number of centimetres in the base and how many rows it takes to cover a rectangle (i.e., the height), we can find the area by multiplying the base by the height. Finally, have students use rulers to find how many square centimetres would occupy the first row in a rectangle—indicated by its length in centimetres—and how many rows of these square centimetres there would be—indicated by its width in centimetres. Multiplying these two values should then be apparent; therefore, the formula for the area of a rectangle is $A = l \times w$, or $A = w \times l$, or $A = b \times h$ (see explanation below). Students should be able to express this rule in words and as a formula written using variables to represent the changing quantities.

- In words: The area of a rectangle can be found by multiplying its length by its width.
- As a formula: $A = l \times w$, where A = area, l = length, and w = width.

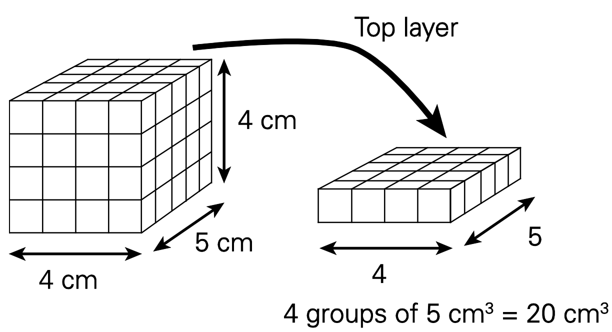
Note: Failure to conceptualize height in 2-D shapes and 3-D objects has been a frequent common error in the understanding of formulae. In a 2-D shape, any side can be called a base. For each base, there is a corresponding height. The height is the distance from the side or point opposite the base drawn perpendicular to the base. For rectangles, the height is exactly the same as the length. Therefore, the formula $A = l \times w$ for rectangles is equivalent to the formula $A = b \times h$. This second formula, $A = b \times h$, will be very useful in later grades in developing the area of parallelograms and triangles and also in the development of many of the volume formulae. Therefore, it is now easier to connect many of our formulae that otherwise might be developed independently.



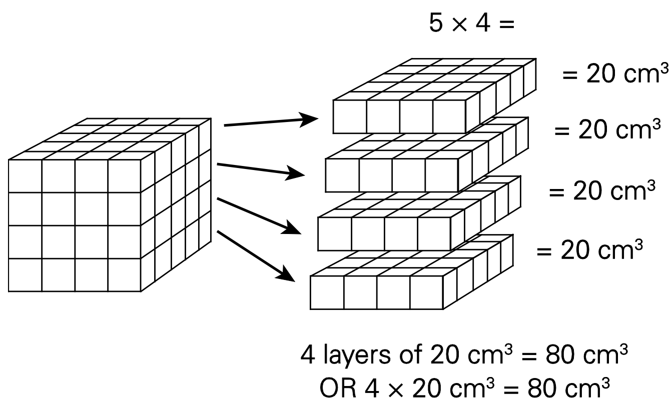
M03.05 and **M03.06** Students have been introduced to volume as the amount of three-dimensional space occupied by an object. Students should be reminded that volume is measured in cubed units, such as mm^3, cm^3, and m^3. Again, because students have not yet been introduced to exponents, they may think of the exponent 3 as meaning three-dimensional.

Use cubic centimetre blocks to model right rectangular prisms. As each cube has a volume of 1 cm^3, the total volume can initially be found by counting the number of cubes in the prism. As this will be very time consuming for larger prisms, it will be necessary to derive a formula to find its volume.

Begin by establishing the three dimensions of the prism. This is done simply by counting the number of cubes along the length, width, and height. Keep in mind that these dimensions are linear distances and are not measured in cubed units. The height of the prism will indicate how many layers of cubes are in the prism. Students will be familiar with the concept of layers in a three-dimensional solid from working with base-ten flats. First, students find the volume of one layer. For example, in the prism below students would determine that the top layer is composed of four rows of five 1-cm^3 blocks. Therefore the volume of the top layer would be 20 cm^3 (4 × 5 = 20).



Students should then draw the conclusion that because there are four layers and each has a volume of 20 cm^3, the total volume of the prism must be 80 cm^3 (four groups of 20 cm^3).



From work with finding volume from the models previously used, students will come to the conclusion that the volume of a given right rectangular prism can be found by finding the number of cubes in one layer (length × width), and then multiplying the volume of one layer by the number of layers (height).

Thus, the general formula $V = l \times w \times h$ can be derived, where V = volume, l = length, w = width, and h = height.

Students should also be able to interpret this formula in words as “The volume of a rectangular prism can be found by multiplying the length by the width by the height.”

Remind students of the commutative property and that the formula may be written with the three dimensions being multiplied in any order.

For example,
$V = l \times w \times h$
$V = w \times h \times l$
$V = h \times l \times w$

M03.07 This will involve the application of concepts previously developed to solve problems. It is important for students to understand that in many cases solving a complex problem involves the process of solving several simpler problems and that it may take several steps to solve the whole problem.

Geometry

SCO G01 Students will be expected to construct and compare triangles, including scalene, isosceles, equilateral, right, obtuse, or acute in different orientations. [C, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

G01.01 Sort a given set of triangles according to the length of the sides.
G01.02 Sort a given set of triangles according to the measures of the interior angles.
G01.03 Identify the characteristics of a given set of triangles according to their sides and/or their interior angles.
G01.04 Sort a given set of triangles and explain the sorting rule.
G01.05 Draw a specified triangle.
G01.06 Replicate a given triangle in a different orientation and show that the two are congruent.

Performance Indicator Background

G01.01 In Mathematics 5, students categorized quadrilaterals according to side length and pairs of parallel lines. Students will expand their knowledge of properties to categorize triangles based on side length and interior angles. Begin by exploring side lengths of triangles and naming the triangles as scalene, isosceles, and equilateral.

To begin classifying triangles, focus on side length. There are three classifications when naming triangles according to their side lengths:

- Scalene—no equal sides
- Isosceles—two equal sides
- Equilateral—three equal sides

Students should learn that not just any combination of three lengths can become the side lengths of a given triangle. The sum of the two shorter sides of a triangle must be greater than the length of the longest side; otherwise the three sides could never connect. If students struggle with remembering the names for each triangle, a strategy such as the following may help. Place the three triangle names in alphabetical order. Then apply the 3, 2, 1 rule: the first triangle (equilateral) has three equal sides, the second (isosceles) has two equal sides, and the third (scalene) has one, or no equal sides.

G01.02 Students will focus on identifying the interior angles of a triangle as right, obtuse, or acute, using 90° as a benchmark. Ask students to identify objects in the classroom that could be used as a 90° reference point (e.g., the corner of a sheet of paper, a textbook, the corner of a ruler). Students can use the object to identify the angles of a triangle as acute (smaller than the 90° angle), obtuse (larger than the 90° angle), or right (equal to the 90° angle).

At this point, introduce another classification of triangles, based on their interior angles:

- a right triangle has one 90° angle.
- an acute triangle has all angles less than 90°.
- an obtuse triangle has one angle greater than 90°.

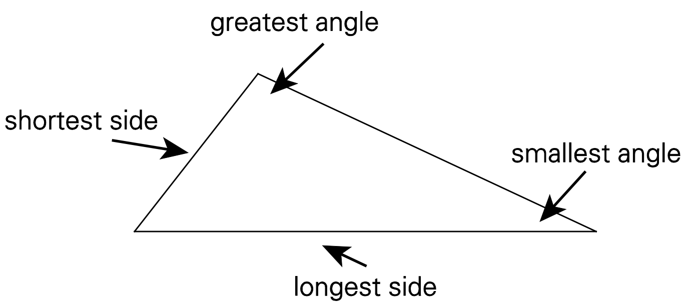
Students can investigate the remaining angles in a right triangle and an obtuse triangle to determine what types of angles they must be.

Exploration should lead to the discovery of the angle relationships in equilateral triangles and isosceles triangles. The definition of equilateral triangles, isosceles triangles, and scalene triangles should then be expanded to include interior angles.

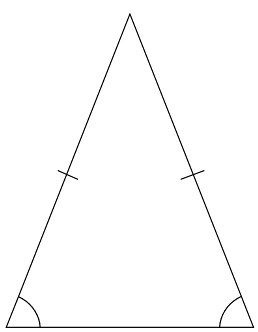
- Equilateral triangle—has all sides equal and all angles equal
- Isosceles triangle—has two equal sides and two equal angles
- Scalene triangle—has no equal sides or equal angles

G01.03 Having an understanding of the properties of triangles is beneficial when drawing triangles. These properties include the following:

- The greatest angle is opposite the longest side, and the smallest angle is opposite the shortest side.



- The sum of the two shorter sides must be greater than the longest side.
- When two angles of a triangle are congruent, then the sides opposite them are congruent (and vice versa).
- The sum of the interior angles of any triangle is 180°.
- A triangle can never have more than one obtuse angle or one right angle.



Students should be encouraged to use appropriate hatch marks on triangles to indicate when sides and angles are equal.

G01.04 Give students the opportunity to sort a set of triangles and complete a table similar to the one below:

Type / Triangle	Equilateral Triangle	Isosceles Triangle	Scalene Triangle	Right Triangle	Acute Triangle	Obtuse Triangle
A						
B						
C						
D						

During this task, students should discover that the triangles will have two classifications. They can be classified according to their side lengths and angles. For example, a right triangle can also be an isosceles triangle, but an equilateral triangle can never be right or obtuse. Students should be allowed time to explore which combinations of right, acute, and obtuse triangles and scalene, isosceles, and equilateral triangles are possible. Those that exist are right isosceles, right scalene, acute isosceles, acute equilateral, acute scalene, obtuse isosceles, and obtuse scalene.

G01.05 Students need step-by-step instruction when they are first learning to draw triangles. It may be beneficial to ask students to begin by drawing a line segment and adding a specified angle to one of its ends. Then progress to asking students to draw a triangle with one angle specified. As students become more comfortable with this, add more specifications to the instructions. For example, give them two line segments and one angle, or two angles and one line segment, to include in the triangle.

When drawing triangles, students should be able to identify the unspecified angles and line segments. They should recognize that being given two angles and one side length results in a unique triangle. For example, if two students are asked to draw a triangle with a side length of 3 cm and angles measuring 40° and 70°, they will draw the same triangle. That is, their resulting triangles will be congruent, acute isosceles triangles. The orientation may be different, and students may need to be reminded that a change in orientation does not make it a different triangle. Unique triangles are also produced when two sides and the contained angle are specified, or when three sides are specified.

It is important that students be able to draw triangles of any specified type (e.g., acute scalene).

G01.06 Students will build upon their existing knowledge of transformational geometry and congruency. Students were exposed to congruent line segments and congruent sides of regular polygons.

Using geo-boards and geo-bands, students could work in pairs to construct the same triangle on each of their geo-boards. Students should notice that after rotating one of the geo-boards a **one-quarter** turn (90°), the rotated triangle has not changed, but its orientation is different. If students have not already suggested that the triangles are congruent, review the concept of congruency with them, referring to their triangles. Congruent figures have exactly the same size and the same shape. They can have different orientations and still be congruent.

There are a number of ways students can demonstrate the congruency of triangles. This concept can include congruency of angles and side lengths. One way to demonstrate congruency is to superimpose an image. This can be done using tracing paper, cut-outs, or a Mira in conjunction with transformations. Another way to demonstrate congruency is to measure the side lengths and angles using a ruler and a protractor.

SCO G02 Students will be expected to describe and compare the sides and angles of regular and irregular polygons. [C, PS, R, V]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

G02.01 Sort a given set of 2-D shapes into polygons and non-polygons and explain the sorting rule.
G02.02 Demonstrate congruence (sides to sides and angles to angles) in a regular polygon by superimposing.
G02.03 Demonstrate congruence (sides to sides and angles to angles) in a regular polygon by measuring.
G02.04 Demonstrate that the sides of a regular polygon are the same length and that the angles of a regular polygon are the same measure.
G02.05 Sort a given set of polygons as regular or irregular and justify the sorting.
G02.06 Identify and describe regular and irregular polygons in the environment.

Performance Indicator Background

G02.01 and **G02.05** Students were exposed to sorting polygons in earlier grades. This should now be extended to include sorting 2-D shapes into polygons and non-polygons. It may be necessary to revisit the definition of a polygon. A polygon is a closed 2-D figure bound by straight line segments that intersect at the vertices. Naming polygons may also require some review.

- Triangles are three-sided polygons.
- Quadrilaterals are four-sided polygons.
- Pentagons are five-sided polygons.
- Hexagons are six-sided polygons.
- Heptagons are seven-sided polygons.
- Octagons are eight-sided polygons.
- Decagons are ten-sided polygons.

During the sorting of polygons, there are instances when students can rely on visual cues when it is clear that a polygon is irregular. However, when determining if a figure is regular, students should be encouraged to measure angles and/or side lengths.

Sorting polygons provides an opportunity for students to revisit the use of Carroll diagrams and Venn diagrams. Provide students with a sorting template and a number of polygons and non-polygons that they can place into their template. Ask students to cut out polygons and glue them into the appropriate section of the sort mat. Ask them to explain their sorting rule.

G02.02 and **G02.03** There are a number of ways students can demonstrate the congruency of regular polygons. This concept can include congruency of angles and side lengths within a single polygon or between sets of polygons. One way is to superimpose an image. This can be done using tracing paper, cut-outs, or a Mira in conjunction with transformations.

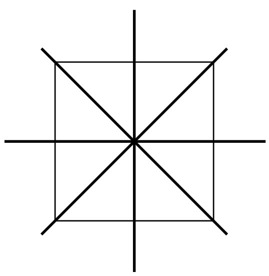
Provide students with two congruent regular polygons of differing orientations and some tracing paper. Students can trace one of the polygons and then place the tracing on top of the second polygon. The two shapes will match.

Another way to demonstrate congruency is to measure the side lengths using a ruler and the angles using a protractor.

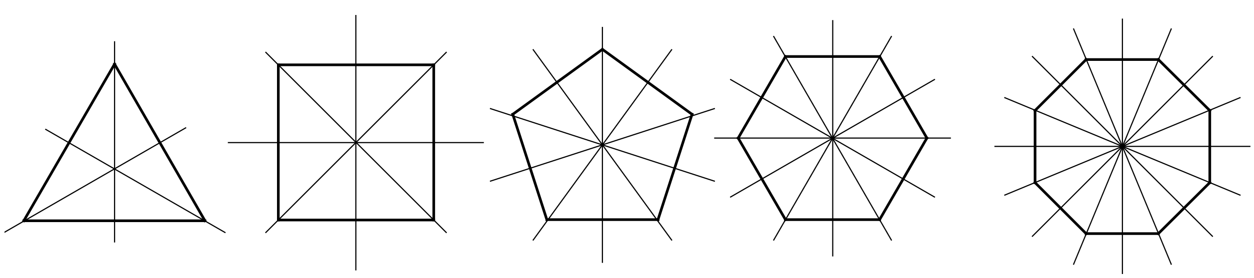
G02.04 The definition of a regular polygon should be developed through a discovery approach. Display several regular polygons and ask students to measure the angles and side lengths, and discuss what they notice. This should highlight that, in regular polygons, all angles are equal and all side lengths are equal. Discussion will lead students to conclude that if all angles of a polygon are equal then all sides are also equal, and vice versa.

Also include symmetry in the definition of a regular polygon. Students were exposed to line symmetry in Mathematics 4. Students should understand that a line of reflective symmetry is the fold line where a polygon is folded onto itself so that each half matches exactly, or is the line where a mirror or Mira, can be placed so that the reflection on one side matches the shape on the other.

In Mathematics 6, students will recognize that reflective symmetry is a characteristic of some polygons, but not others. This can be described by stating how many lines of reflective symmetry a polygon has. After investigations with Miras and folding, students will find that a square has four lines of reflective symmetry as shown below.



Students should discover a pattern showing that the number of lines of reflective symmetry for a regular polygon is the same as the number of sides of that polygon.



G02.06 Initiate a discussion about examples of regular and irregular polygons in the environment. Provide students with a placemat template (regular paper or chart paper to display in the classroom). Ask students to brainstorm and record (by drawing or writing) as many examples of polygons that they see or use in the world around them. Each group can share their “placemat” findings with the rest of the class.

SCO G03 Students will be expected to perform a combination of translation(s), rotation(s), and/or reflection(s) on a single 2-D shape, with and without technology, and draw and describe the image. [C, CN, PS, T, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

G03.01 Demonstrate that a 2-D shape and its transformation image are congruent.
G03.02 Model a given set of successive translations, successive rotations, or successive reflections of a 2-D shape.
G03.03 Model a given combination of two different types of transformations of a 2-D shape.
G03.04 Draw and describe a 2-D shape and its image, given a combination of transformations.
G03.05 Describe the transformations performed on a 2-D shape to produce a given image.
G03.06 Model a given set of successive transformations (translation, rotation, or reflection) of a 2-D shape.
G03.07 Perform and record one or more transformations of a 2-D shape that will result in a given image.

Performance Indicator Background

G03.01 Demonstrate congruency of transformations to students using pattern blocks. Have students choose one shape and a transformation. Ask them to perform the chosen transformation. Ask students to check an image’s congruency by tracing it and overlaying it on the pre-image. Students will see that the size and shape is maintained. If not, this is an indication that there is an error in their creation of the image or they have performed the transformation incorrectly. Students will discover that the images are congruent because the transformations do not change the size or form of the shape.

G03.02 Combining transformations is a new concept for Mathematics 6 students. After working with each of the three transformations, combining transformations of the same type should become a natural progression for students. When students are given a succession of transformations, they should focus on one transformation at a time and recognize that each transformation in succession would be applied to the image resulting from the previous transformation. At this point, students will combine only like transformations.

G03.03 During modelling, place an emphasis on which image has undergone a transformation. Students must be aware that when performing a combination of transformations the second transformation is performed on the first image, not on the original shape.

As students begin working on modelling and performing given combinations of different types of transformations, it is suggested that they limit their work to combining two different transformations only. Generally students will require practice in this area in order to successfully model a combination of transformations on their own.

Ask a student volunteer to identify a single transformation. Another student would perform this transformation on an overhead or an interactive white board. Invite other students to provide different directions to transform the resulting image. Once three or four combinations of different transformations have been used, invite students to brainstorm other transformations that could be used to get the original shape to its final image.

Repeat this task using two other transformations.

G03.04 Students have had practice drawing and describing single and combined like transformations and now will apply these skills and strategies to drawing and describing a 2-D shape when given a combination of different transformations. When students are drawing combined transformations, encourage them to appropriately label their images. For example, when transforming *ABC*, its image after the first transformation should be labelled $A'B'C'$. The second image should be labelled $A''B''C''$, and so on.

G03.05 Students have been describing successive like transformations. Now they will describe successive transformations of all three types. When describing the transformations performed on a shape, students should be encouraged to use appropriate mathematical language. Review with students the appropriate description for each type of transformation.

Some students may be able to describe combinations of differing transformations and can quickly move on to identifying these combinations.

Students may describe any number of combinations of transformations. Students may present differing ways to identify how an image was obtained; a specific number of transformations used cannot always be determined or, in fact, if a combination was used.

Investigate questions, such as
- If a shape undergoes 2 translations, does it matter in which order they take place?
- Could this image have been obtained by a single transformation?

G03.06 Students have worked on single transformations and performing a combination of like transformations. Now, focus is on performing a combination of different transformations on a shape. Combinations should include
- a reflection followed by a translation
- two translations
- two reflections
- a translation followed by a rotation
- two rotations

Remind students when performing a combination of transformations to focus on only one transformation at a time where each new transformation is applied to the previous transformed image. Each new image should be labelled with an additional prime symbol at each vertex.

G03.07 Provide each student with a coordinate grid (first quadrant only) and pattern blocks. Ask each student to carry out two transformations of their choice on the grid and leave only the first and third blocks in place. Ask them to exchange grids with a partner and predict the two transformations that took place. Share their predictions and actual transformations.

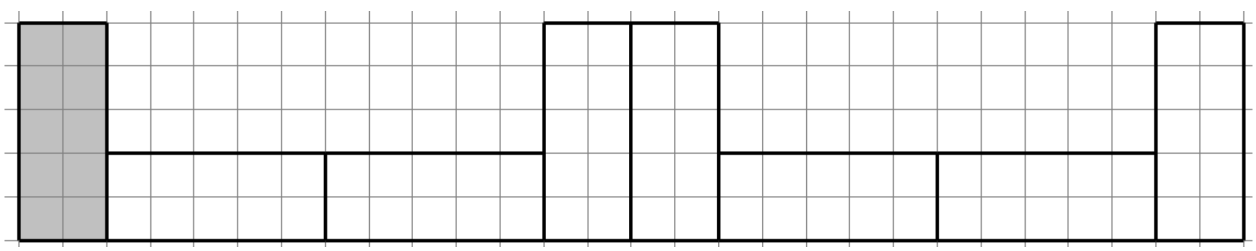
SCO G04 Students will be expected to perform a combination of successive transformations of 2-D shapes to create a design and identify and describe the transformations. [C, CN, T, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

G04.01 Analyze a given design created by transforming one or more 2-D shapes, and identify the original shape and the transformations used to create the design.
G04.02 Create a design using one or more 2-D shapes and describe the transformations used.
G04.03 Describe why a shape may or may not tessellate.
G04.04 Create a tessellation and describe how tessellations are used in the real world.

Performance Indicator Background

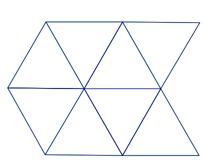
G04.01 Demonstrate a design that could be created using a combination of transformations. Discuss the various transformations that could have been applied to create this design.



Start with a design with a single object and gradually increase in complexity. Include samples found in classroom borders, wallpaper borders, wallpaper, material, mats, rugs, cushion flooring, etc. In these materials, the design is often repeated. Initiate a class discussion about how transformations can be used to create various designs, such as company logos and symbols.

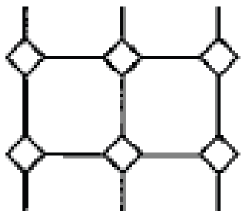
G04.02 Student should create designs, using pattern blocks or other 2-D materials, which involve a number of transformations. The student would then describe the transformations they used to create the design including direction, distance, turn, and/or line of reflection. Students could record the design using dot paper.

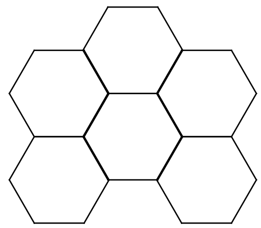
G04.03 and **G04.04** A 2-D figure is said to tessellate if an arrangement of replications of it can cover a surface without gaps or overlapping. For example,



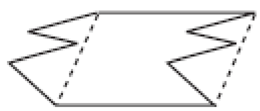
If a number of triangles in the pattern blocks were used, you would be able to use them to cover a surface; therefore, this triangle is said to tessellate.

Students should investigate which pattern blocks tessellate and record and explain how and what transformations are used in the design. Investigations should include regular and irregular polygons that tessellate and those that do not tessellate like pentagons and octagons. The octagon is the shape often used in flooring and tiles where squares fill the gaps because octagonal tiles won't tessellate. Concrete materials work best for these investigations, paper or acetate copies can be used.

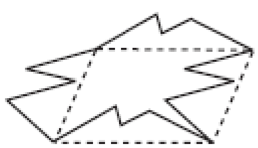




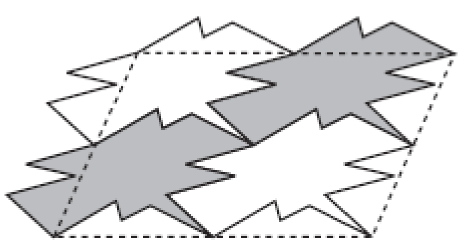
M.C. Escher, as graphic artist, is well-known for his tessellations, which often take the shape of various objects. Explain that these designs are created by using transformations or by combining compatible polygons. Invite students to create their own tessellation designs. For example, ask students to construct a parallelogram, and then construct a random shape on the left side of the parallelogram. Have them slide the random shape so that it is also on the right side, as shown.



Ask students to construct a random shape on the bottom and slide it to the top, as shown.



Translate the new polygon to create tessellations.



This can be done using pencil-and-paper techniques or by using software. This particular task can help students see that mathematics can be used in other areas, such as visual arts.

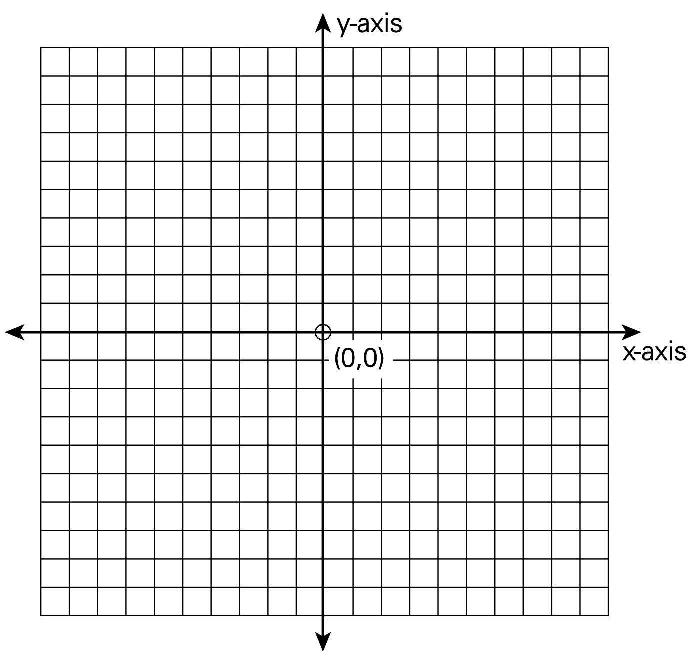
SCO G05 Students will be expected to identify and plot points in the first quadrant of a Cartesian plane using whole number ordered pairs. [C, CN, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

G05.01 Label the axes of the first quadrant of a Cartesian plane and identify the origin.
G05.02 Plot a point in the first quadrant of a Cartesian plane given its ordered pair.
G05.03 Match points in the first quadrant of a Cartesian plane with their corresponding ordered pair.
G05.04 Plot points in the first quadrant of a Cartesian plane with intervals of 1, 2, 5, or 10 on its axes, given whole number ordered pairs.
G05.05 Draw shapes or designs in the first quadrant of a Cartesian plane, using given ordered pairs.
G05.06 Determine the distance between points along horizontal and vertical lines in the first quadrant of a Cartesian plane.
G05.07 Draw shapes or designs in the first quadrant of a Cartesian plane, and identify the points used to produce them.

Performance Indicator Background

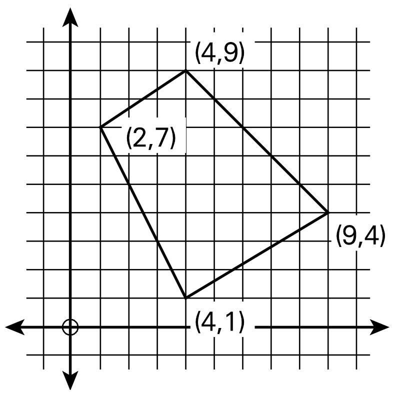
G05.01 It is important that you draw attention to the Cartesian plane. Students will be working with one quadrant of four quadrants on the Cartesian plane, making the link to the notion that a Cartesian plane is created when two lines, one horizontal and one vertical, meet at a point called the origin.



Lead a discussion on the concept of a coordinate grid, focusing on the visual aspect of the grid. Explain to students that a coordinate grid is another name for the quadrants of a Cartesian plane. It is used to locate points on the plane.

Model for students that in order to construct a picture of a coordinate grid, they need to draw a horizontal number line, called the *x*-axis and a vertical number line called the *y*-axis. These two lines intersect at a point called the Origin (0, 0). Use a coordinate grid on the board while you discuss these concepts with the class. Be sure to label the axes correctly. A common error students make when labelling the axes is putting the number in the middle of the blocks. This causes problems when plotting points.

G05.02 Model plotting and identifying the coordinate points corresponding to the vertices of a given 2-D shape. Students are expected to identify the coordinates of the vertices of shapes drawn on a coordinate plane. Identify the coordinates of vertex *A*. Vertex *A* is named by (2,3).



The first number in an ordered pair tells the horizontal distance from the origin and the second number in the ordered pair tells the vertical distance from the origin.

A common error when identifying and plotting points is to reverse the order of the *x*-coordinate and the *y*-coordinate. Encourage students to always label the *x*- and *y*-axes of a Cartesian plane to avoid making this mistake.

G05.03 An ordered pair on a quadrant is sometimes given a letter name to identify the point in a quadrant. That is, a letter name is used in place of the coordinate pair. You may want to ask students to plot points on a grid using coordinate pairs and then give the points letter names. This can be completed as a whole group using the overhead projector or interactive white board.

This work may be connected to SCO G06.

G05.05 and **G05.07** Students are now familiar with finding and plotting points on a Cartesian plane. Students will learn to draw designs, shapes, or block letters on a grid. Encourage students to be creative in their design. They will plot the points then connect them to complete the image. As an extension, they can give other students the coordinates for the points on their image and have others create the same image on their own grid.

Using the interactive white board or an overhead projector, plot points on a grid and join the points together to create a closed figure. Ask students to label the points and to name the figure. You could also recite coordinates and ask students to plot the points on their own grid paper.

When you join points together on the graph, you are creating a line segment. Draw the students' attention to this fact. We can use line segments to measure the distance between two points on a Cartesian plane.

This is a good way to encourage interaction and to motivate the class to focus on points on the grid and finding coordinates easily.

G05.06 Students need to focus on the distance between points along each of the horizontal and vertical lines. They often make the mistake when counting spaces to include the points of each square, instead of the number of squares between the points. Some practice may be needed.

Relate the movement along the axis as being similar to making jumps along a number line.

Give grids to the students with several points along horizontal and vertical lines. Ask them to count the distance between these points. They can work in pairs. This may not take a lot of practice, but it is important that they understand how to measure the distance.

SCO G06 Students will be expected to perform and describe single transformations of a 2-D shape in the first quadrant of a Cartesian plane (limited to whole number vertices). [C, CN, PS, T, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

G06.01 Identify the coordinates of the vertices of a given 2-D shape (limited to the first quadrant of a Cartesian plane).

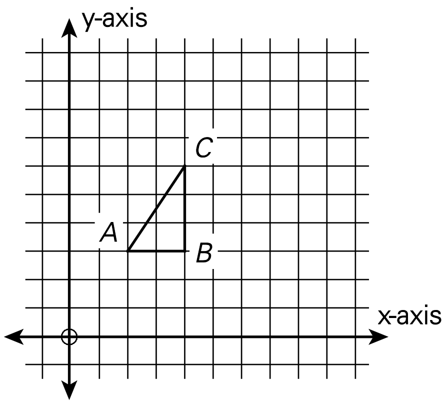
G06.02 Perform a transformation on a given 2-D shape, and identify the coordinates of the vertices of the image (limited to the first quadrant).

G06.03 Describe the positional change of the vertices of a given 2-D shape to the corresponding vertices of its image as a result of a transformation (limited to first quadrant).

Performance Indicator Background

G06.01 As students begin working on various transformations, they will benefit from working with hands-on materials, such as pattern blocks or attribute blocks, to physically manipulate each block as indicated by the transformation. When using symmetrical shapes to apply transformations, it may be a good idea to highlight or mark one of the vertices so students can indicate the orientation of the image. Students should also be given ample opportunities to work with less symmetrical shapes where it is easier to identify the effect of the transformations.

Students will continue their study of transformations by learning about transformations on a coordinate grid. Students should already be familiar with key terms such as **translation**, **reflection**, and **rotation**, and with terms such as **coordinate plane**, **ordered pairs**, **origin**, ***x*-axis** (horizontal axis), ***y*-axis** (vertical axis), ***x*-coordinates**, and ***y*-coordinates** from previous work on data relationships. Continued use of this terminology is very important. Model plotting and identifying the coordinate points corresponding to the vertices of a given 2-D shape. Students are expected to identify the coordinates of the vertices of shapes drawn on a coordinate plane. For example, vertex *A* is named by (2, 3).



A common error when identifying and plotting points is to reverse the order of the *x*-coordinate and the *y*-coordinate. Encourage students to always label the *x*- and *y*-axes of a Cartesian plane to avoid making this mistake.

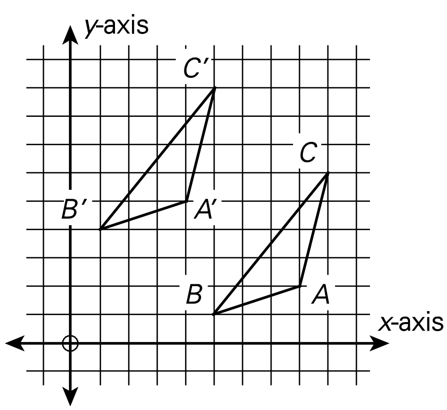
G06.02 Remind students to label the vertices of the 2-D shapes (pre-image), for example, A, B, C, D, and the corresponding vertices of the image using prime notation, A', B', C', D'. Students should then identify the coordinates of the vertices in the image and the pre-image using this notation.

G06.03 When describing the positional change of the vertices of a given 2-D shape (pre-image) to the corresponding vertices of its image as a result of a translation, students should keep in mind the following:

- the shape and its image will have the same orientation
- all vertices move together
- each vertex moves the same way
- if the translation is
  - to the left, the *x*-coordinate will decrease
  - to the right, the *x*-coordinate will increase
  - downward, the *y*-coordinate will decrease
  - upward, the *y*-coordinate will increase

Based on the transformation shown below, students should be able to

- describe the translation (left 4, up 3)
- describe the change in the *x*-coordinates of the vertices (They decreased. They are 4 less.)
- describe the change in the *y*-coordinates of the vertices (They increased by 3.)



When describing the positional change of the vertices of a given 2-D shape (pre-image) to the corresponding vertices of its image as a result of a reflection, students will note that

- the shape (pre-image) and its image are of opposite orientation
- a 2-D shape (pre-image) and its image are congruent
- there is an equal distance from the mirror line to each of the vertices of a 2-D shape (pre-image) and its reflected image
- when reflecting a 2-D shape in a horizontal line of reflection, the *x*-coordinates of the vertices of the image do not change, but the *y*-coordinates do
- when reflecting a 2-D shape in a vertical line of reflection, the *y*-coordinates of the vertices of the image do not change, but the *x*-coordinates do
- when reflecting a 2-D shape in a diagonal line of reflection, both the *x*- and *y*-coordinates of the vertices of the image change

When describing the positional change of the vertices of a given 2-D shape (pre-image) to the corresponding vertices of its image as a result of a rotation on a vertex, students will note that

- all vertices move together $\frac{1}{4}$ (90°), $\frac{1}{2}$ (180°), or $\frac{3}{4}$ (270°) of a turn in the same direction, either clockwise or counter-clockwise
- the shape (pre-image) and its resulting image are congruent
- the shape (pre-image) and its image have a different orientation

Statistics and Probability

SCO SP01 Students will be expected to create, label, and interpret line graphs to draw conclusions. [C, CN, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

SP01.01 Determine the common attributes (title, axes, and intervals) of line graphs by comparing a given set of line graphs.
SP01.02 Determine whether a given set of data can be represented by a line graph (continuous data) or a series of points (discrete data) and explain why.
SP01.03 Create a line graph from a given table of values or a set of data.
SP01.04 Interpret a given line graph to draw conclusions.

Performance Indicator Background

SP01.01 and **SP01.02** Students often see things differently and they may not use the same scales and/or titles for the same graphs. The information may be the same but the way in which they represent it may be different.

Discuss with the class the importance of the title, axis, and intervals when working with graphs. Discuss with students that they may have to adjust these attributes to fit the data they are analyzing. When constructing a graph, ask students to determine if the attributes they have chosen are appropriate and ask them to justify their choices.

Discuss with students the importance of scale. An inappropriate scale can skew (distort or depict unfairly) the data and be misleading. Provide examples to reinforce the importance of scale. Stress that although different graphs can show the same data, one graph may be a better choice to answer a particular question. A scale has to represent the data accurately. Remind students that the scale is related to number lines. They should look at the smallest value and the largest value in their data and then determine the scale to be used for the graph. Expose students to the fact that the horizontal axis is called the *x*-axis and the vertical axis is called the *y*-axis.

Provide opportunities for students to learn the difference between continuous data and discrete data. If the data are continuous, the points on the graph are joined. The points on a line are connected when all the values between the points are permitted. Discrete data is a series of points that are not joined. When data are discrete, there are numbers between those given that are not meaningful in the context of a problem. For example, consider the points (1,3) and (2,6) plotted on a graph. These points can be joined if they represent distance against time, since distance could include values between 3 and 6 and time could include values between 1 and 2, such as 1.5. However, if the graph represents costs against the number of DVDs rented, the points should not be joined since it is not possible to rent 1.5 DVDs.

- Examples of continuous data—The temperature of a given day at each hour of the day. You can predict what the temperature would be at 9 p.m. based on the temperature trend over the day.
- Examples of discrete data—The average monthly temperatures of a particular place over a year. You cannot predict the temperature for next month based on the temperature for the previous month because it changes.

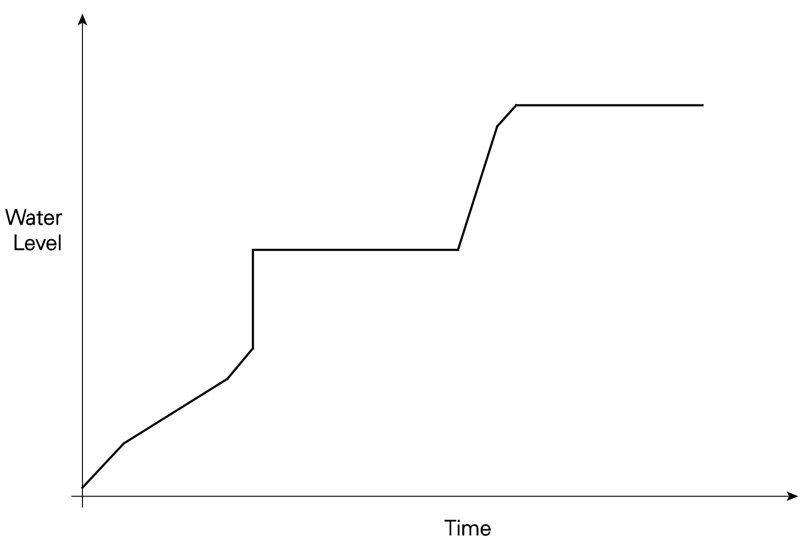
SP01.03 After you have exposed students to various graphs, ask them to create their own. Points are plotted to represent two related pieces of data, one of which is often time. Line graphs should have a title that clearly explains what the graph shows, clearly labelled axes, and a clear scale. A dashed line indicates that the data is discrete, that is the data between the points has no meaning, and a solid line indicates that the data is continuous or discrete.

If students measure the temperature outside every hour during a school day, they could create a graph where the ordered pair is (hour number, temperature). By connecting the points with lines, they see the trend in the temperatures throughout the day.

Once students have a conceptual understanding of graphs, they should use technology to construct graphs. The technology will allow them to focus on the data instead of worrying about plotting points and drawing straight lines.

SP01.04 Students should look at and analyze various line graphs that display data on different topics. They should distinguish between continuous and discrete graphs. Have a whole-group discussion about the differences between continuous and discrete data and when to use each type of graph.

The analysis of graphs should include creating “stories” or real-world situations that describe the relationship depicted. Similarly, when constructing graphs, a story that matches the changes in related quantities should be included. When students are describing a relationship in a graph they should use language such as, “as this increases that decreases”; “as one quantity drops, the other also drops.” Students are usually intrigued by unusual graphs. For example, the graph below displays the level of water in a bathtub as someone takes a bath.



In this type of situation, students should each tell a “story” about the graph by describing what they think happened to lead to the shape of the graph.

Students should be drawing inferences from conventional graphs and tables. These could include, among other things, predictions of values not actually gathered but in intervals between values that were gathered and predictions of values in intervals before and/or after values that were gathered. If, for example, a line graph displays the number of millilitres of rainfall between 12 noon and 4 p.m. for actual measurements made every 30 minutes, students might be asked to find the rainfall at 2:45 p.m.

and to find the rainfall at 4:30 p.m. They should be aware of the assumption(s) being made when they find these values.

Teachers need to emphasise the analysis of the graphs in ways that will cause students to do more than merely read information from them. They should be starting to analyze data displays to draw conclusions, to make decisions, or to stimulate other questions.

In reading and making inferences from graphs, students need to pay particular attention to things such as the actual range of the scale and whether or not the scale begins at zero. The nature of the vertical or horizontal scale can profoundly affect the conclusions drawn. Students can also explore how different scales used for the same set of data can create very different impressions.

SCO SP02 Students will be expected to select, justify, and use appropriate methods of collecting data, including questionnaires, experiments, databases, and electronic media. [C, PS, T]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

SP02.01 Select a method for collecting data to answer a given question, and justify the choice.
SP02.02 Design and administer a questionnaire for collecting data to answer a given question, and record the results.
SP02.03 Answer a given question by performing an experiment, recording the results, and drawing a conclusion.
SP02.04 Explain when it is appropriate to use a database as a source data.
SP02.05 Gather data for a given question by using electronic media, including selecting data from databases.

Performance Indicator Background

SP02.01 Students have had many experiences collecting data in their early years in school; most will have found the favourite foods of other students, lengths of names, or what pets their classmates have. This type of data collection should continue. Students should be aware that there are many ways to collect data and that these various methods may provide slightly different results. For example, students could consider the difference in data collected about favourite foods if they simply ask each classmate to list his/her favourite, as opposed to offering a choice of three foods and asking students which of the three they prefer; they might also consider the difference in results concerning favourite foods if they collect data right before lunch rather than at another time of the day.

Many of the data collection experiences in the earlier grades included surveys of entire populations. Now students should recognize that sampling procedures might be necessary in collecting data. To gather information about a large population, when it is impossible to check every person involved, samples are used. Using information gathered from such samples, generalizations are made about the entire population. However, it is recognized that conclusions drawn from samples may not always be true for the population, but in order to minimize the degree of error, care is taken in the selection of samples. For example, we cannot generalize about the average income of the working population by surveying doctors and lawyers. A random sample is a sample collected from a population so that every member of the population has an equal chance of being selected and the members of the sample are chosen independently of each other. Class discussion should focus on when it would be appropriate to survey a sample versus the entire population.

Discussion should address aspects of the concept of bias in sampling, the idea of a sample being representative of the entire population, and the ways sample size might affect the data. Students should consider both how to choose samples and how safe it is to generalize to the full populations. For example, suppose students wanted to determine people's favourite take-out food. They should understand that it would not be wise to choose a sample of patrons of Pizza Palace because that sample could be biased in favour of pizza. In choosing a sample, students should carefully consider the information being sought and how people answering questions could be biased. For example, if students want to find which radio station is most popular, they should probably consider the mix of ages within the sample, the gender distribution within the sample, the availability of a variety of stations to those sampled, and the time of day. A sample should be constructed to deal with potential biases. Discussion should also focus on whether a particular group is biased or unbiased and be able to justify the decision. When provided with a set of data, students should take time to consider the best way to organize it. For example, if the information collected is about pets, they might have to consider whether to have a category for each different exotic pet or one category called “other”; they might have to decide whether to list the number of different pet owners as opposed to the number of different pets, depending on the proposed use of the data. Choices must also be made about the format of presentation (e.g., tables, graphs or descriptive displays), which might also influence the way students decide to organize the data in the first place.

First-hand data is data that is collected and used for the purpose for which it was collected. Data that may have been collected for one purpose but is used for some secondary purpose is second-hand data. Students should understand that when data is needed for decision making, it is not always necessary to collect it from original sources as the data may already exist to meet the need. Good sources of data include Statistics Canada, government records and published reports, and town offices. When using secondary source data, students should still be encouraged to consider the nature of the sample used so that they can check for bias.

Class discussion should focus on the possible ways of collecting data and the advantages and disadvantages of various hypothetical situations. Such advantages/disadvantages include cost, availability of target groups, and suitability of the collection process given the nature of the desired data. The following are some possible data collection methods: questionnaire, phone interview, personal interview, probability experiment, extraction of second-hand data, and timed sampling. These techniques should be applied as part of small-group projects. Students should be able to justify their selection of a means of data collection by identifying and comparing the advantages and disadvantages of various methods.

Give students opportunities to experiment with organizing and displaying data in a wide variety of ways. This can lead to discussions about which methods of data organization and display are the most effective and the easiest to understand.

In previous grades, students were exposed to collecting information through first-hand data and second-hand data using questionnaires. Remind students that questionnaires are one way to gather information and brainstorm other methods for gathering information. They will come up with several examples from previous years of working with data. Some ideas include observation, surveys, interviews, polls, past records, searching the Internet, and simulations.

Have a discussion about whether a questionnaire has multiple choice responses or Yes/No responses. Another way to gather information is through an interview, where the person doing the interview can probe for more information. Students may also suggest that they could also do an experiment or use databases and/or electronic media.

Write various questions on chart paper and have a discussion about whether or not these are good questions and justify the answers.

Encourage students to think about what they already know about posing questions. Your questions could be related to social studies with reference to community—numbers of police officers, restaurants, recycling centres, etc.

Use teacher-created graphs or graphs from different sources (newspapers, Statistics Canada, social studies textbook, etc.) and ask students to generate questions based on that data. These questions will help students to think about the information on the graph and guide them in analyzing that information.

SP02.02 Brainstorm different topics for which they would like to create a survey or questionnaire. Ask students to select one topic on which they will gather information. They will create the survey questions or questionnaire, carry out the survey, and record the results. Where possible, students should be encouraged to carry out school-wide surveys or even community surveys. These surveys should be shared with the class. Discuss the various questionnaires and how whether their method of gathering information was effective.

SP02.03 The focus is on using experiments to collect data. Have a discussion about experiments and when and why we would carry out experiments. Most students would think of science experiments, which explain particular scientific processes.

Make the association that experiments can be conducted to investigate concepts such as which brands of particular products are best (e.g., which paper towel absorbs the most water).

We can use experiments to gather information. We can then analyze this information to make choices or to determine if one factor affects another (the age of a hockey player to the number of goals scored in a season). Students need to look at graphed data and be able to make inferences and draw conclusions about the information. For this performance indicator, teachers could show students several graphs that represent the results of experiments and ask them to answer specific questions about what they see.

Sample Tasks:
- What do you notice about the results? Is there another method you could use to answer the same question?
- Why was this experiment a good method to find out the answer to your question?
- Why would you not use another method of collecting your data?

SP02.04 Have a discussion about databases and what kinds of information would be contained within a particular database. Students need to know that a database is used for large amounts of data and that the type of database they choose to explore will depend on the type of question they want answered (e.g., NHL, music data, Statistics Canada). Students could use a database to look for information from the past or to look for information that covers a particular period of time.

Brainstorm topics where students might need to search a database to gather information. Allow students to visit particular databases so they can see how the information is stored, and how it is organized.

Ask students where they might search to find data about the number of school-aged children in their province.

SP02.05 Present students with a variety or topics for which they can research information. For example, use the Weather Network website to investigate how the high and low temperatures in a given area have changed over the last ten years.

Encourage the use of the Statistics Canada website to find information on a given topic. A sample topic could be the number of immigrants that came to Canada in each of the last five years.

SCO SP03 Students will be expected to graph collected data and analyze the graph to solve problems. [C, CN, PS]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

SP03.01 Determine an appropriate type of graph for displaying a set of collected data and justify the choice of graph.

SP03.02 Solve a given problem by graphing data and interpreting the resulting graph.

Performance Indicator Background

SP03.01 Students often have little difficulty with creating graphs; however, they may be challenged to analyze the information depicted in the graphs.

Continue to provide opportunities for students to practice making graphs from previously collected data. Present them with some data that you create and ask them to graph it. Students should now be able to justify the reasons why they create the graph as they do. Students will need to be reminded to choose an appropriate scale and an appropriate graph.

Students can work in pairs to analyze the graphs, asking questions about what the graph shows and what kinds of predictions they can make from the data.

Consider writing some questions on the board to guide students' thinking. Some possible choices for discussion are as follows:

- State the facts that the graph shows.
- Which element on the graph is the greatest?
- Which element on the graph is the least?
- What trend does the graph show?
- Can you make a prediction based on the information provided in the graph?
- What could influence the trend in the data?
- Pose "What if ... " types of questions.

Students should be able to justify why they select one type of graph over another. There are many reasons as to why one type of graph is better suited.

Have a discussion with the class on the benefits of using one type of graph over another. Ask them which type of graph they prefer and ask them to explain why. Ask students to explain which type of graphs they see the most in magazines and newspapers and explain why they think that is the case.

SP03.02 Students need to be aware that graphs give us all types of information. It is one way of sharing information other than using the written language. Students have to analyze graphs to get the information they are looking for. Students, in pairs, could create a problem, collect the data, and graph the results. Ask them to write three questions based on the graph for other groups to answer.

SCO SP04 Students will be expected to demonstrate an understanding of probability by ▪ identifying all possible outcomes of a probability experiment ▪ differentiating between experimental and theoretical probability ▪ determining the theoretical probability of outcomes in a probability experiment ▪ determining the experimental probability of outcomes in a probability experiment ▪ comparing experimental results with the theoretical probability for an experiment [C, ME, PS, T]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

SP04.01 List the possible outcomes of a probability experiment, such as
- tossing a coin
- rolling a die with a given number of sides
- spinning a spinner with a given number of sectors

SP04.02 Determine the theoretical probability of an outcome occurring for a given probability experiment.

SP04.03 Predict the probability of a given outcome occurring for a given probability experiment by using theoretical probability.

SP04.04 Conduct a probability experiment, with or without technology, and compare the experimental results to the theoretical probability.

SP04.05 Explain that as the number of trials in a probability experiment increases, the experimental probability approaches the theoretical probability of a particular outcome.

SP04.06 Distinguish between theoretical probability and experimental probability, and explain the differences.

Performance Indicator Background

SP04.01 Students should be given opportunities to identify the possible outcomes of an experiment. For example, when flipping a coin, two outcomes are possible; the coin will land on heads or the coin will land on tails.

SP04.02 Theoretical probability is based on what *should* theoretically happen. Explain to students that, in order to find theoretical probability, they must first determine the total number of possible outcomes. For example, show a spinner with numbers 1 to 5. Students should observe that each section of the spinner is of equal area and each number shows up only once. Therefore, each digit will have an equal chance of being spun. This means the possible outcomes are 1, 2, 3, 4, or 5. To determine the theoretical probability of spinning a 3, students should see that 3 shows up once on the spinner and there are 5 possible outcomes in total. Therefore, the probability of spinning a 3 would be 1 out of 5, or $\frac{1}{5}$. So, if the spinner is spun 100 times, theoretically it should land on “3” 20 times.

$$\text{Theoretical Probability of Event A} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$

Emphasize that probabilities range from 0 (impossible) to 1 (certain). Probabilities less than 0 and greater than 1 do not make sense because events either happen or they do not. In the example above, the theoretical probability is $\frac{1}{5}$ which is closer to 0 (impossible) than to 1 (certain). Rolling a 3 is possible, but not certain.

SP04.03 Students should be able to make predictions of the outcome of an experiment based on what they know about the materials (coins, dot cubes, spinners, etc.) they are using. Students can be encouraged to create their own tools for determining probability. Creating spinners is motivating for many students.

SP04.04 Carry out basic experiments using spinners, dot cubes, or coins to help students gain a better understanding of probability. For example, What is the likelihood of flipping a coin and having it land on heads? What is the likelihood of rolling a dot cube and having it land on a 6?

It is important that students conduct experiments to help them make the connection between experimental and theoretical probability. If they are aware of the theoretical probability of an experiment, they will be better able to make predictions as to what will happen once they carry out an experiment.

Experimental probability is determined by the results of an experiment that has already occurred. In contrast, theoretical probability involves analyzing possible outcomes in advance and using logic and reason to predict what is likely to happen.

Give students examples of experiments that could be carried out. Ask them to identify the experimental and the theoretical probabilities for each experiment.

SP04.05 Students are expected to learn that the more often they carry out the trials for an experiment, the better they will be able to predict a particular outcome. When making important decisions based on an experiment, it is important to carry out an experiment enough times so that you can make a firm prediction of the outcome. As the number of trials increase, the better their predictions will be.

It is important to emphasize the importance of a reasonably large sample size. The results of a small number of trials or just one experiment can be misleading, but when multiple trials are used and/or experiments repeated many more times, the experimental probability will gradually approach the theoretical probability. A good way to investigate this is to carry out an experiment in which students record results of a probability experiment individually, and then combine results from the entire class. The larger number of trials should result in a probability closer to the theoretical probability. For example, students could be asked to determine the theoretical probability of rolling a 3 on a six-sided die. Then, they could be asked to each roll a six-sided die ten times and record their result each time. Students could be asked to determine the experimental probability of rolling a 3 based upon their ten rolls? Students could then be asked to combine all their trials to create class data, and again determine the experimental probability of rolling a 3 based upon all the trials in the class. Students would then analyze which result is closer to the theoretical probability.

SP04.06 To have a strong understanding of probability, students must be able to differentiate between experimental probability and theoretical probability. Coin flipping experiments can be used to facilitate this. For example, students could be asked to predict the probability of a coin landing on heads. The theoretical probability of landing on heads is $\frac{1}{2}$. Ask students to flip a coin 100 times and record the results. When they carry out the experiment, they may discover that the result is not necessarily heads being tossed 50 times out of 100, and therefore, the experimental probability may not be $\frac{1}{2}$. This is a great way to demonstrate the difference between experimental and theoretical probability.

To help students understand the difference between theoretical and experimental probability is to explain to students that theoretical probability deals with what is expected to happen in an ideal world within a given event. For example, “in theory,” given there are two equal colours on a spinner, each colour should have a chance of being spun 5 times out of 10 or $\frac{1}{2}$ of the time. When experiments are completed, the resulting experimental probability may state that red, for example, was spun 6 out of 10 times, whereas blue was spun only 3 out of 10 times. Experimental probability, then, is telling what actually happened within a given event. Reinforce that theoretical probability occurs before the experiment happens, and experimental probability occurs once the experiment is completed.

Experimental probability differs from theoretical probability in that it depends on the actual results of experiments. Sometimes the experimental probability is not the same as the theoretical probability even though, theoretically, the likelihood of the events happening is equal. (getting heads or tails when flipping a coin).

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