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Mathematics 7

CURRICULUM GUIDE

NOVA SCOTIA

Mathematics 7

Implementation Draft
June 2015

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Mathematics 7, Implementation Draft, June 2015

Please note that all attempts have been made to identify and acknowledge information from external sources. In the event that a source was overlooked, please contact Education Program Services, Nova Scotia Department of Education and Early Childhood Development, eps@EDnet.ns.ca.

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Contents

Introduction

Background and Rationale

Mathematics curriculum is shaped by a vision that fosters the development of mathematically literate students who can extend and apply their learning and who are effective participants in society. It is essential that the mathematics curriculum reflect current research in mathematics instruction. To achieve this goal, Western and Northern Canadian Protocol's (WNCP) *The Common Curriculum Framework for K–9 Mathematics* (2006) has been adopted as the basis for the new mathematics curriculum in Nova Scotia.

The Common Curriculum Framework was developed by the seven ministries of education in the Western provinces (Alberta, British Columbia, Manitoba, Northwest Territories, Nunavut, Saskatchewan, and Yukon Territory) in collaboration with teachers, administrators, parents, business representatives, post-secondary educators, and others. The framework identifies beliefs about mathematics, general and specific student outcomes, and performance indicators agreed upon by the seven jurisdictions. The outcomes and performance indicators have been adapted for Nova Scotia. This document is based on both national and international research by the WNCP and the National Council of Teachers of Mathematics (NCTM).

There is an emphasis in the Nova Scotia curriculum on particular key concepts at each grade that will result in greater depth of understanding and, ultimately, stronger student achievement. There is also a greater emphasis on number sense and operations concepts in the early grades to ensure students develop a solid foundation in numeracy.

Purpose

This document provides sets of outcomes and performance indicators to be used as a mandated common base for defining mathematics curriculum expectations. This common base should result in consistent student outcomes in mathematics within the Province of Nova Scotia. It should also enable easier transfer for students moving within the Province or from any jurisdiction that has adopted the WNCP framework. This document is intended to clearly communicate to all education partners across the province the high expectations for students' mathematical learning.

Program Design and Components

Assessment

Ongoing assessment for learning is essential to effective teaching and learning. Research has shown that assessment *for* learning (formative assessment) practices produce significant and often substantial learning gains, close achievement gaps, and build students' ability to learn new skills (Black and Wiliam 1998; OECD 2006). Student involvement in assessment promotes learning. Timely and effective teacher feedback and student self-assessment allow students to reflect on and articulate their understanding of mathematical concepts and ideas.

Assessment in the classroom includes

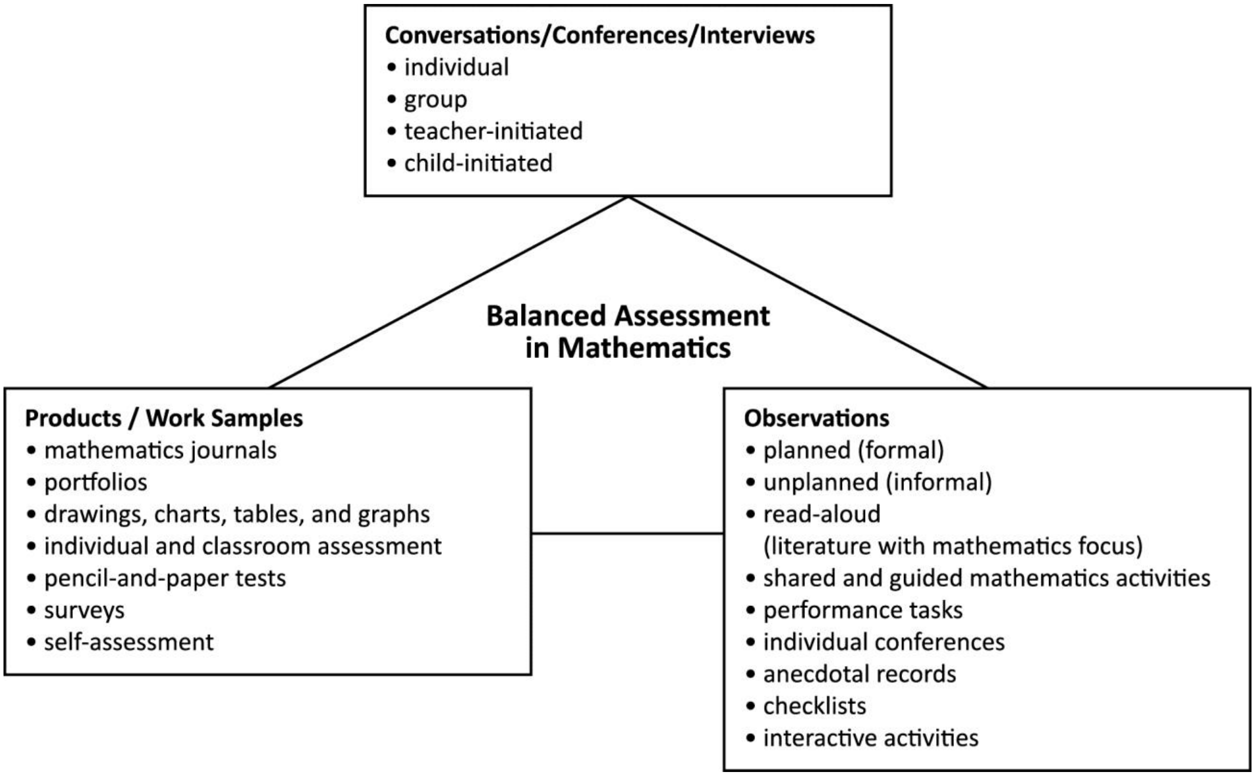
- providing clear goals, targets, and learning outcomes
- using exemplars, rubrics, and models to help clarify outcomes and identify important features of the work
- monitoring progress towards outcomes and providing feedback as necessary
- encouraging self-assessment
- fostering a classroom environment where conversations about learning take place, where students can check their thinking and performance and develop a deeper understanding of their learning

(Davies 2000)

Assessment for learning practices act as the scaffolding for learning, which only then can be measured through assessment of learning (summative assessment). Assessment of learning tracks student progress, informs instructional programming, and aids in decision making. Both forms of assessment are necessary to guide teaching, stimulate learning, and produce achievement gains.

Assessment *of* student learning should

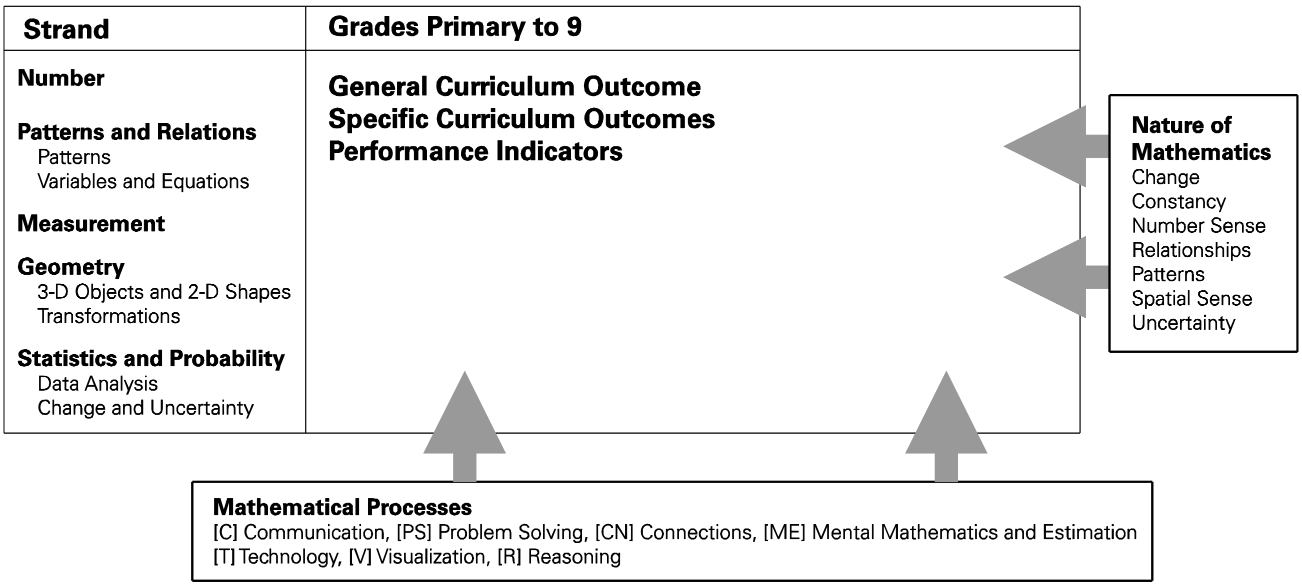
- align with curriculum outcomes
- clearly define criteria for success
- make explicit the expectations for students' performance
- use a wide variety of assessment strategies and tools
- yield useful information to inform instruction


Conversations/Conferences/Interviews
• individual
• group
• teacher-initiated
• child-initiated
Balanced Assessment in Mathematics
Products / Work Samples
• mathematics journals
• portfolios
• drawings, charts, tables, and graphs
• individual and classroom assessment
• pencil-and-paper tests
• surveys
• self-assessment
Observations
• planned (formal)
• unplanned (informal)
• read-aloud
(literature with mathematics focus)
• shared and guided mathematics activities
• performance tasks
• individual conferences
• anecdotal records
• checklists
• interactive activities

Outcomes

Conceptual Framework for Mathematics Primary–9

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



Adapted from *The Common Curriculum Framework for K–9 Mathematics* (Western and Northern Canadian Protocol, 2005, 5). All rights reserved.

Structure of the Mathematics Curriculum

Strands

The learning outcomes in the Nova Scotia Framework are organized into five strands across grades primary to 9.

- Number (N)
- Patterns and Relations (PR)
- Measurement (M)
- Geometry (G)
- Statistics and Probability (SP)

General Curriculum Outcomes (GCO)

Some strands are further subdivided into sub-strands. There is one general curriculum outcome (GCO) per sub-strand. GCOs are overarching statements about what students are expected to learn in each strand/sub-strand. The general curriculum outcome for each strand/sub-strand is the same throughout grades P–9.

NUMBER (N)

GCO: Students will be expected to demonstrate number sense.

PATTERNS AND RELATIONS (PR)

Patterns
GCO: Students will be expected to use patterns to describe the world and solve problems.

Variables and Equations
GCO: Students will be expected to represent algebraic expressions in multiple ways.

MEASUREMENT (M)

GCO: Students will be expected to use direct and indirect measure to solve problems.

GEOMETRY (G)

3-D Objects and 2-D Shapes
GCO: Students will be expected to describe the characteristics of 3-D objects and 2-D shapes and analyze the relationships among them.

Transformations
GCO: Students will be expected to describe and analyze position and motion of objects and shapes.

STATISTICS AND PROBABILITY (SP)

Data Analysis
GCO: Students will be expected to collect, display, and analyze data to solve problems.

Chance and Uncertainty
GCO: Students will be expected to use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

Grade 7 Specific Curriculum Outcomes (SCOs) and Performance Indicators

Specific curriculum outcomes (SCOs) are statements that identify the specific conceptual understanding, related skills, and knowledge students are expected to attain by the end of a given grade.

Performance indicators are statements that identify specific expectations of the depth, breadth, and expectations for the outcome. Teachers use performance indicators to determine whether students have achieved the corresponding SCO.

Process Standards Key

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

NUMBER (N)

N01 Students will be expected to determine and explain why a number is divisible by 2, 3, 4, 5, 6, 8, 9, or 10, and why a number cannot be divided by 0. [C, R]

Performance Indicators

N01.01 Determine if a given number is divisible by 2, 3, 4, 5, 6, 8, 9, or 10, and explain why.
N01.02 Sort a given set of numbers based upon their divisibility using organizers such as Venn and Carroll diagrams.
N01.03 Determine the factors of a given number using the divisibility rules.
N01.04 Explain, using an example, why numbers cannot be divided by 0.

N02 Students will be expected to demonstrate an understanding of the addition, subtraction, multiplication, and division of decimals to solve problems (for more than one-digit divisors or more than two-digit multipliers, the use of technology is expected). [ME, PS, T]

Performance Indicators

N02.01 Use estimation to determine the appropriate place value when calculating the sum or difference.
N02.02 Use estimation to determine the appropriate place value when calculating the product.
N02.03 Use estimation to determine the appropriate place value when calculating the quotient.
N02.04 Represent concretely, pictorially, and symbolically the multiplication and division of decimal numbers.
N02.05 Create and solve a given problem involving the addition of two or more decimal numbers.
N02.06 Create and solve a given problem involving the subtraction of decimal numbers.
N02.07 Create and solve a given problem involving the multiplication of decimal numbers.
N02.08 Create and solve a given problem involving the division of decimal numbers.
N02.09 Solve a given problem involving the multiplication or division of decimal numbers with two-digit multipliers or one-digit divisors (whole numbers or decimals) without the use of technology.
N02.10 Solve a given problem involving the multiplication or division of decimal numbers with more than two-digit multipliers or more than one-digit divisors (whole numbers or decimals) with the use of technology.
N02.11 Check the reasonableness of solutions using estimation.
N02.12 Solve a given problem that involves operations on decimals (limited to thousandths), taking into consideration the order of operations.

N03 Students will be expected to solve problems involving percents from 1% to 100% (limited to whole numbers). [ME, C, CN, PS, R, T]

Performance Indicators

N03.01 Express a given percent as a decimal or fraction.
N03.02 Use mental mathematics to solve percent problems, when appropriate.
N03.03 Use estimation to determine an approximate answer or the reasonableness of an answer.
N03.04 Solve a given problem that involves finding a percent.
N03.05 Determine the answer to a given percent problem where the answer requires rounding, and explain why an approximate answer is needed (e.g., total cost including taxes).

N04 Students will be expected to demonstrate an understanding of the relationship between positive terminating decimals and positive fractions and between positive repeating decimals (with one or two repeating digits) and positive fractions. [C, CN, R, T]

Performance Indicators

N04.01 Predict the decimal representation of a given fraction using patterns.
N04.02 Match a given set of fractions to their decimal representations.
N04.03 Sort a given set of fractions as repeating or terminating decimals.
N04.04 Express a given fraction as a terminating or repeating decimal.
N04.05 Express a given repeating decimal as a fraction.
N04.06 Express a given terminating decimal as a fraction.
N04.07 Provide an example where the decimal representation of a fraction is an approximation of its exact value.

N05 Students will be expected to demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially, and symbolically (limited to positive sums and differences). [C, CN, ME, PS, R, V]

Performance Indicators

N05.01 Use benchmarks to estimate the sum or difference of positive fractions or mixed numbers.
N05.02 Model addition and subtraction of given positive fractions or given mixed numbers, using concrete and pictorial representations, and record symbolically.
N05.03 Determine the sum or difference of fractions mentally, when appropriate.
N05.04 Determine the sum of two given positive fractions or mixed numbers with like denominators.
N05.05 Determine the difference of two given positive fractions or mixed numbers with like denominators.
N05.06 Determine a common denominator for a given set of positive fractions or mixed numbers.
N05.07 Determine the sum of two given positive fractions or mixed numbers with unlike denominators.
N05.08 Determine the difference of two given positive fractions or mixed numbers with unlike denominators.
N05.09 Simplify a given positive fraction or mixed number by identifying the common factor between the numerator and denominator.
N05.10 Simplify the solution to a given problem involving the sum or difference of two positive fractions or mixed numbers.
N05.11 Solve a given problem involving the addition or subtraction of positive fractions or mixed numbers, and determine if the solution is reasonable.

N06 Students will be expected to demonstrate an understanding of addition and subtraction of integers, concretely, pictorially, and symbolically. [C, CN, PS, R, V]

Performance Indicators

N06.01 Explain, using concrete materials such as integer tiles and diagrams, that the sum of opposite integers is zero.
N06.02 Illustrate, using a number line, the results of adding or subtracting negative and positive integers.
N06.03 Add two given integers, using concrete materials and/or pictorial representations, and record the process symbolically.
N06.04 Subtract two given integers, using concrete materials and/or pictorial representations, and record the process symbolically.
N06.05 Illustrate the relationship between adding integers and subtracting integers.
N06.06 Solve a given problem involving the addition and subtraction of integers.

N07 Students will be expected to compare, order, and position positive fractions, positive decimals (to thousandths), and whole numbers by using benchmarks, place value, and equivalent fractions and/or decimals. [CN, R, V]

Performance Indicators

N07.01 Position proper fractions with like and unlike denominators from a given set on a number line, and explain strategies used to determine order.
N07.02 Position a given set of positive fractions, including mixed numbers and improper fractions, on a number line; and explain strategies used to determine order.
N07.03 Position a given set of positive decimals on a number line and explain strategies used to determine order.
N07.04 Compare and order the numbers of a given set that includes positive fractions, positive decimals, and/or whole numbers in ascending or descending order and verify the result using a variety of strategies.
N07.05 Identify a number that would be between two given numbers in an ordered sequence or on a number line.
N07.06 Identify incorrectly placed numbers in an ordered sequence or on a number line.
N07.07 Position the numbers of a given set by placing them on a number line that contains benchmarks, such as 0 and 1 or 0 and 5.
N07.08 Position a given set that includes positive fractions, positive decimals, and/or whole numbers on a number line and explain strategies used to determine order.

PATTERNS AND RELATIONS (PR)

PR01 Students will be expected to demonstrate an understanding of oral and written patterns and their equivalent linear relations. [C, CN, R]

Performance Indicators

PR01.01 Formulate a linear relation to represent the relationship in a given oral or written pattern.
PR01.02 Provide a context for a given linear relation that represents a pattern.
PR01.03 Represent a pattern in the environment using a linear relation.

PR02 Students will be expected to create a table of values from a linear relation, graph the table of values, and analyze the graph to draw conclusions and solve problems. [C, CN, PS, R, V]

Performance Indicators

PR02.01 Create a table of values for a given linear relation by substituting values for the variable.

PR02.02 Create a table of values, using a linear relation, and graph the table of values (limited to discrete elements).

PR02.03 Sketch the graph from a table of values created for a given linear relation, and describe the patterns found in the graph to draw conclusions (e.g., graph the relationship between n and $2n + 3$).

PR02.04 Describe, using everyday language, in spoken or written form, the relationship shown on a graph to solve problems.

PR02.05 Match a given set of linear relations to a set of graphs.

PR02.06 Match a given set of graphs to a given set of linear relations.

PR03 Students will be expected to demonstrate an understanding of preservation of equality by

- **modelling preservation of equality, concretely, pictorially, and symbolically**
- **applying preservation of equality to solve equations [C, CN, PS, R, V]**

Performance Indicators

PR03.01 Model the preservation of equality for each of the four operations, using concrete materials and/or pictorial representations; explain the process orally; and record the process symbolically.

PR03.02 Write equivalent forms of a given equation by applying the preservation of equality, and verify using concrete materials
(e.g., $3b = 12$ is equivalent to $3b + 5 = 12 + 5$ or $2r = 7$ is equivalent to $3(2r) = 3(7)$.

PR03.03 Solve a given problem by applying preservation of equality.

PR04 Students will be expected to explain the difference between an expression and an equation. [C, CN]

Performance Indicators

PR04.01 Identify and provide an example of a constant term, numerical coefficient, and variable in an expression and an equation.

PR04.02 Explain what a variable is and how it is used in a given expression.

PR04.03 Provide an example of an expression and an equation and explain how they are similar and different.

PR05 Students will be expected to evaluate an expression given the value of the variable(s). [CN, R]

Performance Indicator

PR05.01 Substitute a value for an unknown in a given expression and evaluate the expression.

PR06 Students will be expected to model and solve, concretely, pictorially, and symbolically, problems that can be represented by one-step linear equations of the form *x* + *a* = *b*, where *a* and *b* are integers. [CN, PS, R, V]

Performance Indicators

PR06.01 Represent a given problem with a linear equation, and solve the equation using concrete models.
PR06.02 Draw a visual representation of the steps required to solve a given linear equation.
PR06.03 Solve a given problem using a linear equation and record the process.
PR06.04 Verify the solution to a given linear equation using concrete materials and diagrams.
PR06.05 Substitute a possible solution for the variable in a given linear equation into the original linear equation to verify the equality.

PR07 Students will be expected to model and solve, concretely, pictorially, and symbolically, where *a*, *b*, and *c* are whole numbers, problems that can be represented by linear equations of the form

- $ax + b = c$
- $ax = b$
- $\frac{x}{a} = b,\ a \neq 0$

[CN, PS, R, V]

Performance Indicators

PR07.01 Represent a given problem with a linear equation, and solve the equation using concrete models.
PR07.02 Draw a visual representation of the steps used to solve a given linear equation.
PR07.03 Solve a given problem using a linear equation and record the process.
PR07.04 Verify the solution to a given linear equation using concrete materials and diagrams.
PR07.05 Substitute a possible solution for the variable in a given linear equation into the original linear equation to verify the equality.

MEASUREMENT (M)

M01 Students will be expected to demonstrate an understanding of circles by

- **describing the relationships among radius, diameter, and circumference**
- **relating circumference to pi**
- **determining the sum of the central angles**
- **constructing circles with a given radius or diameter**
- **solving problems involving the radii, diameters, and circumferences of circles.**

[C, CN, PS, R, V]

Performance Indicators

M01.01 Illustrate and explain that the diameter is twice the radius in a given circle.
M01.02 Illustrate and explain that the circumference is approximately three times the diameter in a given circle.
M01.03 Explain that, for all circles, pi is the ratio of the circumference to the diameter $\left(\frac{C}{d}\right)$ and its value is approximately 3.14.
M01.04 Explain, using an illustration, that the sum of the central angles of a circle is 360°.
M01.05 Draw a circle with a given radius or diameter, with and without a compass.

M01.06 Solve a given contextual problem involving circles.

M02 Students will be expected to develop and apply a formula for determining the area of triangles, parallelograms, and circles. [CN, PS, R, V]

Performance Indicators

M02.01 Illustrate and explain how the area of a rectangle can be used to determine the area of a triangle.
M02.02 Generalize a rule to create a formula for determining the area of triangles.
M02.03 Illustrate and explain how the area of a rectangle can be used to determine the area of a parallelogram.
M02.04 Generalize a rule to create a formula for determining the area of parallelograms.
M02.05 Illustrate and explain how to estimate the area of a circle without the use of a formula.
M02.06 Generalize a rule to create a formula for determining the area of a given circle.
M02.07 Solve a given problem involving the area of triangles, parallelograms, and/or circles.

Geometry (G)

G01 Students will be expected to perform geometric constructions, including

- **perpendicular line segments**
- **parallel line segments**
- **perpendicular bisectors**
- **angle bisectors**

[CN, R, V]

Performance Indicators

G01.01 Describe examples of parallel line segments, perpendicular line segments, perpendicular bisectors, and angle bisectors in the environment.
G01.02 Identify on a given diagram line segments that are parallel or perpendicular.
G01.03 Draw and construct a line segment perpendicular to another line segment and explain why they are perpendicular.
G01.04 Draw and construct a line segment parallel to another line segment and explain why they are parallel.
G01.05 Draw and construct the bisector of a given angle, using more than one method, and verify that the resulting angles are equal.
G01.06 Draw and construct the perpendicular bisector of a line segment, using more than one method, and verify the construction.

G02 Students will be expected to identify and plot points in the four quadrants of a Cartesian plane, using integral ordered pairs. [C, CN, V]

Performance Indicators

G02.01 Label the axes of a four quadrant Cartesian plane and identify the origin.
G02.02 Identify the location of a given point in any quadrant of a Cartesian plane using an integral ordered pair.
G02.03 Plot the point corresponding to a given integral ordered pair on a Cartesian plane with units of 1, 2, 5, or 10 on its axes.
G02.04 Draw shapes and designs in a Cartesian plane using given integral ordered pairs.
G02.05 Create shapes and designs, and identify the points used to produce the shapes and designs, in any quadrant of a Cartesian plane.

G03 Students will be expected to perform and describe transformations (translations, rotations, or reflections) of a 2-D shape in all four quadrants of a Cartesian plane (limited to integral number vertices). [C, CN, PS, T, V]

Performance Indicators

G03.01 Identify the coordinates of the vertices of a given 2-D shape on a Cartesian plane.
G03.02 Describe the horizontal and vertical movement required to move from a given point to another point on a Cartesian plane.
G03.03 Describe the positional change of the vertices of a given 2-D shape to the corresponding vertices of its image as a result of a transformation, or successive transformations, on a Cartesian plane.
G03.04 Determine the distance between points along horizontal and vertical lines in a Cartesian plane.
G03.05 Perform a transformation or consecutive transformations on a given 2-D shape, and identify coordinates of the vertices of the image.
G03.06 Describe the positional change of the vertices of a 2-D shape to the corresponding vertices of its image as a result of a transformation or a combination of successive transformations.
G03.07 Describe the image resulting from the transformation of a given 2-D shape on a Cartesian plane by identifying the coordinates of the vertices of the image.

Statistics and Probability (SP)

SP01 Students will be expected to demonstrate an understanding of central tendency and range by
- **determining the measures of central tendency (mean, median, mode) and range**
- **determining the most appropriate measures of central tendency to report findings**

[C, PS, R, T]

Performance Indicators

SP01.01 Determine mean, median, and mode for a given set of data, and explain why these values may be the same or different.
SP01.02 Determine the range for a given set of data.
SP01.03 Provide a context in which the mean, median, or mode is the most appropriate measure of central tendency to use when reporting findings.
SP01.04 Solve a given problem involving the measures of central tendency.

SP02 Students will be expected to determine the effect on the mean, median, and mode when an outlier is included in a data set. [C, CN, PS, R]

Performance Indicators

SP02.01 Analyze a given set of data to identify any outliers.
SP02.02 Explain the effect of outliers on the measures of central tendency for a given data set.
SP02.03 Identify outliers in a given set of data, and justify whether or not they are to be included in reporting the measures of central tendency.
SP02.04 Provide examples of situations in which outliers would and would not be used in reporting the measures of central tendency.

SP03 Students will be expected to construct, label, and interpret circle graphs to solve problems. [C, CN, PS, R, T, V]

Performance Indicators

SP03.01 Identify common attributes of circle graphs, such as

- title, label, or legend
- the sum of the central angles is 360^{o}
- the data is reported as a percent of the total, and the sum of the percents is equal to 100%

SP03.02 Create and label a circle graph, with and without technology, to display a given set of data.

SP03.03 Find and compare circle graphs in a variety of print and electronic media, such as newspapers, magazines, and the Internet.

SP03.04 Translate percentages displayed in a circle graph into quantities to solve a given problem.

SP03.05 Interpret a given or constructed circle graph to answer questions.

SP04 Students will be expected to express probabilities as ratios, fractions, and percents. [C, CN, R, T, V]

Performance Indicators

SP04.01 Determine the probability of a given outcome occurring for a given probability experiment, and express it as a ratio, fraction, and percent.

SP04.02 Provide an example of an event with a probability of 0 or 0% (impossible) and an example of an event with a probability of 1 or 100% (certain).

SP05 Students will be expected to identify the sample space (where the combined sample space has 36 or fewer elements) for a probability experiment involving two independent events. [C, ME, PS]

Performance Indicators

SP05.01 Provide an example of two independent events, such as the following, and explain why they are independent.

- spinning a four-section spinner and an eight-sided die
- tossing a coin and rolling a twelve-sided die
- tossing two coins
- rolling two dice

SP05.02 Identify the sample space (all possible outcomes) for each of two independent events using a tree diagram, table, or other graphic organizer.

SP06 Students will be expected to conduct a probability experiment to compare the theoretical probability (determined using a tree diagram, table, or other graphic organizer) and experimental probability of two independent events. [C, PS, R, T]

Performance Indicators

SP06.01 Determine the theoretical probability of a given outcome involving two independent events.

SP06.02 Conduct a probability experiment for an outcome involving two independent events, with and without technology, to compare the experimental probability with the theoretical probability.

SP06.03 Solve a given probability problem involving two independent events.

Mathematical Processes

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics.

Students are expected to
- communicate in order to learn and express their understanding of mathematics (Communication [C])
- develop and apply new mathematical knowledge through problem solving (Problem Solving [PS])
- connect mathematical ideas to other concepts in mathematics, to everyday experiences, and to other disciplines (Connections [CN])
- demonstrate fluency with mental mathematics and estimation (Mental Mathematics and Estimation [ME])
- select and use technologies as tools for learning and solving problems (Technology [T])
- develop visualization skills to assist in processing information, making connections, and solving problems (Visualization [V])
- develop mathematical reasoning (Reasoning [R])

The Nova Scotia curriculum incorporates these seven interrelated mathematical processes that are intended to permeate teaching and learning. The key to these process standards is presented in a box, as shown below, with each specific curriculum outcome within the strands.

Process Standards Key

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics. Communication is important in clarifying, reinforcing, and modifying ideas, knowledge, attitudes, and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics.

Students also need to communicate their learning using mathematical terminology. Communication can help students make connections between and among the different representational modes—contextual, concrete, pictorial, linguistic/verbal, and symbolic—of mathematical ideas. Students must communicate *daily* about their mathematics learning. This enables them to reflect, to validate, and to clarify their thinking and provides teachers with insight into students' interpretations of mathematical meanings and ideas.

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, How would you ... ? or How could you ... ? the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement, perseverance, and collaboration.

Problem solving is also a powerful teaching tool that fosters multiple, creative, and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive, mathematical risk takers.

When students are exposed to a wide variety of problems in all areas of mathematics, they explore various methods for solving and verifying problems. In addition, they are challenged to find multiple solutions for problems and to create their own problem.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to one another or to real-world phenomena, students can begin to view mathematics as useful, relevant, and integrated. Learning mathematics within contexts and making connections relevant to learners can validate past experiences and increase student willingness to participate and be actively engaged. The brain is constantly looking for and making connections.

“Because the learner is constantly searching for connections on many levels, educators need to *orchestrate the experiences* from which learners extract understanding. … *Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching*.” (Caine and Caine 1991, 5).

Mathematics should be viewed as an integrated whole rather than as the study of separate strands or units. Connections must also be made between and among the different representational modes—contextual, concrete, pictorial, linguistic/verbal, and symbolic. The process of making connections, in turn, facilitates learning. Concepts and skills should also be connected to everyday situations and other curricular areas. For example, when developing literacy skills students learn to make text-to-world, text-to-text, and text-to-self connections. Students can also make connections to make mathematics come alive through math-to-world, math-to-math, and math-to-self connections.

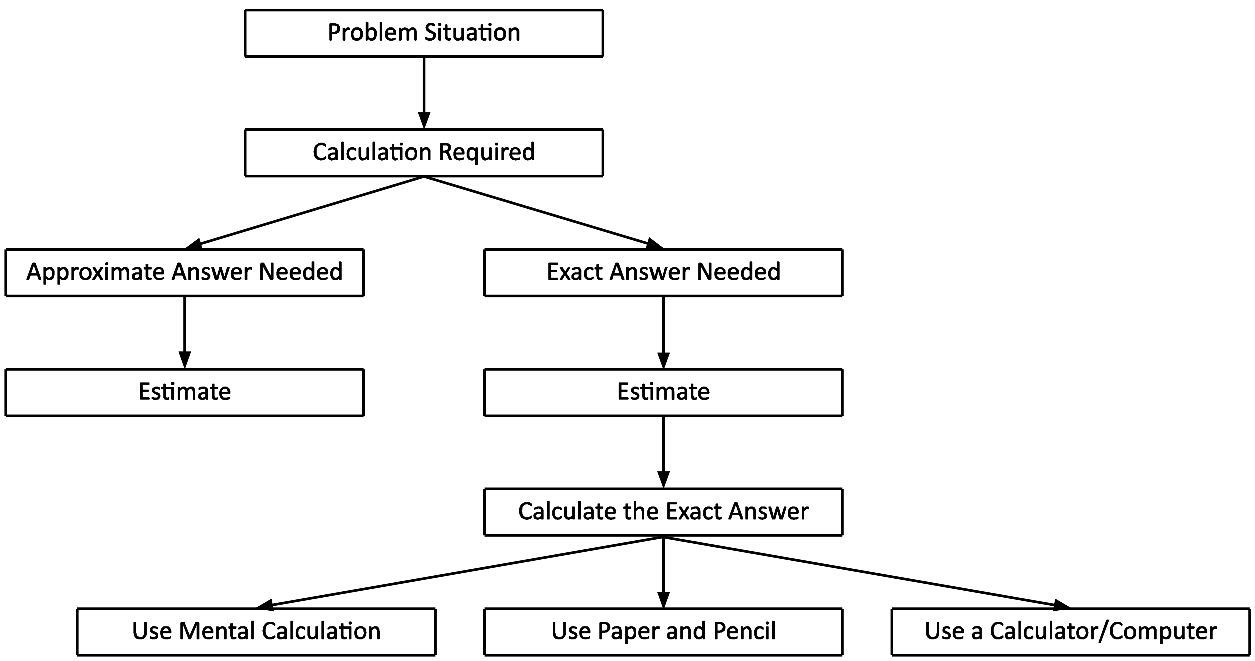
Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally without the use of external aids. Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy, and flexibility. “Even more important than performing computational procedures or using calculators is the greater facility that students need—more than ever before—with estimation and mental math.” (National Council of Teachers of Mathematics, May 2005).

Students proficient with mental mathematics “become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers, and are more able to use multiple approaches to problem solving.” (Rubenstein 2001) Mental mathematics “provides a cornerstone for all estimation processes, offering a variety of alternative algorithms and nonstandard techniques for finding answers." (Hope et al. 1988, v)

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when, and what strategy to use when estimating. Estimation is used to make mathematical judgments and develop useful, efficient strategies for dealing with situations in daily life.

Students need to develop both mental mathematics and estimation skills through context and not in isolation so they are able to apply them to solve problems. Whenever a problem requires a calculation, students should follow the decision-making process as illustrated below.



The skill of estimation requires a sound knowledge of mental mathematics. Both are necessary to many everyday experiences, and students should be provided with frequent opportunities to practise these skills.

Technology [T]

Technology can be effectively used to contribute to and support the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures, and solve problems.

Technology can be used to

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- decrease the time spent on computations when other mathematical learning is the focus
- reinforce the learning of basic facts and test properties

- develop personal procedures for mathematical operations
- create geometric displays
- simulate situations
- develop number sense

The use of calculators is recommended to enhance problem solving, to encourage discovery of number patterns, and to reinforce conceptual development and numerical relationships. They do not, however, replace the development of number concepts and skills. Carefully chosen computer software can provide interesting problem-solving situations and applications.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. While technology can be used in grades primary to 3 to enrich learning, it is expected that students will achieve all outcomes without the use of technology.

Visualization [V]

Visualization "involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world." (Armstrong 1999). The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Images and visual reasoning are important components of number, spatial, and measurement sense. Number visualization occurs when students create mental representations of numbers. These mental images are needed to develop concepts and understand procedures. Images and explanations help students clarify their understanding of mathematical ideas in all strands.

Being able to create, interpret, and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure, when to estimate and to know several estimation strategies. (Shaw and Cliatt 1989)

Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics. Mathematical experiences in and out of the classroom provide opportunities for inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyze observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Mathematics reasoning involves informal thinking, conjecturing, and validating—these help students understand that mathematics makes sense. Students are encouraged to justify, in a variety of ways, their solutions, thinking processes, and hypotheses. In fact, good reasoning is as important as finding correct answers.

Nature of Mathematics

Mathematics is one way of trying to understand, interpret, and describe our world. There are a number of components that define the nature of mathematics, and these are woven throughout this document. These components include change, constancy, number sense, relationships, patterns, spatial sense, and uncertainty.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics. Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, … can be described as

- skip counting by 2s, starting from 4
- an arithmetic sequence, with first term 4 and a common difference of 2
- a linear function with a discrete domain

(Steen 1990, 184)

Constancy

Different aspects of constancy are described by the terms **stability**, **conservation**, **equilibrium**, **steady state**, and **symmetry** (American Association for the Advancement of Science 1993, 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems. Examples of constancy include the following:

- The area of a rectangular region is the same regardless of the methods used to determine the solution.
- The sum of the interior angles of any triangle is 180°.
- The theoretical probability of flipping a coin and getting heads is 0.5.

Number Sense

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (British Columbia Ministry of Education 2000, 146). A true sense of number goes well beyond the skills of simply counting, memorizing facts, and the situational rote use of algorithms. Number sense develops when students connect numbers to real-life experiences and use benchmarks and referents. This results in students who are computationally fluent, flexible with numbers, and have intuition about numbers. The evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections.

Relationships

Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects, and concepts. The search for possible relationships involves the collection and analysis of data, and describing relationships visually, symbolically, orally, or in written form.

Patterns

Mathematics is about recognizing, describing, and working with numerical and non-numerical patterns. Patterns exist in all strands, and it is important that connections are made among strands. Working with patterns enables students to make connections within and beyond mathematics. These skills contribute to students' interaction with an understanding of their environment. Patterns may be represented in concrete, visual, or symbolic form. Students should develop fluency in moving from one representation to another. Students must learn to recognize, extend, create, and use mathematical patterns. Patterns allow students to make predictions and justify their reasoning when solving problems. Learning to work with patterns in the early grades helps develop students' algebraic thinking, which is foundational for working with more abstract mathematics in higher grades.

Spatial Sense

Spatial sense involves visualization, mental imagery, and spatial reasoning. These skills are central to the understanding of mathematics. Spatial sense enables students to reason and interpret among and between 3-D and 2-D representations and identify relationships to mathematical strands. Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 3-D objects and 2-D shapes. Spatial sense offers a way to interpret and reflect on the physical environment and its 3-D or 2-D representations. Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to make predictions about the results of changing these dimensions. For example,

- knowing the dimensions of an object enables students to communicate about the object and create representations
- the volume of a rectangular solid can be calculated from given dimensions
- doubling the length of the side of a square increases the area by a factor of four

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty. Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty. The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation. Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

Curriculum Document Format

This guide presents the mathematics curriculum so that a teacher may readily view the scope of the outcomes that students are expected to achieve during the year. Teachers are encouraged to examine teaching and learning that precedes and following this grade level to better understand how students' learning at a particular grade level is part of a bigger picture of concept and skill development.

The order of the outcomes does not prescribe a preferred order of presentation in the classroom, but provides the specific curriculum outcomes in relation to the overarching general curriculum outcomes (GCOs).

The table on the next page provided the structure of each specific curriculum outcomes section. When a specific curriculum outcome (SCO) is introduced, it is followed by the mathematical processes and performance indicators for that outcome. A scope and sequence is then provided, which relates the SCO to previous and next grade SCOs. Also for each SCO, there is background information, assessment strategies, suggested instructional strategies, suggested models and manipulatives, mathematical language, and a section for resources and notes. For each section, the guiding questions should be used to help with unit and lesson preparation.

Assessment Strategies

Guiding Questions
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Follow-up on Assessment

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcome and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SCO		
Mathematical Processes		
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections
[ME] Mental Mathematics and Estimation		
[T] Technology	**[V]** Visualization	**[R]** Reasoning

Performance Indicators

Describes observable indicators of whether students have achieved the specific outcome.

Scope and Sequence

Previous grade or course SCOs	Current grade SCO	Following grade or course SCOs

Background

Describes the "big ideas" to be learned and how they relate to work in previous grade and work in subsequent courses.

Assessment, Teaching, and Learning

Assessment Strategies

Assessing Prior Knowledge
Sample tasks that can be used to determine students' prior knowledge.

Whole-Class/Group/Individual Assessment Tasks
Some suggestions for specific activities and questions that can be used for both instruction and assessment

Follow-up on Assessment

Planning for Instruction

Choosing Instructional Strategies

Suggested strategies for planning daily lessons.

Suggested Learning Tasks

Suggestions for general approaches and strategies suggested for teaching this outcome.

Suggested Models and Manipulatives

Mathematical Language

Teacher and student mathematical language associated with the respective outcome.

Resources/Notes

Contexts for Learning and Teaching

Beliefs about Students and Mathematics Learning

"Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge." (National Council of Teachers of Mathematics 2000, 20)

The Nova Scotia mathematics curriculum is based upon several key assumptions or beliefs about mathematics learning that have grown out of research and practice. These beliefs include the following:

- Mathematics learning is an active and constructive process.
- Learning is most effective when standards of expectation are made clear with ongoing assessment and feedback.
- Learners are individuals who bring a wide range of prior knowledge and experiences and who learn via various styles and at different rates.
- Learning is most likely to occur when placed in meaningful contexts and in an environment that supports exploration, risk taking, and critical thinking, and that nurtures positive attitudes and sustained effort.

Students are curious, active learners with individual interests, abilities, and needs. They come to classrooms with varying knowledge, life experiences, and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Students develop a variety of mathematical ideas before they enter school. Children make sense of their environment through observations and interactions at home and in the community. Mathematics learning is embedded in everyday activities, such as playing, reading, storytelling, and helping around the home. Such activities can contribute to the development of number and spatial sense in children. Curiosity about mathematics is fostered when children are engaged in activities such as comparing quantities, searching for patterns, sorting objects, ordering objects, creating designs, building with blocks, and talking about these activities. Positive early experiences in mathematics are as critical to child development as are early literacy experiences.

Students learn by attaching meaning to what they do and need to construct their own meaning of mathematics. This meaning is best constructed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of models and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with and translating through a variety of materials, tools, and contexts when constructing meaning about new mathematical ideas. Meaningful discussions can provide essential links among concrete, pictorial, contextual, and symbolic representations of mathematics.

The learning environment should value and respect all students' experiences and ways of thinking, so that learners are comfortable taking intellectual risks, asking questions, and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must realize that it is acceptable to solve problems in different ways and that solutions may vary.

Goals for Mathematics Education

The main goals of mathematics education are to prepare students to

- use mathematics confidently to solve problems
- communicate and reason mathematically
- appreciate and value mathematics
- make connections between mathematics and its applications
- become mathematically literate adults, using mathematics to contribute to society

Students who have met these goals will

- gain understanding and appreciation of the contributions of mathematics as a science, a philosophy, and an art
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical tasks and projects
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity about mathematics and situations involving mathematics

Opportunities for Success

A positive attitude has a profound effect on learning. Environments that create a sense of belonging, encourage risk taking, and provide opportunities for student success help develop and maintain positive attitudes and self-confidence. Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, participate willingly in classroom activities, persist in challenging situations, and engage in reflective practices.

To experience success, students must be taught to set achievable goals or assess their progress as they work toward these goals. Striving toward success and becoming autonomous and responsible learners are ongoing, reflective processes that involve revisiting the setting and assessing of personal goals.

Engaging All Learners

*"*No matter how engagement is defined or which dimension is considered, research confirms this truism of education: *The more engaged you are, the more you will learn."* (Hume 2011, 6)

Student engagement is at the core of learning. This is critical for teachers to take into account when planning and implementing instruction. Effective instruction engages, embraces, and supports all learners through a range of learning experiences. This curriculum is designed to provide learning opportunities that reflect culturally proficient instructional and assessment practices and are equitable, accessible, and inclusive of the multiple facets of diversity represented in today's classrooms.

Engagement in learning occurs when students are provided with opportunities to become more invested in their learning. When teachers know their students as individual learners and as individual people, their students are more likely to be motivated to learn, participate in classroom activities, persist in challenging situations, and engage in reflective practices. Students often become more engaged when teachers demonstrate a genuine belief in each student's potential to learn.

Supportive Learning Environments

A supportive and positive learning environment has a profound effect on students' learning. In classrooms where students feel a sense of belonging, are encouraged to actively participate, are challenged without being frustrated, and feel safe and supported to take risks with their learning, students are more likely to experience success. It is realized that not all students will progress at the same pace or be equally positioned in terms of their prior knowledge of and skill with particular concepts and outcomes. Teachers provide all students with equitable access to learning by integrating a variety of instructional approaches and assessment activities that consider all learners and align with the following key principles:

- Instruction must be flexible and offer multiple means of representation.
- Students must have opportunities to express their knowledge and understanding in multiple ways.
- Teachers must provide options for students to engage in learning through multiple ways.

Teachers who know their students well become aware of individual learning differences and infuse this understanding into planned instructional and assessment decisions. They organize learning experiences to accommodate the many ways in which students learn, create meaning, and demonstrate their knowledge and understanding. Teachers use a variety of effective teaching approaches that may include

- providing all students with equitable access to appropriate learning strategies, resources, and technology
- offering a range of ways students can access their prior knowledge to connect with new concepts
- scaffolding instruction and assignments so that individual or groups of students are supported as needed throughout the process of learning
- verbalizing their thinking to model comprehension strategies and new learning
- balancing individual, small-group, and whole-class approaches to learning activities
- involving students in the co-creation of criteria for assessment and evaluation
- providing students with choice in how they demonstrate their understanding according to learning styles and preferences, building on individual strengths, and including a range of difficulty and challenge
- providing frequent and meaningful feedback to students throughout their learning experiences

Learning Styles and Preferences

The ways in which students make sense of, receive, and process information, demonstrate learning, and interact with peers and their environment both indicate and shape learning preferences, which may vary widely from student to student. Learning preferences are influenced also by the learning context and purpose and by the type and form of information presented or requested. Most students tend to favour one learning style and may have greater success if instruction is designed to provide for multiple learning styles, thus creating more opportunities for all students to access learning. The three most commonly referenced learning styles are

- auditory (such as listening to teacher-presented lessons or discussing with peers)
- kinesthetic (such as using manipulatives or recording print or graphic/visual text)
- visual (such as interpreting information with text and graphics or viewing videos)

While students can be expected to work using all modalities, it is recognized that one or some of these modalities may be more natural to individual students than the others.

A Gender-Inclusive Curriculum

It is important that the curriculum respects the experiences and values of all students and that learning resources and instructional practices are not gender-biased. Teachers promote gender equity and inclusion in their classrooms when they

- articulate equally high expectations for all students
- provide equal opportunity for input and response from all students
- model gender-fair language and respectful listening in their interactions with students

Valuing Diversity: Teaching with Cultural Proficiency

Teachers understand that students represent diverse life and cultural experiences, with individual students bringing different prior knowledge to their learning. Therefore, teachers build upon their knowledge of their students as individuals and respond by using a variety of culturally-proficient instruction and assessment strategies. “Instruction that is embedded in socially meaningful contexts, and tasks that are meaningful and relevant to the lives of students, will engage students in high-level problem-solving and reasoning and enhance students’ engagement (Frankenstein 1995; Gutstein 2003; Ladson-Billings 1997; Tate 1995).” (Herzig 2005)

Students with Language, Communication, and Learning Challenges

Today’s classrooms include students who have diverse backgrounds, abilities, levels of development, and learning challenges. By observing and interacting with students as they work on assigned activities, teachers can identify areas where students may need additional support to achieve their learning goals. Teachers can then respond with a range of effective instructional strategies. Students who have English as an Additional Language (EAL) may require curriculum outcomes at different levels, or temporary individualized outcomes, particularly in language-based subject areas, while they become more proficient in their English language skills. For students who are experiencing difficulties, it is important that teachers distinguish between students for whom curriculum content is challenging and students for whom language-based issues are at the root of apparent academic difficulties.

Students who Demonstrate Gifted and Talented Behaviours

Some students are academically gifted and talented with specific skill sets or in specific subject areas. Most students who are gifted and talented thrive when challenged by problem-centred, inquiry-based learning and open-ended activities. Teachers may challenge students who are gifted and talented by adjusting the breadth, the depth, and/or the pace of instruction. Learning experiences may be enriched by providing greater choice among activities and offering a range of resources that require increased cognitive demand and higher-level thinking at different levels of complexity and abstraction. For additional information, refer to *Gifted Education and Talent Development* (Nova Scotia Department of Education 2010).

Connections across the Curriculum

The teacher should take advantage of the various opportunities available to integrate mathematics and other subjects. This integration not only serves to show students how mathematics is used in daily life, but it helps strengthen the students' understanding of mathematical concepts and provides them with opportunities to practise mathematical skills. There are many possibilities for integrating mathematics in health education, literacy, music, physical education, science, social studies, and visual arts.

Number (N)

GCO: Students will be expected to demonstrate number sense.

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

GUIDING QUESTIONS

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Follow-up on Assessment

The evidence of learning that is gathered from student participation and work should be used to inform instruction.

GUIDING QUESTIONS

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan available at Mathematics Learning Commons: Grades 7–9: http://nsvs.ednet.ns.ca/nsps/nsps26/course/view.php?id=3875.

GUIDING QUESTIONS

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SCO N01: Students will be expected to determine and explain why a number is divisible by 2, 3, 4, 5, 6, 8, 9, or 10, and why a number cannot be divided by 0. [C, R]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N01.01 Determine if a given number is divisible by 2, 3, 4, 5, 6, 8, 9, or 10, and explain why.
N01.02 Sort a given set of numbers based upon their divisibility using organizers such as Venn and Carroll diagrams.
N01.03 Determine the factors of a given number using the divisibility rules.
N01.04 Explain, using an example, why numbers cannot be divided by 0.

Scope and Sequence

Mathematics 6	Mathematics 7	Mathematics 8
N03 Students will be expected to demonstrate an understanding of factors and multiples by ▪ determining multiples and factors of numbers less than 100 ▪ identifying prime and composite numbers ▪ solving problems using multiples and factors	**N01** Students will be expected to determine and explain why a number is divisible by 2, 3, 4, 5, 6, 8, 9, or 10, and why a number cannot be divided by 0.	**N01** Students will be expected to demonstrate an understanding of perfect squares and square roots, concretely, pictorially, and symbolically (limited to whole numbers). **N02** Students will be expected to determine the approximate square root of numbers that are not perfect squares (limited to whole numbers).

Background

Exploration of the divisibility rules serves as an excellent opportunity to extend number sense. Knowledge of divisibility rules provides a valuable tool for mental arithmetic and general development of operation sense. A clear grasp of divisibility helps students identify factors and understand relationships between numbers, solve problems, sort numbers, work with fractions, understand percentages and ratios, and work with algebraic equations. When students can easily identify factors, they can readily identify prime and composite numbers, common factors and multiples, and find both the greatest common factor (GCF) and the least common multiple (LCM). Understanding divisibility enhances students' ability to rename fractions with common denominators and to rewrite fractions in simplest terms, thereby making it easier to compare fractions and to perform operations with fractions. If a dividend can be divided by a divisor to yield a quotient that is a whole number (with no remainder), then the dividend is considered to be divisible by that divisor. 36 is **divisible** by 4 because it gives a quotient of 9, with no remainder. The divisor and dividend are whole numbers.

If a dividend is divisible by a divisor, that divisor is a factor of the dividend. 36 is divisible by 4, therefore 4 is a factor of 36. If the divisor is a factor of the dividend, then the dividend is a multiple of the divisor. 36 is a multiple of 4.

In earlier grades, students built their understanding of sequences and number patterns. These patterns are used to develop division of larger numbers. It is assumed that students can

- recognize number patterns in tables
- extend a table of values using a pattern
- describe the relationships among terms in a table.

If students understand place value and have facility in using mental mathematics strategies and facts, it will be easier for them to find patterns in multiples of factors, to add the digits of multiples, and to recognize numbers that are divisible by a particular factor. Proficiency with these skills will help students to discover the divisibility rules, understand and explain why divisibility rules work, and use divisibility rules effectively to determine divisibility.

Students have explored patterns in a multiplication chart in Mathematics 4 and learned about factors and multiples in Mathematics 6. Building on these, students can discover the divisibility rules for 2, 5, and 10. Also, once students understand divisibility for 2 and 3, they can use this knowledge to develop a means of testing for divisibility by 6. This should be seen as a problem solving opportunity for students. They can also explore whether this strategy will always work for other numbers such as 8 and 10.

In Mathematics 7, students are not expected to do prime factorization. It is a performance indicator in Mathematics 8 in the study of squares and square roots, and a learning outcome in the Mathematics 10 course.

To avoid an arbitrary rule for not being able to divide by 0, use patterns for multiplication and division.

For example, $\frac{12}{4}=3$ because $3\times4=12$

$\frac{10}{5}=2$ because $5\times2=10$

$\frac{0}{4}=0$ because $0\times4=0$

but $\frac{4}{0}=?$ because ? × 0 = 4 (There is no answer)

The divisibility rules described below are for teacher information. It is not expected that students will complete a table like this. The suggested order for instruction of divisibility rules is 2, 5, 10, 3, 6, 9, 4, and 8.

Divisibility Rules for Common Factors—Suggested Teaching Order			
Divisible by	**Rule**	**Explanation**	**Examples and Non-examples**
2	The number is even, that is, the ones digit is 2, 4, 6, 8, or 0.	Even numbers are composed of groups of 2. Therefore, it is necessary only to examine the units (or ones) place when determining divisibility by 2.	238 is divisible by 2 because the digit in the ones place (8) is even 89 is not divisible by 2 because the digit in the ones place (9) is odd.
5	The ones digit is 0 or 5.	Use place value logic. All multiples of 5 end in 0 or 5. (0, 5, 10, 15, 20, 25, 30, 35, …)	130 is divisible by 5 because it ends in 0. 89 is not divisible by 5 because 89 does not end in 0 or 5.
10	The ones digit is 0.	All multiples of 10 have 0 in the ones place.	130 is divisible by 10 because the digit in the ones place (0) is 0. 89 is not divisible by 10 because the digit in the ones place (9) is not 0.
3	The sum of the digits is divisible by 3. Continually add the digits until you end up with a single digit that is divisible by 3 (will ultimately result in a total of 3, 6, or 9).	Use place value and the logic of remainders. Each hundred divides into 33 groups of 3 and leaves 1 unit remaining. Each ten divides into three groups of 3 and leaves 1 unit remaining. The ones are already individual units. Add all the remaining units (or remainders). If this sum divides evenly by 3, the original number is divisible by 3. For example 573 can be written as $5 \times 100 + 7 \times 10 + 3$ $= 5\,(99 + 1) + 7\,(9 + 1) + 3$ Adding the remaining units yields $5 + 7 + 3 = 15$ and we know that 15 is divisible by 3.	354 is divisible by 3 because $3 + 5 + 4 = 12$ $1 + 2 = 3$ The sum of the digit is 3 which is divisible by 3. Therefore 354 is divisible by 3. 238 is not divisible by 3 because $2 + 3 + 8 = 13$ $1 + 3 = 4$ 4 is not divisible by 3. Therefore 238 is not divisible by 3.

Divisibility Rules for Common Factors—Suggested Teaching Order			
Divisible by	**Rule**	**Explanation**	**Examples and Non-examples**
6	The number is even and divisible by 3, that is, the number has both 2 and 3 as factors (is divisible by both 2 and 3).	Every second multiple of 3 is also a multiple of 2. Therefore, if the number is divisible by both prime factors 2 and 3, it must also be divisible by 6 because two groups of 3 make a group of 6.	426 is divisible by 6 because it is divisible by both 2 (it is an even number) and 3 (the sum of its digits is divisible by 3) 153 is not divisible by 6 because it is not divisible by 2 (it is an odd number), but it is divisible by 3 (the sum of its digits is divisible by 3).
9	The sum of the digits is divisible by 9. The number is divisible by 3 twice. (Remember that 9 is 3 × 3.)	Use place value and the logic of remainders. Each hundred divides into 11 groups of 9 and leaves 1 unit remaining. Each ten divides into one group of 9 and leaves 1 unit remaining. The ones are already individual units. Add all the remaining units (or remainders). If this sum divides evenly by 9, or by 3 twice, the original number is divisible by 9.	351 is divisible by 9 because dividing 3 hundreds by 9 leaves 3 units remaining, 5 tens leaves 5 units remaining, and 1 unit is the remainder in the units place. Add up the remainders: 3 + 5 + 1 = 9. Since the remainders are divisible by 9, the entire number is also divisible by 9. 418 is not divisible by 9 because dividing 4 hundreds by 9 leaves 4 units remaining, 1 ten leaves 1 unit remaining, and 8 units are the remainder in the units place. Add up the remainders: 4 + 1 + 8 = 13. Since 13 is not divisible by 9, the entire number is not divisible by 9.
4	The number formed by the tens and ones digits is divisible by 4. That is, the number formed by the tens and ones digits is divisible by 2 twice.	Use place value logic. 100 is the smallest place value position divisible by 4 (100 ÷ 4 = 25). Any number greater than 100 can be expressed as *x* number of hundreds , therefore, only the number formed by the digits in the tens and units places must be examined.	524 is divisible by 4 because 100 is divisible by 4. Therefore 5 x 100 is divisible by 4. 24 is divisible by 2 twice which means it is divisible by 4. 490 is not divisible by 4. Although 4 × 100 is divisible by 4, 90 is not divisible by 2 twice, therefore it is not divisible by 4.
8	The number formed by the hundreds, tens, and ones digits is divisible by 8. That is, the number formed by the hundreds, tens, and ones digits is divisible by 2 three times.	Use place value logic. 1000 is the smallest place value position divisible by 8 (1000 ÷ 8 = 125). Any number larger than 1000 can be expressed as *x* number of thousands Therefore, only the number formed by the digits in the hundreds, tens, and units places must be examined.	7480 is divisible by 8 because 480 is divisible by 2 three times (480 ÷ 2 = 240; 240 ÷ 2 = 120; 120 ÷ 2 = 60). 220 is not divisible by 8 because 220 is not divisible by 2 three times. (220 ÷ 2 = 110, 110 ÷ 2= 55, 55 is not divisible by 2). Therefore 220 is not divisible.

Assessment, Teaching, and Learning

Assessment Strategies

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

Students should be able to select from a repertoire of mental mathematics strategies for addition, subtraction, multiplication, and division.

- Have students describe and apply mental mathematics strategies, such as
  - skip-counting from a known fact
  - using doubling or halving
  - using repeated doubling

- Ask students to identify a number with five factors.

- Have students draw diagrams (such as rectangles or factor trees) to show why a given number is or is not prime (e.g., 10, 17, 27).

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Determine if a given number is divisible by 2, 3, 4, 5, 6, 8, 9, or 10.
- Determine the factors of a given number using the divisibility rules.
- Explain, using an example, why a number cannot be divided by 0.
- Create a 3-digit number that is divisible by both 4 and 5. Is it also divisible by 2 and 8?
- Complete the number by filling in each blank with a digit. Explain how you know your answer is correct:
  - 26_ is divisible by 10
  - 154_ is divisible by 2
  - _6_ is divisible by 6
  - 26_ is divisible by 3
  - 1_2 is divisible by 9
  - 15_ is divisible by 4
- There will be 138 people at a party. Determine if the host can fill tables of 5. Tables of 6? Support your answer by using divisibility rules.
- Choose three numbers that are divisible by both 6 and 9. Find the smallest number, other than 1, by which the chosen numbers are divisible. Share your answers with the class and discuss.
- Each of Eli's four friends has a code number. Keile's number is divisible by 3, 5, and 8. Max's number is divisible by 2 and 3. Jennifer's number is divisible by 4 and 5, but not 3. Ben's number is divisible

by 3 and 5, but not 8. Eli receives a message with the code number 5384 from one of his four friends. Determine who sent the message

- Determine if each statement is true or false and identify an example that supports your answer:
  - All numbers divisible by 6 are divisible by 3.
  - Some, but not all, numbers divisible by 6 are divisible by 3.
  - No numbers divisible by 6 are divisible by 3.
  - All numbers divisible by 3 are divisible by 6.
  - Some, but not all, numbers divisible by 3 are divisible by 6.
  - No numbers divisible by 3 are divisible by 6.
- Explain why it is not possible to calculate 12 ÷ 0.
- The principal of Great School has to determine the number of classes of grade 7 students in her school. Discuss the divisibility rules that can be used to determine the possible number of classes there could be if there are 240 grade 7 students and all classes have an equal number of students.
- Complete the following table and explain how the table shows division by 0 is not possible.

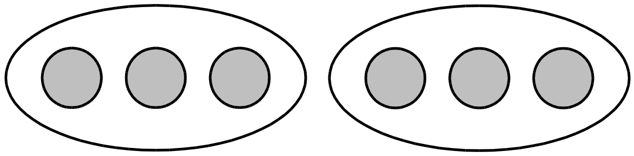
Division Statement	Related Multiplication Statement
6 ÷ 2 = 3	3 × 2 = 6
10 ÷ 5 = 2	2 × 5 = ___
14 ÷ 2 = ___	2 × 7 = 14
15 ÷ ___ = 5	3 × 5 = 15
___ ÷ 8 = 3	3 × 8 = ___
12 ÷ 0 = ____	0 × ___ = 12

Planning for Instruction

Choosing Instructional Strategies

Consider the following strategies when planning daily lessons.

- Organize instruction so that the students develop the divisibility rules themselves through investigations.
- Explore why some divisibility rules work, such as the divisibility rule for 4.
- Show students numbers at random. Ask students to determine by what numbers they are divisible.
- Use a hundred chart to explore patterns of multiples.
- Using the “repeated subtraction” meaning of division will help students understand why numbers cannot be divided by 0. Given 20 ÷ 5, for example, students should understand that 5 can be subtracted from 20 four times (20 – 5 – 5 – 5 – 5 = 0). Given 6 ÷ 0, students should determine that no matter how many times they subtract 0, they will still have 6. Since there is no answer, 6 ÷ 0 is undefined.
- Dividing by 0 can also be visualized using counters. For example, given 6 ÷ 3, how many groups of 3 are in 6? Students separate 6 counters into 2 groups of 3.



Given 6 ÷ 0, students will see that it is not possible to separate the 6 counters into groups of 0. The following are ways to show a number cannot be divided by zero:

- Using a calculator to divide a number by zero results in an error message.
- Applying the action of division results in an impossible situation.

Example:

If you have a quantity x, how many groups of zero can you make? You would be trying to make groups of zero forever. If you had to share a quantity into zero groups, you would have no groups to share the quantity with. Both scenarios are impossible. Using the pattern and logic of related facts provides no solution to dividing by zero. Multiplication and division are inverse operations. Think of related statements such as the following:

4 x 2 = 8 and 8 ÷ 4 = 2

0 x ? = 8 and 8 ÷ 0 = ? (There is no answer.)

SUGGESTED LEARNING TASKS

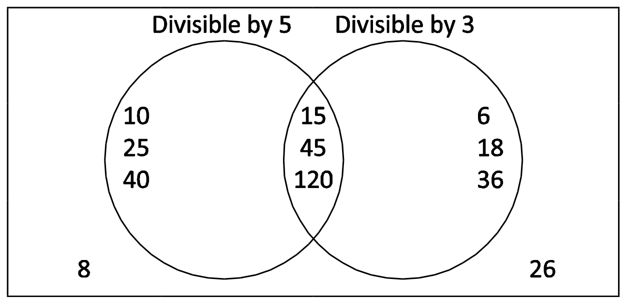
- Explore the use of a calculator as a tool to test for divisibility. Students should realize that the test for divisibility on a calculator involves dividing to see if the quotient is a whole number, not a decimal.
- Explore divisibility rules for 3, 6, and 9. Write the first 10 multiples of 3. What do you notice about the number? If no student mentions the sum of the digits, ask them to find the sum of the digits and describe what they notice. Which numbers in the same list are divisible by 6? What do you notice about these numbers? Test the conclusions, using numbers such as 393, 504, and 5832.
- Sort a given set of numbers based upon their divisibility using organizers, such as Venn and Carroll diagrams.
- Based on what you have discovered, do you think there should be a divisibility rule for zero? Explain your response.

- Have students complete the following table with different numbers:

Number	Is it divisible by 2? How do you know?	Is it divisible by 3? How do you know?	Is it divisible by 4? How do you know?	Is it divisible by 5? How do you know?	Is it divisible by 6? How do you know?	Is it divisible by 8? How do you know?	Is it divisible by 9? How do you know?	Is it divisible by 10? How do you know?

- Create a Carroll diagram or a Venn diagram to sort the following numbers based on the divisibility rules for 3 and 5: 6, 8, 10, 15, 18, 25, 26, 36, 40, 45, 120. Extension: Which numbers are also divisible by 15?

Carroll Diagram	Divisible by 3	Not Divisible by 3
Divisible by 5	15 45 120	10 25 40
Not Divisible by 5	6 36 18	8 26



SUGGESTED MODELS AND MANIPULATIVES

- calculator*
- colour tiles
- hundred chart
- Carroll diagram
- Venn diagram

*also available in *Interactive Math Tools* (Pearson n.d.)

MATHEMATICAL LANGUAGE

Teacher	Student
▪ Carroll diagram ▪ dividend ▪ divisibility ▪ divisible ▪ divisor ▪ factor ▪ multiple ▪ product ▪ quotient ▪ Venn diagram	▪ Carroll diagram ▪ dividend ▪ divisibility ▪ divisible ▪ divisor ▪ factor ▪ multiple ▪ product ▪ quotient ▪ Venn diagram

Resources

Print

Math Makes Sense 7 (Garneau et al. 2007)

- Unit 1: Patterns and Relations (NSSBB #: 2001640)
  - Section 1.1 Patterns in Division
  - Section 1.2 More Patterns in Division
- *ProGuide* (CD; Word Files) (NSSBB #: 2001641)
  - Assessment Masters
  - Extra Practice Masters
  - Unit Tests
- *ProGuide* (DVD) (NSSBB #: 2001641)
  - Projectable Student Book Pages
  - Modifiable Line Masters

SCO N02: Students will be expected to demonstrate an understanding of the addition, subtraction, multiplication, and division of decimals to solve problems (for more than one-digit divisors or more than two-digit multipliers, the use of technology is expected). [ME, PS, T]
[C] Communication **[PS]** Problem Solving **[CN]** Connections **[ME]** Mental Mathematics and Estimation **[T]** Technology **[V]** Visualization **[R]** Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N02.01 Use estimation to determine the appropriate place value when calculating the sum or difference.
N02.02 Use estimation to determine the appropriate place value when calculating the product.
N02.03 Use estimation to determine the appropriate place value when calculating the quotient.
N02.04 Represent concretely, pictorially, and symbolically the multiplication and division of decimal numbers.
N02.05 Create and solve a given problem involving the addition of two or more decimal numbers.
N02.06 Create and solve a given problem involving the subtraction of decimal numbers.
N02.07 Create and solve a given problem involving the multiplication of decimal numbers.
N02.08 Create and solve a given problem involving the division of decimal numbers.
N02.09 Solve a given problem involving the multiplication or division of decimal numbers with two-digit multipliers or one-digit divisors (whole numbers or decimals) without the use of technology.
N02.10 Solve a given problem involving the multiplication or division of decimal numbers with more than two-digit multipliers or more than one-digit divisors (whole numbers or decimals) with the use of technology.
N02.11 Check the reasonableness of solutions using estimation.
N02.12 Solve a given problem that involves operations on decimals (limited to thousandths), taking into consideration the order of operations.

Scope and Sequence

Mathematics 6	Mathematics 7	Mathematics 8
N01 Students will be expected to demonstrate an understanding of place value for numbers greater than one million and less than one thousandth. **N08** Students will be expected to demonstrate an understanding of multiplication and division of decimals (one-digit whole number multipliers and one-digit natural number divisors). **N09** Students will be expected to explain and apply the order of operations, excluding exponents, with and without technology (limited to whole numbers).	**N02** Students will be expected to demonstrate an understanding of the addition, subtraction, multiplication, and division of decimals to solve problems (for more than one-digit divisors or more than two-digit multipliers, the use of technology is expected).	**N03** Students will be expected to determine an understanding of and solve problems involving percents greater than or equal to 0%.

Background

When working with operations involving whole numbers and/or decimals, students should use a mental procedure, an algorithm, or a calculator where appropriate. Students need to understand the relationship between whole number and decimal number operations, including order of operations. Emphasis should be on place value and estimation. Statements that can lead to misconceptions by students and emphasize memorization of rules rather than the understanding of concepts are discouraged. An example of a statement that could lead to misconceptions is, “Dividing by 100 means you move the decimal point two places to the left.” In fact, the decimal point does not move, it is the place value of the digits that changes.

Instruction is not focused on simply mastering procedural rules but on a conceptual understanding. It is important that a problem-solving context be used to help ensure the relevance of the operations. To encourage alternative computational strategies that students have learned in previous grades, present addition and subtraction questions horizontally, as well as vertically. Students should be able to choose algorithms of choice when they calculate with paper and pencil methods. While it is important that the algorithms developed by students are respected, if they are inefficient, guide students toward more appropriate strategies.

The principles that students previously used with operations, estimations, models, and algorithms for whole numbers also apply to decimal numbers. A focus on number sense before addressing procedural operations is advised. If students have difficulty with decimal operations, ensure that they have a good understanding of place value and that they understand the role of the decimal point. Emphasis should be on place value and estimation.

Mental calculations should be encouraged whenever possible. In paper and pencil computations, we usually start at the right and work toward the left. To add mentally, students can start at the left and work to the right.

To calculate 1.7 + 3.6, think:
1 + 3 = 4
7 tenths + 6 tenths = 13 tenths or 1 and 3 tenths
4 + 1 and 3 tenths = 5.3
or
to calculate 1.7 + 3.6 mentally, the make-ten strategy can be used. Partition 3.6 into 0.3 and 3.3.
1.7 + 0.3 = 2.0
2.0 + 3.3 = 5.3

When adding numbers such as 4.2 and 0.23, students are to add corresponding place values. A common error students make is adding digit to digit starting at “the end” but ignoring place value. For example, adding digit to digit with 4.2 and 0.23 will result in the incorrect answer of 0.65 instead of the correct answer of 4.43. Front-end estimation should be used to develop a sense of the size of an answer for any calculations involving decimals. In this simple strategy, students perform operations from left to right using only the whole number part of each value. When determining the sum 9.2 + 3.5 + 12.72, students use 9 + 3 + 12 = 24 to estimate. Similarly, to find the difference 14.31 – 5.2 – 3.6, students use 14 – 5 – 3 = 6 to estimate. Once this estimation is complete, the calculation must be performed.

Have students learn to use estimation to determine whether or not their computations make sense, considering the placement of the decimal. Estimation should be used to develop a sense of the size of the answer for all calculations involving decimals. For example, one might round each of the decimal numbers 2.8 x 8.3 for an estimate of 24 (3 x 8). When estimation is an automatic response students will, when faced with a calculation, not depend on recalling the “counting back decimal places,” rule. For many practical situations involving decimals (e.g., calculating tips on restaurant bills, averaging numbers of points or people, purchasing goods sold by area or volume), estimated answers are often preferred over precise calculations. Situations that involve many decimal points and require precise answers are often technical in nature, and technology is used to calculate these answers. Because estimates are common in everyday use, they provide an effective approach to teach operations with decimals.

Multiplication and division of two numbers will produce the same digits regardless of the position of the decimal point. For example, 12 x 12 = 144, 12 x 1.2 = 14.4, and 1.2 x 1.2 =1.44. As a result, for most practical purposes, there is no reason to develop new rules for decimal multiplication and division. Rather, the computations can be performed as whole numbers and estimation can be used to place the decimal.

If students understand operations as actions, they can represent the operations concretely or pictorially. Examples of the actions are

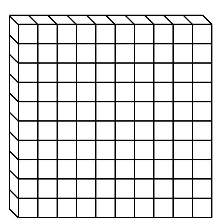
- addition as a “combining” operation
- subtraction as a “taking away” operation
- multiplication as “repeated addition” or “*x* groups of a number”
- division as “repeated subtraction,” “partitioning a number into groups,” or “how many groups of *x* are in a number”

Manipulatives can be used to represent decimal numbers. Operations can be visually represented using number lines, place value mats, and/or grid paper.

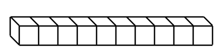
When representing decimals with base-ten blocks, it is necessary to establish which manipulative will represent the value of 1.

Example 1:

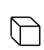
If a flat represents 1



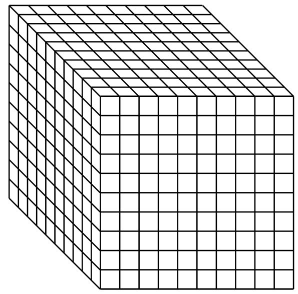
a rod represents one-tenth ($\frac{1}{10}$ or 0.1)



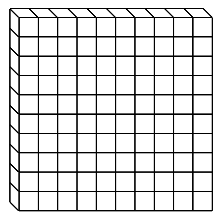
a cube represents one-hundredth ($\frac{1}{100}$ or 0.01)



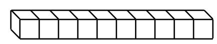
Example 2:
If a large block represents 1,



a flat represents one-tenth ($\frac{1}{10}$ or 0.1)



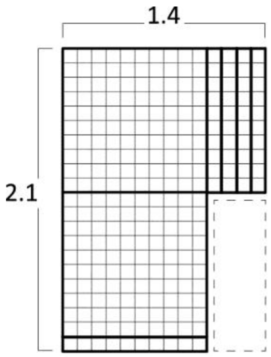
a rod represents one-hundredth ($\frac{1}{100}$ or 0.01)



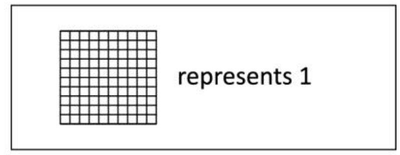
and a cube represents, one-thousandth ($\frac{1}{1000}$ or 0.001)

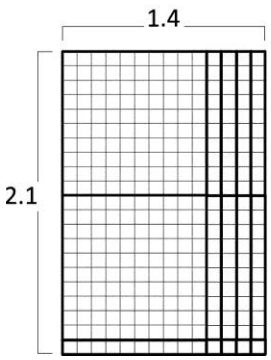
Students have used base-ten blocks in previous grades to model the multiplication of a decimal number by a whole number. In Mathematics 7 base-ten blocks could be used to model the multiplication of two decimal numbers, depending on the numbers. Whereas a problem such as determining the cost of 2.5 kg of cheese at $4.50/kg could be solved using partitioning and mental mathematics, a problem such as "what is the area of a rectangular patio that measures 1.4 m by 2.1 m could be modelled and solved used base-ten blocks.

The base-ten area model will now be extended to 2-digit multipliers with the modelling of 2.1 x 1.4. (the flat represents 1):



The dimensions of the rectangle are 1.4 and 2.1. The area of the rectangle represents the product of these numbers.



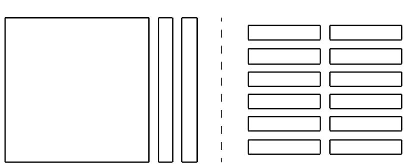


With the region completed, we determine the answer by counting 2 flats where each represents 1, 9 rods where each represents one-tenth, and 4 unit blocks where each represents one-hundredth.

Students could use front-end estimation to check the reasonableness of their answer. When multiplying 2.1 x 1.4, students could arrive at 2.94, 29.4 or 294. Using front-end estimation (2 × 1 = 2) indicates that the answer is close to 2, making 29.4 and 294 unreasonable answers. Students are expected to use manipulatives and algorithms when multiplying decimal numbers with two digits. When solving problems with more than 2-digit multipliers, technology can be used.

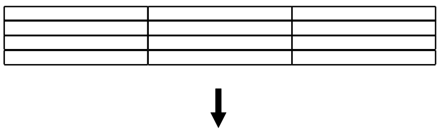
In previous grades, students will have also used base-ten blocks to divide a decimal number by a whole number. The base-ten area model will now be extended to solve problems with one-digit decimal divisors. The focus is on using division of decimals in a problem-solving context. For example, 1.2 m of material was purchased to make banners. If each banner requires 0.4 m of material, how many banners can be made? Students can represent this by dividing 1.2 into groups of 0.4.

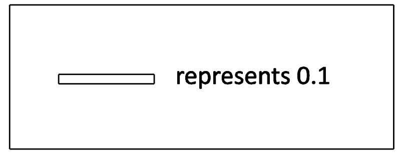
Model 1.2 ÷ 0.4 **Divide the block representing 1 into tenths**



Since a rectangle could not be created with one dimension 0.4 using 1 and 2 tenths, the block representing 1 was traded for 10 tenths.

Arrange the blocks in a rectangular shape that has a height of 0.4





The area of the rectangle is 1.2, and the height is 0.4.
The quotient is represented by the length.
The length is 3, so 1.2 ÷0.4 = 3

Students may find estimation challenging when the divisor is less than 1. Technology can be used to solve division problems with more than a one-digit divisor.

In Mathematics 6, students have used the order of operations, excluding exponents, but limited to whole numbers. This will now be extended to calculations with decimal numbers. Remember that for more than one-digit divisors or two-digit multipliers, the use of technology is recommended. When students are not using technology, “friendly” numbers for which a calculator is not needed should be used.

The order of operations is necessary in order to maintain consistency of results. It is important to provide students with a variety of contexts in which they can recognize the need for the order of operations, such as calculating the total cost for a family with two parents and three children for theatre tickets, where children’s tickets cost $8.50 and adult tickets cost $14.80. Students will write a number sentence such as *C* = 3 × $8.50 + 2 × $14.80. Discuss with students how they would determine the total, and link the discussion to the order of operations.

Students should indicate that it would be necessary to find the total for the adults and the total for the children and then add the totals together. It would not make sense, then, to calculate from left to right:
3 × $8.50 + 2 × $14.80
$25.50 + 2 × $14.80
$27.50 × $14.80
$407.00

An example such as this emphasizes the need to follow the order of operations.

The order of operations for Mathematics 7 is as follows:
- brackets
- division/multiplication (from left to right)
- addition/subtraction (from left to right)

Exponents will be addressed in a later grade.

Instruction should be given on calculator use with regard to the order of operations. Students should recognize the necessity of preparing problems for calculator entry. They should also be aware that different calculators process the order of operations in different ways. Some calculators are programmed to address the order of operations automatically, and others are not. Students could perform one calculation at a time and record the flow of their answers. In this way, if an error is made it is easier to identify where it occurred. They could also insert brackets as a reminder of the correct order to perform calculations.

Assessment, Teaching, and Learning

Assessment Strategies

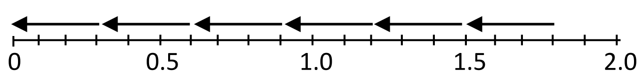
ASSESSING PRIOR KNOWLEDGE

- Ask students the following questions:
  - Estimate 6 × \$6.19.
  - What is the approximate product of 8 and 3.12?
  - Estimate 3 × 4.1.
  - What is the approximate area of a 6 cm × 4.5 cm rectangle?
  - About how much does Yung Kim earn for 7 hours at \$10.45/hour?
- Ask students to solve an order of operations expression and then describe what could have happened if the order of operations steps were not followed. For example, students might describe what an incorrect solution could be.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Create addition, subtraction, multiplication, and division word problems, each with an answer of 4.2.
- Draw or build a model to illustrate 4 x 3.45 and a model to show how to find 5.28 ÷ 4.
- Show how the results of 423 ÷ 3 and 42.3 ÷ 3 related.
- Use a model to show why 4.2 ÷ 0.2 has the same answer as 42 ÷ 2.
- Why might someone find it easier to divide 8.8 by 0.2 than 1.1 by 0.3?
- Respond to the following: Jade said, 3.45 x 4 must be 1.380. There is only one digit before the decimal place in 3.45, so there must be one digit before the decimal place in the product.
- Two decimals are multiplied. The product is 0.48. What might they have been? Give two other pairs of factors.
- Explain how the diagram shows that 1.8 ÷ 0.3 = 6



- Draw a quadrilateral with a perimeter of 16.3 cm, where no sides are whole numbers.
- What does the remainder means when you divide 4.1 by 4?
- Use front-end estimation to determine the position of the decimal in the following products.
  - 7.8 × 3.2 = 2496
  - 28.39 × 2.4 = 68 136
- Solve the following problem using technology. Explain why they think the decimal is in the correct position.

  - Jolene bought 11.8 L of gas for her snowmobile. The gas cost \$1.34 per litre. How much did Jolene pay for her gas?
- Use front-end estimation to determine where the decimal should be placed in the following quotients.
  - $39.06 \div 4.2 = 93$
  - $58.5 \div 3.9 = 15$
- Determine how many times a 0.3 L glass can be used to fill a 1.5 L bottle with water.
- Use technology to solve problems such as the following, and then explain how you know the decimal is in the correct place: John paid \$4.92 for a case of 32 bottles of water to take camping. How much did he pay per bottle?
- Answer the following question: Carole used her calculator to complete each of the following calculations. Should she accept her answer in each case? Why or why not?
  - $24.29 \times 3.8 = 923.02$
  - $8.9 \times 0.4 = 3.56$
  - $36.54 \div 2.9 = 12.6$
  - $8.76 \div 0.4 = 21.9$
- Compare the solution for $4 \times 7 - 3 \times 6$ with the solution for $4 \times (7 - 3) \times 6$. Are the solutions the same or different? Explain your answer.
- Write an expression and then calculate the answer to the following question: Chris found the attendance reports for hockey games at the stadium for the past nine days to be 2787, 2683, 3319, 4009, 2993, 3419, 4108, 3539, and 4602. If tickets were sold for \$12.75 each, and expenses amounted to \$258 712.00, what was the profit for the stadium?
- Write an expression for each of the following and use the expression to answer the question. Ms. Janes bought the following for her project: 5 sheets of pressboard at \$8.95 a sheet, 20 planks at \$2.95 each, and 2 litres of paint at \$9.95. What was the total cost? Three times the sum of \$34.95 and \$48.95 represents the total amount of Jim’s sales on April 29. When his expenses, which total \$75.00, were subtracted, what was his profit?
- Identify where the brackets should be placed in order for the answer to be correct. Show calculations to demonstrate the correctness of your answer.

  $4 + 6 \times 8 - 3 = 77$

  $26 - 4 \times 4 - 2 = 18$

Planning for Instruction

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Use patterns to help students understand the placement of the decimal in the product of two decimal amounts. For example, 9 x 7 = 63 therefore, 9 x 0.7 (or 7 tenths) = 6.3 or 63 tenths.
- Use the “area model” concretely (base-ten blocks) and pictorially (grid paper or array). When considering multiplication by a decimal, students should recognize that, for example, 0.8 of something will be almost that amount, but not quite, and 2.4 multiplied by an amount will be double the amount with almost another one-half of it added on.
- Use story problems to provide students with a relevant context for completing computations.
- Focus on strategies such as rounding and front-end estimation. For example:
  - Rounding: 789.6 ÷ 89, think: "90 multiplied by what number would give an answer close to 800?" Front-end estimation: 6.1 x 23.4 might be considered to be 6 x 20 (120) plus 6 x 3 (18) plus a little more for an estimate of 140, or 6 x 25 = 150.

SUGGESTED LEARNING TASKS

- Describe how to calculate 3 x 1.25 by thinking of it as money.
- Describe the situation by referring to money: 2.40 ÷ 0.1 = 24 (24 dimes in $2.40).
- Work in pairs sharing strategies for estimating in situations, such as
  - 6.1 m of material at $4.95 a metre
  - area of a rectangular plot of land 24.78 m x 9.2 m
  - 0.5 of a length of rope 20.6 m long
  - 9.7 kg of beef at $4.59/kg
  - 4.38 kg of fish at $12.59/kg
- Using a variety of division questions that result in a remainder, investigate and discuss the meaning of the remainders.
- Estimate the following sums or differences using front-end estimation. Then compute the answers and compare them to the estimations.
  - 4.6 + 11.8 + 15.3
  - 19.6 – 15.9 – 1.7
- Display the multiplication of two factors using an open array as shown in the example below. To find the overall product, add the partial products in the array. The numbers used on the outside of the array can be partitioned in flexible ways to create “nice” numbers to multiply.

	2	0.4
3	6	1.2
0.7	1.4	0.28

- Add or subtract the following mentally and explain the process being used.
  - 6.4 + 1.8
  - 4.75 – 1.32
- Germaine has been saving her money to buy skateboard equipment. She has a hundred-dollar bill and three twenty-dollar bills. She is shopping today because the store has a special sale. The store is paying the GST and PST. Germaine has selected a skateboard for $89.99, a helmet for $25.95, kneepads for $9.99, and elbow pads for $10.99. Use your mental mathematics skills to estimate the total cost of Germaine's selections, and determine whether she has enough money to purchase all the items.
  - What is your estimated cost for Germaine's planned purchases?
  - Do you think she has enough money to purchase the selected items? Explain.
  - Calculate the actual total of her bill.
  - Was your estimated cost over or under the actual cost? Explain.
  - How much more money will Germaine need, or how much change will she receive?

SUGGESTED MODELS AND MANIPULATIVES

- area models
- base-ten blocks*
- calculator
- grid paper
- hundred chart
- money
- number lines*
- open array*
- place value chart

*also available in *Interactive Math Tools* (Pearson n.d.)

MATHEMATICAL LANGUAGE

Teacher	Student
▪ addend	▪ addend
▪ addition	▪ addition
▪ array	▪ array
▪ brackets	▪ brackets
▪ decimals	▪ decimals
▪ difference	▪ difference
▪ dividend	▪ dividend
▪ division	▪ division
▪ divisor	▪ divisor
▪ estimation	▪ estimation
▪ front-end estimation	▪ front-end estimation
▪ hundredths	▪ hundredths
▪ multiplicand	▪ multiplicand
▪ multiplier	▪ multiplier
▪ parentheses	▪ parentheses
▪ product	▪ product
▪ quotient	▪ quotient
▪ subtraction	▪ subtraction

▪ sum ▪ tenths ▪ thousandths	▪ sum ▪ tenths ▪ thousandths

Resources

Print

Math Makes Sense 7 (Garneau et al. 2007)

- Unit 3: Fractions, Decimals and Percents (NSSBB #: 2001640)
  - Section 3.3 Adding and Subtracting Decimals
  - Section 3.4 Multiplying Decimals
  - Section 3.5 Dividing Decimals
  - Section 3.6 Order of Operations with Decimals
  - Unit Problem: Shopping with Coupons
- *ProGuide* (CD; Word Files) (NSSBB #: 2001641)
  - Assessment Masters
  - Extra Practice Masters
  - Unit Tests
- *ProGuide* (DVD) (NSSBB #: 2001641)
  - Projectable Student Book Pages
  - Modifiable Line Masters

SCO N03: Students will be expected to solve problems involving percents from 1% to 100% (limited to whole numbers). [ME, C, CN, PS, R, T]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N03.01 Express a given percent as a decimal or fraction.
N03.02 Use mental mathematics to solve percent problems, when appropriate.
N03.03 Use estimation to determine an approximate answer or the reasonableness of an answer.
N03.04 Solve a given problem that involves finding a percent.
N03.05 Determine the answer to a given percent problem where the answer requires rounding, and explain why an approximate answer is needed (e.g., total cost including taxes).

Scope and Sequence

Mathematics 6	Mathematics 7	Mathematics 8
N02 Students will be expected to solve problems involving whole numbers and decimal numbers. **N05** Students will be expected to demonstrate an understanding of ratio, concretely, pictorially, and symbolically. **N06** Students will be expected to demonstrate an understanding of percent (limited to whole numbers) concretely, pictorially, and symbolically.	**N03** Students will be expected to solve problems involving percents from 1% to 100% (limited to whole numbers).	**N03** Students will be expected to demonstrate an understanding of and solve problems involving percents greater than or equal to 0%. **N04** Students will be expected to demonstrate an understanding of ratio and rate. **N05** Students will be expected to solve problems that involve rates, ratios, and proportional reasoning.

Background

Percent is a part-to-whole ratio that compares a number to 100. As such, percents should be introduced as a third way of writing both fractions and decimals. Develop number sense for percent through the use of benchmarks

- 100% is one whole
- 50% is one-half
- 25% is a one-fourth
- 10% is a one-tenth
- 1% is one-hundredth

Before students become skillful at solving problems involving percent, they must have a strong conceptual understanding of fractions, decimals, and percents, and they must be able to interchange equivalent names to represent the concepts.

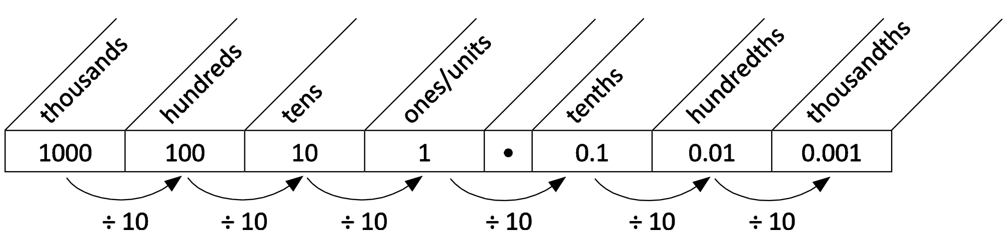
The term fraction has several meanings. An expert blends and separates these meanings for convenience, but this blending can confuse students who lack fluency in applying the different meanings of fraction. Fraction notation is used to represent

- a part of a unit
- a part of a group or set
- a measurement
- a point on a number line
- a ratio or a portion of a turn
- the division operation

Decimals are a convenient means of representing fractional quantities using a place value system.

Fractions may be converted to decimals by using the division operation meaning of fraction and dividing the numerator by the denominator (e.g., $\frac{3}{4}$ may be viewed as 3 ÷ 4 = 0.75). Fractions may also be converted to decimals by finding an equivalent fraction with a denominator of any power of 10 (e.g., 100), and then writing the fraction in standard notation e.g., $\left(\frac{7}{50} = \frac{14}{100} = 0.14\right)$. It is useful to commit to memory some common fraction decimal equivalents, such as halves, fourths, and tenths.

A decimal point is used to separate whole units from parts of units. Each position to the right of the decimal represents a one-tenth of the previous unit. In standard notation, the first position following the decimal represents tenth parts of one whole unit, and the second place represents tenth parts of a tenth, or hundredth parts of one unit, the third position represents tenth parts of a hundredth or thousandth parts of one unit.



When translating standard notation to percents, the decimal point indicates where to read the hundredths in a number. The word percent means per hundred and may be substituted for the word hundredths when reading a number. Therefore, $\frac{7}{100}$ or 0.07 may be read as 7 hundredths and also as 7 percent.

An understanding of place value allows us to express any number as a number of selected units. Just as 141 can represent 14 tens and 1 one, 0.141, which represents a number that is a little larger than one tenth of one whole, may be expressed as 1.41 tenths, 14.1 hundredths, or 141 thousandths. Substituting the word percent as another word for hundredth, the decimal number 0.141 may be considered as 14.1 hundredths, or 14.1 percent (%).

In Mathematics 7, students work with percents from 1% to 100%. The percents may represent a part of one whole item or a part of one set. The quantity represented by the percent depends on the amount in the whole. For example, 1% may be a large or small quantity, depending on the whole. Consider 1% of the money in an individual's piggy bank versus 1% of the money in a bank. Discussion should also focus on the contexts in which 1% would be considered a large quantity and 90% would be considered a small quantity; for example, 1 % of the population in Canada vs 90 % of the population of a rural school. Sometimes the same quantity may also represent different percent values. For example, 20 is 20% of 100, but it is also 100% of 20. Everything is relative to the size of the whole. Identifying which number in a situation represents the whole and which number represents the part is important when solving problems involving percent.

When exact answers are required, students should be able to employ a variety of strategies in calculating percent of a number. When students understand that percent means per hundred, they should be able to write the percent as a fraction with a denominator of 100 (which should be simplified if possible). Once students have a fraction with a denominator of 100, they can write the decimal since they already have experience with this. Students have previously written a fraction as a decimal and a decimal as a fraction.

When students have developed multiple views of fractions and percents, they will benefit from having multiple meaningful approaches to find percent values.

- In the grade 7 student council election, Blair won 25% of the votes. If 80 grade 7 students voted, how many votes did Blair receive?

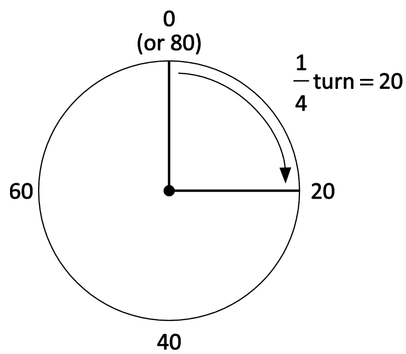
  To find 25% of 80, students may approach the problem various ways:

  - Represent a percent as an equivalent fraction.
    Think of 25% as $\frac{25}{100}$, and the equivalent fraction one-fourth , $\frac{1}{4}$. Find $\frac{1}{4}$ of 80 by dividing by 4:
    80 ÷ 4 = 20, so 25% of 80 is 20.

  - Use the 1% model:
    If 1% of 80 is 80 ÷ 100 = 0.8
    then 10 % of 80 is 10 x 0.8 = 8
    and 25 % of 80 is 10% + 10% + 5% or 8 + 8 + 4 = 20

  - Think of a part-to-whole ratio and set up a proportion: $\frac{25}{100} = \frac{?}{80}$

  - Think of a circle with beginning and end points 0 and 80, and think of 25% as $\frac{1}{4}$ of a turn.



– Think of the decimal equivalent, and represent the word expression as a number expression.

  25% of 80 is ____

  0.25 ×80 = ____

– Use mental mathematics and the distributive property to solve problems involving percent.

  To find 35% of 80, think of 35% as 25% + 10%

  35 % of 80 = (25 % + 10 %) of 80

  25% of 80 is 20 and 10% of 80 is 8.

  20 + 8 = 28

  so 35% of 80 is 28

– Use related fractions to solve percent problems.

  To find $\frac{3}{4}$ of 80, think

  If $\frac{1}{4}$ of 80 is 20, then $\frac{3}{4}$ of 80 must be 3×20 or 60.

With multiple approaches to finding percents, students should choose the most efficient approach for each problem. When setting up examples and creating problems for students, frequently choose numbers that are convenient to work with, so that students will be able to concentrate on the processes involved, rather than on the arithmetic. Also encourage students to use a variety of approaches and not to over-rely on one specific method. For example, they may develop a habit of using factors of 10, and forget to use equivalent fractions or the commonly used, very effective part-to-whole ratio approach. To find 25% of a number, you may think 25% is 10% + 10% + half of 10%, but it may be much more convenient to think of 25% as $\frac{1}{4}$, and divide the whole by 4.

Students should learn to make immediate connections between other percentages and their fraction equivalents, such as 50% and 75% and 20%, 30%, 40%, etc. Encourage students to recognize that percents such as 51% and 12% are close to benchmarks, which could be used for estimation purposes. Students should be able to calculate 1%, 5% (one-half of 10%), 10%, and 50% mentally using their knowledge of benchmarks. When exact answers are required, students should be able to employ a variety of strategies in calculating percent of a number. Students should be able to solve problems that involve finding *a*, *b*, or *c* in the relationship a% of *b* = *c*, using estimation and calculation.

Have students solve problems that involve finding a percent in situations such as calculating sales tax, discounts, commissions, tips, etc. They can employ a variety of strategies when exact answers are required to calculate the percent of a number:

- express the percent as an equivalent decimal and multiplying 12% of 80 = 0.12 × 80 = 9.6
- find 1% and then multiplying
  1% of 80 = 0.8, so 12% of 80 = 0.8 × 12 = 9.6
- express this as a fraction and divide
  25% of $60 = \frac{1}{4} \times 60 = 60 \div 4 = 15$ (This method works best with percentages that are more common.)
- Finding equivalent fractions:
  30% of 85
  30% is $\frac{30}{100} = \frac{3}{10}$
  $\frac{3}{10} = \frac{?}{85}$
  ? = 25.5

It is not necessary that students become proficient in all four methods. The important thing is that they have a method which works well for them.

Students should realize when answers must be rounded in order to make sense in the context presented. For example, when calculating sales tax on a purchase, students may have an answer with as many as four decimal places. Students should understand that in real life, money is only presented as two decimal places, so the answer must be rounded to the nearest hundredth. Answers to problems that involve finding the number of people must be rounded to the nearest whole number, since it makes no sense to speak of a part of a person.

The conceptual understanding developed for this outcome should flow from meaningful problem-solving contexts.

Assessment, Teaching, and Learning

Assessment Strategies

Assessing Prior Knowledge

Tasks such as the following could be used to determine students' prior knowledge:

- Ask students what the ratio of legs to heads would be in a group of bears. Of people? Of spiders?
- Ask students which is the least? The greatest? Explain their answers.
  $\frac{1}{20}$ 20% 0.02
- Ask students what percent of a metre stick is 52 cm
- Ask students to name percents that indicate
  - almost one whole (all) of something
  - very little of something
  - a little less than one-half of something

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- A jacket is now selling for $64. The sign above it indicates that the price was reduced by 20%. What was the original selling price?
- If 30 is close to 80% of a number, what do you know about the number?
- If 60% is a good estimate for $\frac{30}{70}$? Explain your reasoning.
- Explain how you would estimate 48% of something.
- If 2% of a certain number is 0.46, what would 10% of the number be? What is the number?
- What percentage of the total arena capacity was used if $\frac{7}{8}$ of the tickets were sold for a concert?
- Explain why 70% is not a good estimate for 35 out of 80.
- Explain how to estimate the percentage when a test score is 26 correct out of 55.
- Change each of the following to a percent mentally and explain your thinking:

  $\frac{2}{5}\ \frac{4}{24}\ \frac{6}{50}\ \frac{8}{20}$
- Estimate the percent for each of the following and explain your thinking:

  $\frac{7}{48}\ \frac{5}{19}\ \frac{7}{20}$
- Indicate what percent of a book is left to read after 60 out of 150 pages have been read. Explain your thinking.
- Create problems that utilize percent. Use flyers from local supermarkets and/or department stores to create problems that involve calculating the total savings when certain items are purchased at the sale price.
- Answer the following question: Byron took $85 to the mall to buy gifts. He wants to purchase a book for $13, a video game for $18, and a laptop bag for $40. Sales tax is charged at 15%. Does he have enough money with him to make these purchases?

Planning for Instruction

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Employ a variety of strategies when exact answers are required to calculate the percent of a number:
  - express a percent as an equivalent decimal, and then multiply

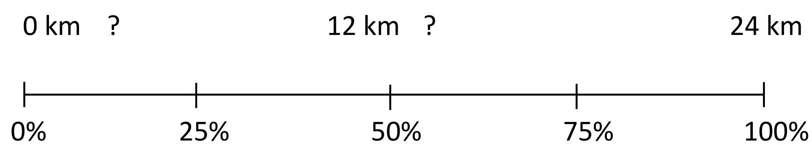
    12% of 80 = 0.12 × 80 (9.6)
  - find 1% and then multiply
  - 12% of 80, find 1% of 80 = 0.8 × 12 (9.6)
  - express the percent as a fraction and divide

25% of 60 = $\frac{1}{4}$ x 60 (60 ÷ 4)

- calculate the missing term in a proportion

  12% of 80 → $\frac{12}{100} = \frac{?}{80}$

- Provide a 10 × 10 grid so students have a visual image of the 1% method. To find 6% of 400, tell students you have $400 and you want to share it equally among the 100 cells. Ask them how much would be in each cell? In 2 cells? In 6 cells? Students can also use this method to estimate; for example, they can estimate 8% of 619 by first mentally finding 8% of 600.
- Have students create problems that utilize percent. They can be given flyers from local supermarkets and/or department stores and use these to create problems that involve calculating the total savings when certain items are purchased at the sale price.
- Use a double number line as a useful tool for understanding percentage. For example: During a 24 km walk/run to raise money for a local children’s hospital, organizers would like to put up markers to tell participants when they are 25%, 50%, and 75% finished. Where will they put the signs?



SUGGESTED LEARNING TASKS

- Present the following three types of problem situations that students will encounter when solving problems that involve finding the whole, the part, or the percent. Include several examples of each type of situation. Present one situation at a time. Ask students to identify the whole, the part, and the percent. Next, they write a word phrase or a number expression to represent the situation. Ask students to find the solutions after all word expressions and number expressions have been represented.
  - A designated percent of a designated number is what number?
    There are 80 cars in a shipment, and 40% of them are silver. How many are silver?
    40% of 80 is ____.
  - A designated number is a designated percent of what number?
    There are 60 cars in a shipment, and 15 of them are red. What percent are red?
  - A designated number is a designated percent of what number?
    25% of the cars in a shipment are blue. There are 50 blue cars. How many cars are in the shipment?

- Solve a problem that may or may not require rounding, such as the following:
  - Your grandmother is going to buy a hooded sweatshirt for your birthday. She heads down to the Pretty Trendy Clothing Store, and finds that the store has an anniversary special. All regular prices are reduced by 20%. She selects a sweatshirt that is regularly priced at $59.99 and pays HST of 15%. How much does she pay for the sweatshirt?
  - After students have had sufficient time to work on the problem, ask individuals to share the strategies they used to solve the problem. Compare students’ solutions, noting whether or not students used rounding, and discuss reasons for their decisions. Discuss which strategies students prefer for this problem. Ensure that students consider the option of calculating the

remaining cost versus calculating the value of the sale and subtracting it from the original price (80% of $59.99 versus $59.99 – 20% of $59.99).

- The manager of a concert hall indicated that, in order to make a profit, the hall must be filled to at least 70% capacity or else the price of each ticket will need to increase. The seating capacity is 1200, and advance ticket sales are at 912. Have enough tickets been sold so that ticket prices will remain low?
- Describe more than one method that could be used to mentally estimate 22% of 310. How could you find the exact answer by calculating mentally?

SUGGESTED MODELS AND MANIPULATIVES

- 10 x 10 grid
- calculator *
- coins
- double number line
- hundredths circle
- open number lines
- various objects for counting (e.g., beans, counters)

*also available in *Interactive Math Tools* (Pearson n.d.)

MATHEMATICAL LANGUAGE

Teacher	Student
▪ commission ▪ decimal ▪ equivalent ▪ fraction ▪ percent ▪ percentage ▪ sales ▪ simplify ▪ skip counting patterns ▪ tax	▪ commission ▪ decimal ▪ equivalent ▪ fraction ▪ percent ▪ percentage ▪ sales ▪ simplify ▪ skip counting patterns ▪ tax

Resources

Print

- *Math Makes Sense 7* (Garneau et al. 2007)
  - Unit 3: Fractions, Decimals and Percents (NSSBB #: 2001640)
    - > Section 3.7 Relating Fractions, Decimals, and Percents
    - > Section 3.8 Solving Percent Problems
    - > Unit Problem: Shopping with Coupons
  - ProGuide (CD; Word Files) (NSSBB #: 2001641)
    - > Assessment Masters
    - > Extra Practice Masters

    - > Unit Tests
  - *ProGuide* (DVD) (NSSBB #: 2001641)
    - > Projectable Student Book Pages
    - > Modifiable Line Masters

Digital

- "Matching Fractions, Decimals, Percentages," *NRICH Enriching Mathematics* (University of Cambridge 2015): http://nrich.maths.org/1249

SCO N04: Students will be expected to demonstrate an understanding of the relationship between positive terminating decimals and positive fractions and between positive repeating decimals (with one or two repeating digits) and positive fractions. [C, CN, R, T]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N04.01 Predict the decimal representation of a given fraction using patterns.
N04.02 Match a given set of fractions to their decimal representations.
N04.03 Sort a given set of fractions as repeating or terminating decimals.
N04.04 Express a given fraction as a terminating or repeating decimal.
N04.05 Express a given repeating decimal as a fraction.
N04.06 Express a given terminating decimal as a fraction.
N04.07 Provide an example where the decimal representation of a fraction is an approximation of its exact value.

Scope and Sequence

Mathematics 6	Mathematics 7	Mathematics 8
N01 Students will be expected to demonstrate an understanding of place value for numbers greater than one million and less than one-thousandth. **N04** Students will be expected to relate improper fractions to mixed numbers and mixed numbers to improper fractions.	**N04** Students will be expected to demonstrate an understanding of the relationship between positive terminating decimals and positive fractions, and between positive repeating decimals (with one or two repeating digits) and positive fractions.	—

Background

“Decimal numbers are simply another way of writing fractions. … Maximum flexibility is gained by understanding how the two symbol systems are related.” (Van de Walle and Lovin 2006b, 107) All fractions can be expressed as terminating or repeating decimals and vice versa. Some students will already know the decimal equivalents of some simple fractions (e.g., $\frac{1}{2}$ = 0.5, $\frac{1}{4}$ = 0.25, $\frac{1}{5}$ = 0.2) as well as any fraction with a denominator of 10, 100, or 1000. For example, to locate 0.75 on a number line, many students think of 0.75 as being three-fourths of the way from 0 to 1. Many students, however, believe that the only fractions that can be described by decimals are those with denominators, which are a power of 10 or a factor of a power of 10. By building on the connection between fractions and division, students should be able to represent any fraction in decimal form, using the calculator as an aid.

All fractions have equivalent decimal names. The decimal names may refer to a definite number of digits. These are **terminating decimals**. A terminating decimal can be easily renamed as a fraction with a denominator that is a power of 10 (e.g., 0.125, read as 125 thousandths, and written as a fraction $\frac{125}{1000}$, which can be simplified to $\frac{1}{8}$).

Knowing common fraction-decimal relationships can help students interpret decimals meaningfully. For example, they see 0.23 and realize that it is almost $\frac{1}{4}$. It is important that students become proficient at correctly reading a decimal number. Reading 0.37 as thirty-seven hundredths, makes the conversion to $\frac{37}{100}$ easy. Students often read 0.37 as "decimal three seven" or "point three seven," which does not provide context or frame of reference and should be avoided. Reinforce the importance of placing zero in front of the decimal to emphasize that it is less than 1.

When some fractions are renamed as decimals, the decimal number contains one or more digits that repeat in a continuous pattern indefinitely (e.g., 1/ 3 = 0.333 . . .). These are **repeating decimals**. The three dots indicating the digits continue without end are called an ellipsis. In North America, the common representation for repeating decimals is to write the number with one set of the repeating digits, and then draw a bar over the digits that form the repeating pattern ($0.\overline{3}$). The series of digits that repeat may be called a period. The bar is called a vinculum. Repeating decimals may also be renamed as fractions (e.g., $\frac{1}{3}$). Characteristic patterns may be used to predict the decimal representation of these fractions and to predict the fraction representation of repeating decimals. Students should be introduced to the terminology "repeating" and "period" as well as bar notation used to indicate repeating periods. The patterns produced by fractions with a variety of denominators should be explored since many have particularly interesting periods.

To express an exact value for a repeating decimal, indicate the repeating section with a vinculum, or write the fraction equivalent. To indicate that the number is an approximation of the true value, use an equal sign with a dot over it ($\doteq$).

Students should use calculators to explore both terminating and repeating decimals and when appropriate to find the decimal form for some fractions and predict the decimal for other fractions. Students should also be aware of the effect of calculator rounding (i.e., automatic rounding caused by the limit on the number of digits that the calculator can display). Where possible, students should use their knowledge of the patterns to determine the fractional form of repeating decimals.

Students should investigate the difference in finding the decimal equivalents for sevenths and eighths:

On a calculator we find:	Using a pattern we find:
$\frac{1}{7} = 0.\overline{142857}$ $\frac{2}{7} = 0.\overline{285714}$ $\frac{3}{7} = 0.\overline{428571}$ Although there is a pattern here, it is not easily observable.	$\frac{1}{8} = 0.125$ $\frac{2}{8} = 0.250$ Therefore: $\frac{3}{8}$ = ? (0.375)

Students should be encouraged to use mental calculation and prior knowledge where possible. For example, the fraction $\frac{4}{25}$ can easily be changed to a decimal by first finding the equivalent fraction with a denominator of 100. Using calculators is encouraged when necessary to find the decimal form for some fractions before predicting the decimal for other fractions. Students are expected to find the decimal representation of a set of fractions such as $\frac{1}{9}$, $\frac{2}{9}$, $\frac{3}{9}$ find a pattern and then use the pattern to predict the decimal for other fractions such as $\frac{4}{9}$, $\frac{5}{9}$, $\frac{10}{9}$. Draw students' attention to fractions in such patterns that will result in a whole number (e.g., $\frac{9}{9}$ = 1, not 0.9). Decimal representations of sets of fractions such as $\frac{1}{12}$ and $\frac{1}{120}$ should also be explored. These patterns can be used to predict the decimal representation of other similar sets.

Students could then be provided with a set of fractions and asked to determine whether the decimal equivalents are terminating or repeating, and re-write repeating decimals using the bar notation. A graphic organizer, such as a T-Chart, may be useful in helping students sort fractions.

Expressing repeating decimals as fractions is more challenging since denominators of 10, 100, 1000 cannot be used. Repeating decimals can be expressed as fractions using denominators of 9, 99, 999, etc., depending on the number of decimal places in the period. Student understanding of this should evolve through discussions of familiar examples, such as $0.\overline{3}$. Students know it is equivalent to $\frac{1}{3}$, not $\frac{3}{10}$. Ask students which denominator could be used for the numerator 3, since the 3 is in the decimal form. Students should easily identify $\frac{3}{9}$. In the example $0.\overline{7}$, the 7 is in the tenths place, but tenths cannot be used since it is not exactly seven tenths. In this case ninths would be used, giving the fraction $\frac{7}{9}$. In the example $0.\overline{18}$, hundredths cannot be used since it is not exactly 18 hundredths, so 99 is used as the denominator, resulting in the fraction $\frac{18}{99}$, which can be simplified to $\frac{2}{11}$.

Students should realize that fractions such as $\frac{1}{6}$ = $0.\overline{16}$ are exact values whereas a calculator display that shows 0.166666667 is an approximation. When students round such values to 0.17 or 0.2, for example, it is important that they recognize that these are approximations, not exact values. Discussion may include real-life situations for which it might make sense to use approximations, such as the distance between towns, the amount of gas in a dirt-bike, mental calculation of discount amounts, etc.

Assessment, Teaching, and Learning

Assessment Strategies

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to use the following numbers to answer the questions below:
  8.0254   2.086   0.83   24.2191
  - In which number does 8 represent a value of eight-hundredths?
  - In which number does 2 represent a value of two-tenths?
  - In which number does 1 represent a value of one ten-thousandth?

- Review fractions such as $\frac{1}{2}, \frac{1}{4}, \frac{3}{4}, \frac{1}{5}, \frac{2}{5}$ their decimal equivalents.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Find the following decimal representations of fractions: $\frac{1}{11}, \frac{2}{11}, \frac{3}{11}$

  And then:
  - Predict the decimals for $\frac{5}{11}, \frac{9}{11}$
  - Predict the fraction that will have 0.636363 ... as a decimal.
  - Predict what the decimal for $\frac{8}{11}$ would look like on a calculator display if the calculator is set to display 8 places after the decimal.
  - Predict the fraction which will have 0.909090…as a decimal.
- How does knowing that $\frac{1}{4}$ = 0.25 helps find the decimal form of $\frac{3}{4}$ and $\frac{5}{4}$?
- Chris had a calculator that displayed 2.3737374. Chris concluded that it was not a repeating decimal. Explain why Chris drew this conclusion and whether or not you believe it to be a correct conclusion.
- Which is larger 0.7 or $0.\overline{7}$? Explain your reasoning.
- Describe a fraction that is a bit less than 0.4 and justify the selection. Determine another fraction that is between these two?
- About 0.4 of a math class will be going on a field trip. Write the decimal in words, and as a fraction in simplest form.
- Sort this set of fractions into repeating and terminating decimals

  $\frac{4}{5}, \frac{2}{3}, \frac{1}{7}, \frac{5}{8}, \frac{5}{9}, \frac{7}{10}, \frac{3}{4}$
- Of all life on Earth, 0.72 live below the ocean's surface. Write this as a fraction in simplest form.
- The following numbers appear on three calculator screens. Match the correct displays to the correct fractions. Use your knowledge of repeating decimals and estimation.

0.55555556	0.28571429	0.30769231
$\frac{2}{7}$	$\frac{5}{9}$	$\frac{4}{13}$

Planning for Instruction

Choosing Instructional Strategies

Consider the following strategies when planning daily lessons.

- Use both proper fractions and mixed numbers in activities.
- Explore patterns of different fraction families, both terminating and repeating, (e.g., $\frac{1}{11}, \frac{2}{11}, \frac{3}{11}$) with a calculator. Some fractions and their repeating decimal equivalents are listed in the following table for reference. Give students opportunities to discover these patterns.

Denominator of the Fraction	Pattern in the Repeating Decimal	Example
7	▪ six repeating digits ▪ digits are 142857 in a cyclic pattern	$\frac{2}{7} = 0.\overline{285714}$
9	▪ single repeating digit ▪ the numerator is the repeating digit	$\frac{7}{9} = 0.\overline{7}$
99	▪ two repeating digits ▪ the numerator is the repeating sequence	$\frac{85}{99} = 0.\overline{85}$
999	▪ three repeating digits ▪ the numerator is the repeating sequence	$\frac{1}{999} = 0.\overline{001}$
11	▪ two repeating digits that are a multiple of 9 ▪ the numerator is the factor × 9 that equals the repeating sequence	$\frac{3}{11} = 0.\overline{27}$

- Have students recognize that only fractions that can be expressed with base-ten denominators (10, 100, and 1000) will be terminating when written in decimal form. For example:

  $3\frac{2}{5} = 3\frac{4}{10} = 3.4$ $\qquad$ $2\frac{3}{8} = 2\frac{375}{1000} = 2.375$

- Provide frequent opportunities to read terminating decimals (e.g., 0.312, is read as three hundred twelve thousandths). When a student reads a terminating decimal, it should be clear how to write it in fractional form.

Suggested Learning Tasks

- Using a set of fractions such as $\frac{1}{13}, \frac{2}{13}, \frac{3}{13}$, find a pattern and then use the pattern to predict the decimal for other fractions such as $\frac{4}{13}, \frac{5}{13}, \frac{10}{13}$.
- Compare the decimals using a calculator for the following pairs and discuss the similarities and differences you observe.

  (a) $\frac{1}{12}$ and $\frac{1}{120}$ $\qquad$ (b) $\frac{3}{8}$ and $\frac{3}{80}$

  (c) Since the decimal for $\frac{3}{16}$ is 0.1875, predict the fraction that would produce a decimal of 0.01875163.

- Use base-ten blocks to explain the decimal equivalents to fractions, even when these decimals repeat. For example, 1÷3 could be modelled by having 1 block shared by 3 people and decide how to share the remaining piece(s). Decimal squares can be used similarly by having students shade, for example, one-third of the square and decide how to shade the remaining square(s). Lead a class demonstration and discussion about representing a fraction quantity with a base-ten place value system. Use base-ten blocks, and include the concepts of terminating and repeating decimals, using division to represent a fraction, notations that represent exact quantities, and approximation.
- Share a candy bar equally between two students. How much of the candy bar each one received? Write $\frac{1}{2}$ on the board. One-half of the candy bar describes exactly what each student received. It's an exact number. Is it possible to name that number with the base-ten place value system?
- A certain candy bar can easily be broken into 8 equal square pieces. There are 27 students in Suri's class, and she has $3\frac{1}{2}$ candy bars. Suri found how many eighths there were in the bars, using equivalent fractions. Are there enough pieces for each student to have a square of the candy? Explain how the answer was found. Represent as a decimal the fractional part that is left over.
- Create cards with fractions and their decimal equivalents. Each student receives a card with either a decimal or a fraction. They circulate around the room to find the card that is equivalent to their own. Each group must explain why their cards belong together.

SUGGESTED MODELS AND MANIPULATIVES

- 10 x 10 grid
- base-ten blocks
- calculator
- fraction pieces
- hundredth circles

MATHEMATICAL LANGUAGE

Teacher	Student
▪ bar notation ▪ decimal equivalent ▪ denominator ▪ ellipsis ▪ numerator ▪ period ▪ repeating decimal ▪ simplify (a fraction to lowest terms) ▪ terminating decimal ▪ unit fraction ▪ viniculum	▪ bar notation ▪ decimal equivalent ▪ denominator ▪ numerator ▪ period ▪ repeating decimal ▪ simplify (a fraction to lowest terms) ▪ terminating decimal ▪ unit fraction

Resources

Print

Math Makes Sense 7 (Garneau et al. 2007)

- Unit 3 Fractions, Decimals and Percents (NSSBB #: 2001640)
  - Section 3.1 Fractions and Decimals
  - Section 3.4 Multiplying Decimals
  - Section 3.5 Dividing Decimals
  - Section 3.6 Order of Operations with Decimals
  - Unit Problem: Shopping with Coupons
- *ProGuide* (CD; Word Files) (NSSBB #: 2001641)
  - Assessment Masters
  - Extra Practice Masters
  - Unit Tests
- *ProGuide* (DVD) (NSSBB #: 2001641)
  - Projectable Student Book Pages
  - Modifiable Line Masters

SCO N05: Students will be expected to demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially, and symbolically (limited to positive sums and differences). [C, CN, ME, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N05.01 Use benchmarks to estimate the sum or difference of positive fractions or mixed numbers.
N05.02 Model addition and subtraction of given positive fractions or given mixed numbers, using concrete and pictorial representations, and record symbolically.
N05.03 Determine the sum or difference of fractions mentally, when appropriate.
N05.04 Determine the sum of two given positive fractions or mixed numbers with like denominators.
N05.05 Determine the difference of two given positive fractions or mixed numbers with like denominators.
N05.06 Determine a common denominator for a given set of positive fractions or mixed numbers.
N05.07 Determine the sum of two given positive fractions or mixed numbers with unlike denominators.
N05.08 Determine the difference of two given positive fractions or mixed numbers with unlike denominators.
N05.09 Simplify a given positive fraction or mixed number by identifying the common factor between the numerator and denominator.
N05.10 Simplify the solution to a given problem involving the sum or difference of two positive fractions or mixed numbers.
N05.11 Solve a given problem involving the addition or subtraction of positive fractions or mixed numbers, and determine if the solution is reasonable.

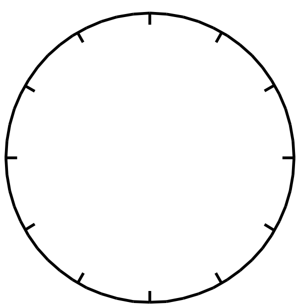
Scope and Sequence

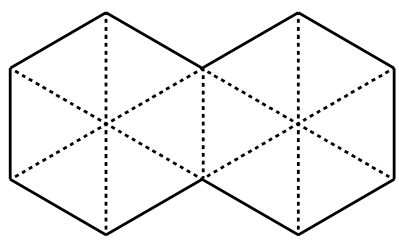
Mathematics 6	**Mathematics 7**	**Mathematics 8**
N03 Students will be expected to demonstrate an understanding of factors and multiples by ▪ determining multiples and factors of numbers less than 100 ▪ identifying prime and composite numbers ▪ solving problems using multiples and factors **N04** Students will be expected to relate improper fractions to mixed numbers and mixed numbers to improper fractions.	**N05** Students will be expected to demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially, and symbolically (limited to positive sums and differences).	**N06** Students will be expected to demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially, and symbolically. **N04** Students will be expected to demonstrate an understanding of ratio and rate.

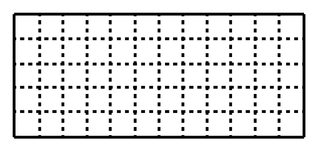
Mathematics 6 (continued) **N05** Students will be expected to demonstrate an understanding of ratio, concretely, pictorially, and symbolically.		

Background

Although, students are first introduced to fractions in Mathematics 3 and conceptual development has continued in Mathematics 4, 5, and 6, it is very important that students have ample opportunity to show they have strong fraction number sense before focusing on common denominators and other rules of computation (Van de Walle and Lovin 2006b, 87). Students must first explore the meaning of fractions, using different models including region, set, length or measurement, ratio, and division. Manipulatives allow students to visualize fractions concretely. As their conceptual understanding develops, students connect their drawings and symbolic representations of these abstract concepts. Be aware that inaccurate drawings can lead to inaccurate results. Tracing the models will help. Minimize inaccuracies by supplying grid paper for drawing rectangles and templates to represent circles or triangular dot paper for fraction pattern blocks. Outlines of the fraction pieces, fraction pattern blocks or the clock circle are other possibilities. Rulers or number lines with equal intervals are also useful as is tracing the models. When students draw their solutions a key should be present stating what represents one whole.


Clock Circle


Fraction Pattern Blocks


Fraction Pieces

A variety of fraction manipulatives should be used in your diagrams and both operations should be modelled.

To help students add and subtract fractions correctly, and with understanding, teachers must help them develop an understanding of the numerator and denominator, equivalence and the relation between mixed numbers and improper fractions (NCTM 2000, 218). The number of equal parts in one whole is the denominator of the fraction, and the number of parts being referred to forms the numerator. So a fraction such as $\frac{5}{7}$ would mean the one whole was divided into 7 equal parts and 5 have been selected. It should be read as five-sevenths and never read as “five over seven.”

Fractions are an extension of the whole number system, and the same principles for adding and subtracting whole numbers apply to adding and subtracting fractions. The meanings of each operation with fractions are the same as the meanings for the operations of whole numbers. Throughout primary and elementary mathematics, students developed conceptual and procedural understandings of operations with whole numbers and decimals. This understanding of operations should be used to give

meaning to fraction computations and should begin by applying these same meanings to fractional parts. For addition and subtraction, it is critical to understand that the numerator tells the number of parts and the denominator the type of part.
"Estimation should be an integral part of computation development to keep students' attention on the meanings of the operations and the expected size of the results." (Van de Walle and Lovin 2006b, 66).

Benchmarks of 0, $\frac{1}{2}$, 1, $1\frac{1}{2}$, 2 etc., should be a focus of the estimation of sums and differences of fractions. Encourage flexibility of thinking by providing learning opportunities that connect

- operations with whole numbers to operations with fractions;
- subtraction of fractions to addition of fractions
- concrete, pictorial and symbolic representations
- operations with fractions to real world problems

(Alberta Education 2004)

Encourage the addition or subtraction of fractions mentally where appropriate. Some equivalent fractions, such as $\frac{1}{2}$ and $\frac{2}{4}$, are easily recognizable. Proficiency in identifying common factors and multiples facilitates renaming less recognizable fractions. Students will have a much easier time identifying factors and multiples when they have quick recall of their multiplication and division facts, and when they can apply divisibility rules. Use friendly fractions that can easily be represented with manipulatives or drawings, and fractions that can easily be related to one another, such as fourths and eighths. An example of a friendly fraction combination is thirds and sixths. An example of an unfriendly fraction combination is fifths and twelfths. Using friendly fractions makes it easier for students to find equivalent fractions and helps them build confidence in the strategies they are developing.

When teaching students addition of fractions, an appropriate sequence begins with fractions with like denominators, then fractions with unlike denominators, and finally improper fractions and mixed numbers. Students need to make the connection between addition and subtraction of fractions. Students should apply prior knowledge of factors to assist them in determining common denominators and simplifying fractions and mixed numbers.

Manipulatives help students construct mental referents that enable them to perform fraction tasks meaningfully. Use a variety of manipulatives, such as fraction blocks and pattern blocks, fraction pieces, fraction strips, and fraction circles to model addition of fractions with like denominators. Pattern blocks are a good model for addition when the fractions have denominators 2, 3, or 6. Fraction circles lend themselves to more denominators, including 2, 3, 4, 5, 6, 8, 10, or 12.
(Fraction circles may be found on the *iTools: Fractions* website at http://www-k6.thinkcentral.com/content/hsp/math/hspmath/na/common/itools_int_9780547584997_/fractions.html or on the Utah State University website at http://nlvm.usu.edu/en/nav/frames_asid_274_g_2_t_1.html?open=activities.)

One whole can be defined using other shapes, such as rectangles. Encourage students to use and reflect on their drawings. Determining the strengths and weaknesses of various representations for a particular problem enhances students' understanding. Once students have modelled addition of fractions with like denominators they should have little difficulty determining their sum. Building from whole number addition, students can generalize addition of parts of one whole. For example, they can think of $\frac{2}{8}+\frac{5}{8}$ as adding two-eights and five-eights to total seven-eights, or $\frac{7}{8}$.

Throughout the unit, encourage students to simplify fractions to make it easier for them to compare fractions and to perform operations with fractions**.** They have previously worked with factors of whole numbers. The use of models can facilitate an understanding of fractional equivalents and expressing fractions in simplest form. Once students use models to add fractions with like denominators, they can begin using fraction pattern blocks, fraction pieces, fraction circles, fraction strips, and number lines to add fractions with related denominators (e.g., thirds and sixths, or fifths and tenths), and then to move to fractions with unrelated denominators (e.g., thirds and fourths). Most of the tasks should involve fractions with "friendly" denominators no greater than 12. At this stage, avoid adding numbers that cannot be easily represented with any model or drawing. Using models to add fractions with unrelated denominators leads students to discover the need for common denominators. For example, when students model $\frac{1}{3}+\frac{1}{2}$, they should quickly realize that the manipulative being used should be divided into sixths to find the sum. Students look for fraction pieces that can exactly cover one-third and one-half, in this case sixths.

They may try several possibilities using their manipulatives first before arriving at this conclusion. Giving them the opportunity to come to this conclusion allows them to develop fraction number sense prior to being introduced to common denominators and other rules of computation. Ideally, the least common multiple of the unlike denominators should be the common denominator used. Progression can then be made to the symbolic level.

$$\frac{1}{3}+\frac{1}{2}$$

$$\frac{1}{3} \xrightarrow{\times 2} \frac{2}{6} \quad \text{and} \quad \frac{1}{2} \xrightarrow{\times 3} \frac{3}{6}$$

$$\frac{2}{6}+\frac{3}{6}=\frac{5}{6}$$

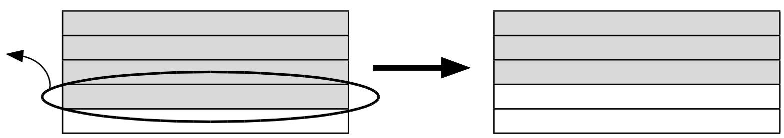
It is important for students to keep focus on the meanings of the numbers and the operations. Estimation should play a role in the development of strategies for working with fractions. Through the use of benchmarks (close to 0, $\frac{1}{2}$, 1, $1\frac{1}{2}$, 2, etc.) developed earlier, students should be encouraged to estimate the solution and use their estimate to verify the reasonableness of the answer obtained. When calculating $\frac{1}{3}+\frac{5}{8}$, students should be able to reason that $\frac{1}{3}$ is a little bit less than $\frac{1}{2}$ and $\frac{5}{8}$ is more than $\frac{1}{2}$, so the answer should be close to 1.

Then they can then use a common denominator to determine the sum.

Students should always check the reasonableness of their answer by comparing it to their initial estimate. It is important for students to work with problems such as $\frac{1}{4}+\frac{1}{6}$.
Many students quickly conclude that the lowest common denominator is the product of the given denominators because in so many questions this turns out to be correct. They need to see that the lowest common denominator is often smaller than the product of the two given denominators. Exposing students to an extreme case often makes this point well. For example, when adding $\frac{1}{2}$ and $\frac{1}{18}$, the product of the denominators is 216, whereas the LCD is 36. The teacher can ask students which would be easier to calculate: $\frac{18}{216}+\frac{12}{216}$ = ? or $\frac{3}{36}+\frac{2}{36}$ = ?

Subtraction of fractions, as with whole numbers, is the inverse operation of addition. Subtraction should be visualized through the use of various models, including fraction pattern blocks, fraction pieces, fraction circles, fraction strips, and number lines. Modelling subtraction of fractions with like denominators, students can physically remove a fraction piece to determine the difference.
For example: $\frac{4}{5} - \frac{1}{5} = \frac{3}{5}$

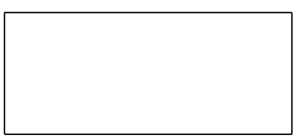
Let  **represent one whole**

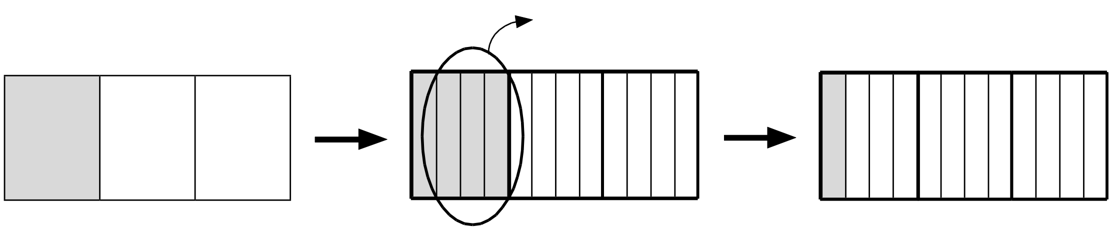


Note: Encourage students to only draw the minuend and use an arrow to model the take away action represented by the subtrahend.

For example,

$$\underbrace{\frac{1}{3}}_{\text{minuend}} - \underbrace{\frac{1}{4}}_{\text{subtrahend}} = \underbrace{\frac{1}{12}}_{\text{difference}}$$

Let  **represent one whole**



When using models, ensure all subtractions have the minuend larger than the subtrahend to result in a positive difference.

Addition and subtraction of mixed numbers is developed from the models and algorithms for addition and subtraction of positive fractions. Various models, such as but not limited to Cuisinaire rods and fraction strips, can help students visualize working with mixed numbers. (Examples of these can be found on the *PBS Learning Media* website at www.pbslearningmedia.org/resource/rttt12.math.cuisenaire/modelling-fractions-with-cuisenaire-rods/ or the University of Cambridge's *NRICH* website at http://nrich.maths.org/4348.)

There are two numerical approaches for adding and subtracting mixed numbers. For addition and subtraction, students may convert mixed numbers to improper fractions and add these as they would proper fractions, returning the final answer to a mixed number. Alternately, for addition they may add the whole number portions separately from the fraction portions and then express the answer as a mixed number. For subtraction, students may subtract the fraction portions separately if the fractional portion of the minuend is larger than the fractional portion of the subtrahend (e.g., $3\frac{6}{7} - 2\frac{1}{3}$; note $\frac{6}{7} > \frac{1}{3}$). Otherwise, students will have to regroup from the whole portion (e.g., $2\frac{4}{7} - 1\frac{2}{3} = 1\frac{33}{21} - 1\frac{14}{21} = \frac{19}{21}$). Encourage students to simplify to lowest terms as a proper fraction or mixed number. Connections to real-world applications should be used throughout the development of adding and subtracting positive fractions and mixed numbers. Various examples include recipes involving cups, timed tasks involving hours, or capacity involving portions.

Assessment, Teaching, and Learning

Assessment Strategies

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

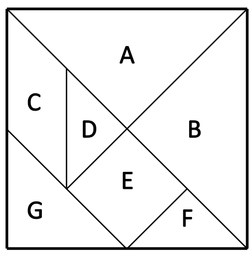
- Tell students you were trying to figure out the multiples of 8 and this is the list you came up with: 0, 8, 16, 23, 32, and 40. Ask if the list is complete. Ask if it is correct. Ask them to explain their thinking.
- Ask students to list all of the factors of 12 and the first six multiples of 12.
- Have students write as many improper fractions as they can with the numbers 3, 6, 7, and 8. Have them represent one of the improper fractions using a model or a picture.
- Provide students with several mixed numbers and improper fractions. For example,
  $2\frac{1}{3}, \frac{7}{4}, \frac{5}{3}, 2\frac{3}{4}, 1\frac{4}{5}$
  Have students place the numbers on an open number line to demonstrate their relative magnitude.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Explain why this doesn't make sense: Sam wrote $\frac{3}{4}$ hr + $\frac{1}{2}$ hr = $\frac{4}{6}$ hr, saying that he had worked on the computer for 45 minutes and watched television for half an hour. Explain his mistake in determining the total time he spent on these activities. What is the correct answer (expressed as a fraction)?
- Create three addition and three subtraction sentences that would have an answer of $\frac{3}{4}$.
- Answer the following and justify your responses:
  - can an answer be sixths when you add fourths and thirds?
  - can an answer be sevenths when you add fourths and thirds?

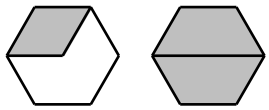
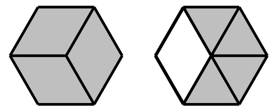
- A container is half full. When half a cup of juice is added to it, the container is three-quarters full. How much liquid can the container hold? Model or draw your answer.
- Determine, with the aid of drawings, if the following is correct: $\frac{1}{4} + \frac{1}{4} = \frac{2}{8}$. Explain your reasoning.
- A tangram is a square puzzle that is divided into seven shapes. Based on the tangram below, answer the following questions:
  - Given that piece A is $\frac{1}{4}$ of the whole square, what are the values of pieces B, C, D, E, F, and G?
  - What is the sum of A and B? B and G? E and F?
  - Which two tangram pieces add up to the value of B? C?
  - Invent a problem on your own and solve it.



- Create three addition expressions with unlike denominators that are equivalent to $\frac{6}{12} + \frac{3}{12}$.
- Use the provided magic square below. The sum of each row, column and diagonal in this magic square must equal 1. Find the missing values.

		$\frac{5}{12}$
$\frac{7}{12}$	$\frac{1}{3}$	
$\frac{1}{4}$		

- Write an addition sentence to represent the fraction of each hexagon that is shaded and use the addition sentence to find the total value of the shaded hexagons.

a)  b) 

- Use pattern blocks to create a design on triangular grid paper and then use fraction addition to name the design. It is possible to use several different addition sentences to name the same design.
- Ask students if
  - adding fourths and thirds results in sixths
  - adding fourths and thirds results in sevenths

  They should justify their answers.
- "Connect Three—Addition of Fractions"

  This two-player game provides an opportunity to practice fraction addition.

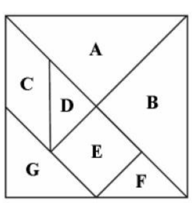
  Materials: game board (Appendix B), two-coloured counters, paper clips

  How to Play:
  - The first player chooses two numbers on the bottom strip and places a paper clip on each. The first player then adds those two numbers, and places a counter on the answer on the game board.
  - The second player moves ONLY ONE of the paper clips on the bottom strip to make a second operation. The second player then places a counter on the answer.
  - Play continues until a player connects three answers in a horizontal, vertical, or diagonal row.

  This game can be modified for subtraction of fractions.
- Respond to the following: Your friend missed yesterday's lesson. When solving a problem today, he suggested that $\frac{5}{6}+\frac{5}{8}=\frac{10}{14}$. How can you convince him that this is not a reasonable solution?
- Use concrete materials or diagrams to show why the following is an incorrect procedure:

$$\frac{3}{8}-\frac{1}{4}=\frac{3-1}{8-4}=\frac{2}{4}=\frac{1}{2}$$

- Use concrete materials of your choice to make up two subtraction questions. Draw diagrams to show your questions. Challenge a classmate to answer your questions using the materials you chose.
- The tangram piece labelled "A" is removed from a finished tangram. Write a subtraction sentence to show the fraction of the completed tangram that remains.



- Using the tangram above, write and answer subtraction questions for:
  - A – D
  - B – E

- Is it possible to find two mixed numbers that add together to form a whole number? You should explain your answer and, if possible, give an example.
- Answer questions such as the following:
  - Andrew plays guitar in a rock band. For a song that is 36 measures long he plays for $4\frac{1}{2}$ measures, rests for $8\frac{3}{8}$ measures, plays for another 16 measures, rests for $2\frac{1}{4}$ measures, and plays for the last section. How many measures are in the last section?
  - This week, Mark practised piano for $3\frac{1}{2}$ hours, played soccer $6\frac{1}{4}$ hours, and talked on the phone for $4\frac{1}{3}$ hours.
    - > How many hours did Mark spend practising piano and playing soccer?
    - > Hour many more hours did Mark spend playing soccer than talking on the phone?

Planning for Instruction

Choosing Instructional Strategies

Consider the following strategies when planning daily lessons.

- Use concrete models to demonstrate the meaning of fractions. The numerator of a fraction counts or tells how many of the fractional parts (of the type indicated by the denominator) are under consideration. The denominator names the kind of fractional part that is under consideration (what is being counted).
- Students find it challenging to conceptualize what represents one whole so it is important to use a variety of materials so their understanding is not associated with a single model.
- Give students opportunity to vary the shape used to represent one whole. Use different shapes within a model/particular manipulative to represent one whole. For example, the hexagon may represent one whole, but so too may the trapezoid, the chevron, rhombus, etc.
- Use a problem-solving context that is relevant to students.
- Connect problems applying the addition and subtraction of fractions and mixed numbers to similar problems with whole numbers. Include various structures of problems for addition and subtraction such as, part-part-whole and comparison from previous grades, such as $\frac{1}{2} + \square = \frac{5}{8}$ or $\square + \frac{1}{4} = \frac{2}{3}$.
- Connect the subtraction of fractions to the addition of fractions.
- Estimate sums and differences of fractions before calculating by using benchmarks (0, $\frac{1}{2}$, 1, $1\frac{1}{2}$, 2, etc.)
- Have students explore sums and differences of fractions by using a variety of models.
- Emphasize that students connect the concrete, pictorial, and symbolic representations for sums and differences of fractions. Once students internalize the fact that fractions can be added and subtracted symbolically, they become less reliant on concrete and pictorial models.
- Ensure that students record all solutions to fraction computations in the simplest form.
- Give students a variety of addition and subtraction expressions including positive fractions and mixed numbers. Ask them to explain how to determine the sum or difference using concrete materials, drawings, or descriptions.
- Give students solutions to a variety of addition or subtraction expressions, some of which have errors in them. For example,

$12\frac{1}{4} - 9\frac{2}{3} = 3\frac{5}{12}$

$\frac{7}{8} + \frac{1}{3} = \frac{8}{11}$

$2\frac{1}{5} - 1\frac{3}{5} = 1\frac{2}{5}$

Find the errors and explain how you would correct them.

SUGGESTED LEARNING TASKS

- Have students solve contextual problems using concrete models. As they develop an understanding, students can record the steps symbolically as they solve the operations. Provide a variety of contexts: baking, mowing lawns, time, etc.
- Create cards with addition expressions and their equivalent manipulative representations. Each student receives a card with either the addition expression, or the representation. Ask the students to find their partner in the class. Each group must then explain why their cards match.
- Create cards with subtraction expressions and their equivalent manipulative representations. Each student would receive a card with either the subtraction expression or the representation. They find their partner in the class. Each group must then explain why their cards match.
- Jody added fractions and found the answer to be $\frac{5}{8}$. What could the fractions have been? How many different answers could there be?
- Use diagrams to model addition and subtraction of fractions.

  $\frac{3}{5} + \frac{1}{2}$ $\qquad$ $\frac{6}{10} + \frac{5}{10}$ $\qquad$ $\frac{3}{8} - \frac{1}{4}$
- When one fraction is subtracted from another fraction, the difference is zero. The fractions have different denominators. Determine what the fractions could be. Give two possible answers.

SUGGESTED MODELS AND MANIPULATIVES

- circle templates/graphs
- counters
- Cuisenaire rods
- fraction blocks
- fraction circles*
- fraction pieces
- fraction strips*
- grid paper
- number line*
- pattern blocks*
- various objects for counting (e.g. beans, counters)

*also available in *Interactive Math Tools* (Pearson n.d.)

MATHEMATICAL LANGUAGE

Teacher	Student
▪ addend	
▪ denominator	▪ denominator
▪ difference	▪ difference
▪ equivalent fractions	▪ equivalent fractions
▪ factor	▪ factor
▪ greatest common factor	▪ greatest common factor
▪ fraction	▪ fraction
▪ improper fraction	▪ improper fraction
▪ least common multiple	▪ least common multiple
▪ lowest terms	▪ lowest terms
▪ minuend	
▪ mixed number	▪ mixed number
▪ multiple	▪ multiple
▪ numerator	▪ numerator
▪ proper fraction	▪ proper fraction
▪ simplify	▪ simplify
▪ sum	▪ sum
▪ subtrahend	

Resources

Print

Making Math Meaningful to Canadian Students K–8, Second Edition (Small 2013), 264–269

Math Makes Sense 7 (Garneau et al. 2007)

- Unit 5: Operations with Fractions (NSSBB #: 2001640)
  - Section 5.1 Using Models to Add Fractions
  - Section 5.2 Using Other Models to Add Fractions
  - Section 5.3 Using Symbols to Add Fractions
  - Section 5.4 Using Models to Subtract Fractions

  - Section 5.5 Using Symbols to Subtract Fractions
  - Section 5.6 Adding with Mixed Numbers
  - Section 5.7 Subtracting with Mixed Numbers
  - Reading and Writing in Math: Writing a Complete Solution
  - Unit Problem: Publishing A Book
- *ProGuide* (CD; Word Files) (NSSBB #: 2001641)
  - Assessment Masters
  - Extra Practice Masters
  - Unit Tests
- *ProGuide* (DVD) (NSSBB #: 2001641)
  - Projectable Student Book Pages
  - Modifiable Line Masters

Digital

- "Cuisenaire Environment," *NRICH Enriching Mathematics* (University of Cambridge 2015): http://nrich.maths.org/4348
- "Fraction Pieces," *Utah State University* (Utah State University 2015): http://nlvm.usu.edu/en/nav/frames_asid_274_g_2_t_1.html?open=activities
- "iTools: Fractions," *Houghton Mifflin Harcourt School Publishers* (Houghton Mifflin Harcourt School Publishers 2015): www-k6.thinkcentral.com/content/hsp/math/hspmath/na/common/itools_int_9780547584997_/fractions.html
- "Mathematics Blackline Masters Grades P to 9, Table of Contents," *Nova Scotia Department of Education and Early Childhood Development* (Province of Nova Scotia 2015): http://lrt.ednet.ns.ca/PD/BLM/table_of_contents.htm
- "Modeling Fractions with Cuisenaire Rods," *PBS Learning Media* (PBS and WGH 2015): www.pbslearningmedia.org/resource/rttt12.math.cuisenaire/modelling-fractions-with-cuisenaire-rods

SCO N06: Students will be expected to demonstrate an understanding of addition and subtraction of integers, concretely, pictorially, and symbolically. [C, CN, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N06.01 Explain, using concrete materials such as integer tiles and diagrams, that the sum of opposite integers is zero.
N06.02 Illustrate, using a number line, the results of adding or subtracting negative and positive integers.
N06.03 Add two given integers, using concrete materials and/or pictorial representations, and record the process symbolically.
N06.04 Subtract two given integers, using concrete materials and/or pictorial representations, and record the process symbolically.
N06.05 Illustrate the relationship between adding integers and subtracting integers.
N06.06 Solve a given problem involving the addition and subtraction of integers.

Scope and Sequence

Mathematics 6	Mathematics 7	Mathematics 8
N07 Students will be expected to demonstrate an understanding of integers contextually, concretely, pictorially, and symbolically.	**N06** Students will be expected to demonstrate an understanding of addition and subtraction of integers, concretely, pictorially, and symbolically.	**N07** Students will be expected to demonstrate an understanding of multiplication and division of integers, concretely, pictorially, and symbolically.

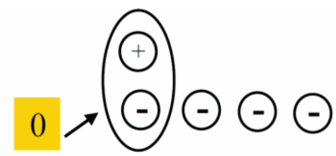
Background

Integers are the set of numbers consisting of the natural numbers (1, 2, 3, …), their opposites (–1, –2, –3, …), and zero. They are also referred to as the whole numbers and their opposites. Integers indicate both a quantity (magnitude) and a direction from zero. Positive integers are greater than zero and are located to the right of zero on the number line. They are represented by a positive symbol (+) before the integer, such as (+5). Negative integers are less than zero and are located to the left of zero on the number line. They are represented by a negative symbol (–) before the integer, such as (–3). There are two common notations for integers. The symbols are written with the + or – sign preceding the integer, as in –5 or +3 or enclosed within parentheses, as in (+5), (–3). The parentheses are commonly used in student materials to avoid any confusion between the integer sign and the notations for addition and subtraction. In the expression (+5) – (–3) the parentheses indicate the numbers inside are integers and distinguish the integer symbols from the subtraction symbol.

Understanding and working with integers is important in daily life. Integers are regularly encountered in contexts such as finances (net worth, balance sheets, and profit or loss), investments, temperatures, elevations, time relevant to events, and sports. Provide students with contextualized problems of this nature. Proficiency with integers is necessary when evaluating algebraic expressions and solving equations. It allows students to graph relations using all four quadrants. Work with integers will be applied to future study of rational expressions, and extended to irrational and real numbers. It continues to build number sense, preparing students for a wide range of problem-solving activities. Operations with integers build on operations with whole numbers.

The balance of positive and negative values is known as the zero principle, and it is the foundation for many computations involving integers. Emphasis must be placed on the zero principle and its application in addition and subtraction situations. For example, (–1) + (+1) = 0, (–3) + (+3) = 0, (–17) + (+17) = 0. The sum of any pair of opposite integers is zero. Based on this principle, zero and other integers can be expressed in multiple ways.

A tile or counter representing (+1) and one representing (–1) form a zero pair. When combined, these tiles model the number zero. A given integer can be modelled in many different ways. For example, one way to represent –3 using integer tiles is shown below.



Using integer tiles or counters and the zero principle to model the various ways a number can be represented will help students conclude that adding a zero pair does not change the value of the integer being modelled. Work with zero pairs will be important as students concretely add and subtract integers. Students will progress to adding and subtracting integers symbolically. They will generalize and apply rules for these integer operations.

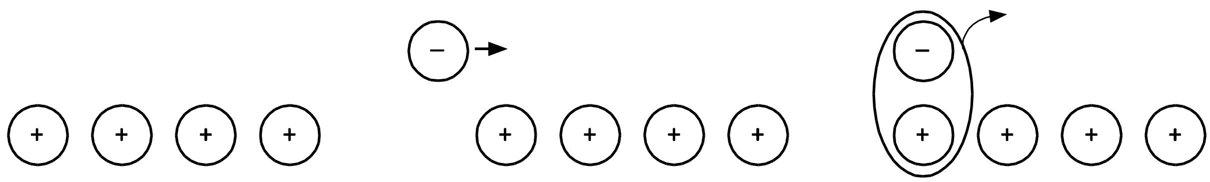
Concrete models commonly used for representing integers are coloured counters, alge-tiles, and number lines. Digital models also exist and simulations allow students to work with representations of positive and negative counters. Students should be exposed to multiple models. Parallel development, using both models at the same time, may be the most conceptual approach. Integers involve two concepts, “quantity” and “opposite.” Quantity is modelled by the number of counters or length of the arrows in a vector model. Opposite is represented as different colours or different directions. An example of adding integers using each model is provided.

(+4) + (–1) = +3

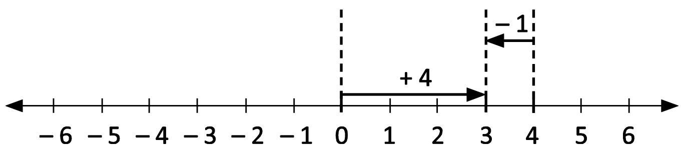
John was a +4 in his first game of a hockey tournament. He was a –1 in the second game of the tournament. What is his plus/minus at the end of the second game?

Bill was +4 at the end of the first 9 holes of golf. In the next 9 holes, his score was –1. What is his score at the end of 18 holes of golf?

After a heavy rain, the Sackville River was 4 metres above its usual level. After one day, the river dropped 1 m. What was the level of the river on the second day?



Start at 0. Move 4 units to the right to represent +4. From there, move 1 unit to the left to represent –1.

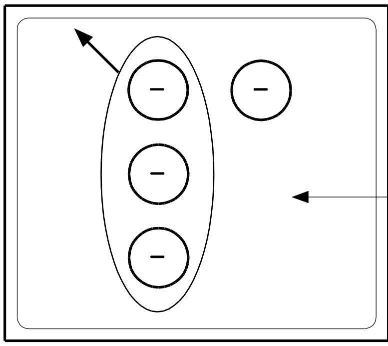


It was a dry summer in Shubenacadie. The river was 2 m below its normal level. During August, there was no rain, and the water level went down another metre.
How far is the river below the normal level now?
(–2) + (–1) = (–3)

The distance an integer is from zero represents the magnitude of the integer, and the direction from zero represents whether the integer is positive or negative. While teachers may model correct use of absolute value (the distance from an integer to zero on a number line), it is not an expectation for students to know the term. On a vertical number line, the distance above zero represents positive integers. Distances below zero represent negative integers. Numbers increase in value above zero on a vertical number line, and decrease in value below zero on a vertical number line. On a horizontal number line, numbers increase in value to the right of zero, and decrease in value to the left of zero. Values always increase from left to right, and decrease from right to left.

The use of models helps students develop a conceptual understanding of the following principles for adding integers.
1. The sum of two positive integers is positive.
2. The sum of two negative integers is negative.
3. The sum of a negative integer and a positive integer can be negative or positive. The sum has the sign of the number that is further from zero.

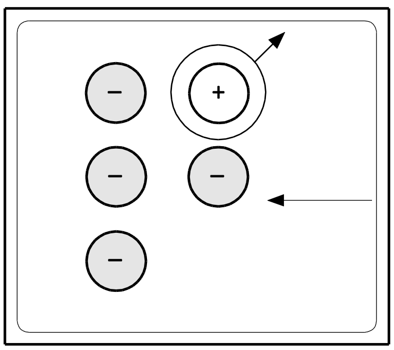
As with addition, it is important for students to have a conceptual understanding of integer subtraction. Students should model subtraction of integers using coloured counters and number lines. One possibility when subtracting integers is to use a “take away” meaning. This is easily manageable when the integers have the same sign, and the subtrahend is closer to zero than the minuend, or starting amount. For example, (–4) – (–3) can be modelled with counters as follows.



Place 4 negative counters on the mat to show – 4. Remove 3 negative counters to show subtracting (or take away) – 3.

So, – 4 – (– 3) = – 1

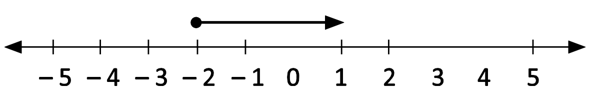
When taking away is not immediately possible, it is necessary to add zero pairs to re-represent the minuend and allow the student to be able to remove the tiles that represent the subtrahend. Guide this developmental understanding through the use of good questions. To solve (–3) – (+1) ask students how they could represent (–3) that would allow them to take away (+1).



Place 3 negative counters on the mat to show – 3. To subtract + 1, you must remove 1 positive counter. But there are no positive counters on the mat. You must add 1 zero pair to the mat. The value of the counters on the mat does not change. Then you can remove 1 positive counter.

When 1 is taken away from – 3, the result is – 4. So (– 3) – (+1) = – 4

To subtract integers, you can also use a "think addition" meaning. To determine (+1) – (–2), ask "How much must be added to (–2) to get to (+1)?" Using a number line, begin at (–2) and draw an arrow to (+1). It has a length of 3 pointing right. ∴ (+1) – (–2) = +3.



While the rule "to subtract an integer, add the opposite" allows students to reach the correct answer, it does not promote conceptual understanding. Students should be led to this conclusion through the use of models. A good example would involve using the number line to subtract a negative from a positive. Such a situation should make it easier for students to see why you can add the opposite to subtract. From the previous example, (+1) – (–2) tells what to add to (–2) to get to (+1). To go from (–2) to (+1), move 2 to the right to get to 0, and then another 1 to the right to get to (+1). The total amount to be added is (+2) + (+1) or, since addition is commutative, (+1) + (+2). Students should now see that (+1) – (–2) = (+1) + (+2). Patterning can be used to develop this as well. Ask students to study a pattern such as the one given here.

(+4) – (+2) = 2	(+4) + (–2) = 2
(+4) – (+1) = 3	(+4) + (–1) = 3
(+4) – (0) = 4	(+4) + (0) = 4
(+4) – (–1) = 5	(+4) + (+1) = 5

(+4) – (–2) = 6   (+4) + (+2) = 6

As they compare each column, they should conclude that subtracting results in the same answer as adding the opposite. Students should be aware that while addition of integers is commutative, subtraction is not. In fact, if the order of the integers in the subtraction statement changes, the differences are opposite integers.

Students should eventually move away from models as they practice integer arithmetic. However, it is important that they not view the procedural rules as arbitrary. While the correct answer is important, emphasis should be on the rationale and not on how quickly they can get answers.

Assessment, Teaching, and Learning

Assessment Strategies

Assessing Prior Knowledge

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students: How many negative integers are greater than –9?
- Tell students that a number is 14 jumps away from its opposite on a number line. Ask, what is the number?
- Ask students to explain whether it is true that
  - a negative number further from 0 is less than a negative number that is close to 0
  - a negative number is always less than a positive number
  - a positive number is always greater than a negative number

Whole-Class/Group/Individual Assessment Tasks

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Use a number line or two colour counters to explain why the following calculations are correct.
  (–3) + 8 = 5
  (–5) – 3 = –8
  (–4)– (–6) = 2
  9 + (–2) = 7
  6 – 4 = 2
  8 – (–3) = 11
- Jon saved $50 during the fall. He owes $15 to his friend. He earned $20 mowing lawns. What is Jon's net worth?
- Find integer pairs with a sum of –16 where
  - one number is less than –16
  - one number is greater than +16

  - one number is greater than 0 and less than +5
- Can you model +2 with an odd number of counters? Explain why or why not.
- Model (– 2) – (– 4) using a number line.
- Is the sum of a negative integer and a positive integer always negative? Explain why or why not.
- Can +2 be modelled with an odd number of counters? Explain your reasoning.
- Answer the following: Tim has a debt of $55. He earns $30 on each of two days and spends $39 on a pair of pants. How much money or debt does he now have? Include a diagram and a number sentence.
- Is the sum of a negative number and a positive number always negative? Explain your reasoning.
- When you add two negative integers, you always get a negative sum. When you subtract two negative integers, do you always get a negative difference? Explain with the aid of examples.
- Respond to the following:
  - Is the difference between a negative number and a positive number always negative? Explain your reasoning.
  - How do you subtract integers using tiles? Explain and give an example.
  - Without actually calculating the difference between two integers, how do you know whether the answer will be positive, negative, or zero? Explain with the aid of examples.
- Write an addition equation to describe each situation, and explain its meaning.
  - Before you went to sleep last night the temperature was –3°C. During the night the temperature dropped by 5°C. What was the temperature in the morning?
  - Mrs. Brown parked in the parking garage 10 m below street level. She then got on an elevator and went up 27 m to her office. How far above the street is her office?
- Write a subtraction equation for each situation.
  - A soccer ball was kicked 5 m in the forward direction on the first play of the game. The opposing team then kicked it back 6 m. What was the total change in distance?
  - If the average temperature in Yarmouth is 19°C in July and –6°C in January, what is the difference between the average highest and lowest temperatures?
  - Adam and André were digging in the sand at the beach. Archie dug a hole that was 22 cm below the surface, and André dug a hole that was 13 cm below the surface. What is the difference in the depths of the holes?

Planning for Instruction

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Emphasize that students begin with concrete, move to pictorial and finally to symbolic representations for sums and differences of integers.
- Give students opportunities to use a variety of shapes within a model to represent one whole. For example, when using pattern blocks, the yellow hexagon could represent one whole or when combining pattern blocks with fraction blocks, the pink decagon piece could be used to represent one whole and thus the yellow hexagon would now represent one-half.
- Connect the subtraction of integers to the addition of integers.

- Use problem-solving contexts that are relevant to students.
- Connect problems applying addition and subtraction of integers to similar problems with whole numbers.
- Have students explore sums and differences of integers by using a variety of models, such as algebra tiles, two-colour counters, and arrows on a number line. It should be noted that it is important to always use a key in pictorial representations of integers, so students know which colour represents positive integers and which represent negative integers. There is no set standard for these and students should be flexible in their thinking.
- Include various structures of problems such as, “part-part-whole” and “comparison” from previous grades. For example, (–6) – ☐ ⍰ = 3 ☐ + 2 = (–5) (–9) = (–4)+ (– ☐).
- Have students justify the strategies they use in finding sums and differences of integers and provide opportunities to discuss strategies used by others.

SUGGESTED LEARNING TASKS

- Solve addition and subtraction of integers through the use of magic squares. For example,

	–7	
		–11
–9	–1	–2

- Roll two dice of different colours. Assign negative to one colour and positive to the other, and find the sum of the numbers rolled. Roll the two dice again, find the sum and add the result to your previous score. Exchange turns until one person reaches +20 or –20. Why would it be fair to accept +20 or –20 as the winning score?
- Solve and create problems using real-life situations such as: time zones, temperature, heights above and below sea level, profits and losses, games, sports, shares of stocks, etc.
- Create cards with opposite integers written on them. Distribute these cards to students. Locate their opposite and sit in zero pairs.
- Using integer tiles, demonstrate three ways to model an integer such as –7, 8, –2, etc. Share your models with the class.
- Model as many integers as you can using exactly nine integer counters.
- You earn $5 and then spend $5. How much is your profit or loss? Draw diagrams with integer counters to represent this problem.
- Arrange students in teams. Begin with each team standing. Write an addition expression on the board and have students write the answer. Students with correct answers remain standing. All others sit. After three questions, the team with the most people standing receives 10 points. One member of the team is selected to attempt a fun activity (e.g., shooting a foam basketball into a net) for an additional 5 points for the team. Then a new round of play begins. The team that reaches 100 points first wins.
- Find three pairs of integers with a sum of –29.
- Create and solve their own problems using real–life situations such as time zones, temperature, heights above and below sea level, profit/loss, etc.

- Find charts in the daily newspaper showing the low and high temperatures for the day for cities around the world. Use information from the charts to make up two problems involving subtraction of positive and negative numbers. Find the solutions and then exchange problems with other students and solve.
- Teachers can use masking tape (or other materials) to make a large number line on the floor. Students can discuss how a number line can be used for subtraction, as well as for addition of integers. Students can walk on the number line to show subtractions such as (+7) – (–3) or (–4) – (–2).

SUGGESTED MODELS AND MANIPULATIVES

- algebra tiles*
- horizontal number lines
- thermometers
- two-colour counters
- vertical number lines

*also available in *Interactive Math Tools* (Pearson n.d.)

MATHEMATICAL LANGUAGE

Teacher	Student
▪ absolute value	
▪ integer	▪ integer
▪ minuend	
▪ negative integer	▪ negative integer
▪ positive integer	▪ positive integer
▪ sign	▪ sign
▪ subtrahend	
▪ zero principle	▪ zero principle
▪ magnitude	

Resources

Print

Math Makes Sense 7 (Garneau et al. 2007)
- Unit 2: Integers (NSSBB #: 2001640)
  - Section 2.1 Representing Integers
  - Section 2.2 Adding Integers with Tiles
  - Section 2.3 Adding Integers on a Number Line
  - Section 2.4 Subtracting Integers with Tiles
  - Section 2.5 Subtracting Integers on a Number Line
  - Unit Problem: What Time Is It?
- *ProGuide* (CD; Word Files) (NSSBB #: 2001641)
  - Assessment Masters
  - Extra Practice Masters

  - Unit Tests
- *ProGuide* (DVD) (NSSBB #: 2001641)
  - Projectable Student Book Pages
  - Modifiable Line Masters

Digital

- "National Library of Virtual Manipulatives: Number and Operations (Grades 3–5)," *Utah State University* (Utah State University 2015): http://nlvm.usu.edu/en/nav/category_g_2_t_1.html (Select Color Chips—Addition or Color Chips—Subtraction from the list of virtual manipulatives provided.)

SCO N07: Students will be expected to compare, order, and position positive fractions, positive decimals (to thousandths), and whole numbers by using benchmarks, place value, and equivalent fractions and/or decimals. [CN, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N07.01 Position proper fractions with like and unlike denominators from a given set on a number line, and explain strategies used to determine order.

N07.02 Position a given set of positive fractions, including mixed numbers and improper fractions, on a number line; and explain strategies used to determine order.

N07.03 Position a given set of positive decimals on a number line and explain strategies used to determine order.

N07.04 Compare and order the numbers of a given set that includes positive fractions, positive decimals, and/or whole numbers in ascending or descending order and verify the result using a variety of strategies.

N07.05 Identify a number that would be between two given numbers in an ordered sequence or on a number line.

N07.06 Identify incorrectly placed numbers in an ordered sequence or on a number line.

N07.07 Position the numbers of a given set by placing them on a number line that contains benchmarks, such as 0 and 1 or 0 and 5.

N07.08 Position a given set that includes positive fractions, positive decimals, and/or whole numbers on a number line and explain strategies used to determine order.

Scope and Sequence

Mathematics 6	Mathematics 7	Mathematics 8
N01 Students will be expected to demonstrate an understanding of place value for numbers greater than one million and less than one-thousandth. **N04** Students will be expected to relate improper fractions to mixed numbers and mixed numbers to improper fractions. **N07** Students will be expected to demonstrate an understanding of integers contextually, concretely, pictorially, and symbolically.	**N07** Students will be expected to compare, order, and position positive fractions, positive decimals (to thousandths), and whole numbers by using benchmarks, place value, and equivalent fractions and/or decimals.	—

Background

To be efficient at comparing and ordering fractions and decimals students must understand the magnitude of these numbers in our number system and how they can be represented. They must realize that fractions and decimals are interchangeable names for the same quantity and must be able to convert one to the other. They must be proficient at renaming and simplifying fractions and use multiple strategies for comparing them.

Have students continue to use conceptual methods to compare fractions and decimals, such as context problems and models. Students tend to think of fractions as sets or regions whereas they think of decimals as being more like whole numbers. A significant goal of instruction in decimal and fraction numeration should be to help students see that both systems can be used to represent the same concepts. For this reason it is important that a variety of models and benchmarks be used. Money should not be used as the exclusive model for decimals as it is very limiting (typically only extends to hundredths).

Students should develop a variety of strategies to compare fractions. Students need experiences comparing fractions with the same denominator, with the same numerator, and with unlike denominators. If both fractions have the same denominator, the larger numerator represents the larger fraction (e.g., $\frac{5}{8} > \frac{3}{8}$). If the denominators are different, students will write equivalent fractions with like denominators and then compare the numerators. If both fractions have the same numerator, the fraction with the smallest denominator is larger (e.g., $\frac{2}{7} > \frac{2}{9}$).

A rich understanding of place value allows students to compare and order decimals using strategies similar to those used with whole numbers. Once students develop a sense of the benchmark fractions (0, $\frac{1}{2}$, 1, 1 ½ , 2) and know the decimal equivalents (0, 0.5, 1, 1.5, 2), they are able to use them interchangeably as a powerful strategy for comparing and ordering fractions and decimals.

Students can change fractions greater than one, such as $\frac{10}{8}$ or $\frac{7}{5}$, to mixed fractions if they choose. Repeated addition can be used as a strategy to write mixed numbers. Recognizing what makes one whole, $\frac{10}{8}$ can be rewritten as $\frac{8}{8} + \frac{2}{8}$.

Have students apply their prior knowledge of equivalent fractions to help determine a fraction that is between two given fractions in an ordered sequence. Some students may have difficulty identifying a number that is between two given numbers, especially when the numbers given are fractions with the same denominator and are close in value (e.g., $\frac{3}{10}$ > ? > $\frac{4}{10}$). These two fractions could be rewritten as $\frac{6}{20}$ and $\frac{8}{20}$. Students can now more easily identify that $\frac{7}{20}$ is a possible answer. If the fractions are rewritten as $\frac{12}{40}$ and $\frac{16}{40}$, students can see that there are more options.

When working with decimal numbers, students can use place value for comparison. With numbers such as 0.3 and 0.4, they can use the hundredths instead of tenths (e.g., 38 hundredths is between 3 tenths and 4 tenths). Students can use similar strategies learned from placing fractions on a number line to identify a fraction between two decimals (e.g., 0.4 > $\frac{?}{8}$ > 0.7), a decimal between two fractions, or a number between a given decimal and a given fraction.

Students can use these same strategies (decimal or fraction equivalents, benchmarks, place value) to identify an incorrectly placed number in a given ordered sequence or on a given number line. Decimal numbers are simply another way of writing fractions. Both notations have value. Maximum flexibility is gained by understanding how the two symbols are related (Van de Walle and Lovin 2006b, 107).

Assessment, Teaching, and Learning

Assessment Strategies

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to describe how the bold-faced digits in the following two numbers are the same and how they are different.

  0.00**6**0 0.000**6**

- Ask students to write decimals using place-value language and expanded notation to help explain equivalence of decimals.
  For example,

  0.4 means four- tenths
  0.40 means four- tenths + 0 hundredths
  0.400 means four- tenths + 0 hundredths + 0 thousandths

- Write and model a mixed number, , that is greater than $\frac{3}{3}$, less than $\frac{6}{3}$ and has a denominator of 3
- Provide students with several mixed numbers and improper fractions.
  For example, $2\frac{1}{3}$, $\frac{7}{4}$, $\frac{5}{3}$, $2\frac{3}{4}$, $1\frac{4}{5}$.
  Have students place the numbers on an open number line to demonstrate their relative magnitude.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Estimate a value to make each of the following true:

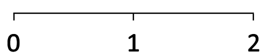
$0.4 < \frac{?}{8} < 0.7$ $\frac{3}{10} < 0.? < \frac{4}{10}$

- Pose questions such as the following:
  - Which is greater, $\frac{3}{10}$ or $\frac{3}{8}$? Which is greater $\frac{3}{8}$, or $\frac{7}{10}$? Which is less, $\frac{4}{5}$ or $\frac{3}{4}$?"
  - Explain each selection.
  - Explain how you would compare fractions using your understanding of benchmarks.

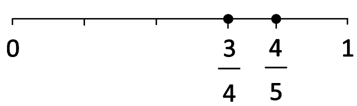
- Place the following numbers on a number line that has a few benchmarks labelled:

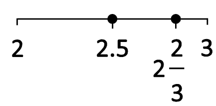
  $\frac{3}{7}, 1\frac{1}{3}, \frac{5}{9}, \frac{13}{12}, 1\frac{4}{9}$, 0.45, 0.93

- Place the numbers 2.3, 2.4, 2.32, 2.36, 2.327 on a number line.
- Arrange the numbers 0.96, 0.9, 0.9, 0.96, 0.09 from greatest to least.
- Write each of the following numbers in an approximate location on the number line provided. You should explain the strategies you used to approximate each point on the line.
  $\frac{3}{7}, 1\frac{1}{3}, \frac{5}{9}, \frac{13}{12}, 1\frac{4}{9}$, 0.45, 0.93



- Suzie and Polly both worked very hard and have nearly completed their math assignment. Suzie has completed $\frac{5}{6}$ of the project and Polly has completed 0.8 of her project. Who was closer to completing the assignment? How do you know?
- Choose three values that are not whole numbers and explain how to write them in order using benchmarks.
- Identify a number that would fit between the plotted points on these number lines and explain your choice.





- Identify which number(s) are not in the correct position in the sets of numbers below. You should record and justify your responses.
  - 0.75, $0.\overline{7}$, $\frac{8}{9}, \frac{3}{5}, \frac{10}{11}$
  - $\frac{4}{5}$, $0.\overline{81}$, $\frac{9}{10}, \frac{13}{15}$, $1.\overline{1}$

Planning for Instruction

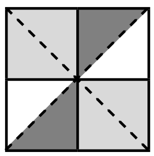
Choosing Instructional Strategies

Consider the following strategies when planning daily lessons.

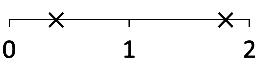
- Have students initially construct models and drawings (e.g., hundredths grid and number lines) to compare decimals before moving to comparing them with other strategies.
- Encourage students to compare fractions greater than one by considering them as mixed numbers. For example, which is greater $\frac{10}{8}$, or $\frac{7}{5}$? A possible answer: "I know that $\frac{7}{5}$ is greater because $\frac{10}{8}$ is $1\frac{2}{8}$, and $\frac{7}{5}$ is $1\frac{2}{5}$, and I know that since $\frac{2}{5}$ is greater than $\frac{2}{8}$, then $1\frac{2}{5}$ is greater than $1\frac{2}{8}$."
- Have students choose the greater fraction or decimal in a given a pair. Have them defend their choice. They must then prove their answer is correct using a model of their choice.

SUGGESTED LEARNING TASKS

- Create patches (made of paper) for a class patchwork quilt in which the colours on their patches show a particular comparison. For example, this patch could be used to illustrate that $\frac{2}{4} > \frac{2}{8}$.



- Order a set of unit fractions from least to greatest. You should be able to defend their order.
- Estimate the fraction (or decimal) that best represents each *x*.



- Give students five decimal numbers that have friendly fraction equivalents. Keep the numbers between two consecutive whole numbers. For examples 2.5, 2.125, 2.4, 2.75, 3.66. Give them a number line encompassing the same whole numbers and use subdivisions that are only fourths, only thirds or only fifths but without labels. Locate each decimal on the number line and provide the fraction equivalent for each (Van de Walle and Lovin, 2006b, 115).
- Place positive decimal numbers and fractions on number lines, such as 2.3, 2.4, 2.32, 2.36, 2.327.
- Create a human number line. Each student is given a card with a fraction or decimal number. Have students order themselves into a line based on the relative size of their number. Ask students why they chose their position. (Alternate version: Use a skipping rope or clothesline as the number line. Students attach their number to the line in an appropriate position.)
- Create a set of cards with a variety of fractions and decimals. Each student gets five cards and must lay them out in the order they receive them. Students take turns trading one of the cards for a new one from the pack. They place the new card in whichever location best helps get the cards in order. The object is to be the first to get their cards in order. The pack should have sufficient cards to allow the game to run smoothly: for groups of 3, there should be at least 30 cards per group, for groups of 4, at least 40 cards per group.

SUGGESTED MODELS AND MANIPULATIVES

- base-ten blocks
- counters
- Cuisenaire rods
- fraction bars
- fraction pieces
- hundredth grids
- number lines
- pattern blocks
- place-value chart

MATHEMATICAL LANGUAGE

Teacher	Student
▪ ascending ▪ denominators ▪ equivalent fractions ▪ horizontal ▪ improper fractions ▪ mixed numbers ▪ numerators ▪ proper fractions ▪ repeating decimal ▪ sequence ▪ terminating decimal ▪ unlike denominators ▪ verify ▪ vertical	▪ ascending ▪ denominators ▪ equivalent fractions ▪ horizontal ▪ improper fractions ▪ mixed numbers ▪ numerators ▪ proper fractions ▪ repeating decimal ▪ sequence ▪ terminating decimal ▪ unlike denominators ▪ verify ▪ vertical

Resources

Print

Making Math Meaningful to Canadian Students K–8, Second Edition (Small 2013), 249–277

Teaching Student-Centered Mathematics, Grades 5-8, Volume Three (Van de Walle and Lovin 2006b), 107–130

Math Makes Sense 7 (Garneau et al. 2007)

- Unit 3: Fractions, Decimals and Percents (NSSBB #: 2001640)
  - Section 3.2 Comparing and Ordering Fractions and Decimals
- *ProGuide* (CD; Word Files) (NSSBB #: 2001641)
  - Assessment Masters
  - Extra Practice Masters
  - Unit Tests
- *ProGuide* (DVD) (NSSBB #: 2001641)
  - Projectable Student Book Pages
  - Modifiable Line Masters

Digital

- "Equivalent Fractions," *Illuminations: Resources for Teachers* (National Council of Teachers of Mathematics 2015): http://illuminations.nctm.org/Activity.aspx?id=3510

Patterns and Relations (PR)

GCO: Students will be expected to use patterns to describe the world and solve problems.

GCO: Students will be expected to represent algebraic expressions in multiple ways.

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

GUIDING QUESTIONS

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Follow-up on Assessment

The evidence of learning that is gathered from student participation and work should be used to inform instruction.

GUIDING QUESTIONS

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan available at Mathematics Learning Commons: Grades 7–9: http://nsvs.ednet.ns.ca/nsps/nsps26/course/view.php?id=3875.

GUIDING QUESTIONS

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SCO PR01: Students will be expected to demonstrate an understanding of oral and written patterns and their equivalent linear relations. [C, CN, R]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

PR01.01 Formulate a linear relation to represent the relationship in a given oral or written pattern.
PR01.02 Provide a context for a given linear relation that represents a pattern.
PR01.03 Represent a pattern in the environment using a linear relation.

Scope and Sequence

Mathematics 6	Mathematics 7	Mathematics 8
PR01 Students will be expected to demonstrate an understanding of the relationships within tables of values to solve problems. **PR02** Students will be expected to represent and describe patterns and relationships, using graphs and tables.	**PR01** Students will be expected to demonstrate an understanding of oral and written patterns and their equivalent linear relations.	**PR01** Students will be expected to graph and analyze two-variable linear relations.

Background

Mathematics is often referred to as the science of patterns, as patterns permeate every mathematical concept and are found in everyday contexts. Patterns are prevalent in plant and animal life, as well as in the physical world. They are evident in the arts, music, structures, movement, time, and space. Our number system is rooted in pattern, and an understanding of pattern is the basis of mathematical concepts in every strand of mathematics. Someone who possesses the ability to identify a pattern and its symbolic relation can solve a problem that previously seemed insurmountable.

Some characteristics of patterns include the following. Patterns include repeating patterns and increasing and decreasing patterns. Repeating patterns have a “core,” which is the part that repeats. Increasing and decreasing patterns are evident in a wide variety of contexts, including arithmetic and geometric situations. Arithmetic patterns, such as 11, 8, 5, 2, … are patterns that are formed by adding or subtracting the same number each time. Geometric patterns, such as 2, 6, 18, 54, … are patterns that are formed by multiplying or dividing by the same number each time.

Patterns represented concretely or pictorially can also be written as number patterns, where numbers represent the quantity in each step of the pattern. In Mathematics 5 students used patterns to generalize relationships. How a term value changes from one term to the next defines the **recursive relationship** in the pattern. It explains what is done to one number to determine the next number in the pattern. An expression that explains what you do to the term number to get the corresponding term value in the pattern describes the **functional relationship** in the pattern and is known as the pattern rule. Students continue to investigate these relationships in Mathematics 6 through tables and graphs. Patterns can be represented in different ways. The recursive and functional relationships can be seen in the different representations of a pattern. Determining the expression that represents the functional relationship can be challenging, and will require persistence to be able to determine the term value from the term number. Each type of representation provides a different view and a different way to think about the relationships. Encourage students to work toward identifying the functional relationship in each type of representation.

To increase students' ability to think symbolically, begin with more obvious relations and teach students to ask themselves increasingly complex questions about the relations (e.g., What remains the same? What changes? By how much does it change? Is this true for every term in the sequence? How can that idea be represented? What happens if … ?).

The following process for representing patterns flows from the concrete or pictorial to the symbolic. It is important to guide students through the process.

Concrete or pictorial representation: The context of the pattern exists in the physical pattern itself, and can be represented concretely or pictorially. Students can examine the physical terms to determine what remained the same in each term and what changed. Playing with colour or arrangement of patterns can often help students to see the constant and the changing aspects of a pattern.

Example:

Pictorial representation of a pattern	★★ ★	★★ ★★ ★	★★ ★★ ★★ ★	★★ ★★ ★★ ★★ ★		

In this example, two stars are added to the top of each new term. This pattern can be extended by adding 2 stars each time. Students should be asked to describe the patterns they see in words and share their thinking orally with the class. Include descriptions that represent both recursive thinking and functional thinking. Pictorial representations can be represented in a table of values.

Tables of values: Charts or tables of values display numeric representation of the pattern values, and may also be used to record the changes between the terms. These representations, as illustrated in the following example, facilitate numeric comparisons. They can be presented in horizontal or vertical format.

Example:

Term number (*n*) (the position for the term in the sequence)	1	2	3	4	5	6
Term value (*v*) (the number of items in the term)	3	5	7	9		

Reading across in the table above and seeing that each term value increases by 2 is *recursive thinking.* Using the term number and the corresponding term value to determine the pattern rule is *functional thinking.* Ask, “What can be done to the term number to get the term value?” “What changes?” “What stays the same?” In the pictorial representation above the single star is the same in each term and the number of groups of two stars changes. The pattern rule here is, multiply the term number by 2 and add 1. Ask students what the 2 represents in the pattern and what the 1 represents.

The relationship can be expressed as the expression $2n + 1$ or as the equation $2n + 1 = v$, where n = the term number and v = the term value.

This outcome should be done in conjunction with PR02.

Assessment, Teaching, and Learning

Assessment Strategies

ASSESSING PRIOR KNOWLEDGE

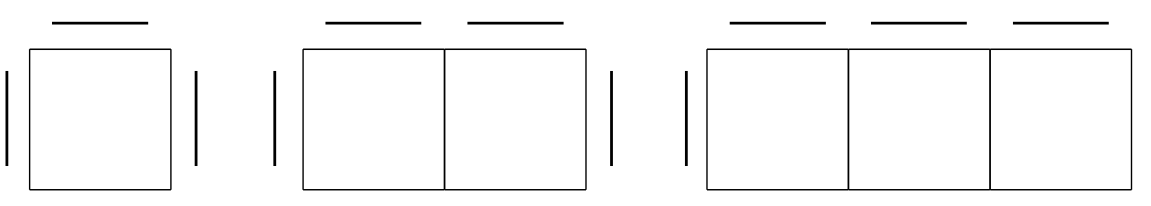
Tasks such as the following could be used to determine students’ prior knowledge.

- Ask students to fill in the missing values in a given table and explain their thinking.

Term number	1	2	3	?
Term value	4	8	?	16

- Mary walked 3 km on Monday. Each day that week, she walked 2 more km than the day before. Ask students to create a table of values for this data, describe the pattern, and make a graph.
- Ask students to refer to the following table of values to answer these questions:

Number of tables	Number of chairs
1	4
2	6
3	8
4	10
5	12



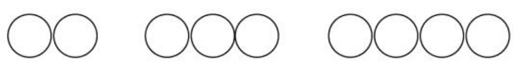
  – Describe the pattern rule for the number of chairs you would need for the tables. Explain your thinking.
  – Use this rule to predict the number of chairs for 10 tables.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

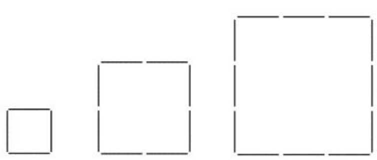
Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Describe a real-life situation that could be represented by $3p + 4$.
- Create a pattern and represent it in five ways.
- For the following diagrams,
  - construct and extend the pattern with concrete objects
  - describe the pattern rule in their own words (for example, Start with 2 and add 1 each time)
  - Describe how to determine any term value using the pattern rule
  - use the pattern to determine the 30^{th} term value
  - develop a table
  - generate a graph
  - write an equation to describe the linear relation

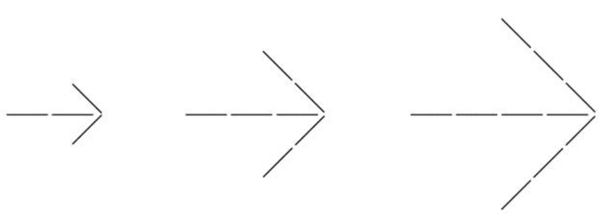
a) (The pattern rule is used to describe the number of circles in each term.)



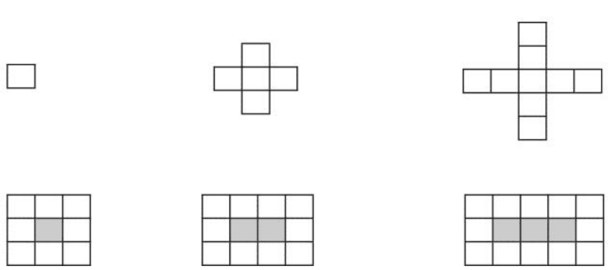
b) (The pattern rule is used to describe the number of toothpicks in each term.)



c) (The pattern rule is used to describe the number of toothpicks in each term.)

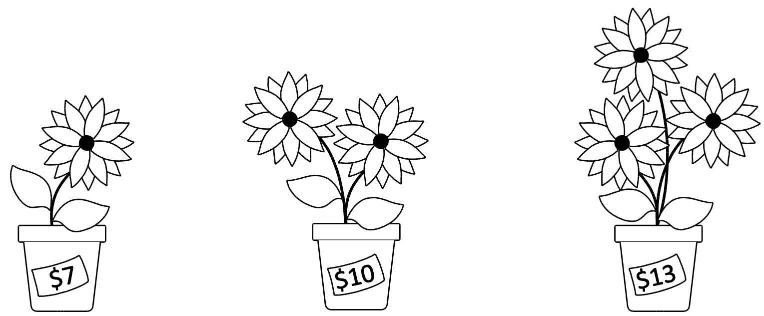


d) (The pattern rule is used to describe the number of white squares in each term.)



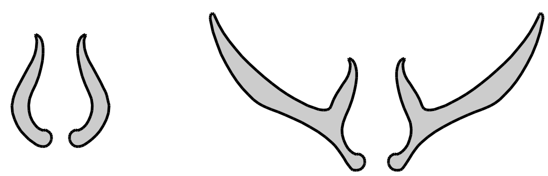
- Continue the pattern and complete the table to show the pattern. Describe the relationship between the variables, write an equation for the linear relation, and graph the table of values.

Cost of Flowers in a Vase

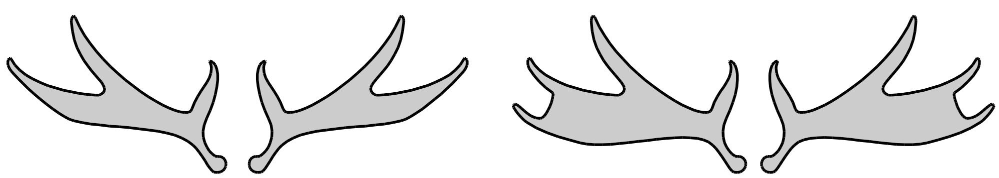


Number of flowers (f)	1	2	3	4	5	6	7
Cost in dollars (c)							

Number of Tips on Moose Antlers



1 Year Old **2 Years Old**



3 Years Old **4 Years Old**

Age of moose in years (*a*)	1	2	3	4	5	6	7
Number of tips on antlers							

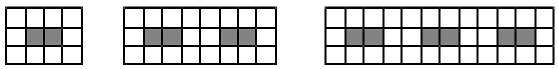
Planning for Instruction

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Ask students to describe patterns and rules orally and in writing before using algebraic symbols. Provide opportunity to connect the concrete and pictorial representations to symbolic representations as well as connecting the symbolic representations to pictorial and concrete representations.
- Provide examples of growth patterns that are arithmetic and geometric.

- Encourage students to draw diagrams and create tables of values to assist them in visualizing the relationship when formulating linear relations representing oral or written patterns.
- Give examples, prior to asking students to provide contexts to represent linear relations. The expression 10h + 2, for example, could represent the amount of money a person makes if they are paid $10 per hour plus a $2 bonus.
- Students could investigate a number of patterns that may be expressed using linear relations, such as the black-and-white tile pattern for kitchen flooring, as shown below. Students should be able to construct a table of values showing the number of black tiles and the number of white tiles in the first five designs, describe the pattern, and write an equation.



Number of black tiles (*b*)	Number of white tiles (*w*)
2	10
4	20
6	30
8	40
10	50

SUGGESTED LEARNING TASKS

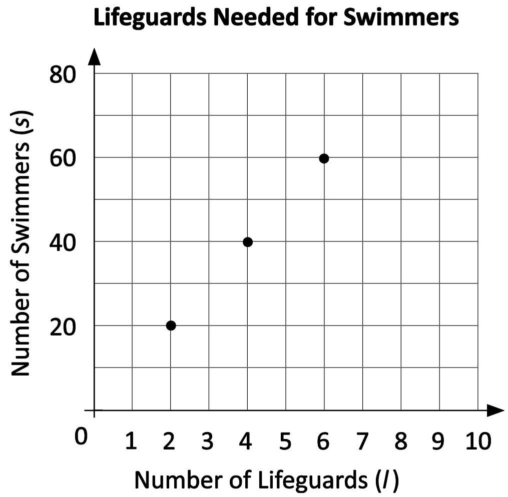
- Study the tables below, describe the relationship between the variables, write an equation, and then graph the relation and describe the graph.

Cost of renting a scooter

Number of hours (*n*)	1	2	3	4	5
Cost (*c*)	50	70	90	110	130

- A taxi charges a base fare of $4, plus $1 for every kilometre travelled. This can be represented by the equation $c = n + 4$ (c = cost, n = number of km). Make a table of values showing the total cost for the first 5 km. You can graph the table of values and describe the pattern. How much would a 10 km taxi ride cost?

- Using the graph below, have students perform the following tasks:



  - How many swimmers would be allowed for 10 lifeguards?
  - How many lifeguards would be needed for 50 swimmers?
  - Describe the pattern in words.
  - Write an equation for the number of (*l*) lifeguards needed for (*s*) swimmers.
- Create a context that can be represented by the following number pattern.
  89, 74, 62, 53, 47, …, …, …

SUGGESTED MODELS AND MANIPULATIVES

- colour tiles*
- linking cubes
- counters
- toothpicks

*also available in *Interactive Math Tools* (Pearson n.d.)

MATHEMATICAL LANGUAGE

Teacher	Student
algebraic expression	algebraic expression
arithmetic patterns	arithmetic patterns
decrease	decrease
discrete data	discrete data
explicit relationship	explicit relationship
increase	increase
geometric patterns	geometric patterns
increasing	increasing
decreasing	decreasing
linear graph	linear graph
linear relation	linear relation
	linear relationship
predict	predict
recursive relationship	recursive relationship
relationships	relationships

Teacher	Student
▪ repeating	▪ repeating
▪ table of values	▪ table of values
▪ term	▪ term
▪ term number	▪ term number
▪ term value	▪ term value
▪ unknown term	▪ unknown term
▪ values	▪ values

Resources

Print

Math Makes Sense 7 (Garneau et al. 2007)

- Student Book Unit 1 Patterns and Relations (NSSBB #: 2001640)
  - Section 1.3 Algebraic Expressions
  - Section 1.4 Relationships in Patterns
  - Unit Problem: *Fund Raising*
- *ProGuide* (CD; Word Files) (NSSBB #: 2001641)
  - Assessment Masters
  - Extra Practice Masters
  - Unit Tests
- *ProGuide* (DVD) (NSSBB #: 2001641)
  - Projectable Student Book Pages
  - Modifiable Line Masters

SCO PR02: Students will be expected to create a table of values from a linear relation, graph the table of values, and analyze the graph to draw conclusions and solve problems. [C, CN, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

PR02.01 Create a table of values for a given linear relation by substituting values for the variable.
PR02.02 Create a table of values, using a linear relation, and graph the table of values (limited to discrete elements).
PR02.03 Sketch the graph from a table of values created for a given linear relation, and describe the patterns found in the graph to draw conclusions (e.g., graph the relationship between n and $2n + 3$).
PR02.04 Describe, using everyday language, in spoken or written form, the relationship shown on a graph to solve problems.
PR02.05 Match a given set of linear relations to a set of graphs.
PR02.06 Match a given set of graphs to a given set of linear relations.

Scope and Sequence

Mathematics 6	Mathematics 7	Mathematics 8
PR01 Students will be expected to demonstrate an understanding of the relationships within tables of values to solve problems. **PR02** Students will be expected to represent and describe patterns and relationships, using graphs and tables.	**PR02** Students will be expected to create a table of values from a linear relation, graph the table of values, and analyze the graph to draw conclusions and solve problems.	**PR01** Students will be expected to graph and analyze two-variable linear relations.

Background

Remind students that patterns are represented in many equivalent forms and one form can be translated into another. Each different representation provides a different view of the same pattern. The more views students see, the greater their understanding of the pattern will be. A word description of the pattern can be used to make a physical representation of the pattern, as well as a table of values, and/or a graph. The x- and y-values of a graph can also be represented in a word description of a pattern, and can be used to form an algebraic equation.

Student investigations of linear relations should start with models, followed by oral and written descriptions. Where appropriate, use manipulatives to foster a better understanding of the concept.

Work on patterns continues in PR02 as students create a table of values from a linear relation and sketch the graph. The word linear may cause confusion but should become clear as students graph the

table of values and see the linearity of the points. This outcome should be done in conjunction with PR01.
A linear pattern can be described using a table of values. For example, the number pattern 3, 5, 7, 9, 11 … expressed in a table of values is

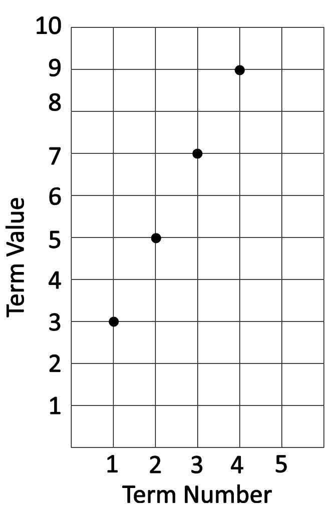
Term number (n)	1	2	3	4	5
Term value (v)	3	5	7	9	11

A variable such as *n* is used to represent an unknown quantity. It is important to note that students have been using a symbol (open frame in the form of a circle, square, triangle, etc.) to represent an unknown value since Mathematics 4, and variables were formally introduced in Mathematics 5.

Students use tables to organize the information that a pattern provides. When using tables, it is important for students to realize that they are looking for the relationship between the term number, represented by a variable, and the term value, represented by an expression.

A graph provides a picture of a relationship in a pattern. It provides clear evidence of whether the values are increasing or decreasing, how quickly the change is happening, and the increment of change. Relationships can be described by articulating these changes. In this outcome, students work with graphs, sketching the graph from a table of values, or analyzing given graphs. The analysis of graphs should include creating stories that describe the relationship depicted and constructing graphs based on a context that involves changes in related quantities. When students are describing a relationship in a graph they should use appropriate mathematical language.

Note: In the previous equation, $v = 2n + 1$, and 2 is the coefficient and 1 is the constant.



In Mathematics 7, students work with discrete data. Since *n* and *v* in the above equation refer specifically to the term number and term value, both of which are natural numbers (1, 2, 3, 4, ...). As a result, the graph of the relationship is the points and no line should be drawn through these points. When assessing student performance, keep in mind that students are limited to discrete elements and linear relations, and do not include powers and exponents.

Students should be given opportunities where appropriate, to find a point between two known points (interpolate), as well as to find a point that lies beyond the known data (extrapolate). This terminology is not the focus here. Students should be able to describe the general pattern of the graph (e.g., it goes upward to the right with the points in a straight line).

As an extension, students could explore if an ordered pair satisfies a given equation by plotting the points to see if it follows the pattern, and substituting it into the equation.

Assessment, Teaching, and Learning

Assessment Strategies

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to create a graph to display the relationship between the number of tricycles and the numbers of wheels. They should represent this data in a table of values as well.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

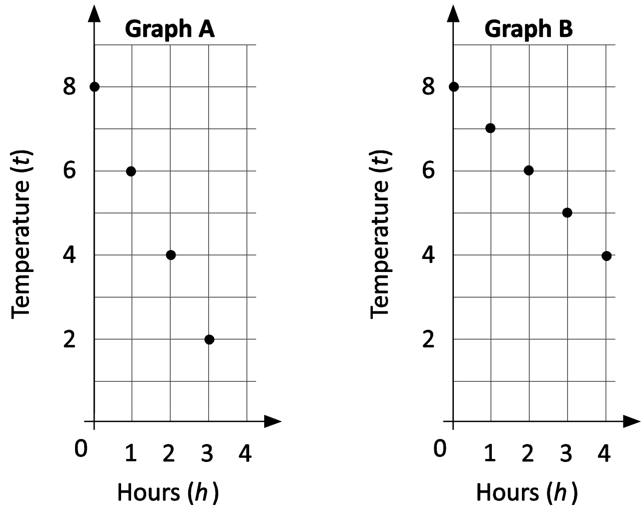
- Study the tables below, describe the relationship between the variables and write a linear relation and then graph the relation and describe the graph.

Songs on an MP3 Player

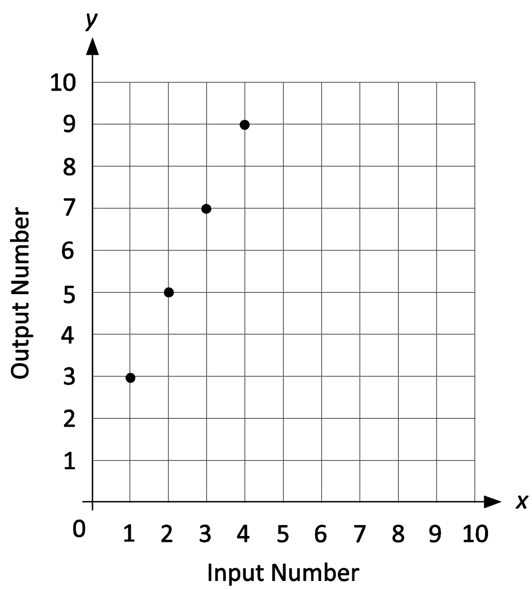
Number of Pop songs (*p*)	1	2	3	4	5
Number of Rock songs (*r*)	3	6	9	12	15

- On Monday, the morning temperature was 8 degrees. The temperature drops 2 degrees each hour. Faith says that Graph A shows this relationship and it can be written as $y = 8 – 2x$. The next day the morning temperature is 8 degrees. The temperature drops 1 degree each hour. Faith says that Graph B shows this relationship and it can be written as $y = 8 – x$.

– Determine if she is correct and explain your reasoning



- Determine which relations can be matched with the graph. Explain your reasoning.



x represents the input number
y represents the output number

$y = 2x + 1$
$y = x + 2$
The y value is equal to double the x value increased by 1.
The y value is equal to double the x value decreased by 1.

- Describe a real-life situation that could be represented by $3p + 4$.

Planning for Instruction

Choosing Instructional Strategies

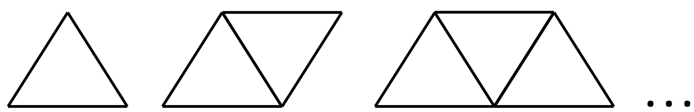
Consider the following strategies when planning daily lessons.

- Encourage students to draw diagrams and create tables of values to help them visualize the linear relations that represent oral or written patterns.

- Provide experiences representing the same pattern in multiple ways; i.e., model, diagram, table of values, graph, and expression.
- Use real-world contexts as much as possible. Have students represent the pattern by formulating a linear relation.

SUGGESTED LEARNING TASKS

- Continue the pattern for up to seven triangles with the diagram below.



Term number: (n)	1	2	3	4	5	6	7	…	10	…	20
Total number of line segments (t)								…		…	

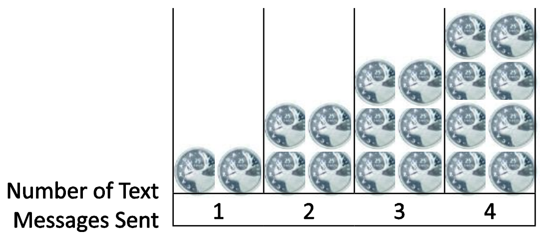
- Complete the chart to show pattern growth.
- Describe in writing how the pattern grows.
- Write an equation to show the relationship between the term number and the number of line segments.
- Draw a graph to show the pattern.
- Does it make sense to join the points? Explain the shape of the graph.
- Provide students with the pattern: 97, 94, 91, 87… Continue the pattern for the next three numbers.
- Describe, in words, how the pattern grows. Write the equation.
- Provide students with the following table which shows the relationship between the number of passengers on a tour bus and the total cost of providing boxed lunches.

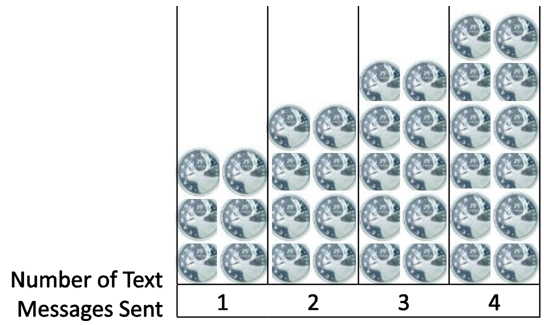
Passengers (p)	1	2	3	4	5
Cost of lunches (c)	5	10	15	20	25

- Explain how the lunch cost is related to the number of passengers.
  - Write an equation for finding the lunch cost (c) for the number of passengers (p).
  - Use the equation to find the cost of lunch if there were 25 people on the tour.
  - Draw a graph to show the relationship in the table of values.
  - How many people were on the bus if the tour-bus leader spent $200 on box lunches?

- Provide students with a table of values and a graph and ask if they match. Have them justify their thinking.

- For the following diagrams,
  - construct and extend the pattern with concrete objects
  - draw a representation of the pattern
  - describe the pattern in their own words
  - develop a table
  - generate a graph
  - write an equation





SUGGESTED MODELS AND MANIPULATIVES

- linking cubes
- colour tiles
- toothpicks,
- counters

MATHEMATICAL LANGUAGE

Teacher	Student
▪ algebraic expression	▪ algebraic expression
▪ arithmetic patterns	▪ arithmetic patterns
▪ decrease	▪ decrease
▪ discrete data	▪ discrete data
▪ explicit relationship	
▪ increase	▪ increase
▪ geometric patterns	▪ geometric patterns
▪ increasing	▪ increasing
▪ linear graph	▪ linear graph
▪ linear relationship	▪ linear relationship
▪ predict	▪ predict
▪ recursive relationship	
▪ relationships	▪ relationships
▪ repeating	▪ repeating
▪ table of values	▪ table of values
▪ term	▪ term
▪ term number	▪ term number
▪ term value	▪ term value
▪ unknown term	▪ unknown term

▪ values ▪ variable	▪ values ▪ variable

Resources

Print

Math Makes Sense 7 (Garneau et al. 2007)

- Student Book Unit 1 Patterns and Relations (NSSBB #: 2001640)
  - Section 1.5 Patterns and Relationships in Tables
  - Section 1.6 Graphing Relations
  - Unit Problem: Fund Raising
- *ProGuide* (CD; Word Files) (NSSBB #: 2001641)
  - Assessment Masters
  - Extra Practice Masters
  - Unit Tests
- *ProGuide* (DVD) (NSSBB #: 2001641)
  - Projectable Student Book Pages
  - Modifiable Line Masters

SCO PR03: Students will be expected to demonstrate an understanding of preservation of equality by ▪ modelling preservation of equality, concretely, pictorially, and symbolically ▪ applying preservation of equality to solve equations [C, CN, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

PR03.01 Model the preservation of equality for each of the four operations, using concrete materials and/or pictorial representations; explain the process orally; and record the process symbolically.

PR03.02 Write equivalent forms of a given equation by applying the preservation of equality, and verify using concrete materials (e.g., $3b = 12$ is equivalent to $3b + 5 = 12 + 5$ or $2r = 7$ is equivalent to $3(2r) = 3(7)$.)

PR03.03 Solve a given problem by applying preservation of equality.

Scope and Sequence

Mathematics 6	Mathematics 7	Mathematics 8
PR04 Students will be expected to demonstrate and explain the meaning of preservation of equality concretely, pictorially, and symbolically.	**PR03** Students will be expected to demonstrate an understanding of preservation of equality by ▪ modelling preservation of equality, concretely, pictorially, and symbolically ▪ applying preservation of equality to solve equations	**PR02** Students will be expected to model and solve problems, concretely, pictorially, and symbolically, where *a*, *b,* and *c* are integers, using linear equations of the form ▪ $ax = b$ ▪ $\frac{x}{a} = b,\ a \neq 0$ ▪ $ax + b = c$ ▪ $\frac{x}{a} + b = c,\ a \neq 0$ ▪ $a(x + b) = c$

Background

To understand equality, students must first understand that equality is a relationship, not an operation. “They should come to view the equals sign as a symbol of equivalence and balance.” (NCTM 2000, 39) However, many students think the equal sign is a symbol that tells them to do something or find the answer. They do not understand that the equal sign is a symbol of equivalence and balance.

For example, if students see $3 \times 4 = n \cdot 6$, some might think they are being asked to multiply 3×4 which would result in $n = 12$. If n were 12, the statement would say $3 \times 4 = 12 \times 6$ or $12 = 72$, which is not a true statement. The equal sign is used to show that the expressions on either side of the equal sign are equivalent and represent the same value. While grade 7 students have little difficulty determining the missing number in 7 + 2 = *n*, some may struggle to determine the value of n in equations such as 3 + 5 = 1 + *n* or 2 x *n* = 3 + 3.

Students are introduced to the concept of equality in Mathematics 2 and worked with it in successive grades. To facilitate conceptual understanding of equality students first learn to read the equal sign (=) as “is the same as.” In Mathematics 6 students begin to learn about preservation of equality. To understand preservation of equality, students must realize is that equality is a relationship, not an operation. Both equality and inequality express relationships between quantities. When the quantities balance, there is equality. The equal sign is a symbol that indicates the quantity on the left side of the equal sign is the same as the quantity on the right side of the equal sign. When there is an imbalance, there is inequality. The work in Mathematics 6 reinforced the idea of balance and the fact that any change to one side of an equation must be matched with an equivalent change to the other side of the equation to maintain the balance. The equal sign is used to express this balance.

A number sentence is called an equation. A number sentence with a variable is an algebraic equation. Equality between quantities can be considered as

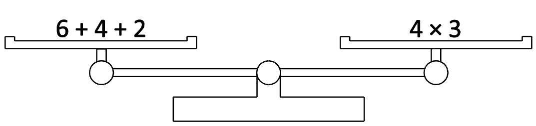
- whole to whole relationships (eight red chips is the same quantity as eight blue chips or 8 = 8)
- part-part to whole relationships (3 + 5 = 8)
- whole to part-part relationships (8 = 3 + 5)
- part-part to part-part relationships (4 + 4 = 3 + 5)

Solving equations requires that the balance of the equation be maintained so that the expressions on either side of the equal sign continue to represent the same quantity. For example, if a quantity is added to one side of the equation then, to maintain equality, the same quantity must be added to the other side of the equation. If $2x = 10$, then it is also true that $2x + 3 = 10 + 3$. This is an application of the preservation of equality.

The equality must be maintained similarly for the other operations. For example, if $2x = 10$ then it is true that $4(2x) = 4(10)$.

The most useful models for demonstrating preservation of equality are the balance-scale model and algebra tiles. Students must have experiences with concrete models and pictorial representations, before moving to the symbolic. Ensure strong connections are made between each of these representations

Students can use the two-pan balance scale approach in solving equations using systematic trial or inspection.



Learning experiences should be presented in such a way that students have ample opportunity to work with a variety of concrete materials when solving linear equations through preservation of equality, to explain the process orally, to represent it pictorially, and to record it symbolically. The skills students develop in solving linear equations through preservation of equality, and the experience they gain representing patterns and contextual situations as relations and linear equations, will give them a good foundation for later mathematics courses.

Assessment, Teaching, and Learning

Assessment Strategies

Assessing Prior Knowledge

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to write an equation to represent the following situations:
  - Bonnie is 4 years older than Mohammed. Mohammed is 12 years old. Write and model an equation to represent the problem. Write an equivalent equation to represent the problem that preserves equality.
  - There are 12 plums in a bowl. There were 32 plums at the start. Some have been eaten. How many plums are missing from the plate? Write and model an equation to represent the problem. Write an equivalent equation to represent the problem that preserves equality.

- Ask students to determine whether the following pairs of equations preserve equality and explain why.

  $4t = 8$ and $4t + 2 = 10$

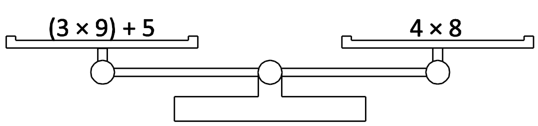
  $8k = 40$ and $2k = 10$

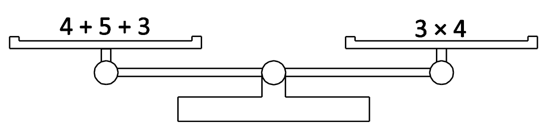
  $9 = 3s$ and $18 = 9s$

Whole-Class/Group/Individual Assessment Tasks

Consider the following **sample tasks** (which can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

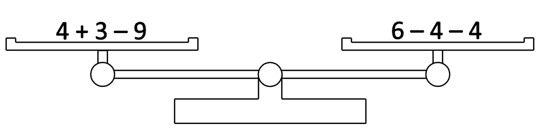
- Determine if the scales below will balance and to explain your thinking.





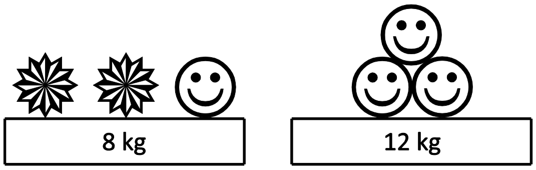
- Write two equations that would be equivalent to $2p + 4 = 6$. Then, use a balance scale model to represent your equations and to prove equivalency.

- Using the diagram below:



  - Are the pans balanced? How do you know?
  - What would happen if you added 5 to the right-hand side of the balance?
  - How can you rebalance the pans to preserve the equality?

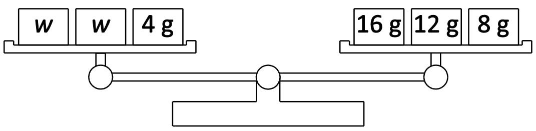
- What is the mass of each shape? How do you know?

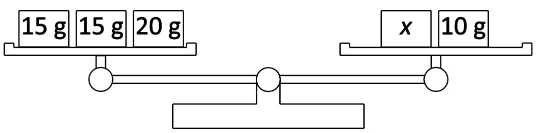


- Determine whether the following pairs of equations are equivalent and to explain your thinking.

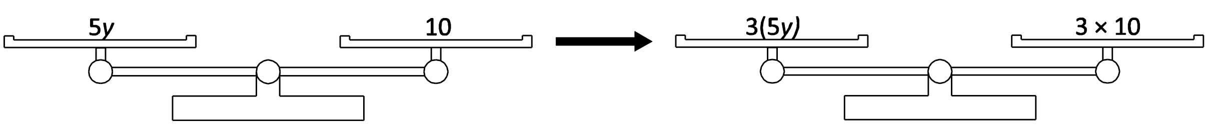
  $3c = 15$ and $3c + 6 = 15 + 6$
  $4n = 9$ and $2(4n) = 2(9)$
  $10 = 5a$ and $10 \div 2 = 5a \div 2$

- Write two equations that are equivalent to $3n = 5$ and verify using a model.
- Find the values of the unknown mass on each balance scale and to sketch the steps used.





- Provide illustrations of pan balances that show equivalent expressions. Draw and record the equation shown, then draw and record the results when adding the same amount to both sides, subtracting the same amount from both sides, multiplying both sides by the same factor, and dividing both sides by the same divisor.



Planning for Instruction

CHOOSING INSTRUCTIONAL STRATEGIES

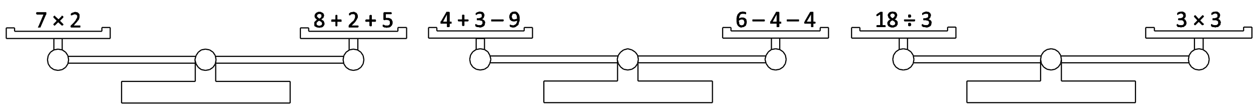
Consider the following strategies when planning daily lessons.

- Build understanding of equality by first using equations and exploring what happens when something is changed on one side of the equation and what has to be done to compensate for the change in order to preserve equality.
- Have students use balance scales to illustrate an equality and then connect the concrete to the pictorial and symbolic representations.

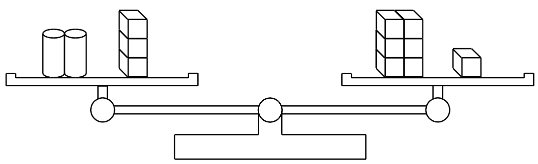
- Use the pictorial representation of algebra tiles on balance scales to solve equations in which the solution is an integer.
- Have students solve problems by writing the appropriate equation, illustrating the solution concretely or pictorially using balance scales, and recording the solution symbolically.

SUGGESTED LEARNING TASKS

- With the balance scales diagrams shown below: Are the scales balanced? Explain how you know. If the pans are not balanced, what would you have to do to make them balance?



- Write the equation that represents the balance scale model below. (The pans are balanced and all pieces are positive.) Then solve the equation both pictorially and symbolically.



- Write two equations that are equivalent to $3n + 1 = 7$.

SUGGESTED MODELS AND MANIPULATIVES

- algebra tiles
- balance scales
- base-ten blocks
- colour tiles
- geometric solids
- linking cubes
- number balance
- objects to represent the variables
- virtual manipulatives*

*also available in *Interactive Math Tools* (Pearson n.d.)

Mathematical Language

Teacher	Student
▪ balance ▪ equality ▪ equivalence ▪ imbalance ▪ inequality ▪ preservation ▪ preservation of equality ▪ relationship ▪ variables	▪ balance ▪ equality ▪ equivalence ▪ imbalance ▪ inequality ▪ preservation ▪ preservation of equality ▪ relationship ▪ variables

Resources

Print

Making Mathematics Meaningful to Canadian Students, K–8 (Small 2009), 587–588

Making Mathematics Meaningful to Canadian Students, K–8, Second Edition (Small 2013), 626–628

Teaching Student-Centered Mathematics, Grades 5–8, Volume Three (Van de Walle and Lovin 2006b), 279–281

Math Makes Sense 7 (Garneau et al. 2007)

- Student Book Unit 6 Equations (NSSBB #: 2001640)
  - Section 6.2 Using a Model to Solve Equations
  - Section 6.3 Solving Equations Involving Integers
  - Section 6.4 Solving Equations Using Algebra
  - Section 6.5 Using Different Methods to Solve Equations
  - Unit Problem: Choosing a Digital Music Club
- *ProGuide* (CD; Word Files) (NSSBB #: 2001641)
  - Assessment Masters
  - Extra Practice Masters
  - Unit Tests
- *ProGuide* (DVD) (NSSBB #: 2001641)
  - Projectable Student Book Pages
  - Modifiable Line Masters

Digital

- *National Library of Virtual Manipulatives* (Utah State University 2015): http://nlvm.usu.edu/en/nav/vlibrary.html

- *Illuminations: Resources for Teaching Math* (National Council of Teachers of Mathematics 2015): http://illuminations.nctm.org

SCO PR04: Students will be expected to explain the difference between an expression and an equation. [C, CN]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

PR04.01 Identify and provide an example of a constant term, numerical coefficient, and variable in an expression and an equation.
PR04.02 Explain what a variable is and how it is used in a given expression.
PR04.03 Provide an example of an expression and an equation and explain how they are similar and different.

Scope and Sequence

Mathematics 6	Mathematics 7	Mathematics 8
PR03 Students will be expected to represent generalizations arising from number relationships using equations with letter variables. **PR04** Students will be expected to demonstrate and explain the meaning of preservation of equality concretely, pictorially, and symbolically.	**PR04** Students will be expected to explain the difference between an expression and an equation.	–

Background

Students first learned the term equation in Mathematics 1 when they were introduced to number sentences. The term expression was introduced in Mathematics 2, and the terms coefficient and constant were introduced in Mathematics 5.

Student's previous experience with representing patterns, equality, and inequality should assist them in understanding the difference between the expressions and equations.

- An expression is a mathematical phrase composed of number(s) and/or variable(s).
- An equation is a statement that two quantities or expressions are equal in value or equivalent.
- The major difference between an equation and an expression is that an equation expresses equality through the use of an equal sign.
- $3 + 4$ and $3 + y$ are examples of expressions
- $3 + 4 = 7$ and $3 + y = 7$ are examples of equations

Variables can have different uses. Students may think that a variable represents one specific unknown value such as in the equation $2n + 12 = 34$. However in an expression, such as $3n – 1$, and an equation, such as $y = 2x + 1$, the variable can have more than one value and therefore varies. It is important for students to understand this distinction. Also students are asked to use variables to make generalizations such as the nth term in a pattern or determining a formula. In this case the variable is used as a pattern generalizer and can have more than one value. It is also important for students to understand that equations like $x + 6 = 10$ can also be written as $10 = x + 6$ without changing the equality or balance. Students need experiences seeing the variable written on either side of the equation.

A numerical coefficient is a quantity (usually a numerical constant), by which a variable is multiplied in an expression or an equation; for example, in the algebraic equation $2p = 10$, the numerical coefficient is 2. Students should work with expressions with a numerical coefficient of 1, as well as with fractional coefficients. Students sometimes have difficulty identifying that the numerical coefficient is 1 in expressions such as $x + 5$. In the equation $\frac{1}{2}$k = 6, k is the variable, 6 is the constant term and $\frac{1}{2}$ is the numerical coefficient. It may be beneficial for some students to rewrite an equation like $\frac{k}{2} + 6 = 10$ as $\frac{1k}{2} + 6 = 10$ so that they can clearly see that the numerical coefficient is $\frac{1}{2}$.

It is important that students understand mathematical conventions, and regularly use lower case letters for variables. Outcome PR04 should be done in conjunction with outcome PR05.

Assessment, Teaching, and Learning

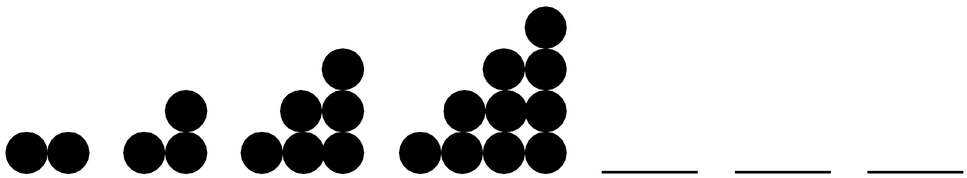
Assessment Strategies

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Provide students with a number of equations, such as 25 + 13 = n + 25, and ask them to determine the value of the variable that makes a true statement. Observe whether the students misinterpret the equal sign, or the commutative property by answering 38. Include the four operations in the equations provided.
- Ask students to use a balance to model the solution to single-variable, one-step equations such as the following:
  $13 + n = 20$ $\quad$ $49 = 7p$ $\quad$ $8t = 40$ $\quad$ $m \div 5 = 7$
- Invite students to write and explain the formula for finding the perimeter of any regular polygon (equilateral triangle, square, regular pentagon, regular hexagon, etc.) using variables.

- Provide students with pictures or models of the first three steps of an increasing pattern. Invite students to extend the pattern for several more steps, record the pattern in a table, and look for the relationship. Ask them to write the relationship as an expression and use the expression to predict entries at any step.

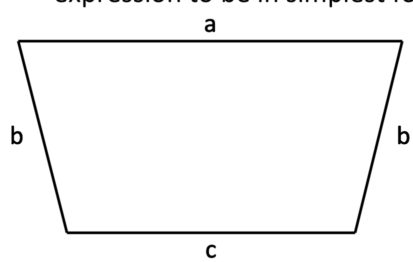


Term Number	Term Value
1	2
2	3
3	6
4	10

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Write an expression to represent the perimeter of the following figure. Do you consider your expression to be in simplest form?



- Complete the following tables.

Algebraic Expression	Expression in Words	Variable	Numerical Coefficient	Constant
$3b - 1$	One less than three times a number	b	3	-1
$x + 3$				
$\frac{n}{2} + 4$				
$5 - 7y$				
$6 + n$				

Algebraic Equation	Equation in Words	Variable	Numerical Coefficient	Constant
$3b + 1 = 7$	One more than three times a number is seven	b	3	1 and 7
$16 = x + 7$				
$\frac{n}{25} + 3 = 7$				

$9 = 15 - 2y$				
$11 = 4 + n$				

- Write each statement as an expression
  - a number increased by 14
  - the quotient of a number and 4
  - the difference between 35 and a number 75 take away a number
  - the product of a number and 7
  - a number decreased by 7
  - four times a number

- Change each statement above so that it could be represented by an equation.
- Write an algebraic expression that has a variable *h*, numerical coefficient 4, and constant term 11.
- Which of the following are expressions? Which are equations? How are they similar? How are they different?

  $2 - x$
  $20 = 5v$
  $\frac{h}{3} = 4$
  $w + 7$
  $10 = b + 5$
  $7 + a = 9(2)$

- Write the algebraic expression that would represent the situation.
  - Chris worked a certain number of hours yesterday and 8 more hours today.
  - Loretta earned $10.50 for each hour she worked.
  - Noel has $20 in his pocket. He earns $11 for each hour he works.

Planning for Instruction

Choosing Instructional Strategies

Consider the following strategies when planning daily lessons.

- Encourage students to describe patterns and rules orally and in writing before using algebraic symbols.
- Provide opportunity for students to connect the concrete and pictorial representations to symbolic representations as well as connecting the symbolic representations to pictorial and concrete representations.
- Ask students to provide examples of algebraic equations and examples of algebraic expressions. Ask them what it is about the expressions that make them algebraic. Ask students to explain what it is that makes the examples they created equations or expressions. Ask whether an algebraic expression or an algebraic equation can be demonstrated using a balance. Ask students to explain or demonstrate.

SUGGESTED LEARNING TASKS

- Create a set of algebraic expressions and equations on index cards. Create a matching set of cards with the "word" forms of each expression or equation. Randomly distribute the cards among the class. Invite students to find the person whose card matches theirs (e.g., $6k + 3$ would match with "three more than six times a number"). The cards could also be used as a "Concentration Game" in which cards are turned over two at a time and the player determines if they "match." If the cards do not match, the cards are turned back over and the next player takes a turn.
- Provide students with some expressions and/or equations in algebraic form, such as

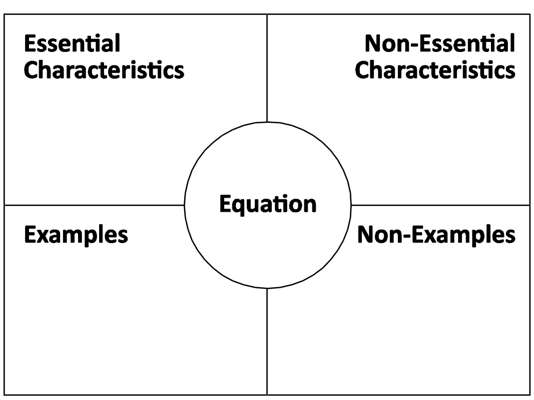
  $4p - 5 = b$ $\quad$ $4p - 5$
  $p + 5$ $\quad$ $4p - 5 = 55$

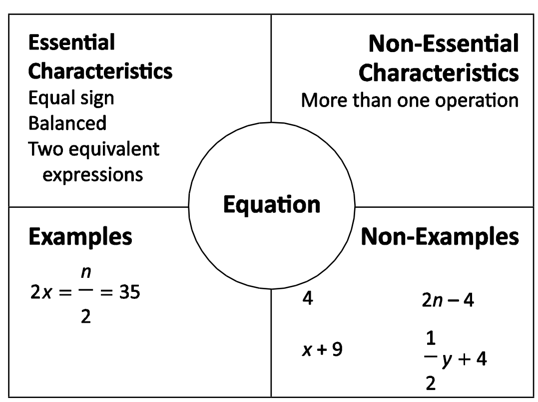
  - Describe the ways in which they are similar and ways in which they differ. Which are equations and which are expressions? Explain why.
  - Create a contextual story problem for each equation or expression.
  - Work collaboratively to create a classroom chart as shown below.

Algebraic Expression	Expression in Words	Variable	Numerical Coefficient	Constant
$3b - 1$	one less than three times a number	b	3	–1

Algebraic Equation	Equation in Words	Variable	Numerical Coefficient	Constant
$3b + 1 = 7$	one more than three times a number is seven	b	3	1 and 7

- Ask students to complete Frayer Model concept maps for expressions and for equations. Once students complete the maps, have them share their ideas with others, modifying their maps as necessary to incorporate new information.





SUGGESTED MODELS AND MANIPULATIVES

- balance*
- virtual balance*

*also available in *Interactive Math Tools* (Pearson n.d.)

MATHEMATICAL LANGUAGE

Teacher	Student
▪ equation ▪ expression ▪ numerical coefficient ▪ unknown ▪ variable	▪ equation ▪ expression ▪ numerical coefficient ▪ unknown ▪ variable

Resources

Print

Making Mathematics Meaningful to Canadian Students, K–8 (Small 2008), 587–588

Making Mathematics Meaningful to Canadian Students, K–8, Second Edition (Small 2013), 626–628

Teaching Student-Centered Mathematics, Grades 5–8, Volume Three (Van de Walle and Lovin 2006b), 279–281

Math Makes Sense 7 (Garneau et al. 2007)

- Unit 1: Patterns and Relations (NSSBB #: 2001640)
  - Section 1. 3 Algebraic Expressions
  - Section 1.7 Reading and Writing Expressions
  - Section 1.8 Solving Equations Using Algebra Tiles
  - Unit Problem: Fund Raising
- Unit 6: Equations (NSSBB #: 2001640)
  - Section 6.1 Solving Equations
  - Unit Problem: Choosing a Digital Music Club
- *ProGuide* (CD; Word Files) (NSSBB #: 2001641)
  - Assessment Masters
  - Extra Practice Masters
  - Unit Tests
- *ProGuide* (DVD) (NSSBB #: 2001641)
  - Projectable Student Book Pages
  - Modifiable Line Masters

Digital

- *National Library of Virtual Manipulatives* (Utah State University 2015): http://nlvm.usu.edu/en/nav/vlibrary.html
- *Illuminations: Resources for Teaching* Math (National Council of Teachers of Mathematics 2015): http://illuminations.nctm.org

SCO PR05: Students will be expected to evaluate an expression given the value of the variable(s). [CN, R]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicator

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

PR05.01 Substitute a value for an unknown in a given expression and evaluate the expression.

Scope and Sequence

Mathematics 6	Mathematics 7	Mathematics 8
PR03 Students will be expected to represent generalizations arising from number relationships using equations with letter variables. **PR04** Students will be expected to demonstrate and explain the meaning of preservation of equality concretely, pictorially, and symbolically.	**PR05** Students will be expected to evaluate an expression given the value of the variable(s).	–

Background

We can use symbols to represent a pattern. A variable is a symbol that can represent one unknown quantity. Students are familiar with variables in formulas, such as area = base *x* height, A = bh. Students might relate variables to things that change over time that are part of their own experiences, such as their height.

Some letters used as variables may be confusing to students as they have more than one meaning. For example, x may be mixed up with the multiplication symbol or m may be confused with metres. It is important that an expression such as 3*m*, with no space between the 3 and the m, is read as “a number multiplied by 3,” or “3 times *m*.” If it is written as 3 m, with a space between the 3 and the m, then the m represents a quantity such as metres and 3 m could be read as 3 metres. When evaluating algebraic expressions, ensure that students understand the meaning of such notations.

It is also possible for students to confuse the placement of variables when writing expressions or equations; for example, if there are 6 notebooks (n) for each student (s), they might write $s = 6n$, instead of $n = 6s$. Reading each of these equations as a sentence will help students determine which equation correctly represents the context.

To evaluate an algebraic expression, students substitute a number for the variable and carry out the computation. Using real-life situations relevant to students will help with this.

Draw students' attention to expressions such as $4h + 8$, where multiplication is used. If $h = 5$, for example, a common student error is writing 45 + 8 instead of 4(5) + 8 or $4 \cdot 5 + 8$.

Students should also be aware that division is often represented as a fraction, such as $8 - \frac{m}{2}$ or 8 – m ÷ 2. Outcome PR04 should be done in conjunction with outcome PR05.

Assessment, Teaching, and Learning

Assessment Strategies

Assessing Prior Knowledge

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to determine the value of the variable to make a true statement.
  $18 + n = 31$ $81 = 9 \times t$ $8 \times w = 56$ $x \div 6 = 7$

Whole-Class/Group/Individual Assessment Tasks

Consider the following **sample tasks** (which can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Determine which expression has the greatest value if $p = 8$.
  $p + 7$
  $2p$
  $10 - p$
  $8 \div p$
  $3p - 12$
  $2 + 2p$

- Evaluate the following:
  $2k + 5$ when $k = 21$
  $5 + 4m$ when $m = 4.2$
  $\frac{y}{4} + 22$ when $y = 60$
  $-3 + 5q$ when $q = 2$

- Explain the steps used to evaluate the following expressions for the given value of the variable:
  $3p + 5$, for $p = 1$
  2 m – 3, for m = 6

Planning for Instruction

Choosing Instructional Strategies

Consider the following strategies when planning daily lessons.

- Provide opportunity to connect the concrete and pictorial representations to symbolic representations, as well as connecting the symbolic representations to pictorial and concrete representations. For example, introduce the concept of algebraic expressions and equations using real-life examples. For example, if you pay a basic fee of \$20 a month for a cell phone and are charged \$0.90 for each text message, your monthly bill could be determined using the expression $20 + 0.90h$.

SUGGESTED LEARNING TASKS

- Evaluate the following expressions if $n = 5$ and $v = 2$.
  - $6n$
  - $3 + v$
  - $4(n + 2)$
  - $10v$
  - $n + 23$
  - $7v + 8$
  - $\frac{25}{n}$
  - $35 - 2v$
  - $6n - \frac{12}{2}$
- Alisha gets paid \$9 an hour to babysit. She gets a bonus of \$5 per hour if she works past 10 pm. The expression $9m + 5$ represents what Alisha got paid last night. What does the variable in this expression represent? What is the constant in this expression? What does it represent? If she babysat from 5:00 pm till midnight, how much did she earn last night?

SUGGESTED MODELS AND MANIPULATIVES

- algebra tiles *
- linking cubes

*also available in *Interactive Math Tools* (Pearson n.d.)

MATHEMATICAL LANGUAGE

Teacher	Student
▪ evaluate ▪ substitution ▪ unknown ▪ variable	▪ evaluate ▪ substitution ▪ unknown ▪ variable

Resources

Print

Math Makes Sense 7 (Garneau et al. 2007)

- Unit 1: Patterns and Relations (NSSBB #: 2001640)
  - Section 1. 3 Algebraic Expressions
  - Section 1.4 Relationships in Patterns
  - Section 1.8 Solving Equations Using Algebra Tiles
  - Unit Problem: Fund Raising
- *ProGuide* (CD; Word Files) (NSSBB #: 2001641)
  - Assessment Masters
  - Extra Practice Masters
  - Unit Tests
- *ProGuide* (DVD) (NSSBB #: 2001641)
  - Projectable Student Book Pages
  - Modifiable Line Masters

SCO PR06: Students will be expected to model and solve, concretely, pictorially, and symbolically, problems that can be represented by one-step linear equations of the form *x* + *a* = *b*, where *a* and *b* are integers. [CN, PS, R, V]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

PR06.01 Represent a given problem with a linear equation, and solve the equation using concrete models.
PR06.02 Draw a visual representation of the steps required to solve a given linear equation.
PR06.03 Solve a given problem using a linear equation and record the process.
PR06.04 Verify the solution to a given linear equation using concrete materials and diagrams.
PR06.05 Substitute a possible solution for the variable in a given linear equation into the original linear equation to verify the equality.

Scope and Sequence

Mathematics 6	Mathematics 7	Mathematics 8
PR03 Students will be expected to represent generalizations arising from number relationships using equations with letter variables. **PR04** Students will be expected to demonstrate and explain the meaning of preservation of equality concretely, pictorially, and symbolically.	**PR06** Students will be expected to model and solve, concretely, pictorially, and symbolically, problems that can be represented by one-step linear equations of the form *x* + *a* = *b*, where *a* and *b* are integers.	**PR02** Students will be expected to model and solve problems, concretely, pictorially, and symbolically, where *a*, *b,* and *c* are integers, using linear equations of the form ▪ $ax = b$ ▪ $\frac{x}{a} = b,\ a \neq 0$ ▪ $ax + b = c$ ▪ $\frac{x}{a} + b = c,\ a \neq 0$ ▪ $a(x + b) = c$

Background

There are many methods for solving a one-step linear equation such as: inspection, systematic trial (guess and test), using balances or illustrations of balances, creating models using algebra tiles and rewriting the equation, to show equality. Students should be encouraged to choose the most appropriate method for solving a given problem. Emphasis at this level should be on solving problems concretely, pictorially, and symbolically.

Concretely: Students should be comfortable representing integers and operations on integers using algebra tiles and should continue to do so when modelling an addition or subtraction equation. The zero principle is an important aspect of preserving equality between the expressions on the left side and right side of the equal sign.

Pictorially: Encourage students to use concrete models when solving problems and then draw pictures of their models in order to move from the concrete to the pictorial.

Symbolically: Students should understand that adding or subtracting the same value from both sides of an equation keeps the equation balanced.

Students should estimate in advance a reasonable solution to the problem, and once they solve the equation to determine the solution, it be verified through substitution into the original equation.

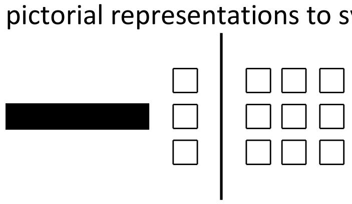
While some students may immediately arrive at the value of the unknown (the variable), it is important to work through the various strategies presented in this unit since work with algebraic equations will become increasingly complex throughout later grades.

When solving equations with the use of algebra tiles, students need to decide which colour will represent positive and which will represent negative, regardless of the colour of the tiles available. Throughout this curriculum document, shaded tiles represent positive values and white tiles represent negative values.

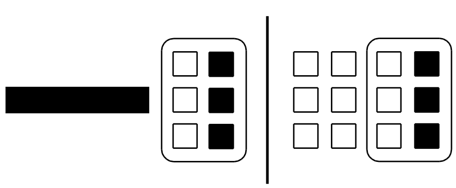
Students have used algebra tiles or a similar manipulative to solve linear equations involving whole numbers, and will extend this knowledge to include all integers. Students will be required to draw upon outcome 7N06 from the Integers unit.

This outcome does not include equations that involve multiplication and division.

To model the solution to an equation that uses subtraction, such as $x - 3 = -9$, students will attempt to isolate the variable. To isolate the variable, zero pairs are made by adding 3 positive tiles to each side. Once the zero pairs are removed, the tiles show that $x = -6$. As equations are modelled, students also record the process symbolically. This will help students navigate the transition from concrete and pictorial representations to symbolic representation.



$x - 3 = -9$



$x - 3 + 3 = -9 + 3$

$x = -6$

Students then verify the solution by replacing the variable tile in the original equation with the appropriate number of unit tiles. In the above example, 6 negative tiles would be used.

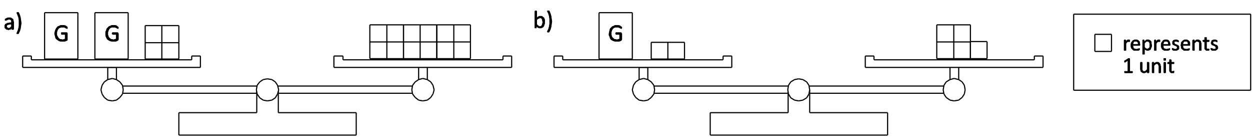
Assessment, Teaching, and Learning

Assessment Strategies

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to write an equation that represents each model:



a) $2G + 4 = 10$ b) $G + 2 = 5$

- Ask them to explain what *G* represents in each equation.
- Ask students to determine if $2g + 3 = 7$ and $2g + 4 = 8$ are equivalent forms of equations. Ask students to explain their thinking using models.
- Invite students to model and write two equations that are equivalent to $4b = 12$. Ask them to explain how they know the two equations are equivalent.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Create and solve an equation for the following contexts
  - I have 14 collector pins from events I attended at the stadium. Together, my friend and I have 19 pins. How many pins does my friend have?
  - There are 23 students in our class. Of the 23 students, 9 walk or get rides to school and the rest come by bus. How many students in the class come to school by bus?
- Identify which of the equations have the solution x = –2.

  $x + 3 = -5$
  $-5 = x - 3$
  $x - 7 = -5$
  $x + 3 = 1$
- Sketch the steps used to solve each equation, and then verify the solution.

  $4 = n - 3$
  $-2 = h + 1$
  $2 = y - 6$
  $w - 4 = 1$

- Explain the steps they would use to determine the value of b in the given equations:

  $b + 8 = -13$
  $(-6) - b = 81$
  $154 + b = 340$
  Explain if each equation has just one value for b or if they think there are others in addition to the one found. Have students draw a pictorial representation of the steps needed to find the value of b and verify their answer by substituting it into the original equation.
- What integer will make this equation true:

  $m + 2 = 12$
- Rewrite the equation so that 2 no longer exists on the left-hand side of the equation and yet equality is still maintained.
- Solve each equation using algebra tiles, and by inspection. Verify each solution by substitution.

  $c + 4 = 9$ $\quad\quad$ $n - 3 = 8$

Planning for Instruction

Choosing Instructional Strategies

Consider the following strategies when planning daily lessons.

- Use a balance scale as a particularly helpful model for illustrating the importance of adding and subtracting like values to each side in order to maintain equality. *The National Library of Virtual Manipulatives* website has an activity to explore the use of balance scales (http://nlvm.usu.edu/en/nav/category_g_3_t_2.html).
- Ask students what other ways an equation can be written. For example, consider how you could rewrite the equation $a + 9 = 14$ (e.g., $14 - 9 = a$). It is through rewriting an equation in alternative forms that students arrive at the notion of solving equations using more formal methods.
- Have students use algebra tiles to represent equations such as $b + (-4) = 6$. Students should use the tiles to solve the equation and then sketch the tiles they used. This will help them move from solving problems concretely to solving them pictorially.
- Explore the "cover-up" method as an extension. The cover-up method is named for the way it is typically applied. For example, using the equation $p + (-5) = 25$, cover up the p and ask the question, what added to –5?

Suggested Learning Tasks

- Use mental mathematics to solve each equation.

  $12 = m - 4$
  $n + 5 = 11$
  $15 = x + 9$
- Solve each equation by isolating the variable.

  $g - 9 = 31$
  $p - 5 = 8$
- Explain how to find the value of x in the given equations.

  $x + 8 = 13$
  $6 + x = 81$

$154 = x + 340$
$x + 4 = 9$
$x - 3 = 8$

- Using a small envelope, place a number of counters inside. On the outside of the envelope write a variable such as w. Write an equation such as $w + 3 = 7$, where w represents the number of counters inside the envelope. Ask students to guess the number of counters in the envelope to make the equation true and then verify by checking the envelope.
- Work in pairs to make equations in the form $x + a = b$ (where a and b are integers). Try the following criteria:
  - a is negative
  - both a and b are negative (or positive)
  - a is positive and b is negative
- Write an equation for each problem and then use algebra tiles to solve and verify.
  - The temperature dropped 5°C to –2°C. What was the original temperature?
  - Frank is 9 years old. He is 4 years older than Joe. How old is Joe?
  - Susan borrowed books from the library. She then returned 4 books. If she still has 3 books at home, how many did she borrow?
- Solve equations by isolating the variable. Include a pictorial or symbolic representation of the steps. Verify your solutions. Record all of the steps in the graphic organizer below.

Pictorial Diagram	Symbolic Steps
Solution: In the Equation	Verification

SUGGESTED MODELS AND MANIPULATIVES

- algebra tiles*
- balance scale*****

*also available in *Interactive Math Tools* (Pearson n.d.)

MATHEMATICAL LANGUAGE

Teacher	Student
▪ balanced ▪ inspection ▪ solve ▪ systematic trial ▪ verify ▪ zero principle	▪ balanced ▪ inspection ▪ solve ▪ systematic trial ▪ verify

Resources

Print

Math Makes Sense 7 (Garneau et al. 2007)

- Unit 1: Patterns and Relations (NSSBB #: 2001640)
  - Section 1.8 Solving Equations Using Algebra Tiles
  - Unit Problem: Fund Raising
- Unit 6: Equations (NSSBB #: TBD)
  - Section 6.2 Using a Model to Solve Equations
  - Section 6.3 Solving Equations Involving Integers
  - Section 6.4 Solving Equations Using Algebra
  - Section 6.5 Using Different Methods to Solve Equations
- *ProGuide* (CD; Word Files) (NSSBB #: 2001641)
  - Assessment Masters
  - Extra Practice Masters
  - Unit Tests
- *ProGuide* (DVD) (NSSBB #: 2001641)
  - Projectable Student Book Pages
  - Modifiable Line Masters

Digital

- "Algebra (Grades 6–8)," *National Library of Virtual Manipulatives* (Utah State University 2015): http://nlvm.usu.edu/en/nav/category_g_3_t_2.html

SCO PR07: Students will be expected to model and solve, concretely, pictorially, and symbolically, where *a*, *b,* and *c* are whole numbers, problems that can be represented by linear equations of the form ▪ $ax + b = c$ ▪ $ax = b$ ▪ $\frac{x}{a} = b,\ a \neq 0$ [CN, PS, R, V]
[C] Communication **[PS]** Problem Solving **[CN]** Connections **[ME]** Mental Mathematics and Estimation **[T]** Technology **[V]** Visualization **[R]** Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

PR07.01 Represent a given problem with a linear equation, and solve the equation using concrete models.
PR07.02 Draw a visual representation of the steps used to solve a given linear equation.
PR07.03 Solve a given problem using a linear equation and record the process.
PR07.04 Verify the solution to a given linear equation using concrete materials and diagrams.
PR07.05 Substitute a possible solution for the variable in a given linear equation into the original linear equation to verify the equality.

Scope and Sequence

Mathematics 6	Mathematics 7	Mathematics 8
PR03 Students will be expected to represent generalizations arising from number relationships using equations with letter variables. **PR04** Students will be expected to demonstrate and explain the meaning of preservation of equality concretely, pictorially, and symbolically.	**PR07** Students will be expected to model and solve, concretely, pictorially, and symbolically, where *a*, *b,* and *c* are whole numbers, problems that can be represented by linear equations of the form ▪ $ax + b = c$ ▪ $ax = b$ ▪ $\frac{x}{a} = b,\ a \neq 0$	**PR02** Students will be expected to model and solve problems, concretely, pictorially, and symbolically, where *a*, *b,* and *c* are integers, using linear equations of the form ▪ $ax = b$ ▪ $\frac{x}{a} = b,\ a \neq 0$ ▪ $ax + b = c$ ▪ $\frac{x}{a} + b = c,\ a \neq 0$ ▪ $a(x + b) = c$

Background

In order for students to solve linear equations in the forms $ax + b = c$; $ax = b$; $\frac{x}{a} = b$, $a \neq 0$ they must understand the idea of "balancing" equations, which is the basis of preservation of equality in an equation (left side = right side). In the form $ax + b = c$, students need to perform a two-step process to solve for the variable whereas in other equations a single-step process is used.

In this outcome, only whole numbers are used for *a, b*, and *c*. In the Integers unit, work was limited to the addition and subtraction operations. Since multiplication and division of integers will not be introduced until Mathematics 8, students are not expected to multiply or divide integers when solving equations in this unit. Therefore, when selecting equations of the form $ax + b = c$, ensure that $b < c$. In the equation $3x + 9 = 6$, although the *a*, *b*, and *c* values are whole numbers, subtracting 9 from both sides will leave $3x = -3$. Since $3x$ is a multiplication statement, students are not expected to be able to determine that $3(-1) = -3$ and, therefore, $x = -3$.

While some students may immediately arrive at the value of the unknown (the variable), it is important to work through the various strategies presented in this unit since work with algebraic equations will become increasingly complex throughout later grades.

When using systematic trial, students are expected to choose a reasonable value as the solution, and then evaluate using the order of operations to determine if the chosen value for the variable maintains the equality of the two expressions. If the chosen value does not work, students should question whether it is too small or too large, and then choose another value. They continue in this way until the correct value is found. At first, students might begin by using the guess-and-check strategy. By observing patterns in their results, they should become more systematic in the guesses they make. To solve $2n + 1 = 201$, students could use guess-and-check:

- 2 x 10 + 1 = 21 (An input of 10 is not large enough.)
- 2 x 50 + 1 = 101 (An input of 50 is not large enough.)
- 2 x 100 + 1 = 201 (Therefore, 100 is the correct input value.)

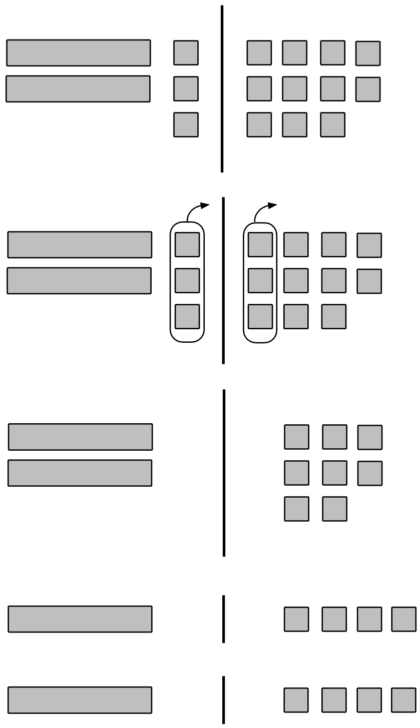
This approach is valuable for students to explore, especially when verifying their solution to an equation.

Inspection differs from systematic trial. It is not a guess-and-check approach. To solve $3x + 7 = 19$, students will replace the $3x$ with the value needed to add to 7 to get 19. They will then determine that the value, 12, is equal to 3(4). Therefore $x = 4$. This can also be thought of as the "cover-up" method. Using the same equation, cover up the $3x$ and ask "What added to 7 makes 19?" Next, cover up the *x* and ask "What multiplied by 3 makes 12?"

The use of concrete models, is essential in developing students' comprehension of solving equations. Students can also draw pictures of their models and explain how they were used to solve the equation. This will help as students move from the concrete representation to the pictorial representation. It is also important that students verify the solutions to equations using their models.

Algebra tiles or other sets of counters are useful manipulatives for modelling algebraic equations. It is important to use tiles or counters of one colour, since only "positives" will be used. For the purposes of this curriculum guide, shaded tiles represent "positive" and unshaded tiles represent "negative." The rectangular tile is used to represent the variable, commonly referred to as *x*. The small squares are used to represent units. A vertical line can be used to represent the equal sign. When working with algebra tiles, it is important to be consistent about what each manipulative represents.

To record the steps symbolically as well as pictorially students may use either the two-pan balance scale or algebra tiles. Students should practice recording the steps symbolically using equations with one operation before recording the steps symbolically using two operations as used in the example $2n + 3 = 11$.



$2n + 3 = 11$

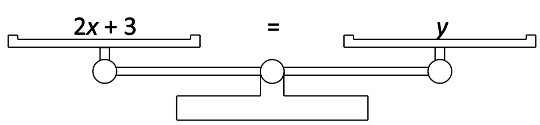
$2n + 3 - 3 = 11 - 3$

$2n = 8$

$\frac{2n}{2} = \frac{8}{2}$

$1n = 4$

The balance model is based on the principle that an equation represents two equal expressions separated by an equal sign. The equal sign is represented by the fulcrum or balance point of a scale, and the expressions on either side represent masses placed in either pan of the balance. The expressions are equal; both represent the same value and can symbolize equal masses.



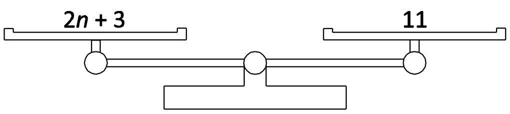
In the balance scale metaphor, changing the mass on one side of the fulcrum will tip the scale. Making an identical change on the opposite side of the fulcrum will rebalance the scale.

In the concrete model, a balance scale is used along with identical objects, such as blocks, cubes, or marbles, to represent numbers, and paper bags or polystyrene cups, to represent variables. Designated objects are added to the bags evenly, and identical changes are made to both sides of the scale until balance is achieved. The objects in the bag could be counted to obtain the value of the variable, or the items can be manipulated until one bag is isolated on one side of the scale. The quantity it represents is isolated on the opposite side, and the scale is at equilibrium.

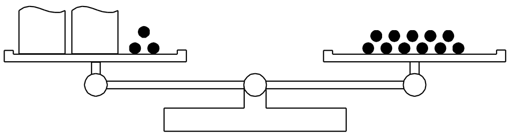
In the symbolic representation of the model, the equation is solved by performing identical operations on either side of the equal sign, until a variable remains on one side and a value on the other.

● represents a counter

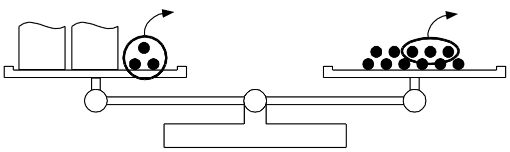
represents a bag containing an unknown number of counters



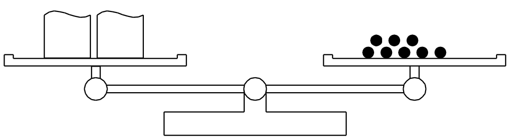
Represent $2n + 3 = 11$ as a balance



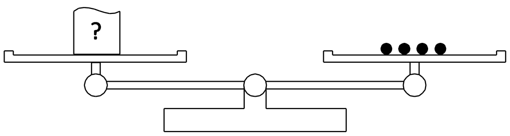
$2n + 3 = 11$
Show this concretely (or pictorially).



$2n + 3 = 11$
$\quad -3 \quad -3$
Maintaining balance, remove 3 chips from each side.

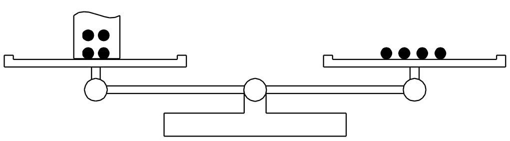


$2n = 8$
Simplify.

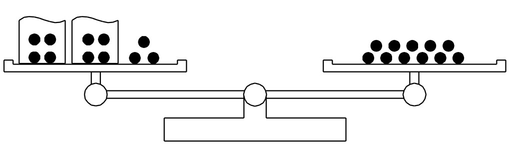


$\frac{2n}{2} = \frac{8}{2}$

Determine the number of counters that would be in each bag.



$n = 4$
Simplify.



$2n + 3 = 11$ (?)
$8 + 3 = 11$ (?)
$11 = 11$ (✓)
Check.

Problem solving is an important skill that we want students to understand and develop. Solving formulas and equations are a regular part of problem solving and it is important for students to understand situations in which they will use, develop, and apply such knowledge.

The use of diagrams and concrete materials to demonstrate the idea of solving for *x* is a natural progression to lead the students to an understanding of the steps needed to isolate the variable. It is after this progression that students will be able to solve for *x* in a linear equation and record the process.

Students should estimate in advance what might be a reasonable solution, and be aware that once they acquire a solution, it can be verified by substitution into the original equation.

Assessment, Teaching, and Learning

Assessment Strategies

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

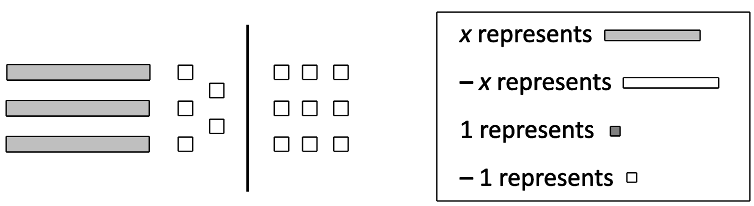
- Ask students to solve single-variable, one-step equations such as the following:
  $18 + n = 31$ $81 = 9p$ $8k = 56$ $m \div 6 = 7$
- Ask students to model and write two other equations that are equivalent to $4b = 12$. Ask them to explain how they know the equations are equivalent.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

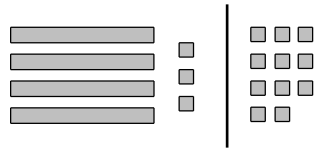
Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Solve the following using algebra tiles and draw pictures to represent the steps taken:
  $\frac{x}{2} + 1 = 5$
  $2x = 16$
  $4x + 8 = 40$
  - What do you notice about the answers for each of the above equations?
  - Analyze the three equations and to determine why the equations had these solutions.

- Provide students with a drawing of algebra tiles, such as shown below and ask them to write the equation that is represented. Solve the equation and draw and record the steps that were taken.



- Susan was given the equation $5j + 7 = 22$ and asked to solve for *j*. She indicated that $j = 15$ but was told that her answer was incorrect. Explain what her error was and how you would correct her thinking to correctly solve for *j*.
- Provide students with an equation written in words. For example, four more than twice a number is fifteen.
  - Write the equation using symbols ($2v + 4 = 15$, or $4 + 2v = 15$).
  - Use tiles to solve the equation.
  - Verify the solution by substitution.
- The algebra tile diagram below represents an equation. Identify the two expressions that make up the equation, then write the equation. Solve the equation, drawing pictures to represent the steps as they work.



- Use tiles to solve each equation and draw pictures to represent each step.

  $7 + x = 10$
  $4x = 16$
- Given the statement three more than twice a number is 19, write an equation that can be solved to find the number, use algebra tiles to solve, and verify the solution.
- Answer the questions below based on the following situation: A hockey school charges a team $800 per day plus $20 per player per day for food, equipment, and lessons. A team raised $1040 for a one-day practice.
  - Write an equation to represent this situation.
  - Solve the equation, first using systematic trial, and then by inspection, to determine how many players are on the team. Which method do you prefer? Why?
- Respond to the following:
  - When solving $4d + 24 = 36$, Sarah chose 3 for her first value for *d* and Billy chose 6. Which number is the better choice? Explain how you made your decision.
  - Ryan was asked to solve the equation $5d + 7 = 22$ for *d*. Using inspection, he found that $d = 15$. He was told that his answer was incorrect. Explain Ryan's mistake and how he should solve the equation correctly.

Planning for Instruction

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Have students work with materials to model and diagram the idea of balancing and preserving equality with the use of algebra tiles, balances, etc., with the natural progression to formulating and solving written solutions and substituting. After this progression, students will be able to solve for *x* in a linear equation and record the process:

  $3x + 2 = 8$
  $3x + 2 - 2 = 8 - 2$
  $=\frac{3x}{3} = \frac{6}{3}$
  $x = 2$

  Have students consider in advance what might be a reasonable solution, and be aware that once they acquire a solution, it can be checked for accuracy by substitution into the original equation.

  Check: $3x + 2 = 8$, where $x = 2$
  $3(2) + 2 = 8$
  $6 + 2 = 8$
  $8 = 8$

- Use the "cover up" method to aid in the initial understanding of the step elimination process. For example, given the equation $4m + 4 = 20$, "cover" the "4*m*" and think, "what amount added to 4 to equals 20?" Since the number is 16 think, "what number multiplied by 4 equals 16?" Recall that 4 x 4 = 16, therefore $m = 4$.

- Have students verify the solution to the equation as shown below.

  $4m + 4 = 20$

  $4(4) + 4 = 20$

  $14 = 4 = 20$

  $20 = 20$

SUGGESTED LEARNING TASKS

- Use a calculator to explore what buttons you would use to find the answer for questions such as, $60 \div -b = 12$.
- Distribute cards to pairs of students that have one- and two-step equations that are shown pictorially, symbolically, and concretely. Have students match up the corresponding cards. As an extension, have students create their own cards.
- Provide a step-by-step pictorial representation of a solved equation. Have students provide the symbolic representation of each step. As an extension, all steps except for the answer can be provided and students will have to solve for the coefficient and represent it both symbolically and pictorially.

SUGGESTED MODELS AND MANIPULATIVES

- algebra tiles*
- bags
- balance scale
- counters*
- linking cubes
- polystyrene cups

*also available in *Interactive Math Tools* (Pearson n.d.)

MATHEMATICAL LANGUAGE

Teacher	Student
▪ balance	▪ balance
▪ equality	▪ equality
▪ evaluate	▪ evaluate
▪ guess-and-check	▪ guess-and-check
▪ inspection (cover-up)	▪ inspection (cover-up)
▪ linear equation	▪ linear equation
▪ one-step elimination process	▪ one-step elimination process
▪ preserve equality	▪ preserve equality
▪ substitution	▪ substitution
▪ systematic trial	▪ systematic trial
▪ two-step elimination process	▪ two-step elimination process
▪ value	▪ value
▪ variable	▪ variable
▪ verify	▪ verify

Resources

Print

Math Makes Sense 7 (Garneau et al. 2007)

- Unit 1: Patterns and Relations (NSSBB #: 2001640)
  - Section 1.8 Solving Equations Using Algebra Tiles
  - Unit Problem: Fund Raising
- Unit 6: Equations
  - Section 6.1 Solving Equations
  - Section 6.2 Using a Model to Solve Equations
  - Section 6.4 Solving Equations Using Algebra
  - Section 6.5 Using Different Methods to Solve Equations
  - Unit Problem: Choosing a Digital Music Club
- *ProGuide* (CD; Word Files) (NSSBB #: 2001641)
  - Assessment Masters
  - Extra Practice Masters
  - Unit Tests
- *ProGuide* (DVD) (NSSBB #: 2001641)
  - Projectable Student Book Pages
  - Modifiable Line Masters

Measurement (M)

GCO: Students will be expected to use direct and indirect measurement to solve problems.

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

GUIDING QUESTIONS

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Follow-up on Assessment

The evidence of learning that is gathered from student participation and work should be used to inform instruction.

GUIDING QUESTIONS

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

GUIDING QUESTIONS

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SCO M01: Students will be expected to demonstrate an understanding of circles by ▪ describing the relationships among radius, diameter, and circumference ▪ relating circumference to pi ▪ determining the sum of the central angles ▪ constructing circles with a given radius or diameter ▪ solving problems involving the radii, diameters, and circumferences of circles. [C, CN, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

M01.01 Illustrate and explain that the diameter is twice the radius in a given circle.
M01.02 Illustrate and explain that the circumference is approximately three times the diameter in a given circle.
M01.03 Explain that, for all circles, pi is the ratio of the circumference to the diameter $\frac{C}{d}$ and its value is approximately 3.14.
M01.04 Explain, using an illustration, that the sum of the central angles of a circle is 360°.
M01.05 Draw a circle with a given radius or diameter, with and without a compass.
M01.06 Solve a given contextual problem involving circles.

Scope and Sequence

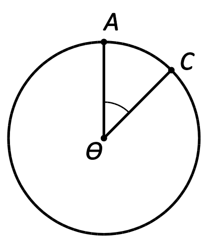
Mathematics 6	Mathematics 7	Mathematics 8
M01 Students will be expected to demonstrate an understanding of angles by ▪ identifying examples of angles in the environment ▪ classifying angles according to their measure ▪ estimating the measure of angles using 45°, 90°, and 180° as reference angles ▪ determining angle measures in degrees ▪ drawing and labelling angles when the measure is specified **M02** Students will be expected to demonstrate that the sum of interior angles is 180° in a triangle and 360° in a quadrilateral.	**M01** Students will be expected to demonstrate an understanding of circles by ▪ describing the relationships among radius, diameter, and circumference ▪ relating circumference to pi ▪ determining the sum of the central angles ▪ constructing circles with a given radius or diameter ▪ solving problems involving the radii, diameters, and circumferences of circles.	**M03** Students will be expected to determine the surface area of right rectangular prisms, right triangular prisms, and right cylinders to solve problems. **M04** Students will be expected to develop and apply formulas for determining the volume of right rectangular prisms, right triangular prisms, and right cylinders.

Mathematics 6 (continued) **M03** Students will be expected to develop and apply a formula for determining the ▪ perimeter of polygons ▪ area of rectangles ▪ volume of right rectangular prisms		

Background

A circle is a plane (2-D) figure that has all its points equidistant from a fixed point called the centre of the circle. The radius is the distance from the centre of the circle to the edge of that circle while the diameter is a line segment passing through the centre of the circle with both endpoints on the circle. The circumference of a circle is the distance around or the perimeter of a circle.

Students come to Mathematics 7 with a background in classifying and measuring angles. In order to illustrate that the sum of the central angles of a circle is equal to 360°, students first need to be aware that a central angle is one with its vertex at the centre of the circle and its rays intersecting the circumference.



Students need to understand that the vertices of all such angles meet at one point, the centre of the circle. The measure of each of the angles is taken from the centre of the circle. Each central angle has its vertex at the centre of the circle, and each ray radiates to a different point on the circumference of the circle. Investigate the sum of the central angles of a circle by having students construct circles with centres marked, and placing pattern blocks, paper cut outs of any quadrilateral, or paper cut outs of angles, so that the vertices meet at the centre of the circle. (Each of the 4 angles of a square measure 90°, so when 4 squares meet at the centre of the circle, the sum of the angles will be 360°.) The close relation between angles and circles can be used as a starting point to develop an understanding of circle concepts.

Students should understand that, in any circle, the ratio of the circumference to the diameter is a constant, and that the Greek letter π (pi) is used to represent the value of this ratio. Pi (π) is an irrational number; it is a non-repeating, non-terminating decimal that cannot be expressed as a fraction $\frac{C}{d}$.

- (π_= 3.14159265358979323846264338327950 ...). The value of π is often **approximated** as 3.14 although most calculators have a π key. Students often believe that 3.14 is pi and not just an approximation. For rough estimates, students may use 3 as an approximate value for π. It is important that students recognize that pi is not so much a special number as it is a special relationship (the relationship of the circumference of a circle divided by its diameter). It is acceptable to use 3.14 for pi or the pi button on the calculator but students need to understand that this will result in different solutions.

Provide hands-on learning opportunities for students to explore and discover the concepts and relationships within circles. Any exploration that is done to investigate pi should include collecting measures of circumference and diameter. Ratios for the circumference to the diameter should also be computed and the information can be recorded in a table similar to the one below.

Circular Object	**Circumference**	**Diameter**	$\frac{C}{d}$

In using any type of measurement, the attribute to be measured must be identified, and the appropriate unit chosen. When measuring the circumference, radius, and diameter of circles, the attribute that is being measured is length, and therefore the appropriate units to use include millimetres, centimetres and metres. When finding the sum of the central angles of a circle, the appropriate unit of measure is the degree, and each degree being $\frac{1}{360}$ of a circle.

Develop the formulas, $C = \pi d$ and $C = 2\pi r$, through exploration activities once the value of π has been established. Students should use these formulas to solve application problems.

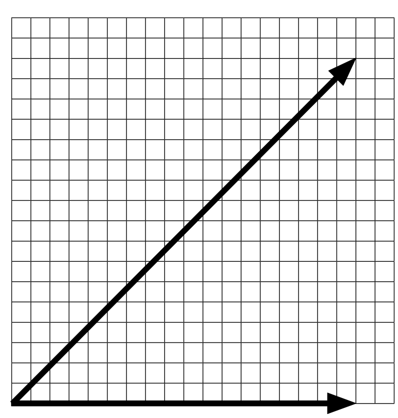
Assessment, Teaching, and Learning

Assessment Strategies

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Review the concept of perimeter by having students estimate the measure of the perimeter of designated surfaces (e.g., a tissue box, desk surface, classroom door, window, ceiling), and having them justify their responses.
- Show students the diagram below and ask why it is easy to tell that the angle measure is 45°.

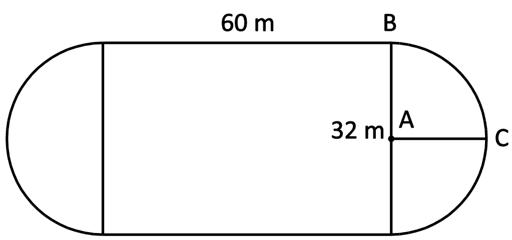


- Give students the measure of one angle in a triangle. Ask them to derive three other pairs of possible angle measures for the remaining two angles. For example, a triangle has a 45° angle. What are three possible sets of measures for the remaining two unknown angles?
- Ask students to determine the measure of the third angle in a triangle and the fourth angle of a quadrilateral when the measures of the other angles are given.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- As an Exit Card, give students a circle with one central angle indicated. Determine the measure of the other central angle.
- Decide which of the following is the best estimate of the circumference of a circle with a radius of 3.5 cm, 10.5 cm, 21 cm, or 42 cm. Ask them to justify their choice.
- Ali's school has a running track that is semi-circular at each end, as shown. The straight sides are 60 metres and the track is 32 metres wide. About how many times does he have to go around the track to run 2 km?



- George's parents are buying a new circular dining-room table. They want the table large enough to seat 8 people so that each person has 60 cm of table space along the circumference. What should the diameter of the table be? By how much would the diameter change if George's parents decide to reduce seating space to 45 cm?
  - If each person requires only 45 cm of space around the table, what are the smallest possible dimensions of the dining room, if each chair requires at least 80 cm of space between the table and the closest wall to allow people easy access to their seating place? What assumptions did you make?
- Construct circles that meet the following criteria
  - A circle with a radius of 3 cm.
  - A circle with a diameter of 8 cm.
- If you know the radius, what can you do to get the diameter?
- If you know the diameter, what can you do to get the radius?
- Estimate how many strokes it would take for Assoun to swim around the edge of a pool, if it took him 30 strokes to swim across the widest part of a circular pool.
- One whole pizza was sitting on top of the stove. Jacinthe cut out a piece of pizza and ate it. The central angle of the missing piece was 45°. Lisette came by, sliced some pizza, ate it, and left. The central angle of the remaining pizza was 120°. How much of the pizza did Lisette eat?
- What is the best estimate for the circumference of a circle with a diameter of 12 cm? Justify your choice.

  (a) 6 cm (b) 18 cm (c) 36 cm

- What is the best estimate for the circumference of a circle with a radius of 10 cm? Justify your choice.

  (a) 30 cm (b) 60 cm (c) 90 cm
- Respond to the following: A manufacturing company is producing dinner plates with a diameter of 30 cm. They plan to put a gold edge around each plate. Determine the length of gold edging they need for an eight plate setting. If gold edging costs $4 per cm, what would it cost to trim all of the plates?
- Respond to the following: A dog is tethered to a stake in a yard and can walk or run in a circle. The largest circumference of his runway is 56.52 m. What is the length of the dog’s tether rope? Explain your thinking.

Planning for Instruction

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

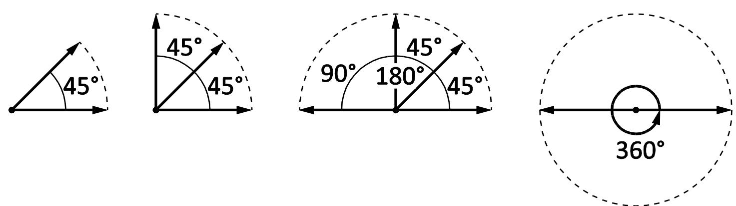
- Introduce circle vocabulary using a human circle with one student as the centre of the circle. (Do not use the word middle.) Ask how they can use the person at the centre to check that the circle made is truly circular. This should lead to discussion of the circle characteristic that any point on the circle is the equidistance from its centre. Provide the student at the centre with a string about 3 metres long. Ask the student in the centre to give the other end of the string to a student on the circle, adjusting their position so the string is taut. Continue until the string has been passed to all students on the circle. Introduce the term radius, and ask students what represents the radius in their circle. After they understand that the length of the string represents the radius, they can predict how many lengths of string are needed to extend from one student to another student directly opposite them on the circle, passing through the centre of the circle. They should conclude that double the radius equals the distance across the circle through the centre. Introduce the term diameter to represent this distance. Introduce the term circumference as the distance around the circle and relate it to perimeter.
- Connect the circumference of circles to perimeters of polygons.
- Have students construct and draw circles of various sizes using a variety of construction options (compass, pencil and string, geometry software, etc.). Consider using a bullseye compass. These look like a ruler that rotates around a circle on one end. The centre of the circle is held in place and the pencil is put in one of the holes in the “ruler” (the radius). These are easier for most students to use to construct circles since the pencil is kept a constant distance from the centre.

- A variety of strategies are available to illustrate that the sum of the central angles always totals equal to 360°.
- Given any quadrilateral, students can “tear off” the vertices and arrange them so that they all meet in the centre of a circle. Pattern blocks or tangram pieces may also be used.
- Given any two triangles, students can “tear off” the vertices and arrange them so that they all meet in the centre of a circle. Pattern block or tangram pieces may also be used.

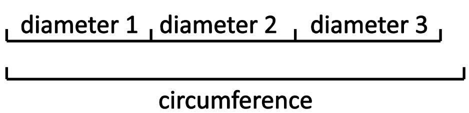
- Draw or use a cut-out of a 45° angle and have students connect the rays with an arc (Figure 1).
- Stack two angles, one next to the other, with a common vertex and common ray (Figure 2).
- Connect the rays with another arc. Ask students what type of angle is created and its measure. Line up another 90° angle alongside the image (Figure 3). What is the measure of the largest angle in Figure 3?
- Complete the task by adding another 180° angle below the image. (Figure 4)

Figure1 Figure 2 Figure 3 Figure 4

- Investigate the concept of π through measurement and chart the value of π for a number of circular 3-D objects. Students can bring round containers from home for this purpose. This activity can be done in groups, and results reported to the whole class. Inform students there are several ways to measure circumference, such as the following:
  - Wrap a measuring tape around the object.
  - Wrap a string or a ribbon around the object, and then measure the string or ribbon.
  - Mark a starting point on the object and roll the circle along a ruler, returning to the starting point.



  - Roll the object along a paper and mark the starting and ending points. Connect the points with a line and measure the line. (Note: For differentiation, rolling the object to get the line of circumference is a useful strategy, as students can then measure the diameter and physically place the diameter over the line and see how many diameters long the line is. This requires no calculation.)



  - Using technology such as Excel or Number create a table such as below to record measurements.

Circular Object	**Circumference**	**Diameter**	$\frac{C}{d}$

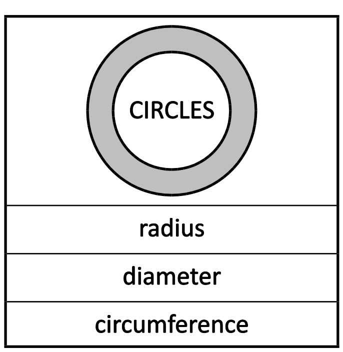
Students' measurements will not be completely accurate, so the calculations to decimal places will not be consistent. Nevertheless, students should have found that, regardless of the size of the circle, the circumference is always a little longer than three diameters of that circle (C = 3*d* and a little more). If students have measured very carefully, they should have calculated values between 3.1

and 3.2. Explain that this relation $\frac{C}{d}$ is a very important mathematical constant. It is commonly approximated as 3.14, and is termed pi (π).

- Provide opportunities for students to connect the concrete, pictorial, and symbolic representations as they explore the properties of circles.
- Have students justify the strategies they use in solving problems related to circles and critique strategies used by others.
- Use literature to stimulate students' thinking about circles and their properties, such as *Sir Cumference and the Dragon of Pi* (1999) by Cindy Neuschwander.

SUGGESTED LEARNING TASKS

- Cut out a quadrilateral, tear off its four angles and place them together. What is the measurement of the four angles together? (360˚) How does this relate to the interior angles of circles?
- Collect a series of circular containers and ask students to sort them into those for which the circumference is about equal to the height, the circumference is less than the height, and the circumference is more than the height. Ask them to explain their choices and then measure to confirm.
- Discuss why it is challenging to draw a perfect circle without any tools.
- Consider whether or not the following statement is true and to give examples to support their answers:
  - If the radius of a circle is doubled to make a new circle, the diameter is also doubled.
- Make a list of sports in which circles play an important role and estimate the radius of each circle described.
- Use words and diagrams to explain how to find the diameter of a circle if the radius is known.
- Draw circles with a 10 cm radius, with a 5 cm radius, and with a 6 cm diameter.
- Write a set of instructions to describe how to draw a circle with a diameter of 8 cm, using your preferred style of compass. Then give your instructions to a classmate who will draw it. Then decide if the drawing of your circle is accurate.
- Research pi and share the information you find.
- Create a 3-tab foldable as an organizational tool when describing the relationships among radius, diameter, and circumference.



- Have groups create presentations with ideas such as the following:
  - The sum of the central angles equals 360°.
  - If the radius is known, the diameter can be determined, and vice versa.

- If the circumference is known, the diameter can be determined, and vice versa. $\frac{C}{d}$ = the constant pi (π), and pi has an approximate value of 3.14. $\frac{C}{d} \doteq 3.14$, so $3.14 \cdot d \doteq C$ and $\frac{C}{3.14} \doteq d$.
- The circumference of a circle can be determined if the radius of the circle is known. 2*r* can replace *d* in every relation, so 3.14 $3.14 \cdot 2 \cdot r \doteq C$ or $6.28r \doteq C$.

SUGGESTED MODELS AND MANIPULATIVES

- bullseye compass
- compass
- graphing software
- measuring tapes
- pattern blocks*
- protractor
- rulers
- string
- tangrams*
- various circular objects

*also available in *Interactive Math Tools* (Pearson n.d.)

MATHEMATICAL LANGUAGE

Teacher	Student
▪ angle (acute, obtuse, reflex, right, straight)	▪ angle (acute, obtuse, reflex, right, straight)
▪ arc	▪ arc
▪ centre (of the circle)	▪ centre (of the circle)
▪ central angle	▪ central angle
▪ circle	▪ circle
▪ circumference	▪ circumference
▪ diameter	▪ diameter
▪ irrational number	▪
▪ pi	▪ pi
▪ radius	▪ radius
▪ rays	

Resources

Print

Sir Cumference and the Dragon of Pi (Neuschwander 1999)

Math Makes Sense 7 (Garneau et al. 2007)

- Unit 4: Circles and Area (NSSBB #: 2001640)
  - Section 4.1 Investigating Circles
  - Section 4.2 Circumference of A Circle
  - Unit Problem: Designing a Water Park
- *ProGuide* (CD; Word Files) (NSSBB #: 2001641)
  - Assessment Masters

  - Extra Practice Masters
  - Unit Tests
- *ProGuide* (DVD) (NSSBB #: 2001641)
  - Projectable Student Book Pages
  - Modifiable Line Masters

"Teacher to Teacher: Playing around with "Mono-pi-ly," *Mathematics Teaching in the Middle School* Volume 11, No. 6, February 2006 (Kroon 2006), 294–297 (Available in PDF: www.nctm.org/Publications/mathematics-teaching-in-middle-school/2006/Vol11/Issue6/Teacher-to-Teacher_-Playing-around-with-Mono-pi-ly/)

Digital

- "Pattern Blocks," *Ginger Booth* (Ginger Booth 2013): http://gingerbooth.com/flash/patblocks/patblocks.php#.VAM4BrxdW7w (Interactive pattern blocks)
- "NRICH Enriching Mathematics, Unnamed [interactive circular geoboard]," *University of Cambridge* (University of Cambridge 2015): http://nrich.maths.org/content/id/2883/circleAngles.swf

SCO M02: Students will be expected to develop and apply a formula for determining the area of triangles, parallelograms, and circles. [CN, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

M02.01 Illustrate and explain how the area of a rectangle can be used to determine the area of a triangle.
M02.02 Generalize a rule to create a formula for determining the area of triangles.
M02.03 Illustrate and explain how the area of a rectangle can be used to determine the area of a parallelogram.
M02.04 Generalize a rule to create a formula for determining the area of parallelograms.
M02.05 Illustrate and explain how to estimate the area of a circle without the use of a formula.
M02.06 Generalize a rule to create a formula for determining the area of a given circle.
M02.07 Solve a given problem involving the area of triangles, parallelograms, and/or circles.

Scope and Sequence

Mathematics 6	Mathematics 7	Mathematics 8
M03 Students will be expected to develop and apply a formula for determining the ▪ perimeter of polygons ▪ area of rectangles ▪ volume of right rectangular prisms **G04** Students will be expected to perform a combination of successive transformations of 2-D shapes to create a design and identify and describe the transformations.	**M02** Students will be expected to develop and apply a formula for determining the area of triangles, parallelograms, and circles.	**M03** Students will be expected to determine the surface area of right rectangular prisms, right triangular prisms, and right cylinders to solve problems. **M04** Students will be expected to develop and apply formulas for determining the volume of right rectangular prisms, right triangular prisms, and right cylinders.

Background

Area can be defined as a measure of the space inside a region or how many square units it takes to cover a region. In using any type of measurement, including area, it is important to discuss the similarities in developing understanding of the different measures:

- first identify the attribute to be measured
- then choose an appropriate unit
- finally compare that unit to the object being measured

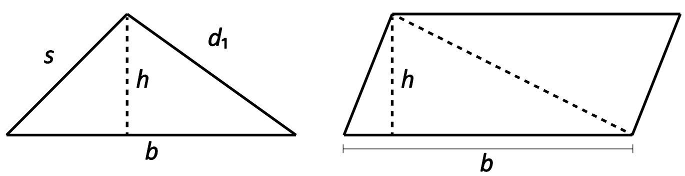
(NCTM, 2000, 171).

An understanding of conservation of area is critical: an object retains its size when the orientation is changed or when it is rearranged by subdividing it in any way. When measuring area, some appropriate units include cm^2 and m^2.

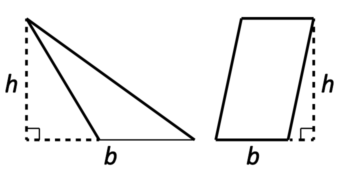
Formulas for finding the area of 2-D shapes provide a method of measuring area by using only measures of length (Van de Walle and Lovin 2006b, 230). The areas of rectangles, parallelograms, triangles, and circles are related, with the area of rectangles forming the foundation for the areas of the other 2-D shapes.

Students have been studying the concept of area since Mathematics 4. Now that students are being asked to make generalizations regarding area, they will need to use their understanding of what area is, and of how to find the area of a rectangle. The formula $A = l \times w$ was introduced as a way of counting the squares contained in the area of a rectangle. It represents area as an array of squares, and provides a way of counting them. That is, the formula $A = l \times w$ represents the number of squares in each row multiplied by the number of rows. Multiplying these two values should then be apparent; therefore, the formula for the area of a rectangle is $A = l \times w$, or $A = w \times l$, or $A = b \times h$.

The ability to identify the base and height of shapes is very important for clear communication about shapes and to develop and apply formulas. Any flat side of a shape can serve as its base. The base can depend on the orientation of the shape. For each base, there is a corresponding height. A shape may have different heights, depending on what side is chosen as the base. Height is the perpendicular distance between the highest point of the shape and its base. Height is measured along a line that is at a right angle to the extension of the base (the perpendicular line). For rectangles, the formula $A = l \times w$ is equivalent to the formula $A = b \times h$. $A = b \times h$ is more useful when developing the area of parallelograms and triangles in Mathematics 7 and determining surface area in Mathematics 8.



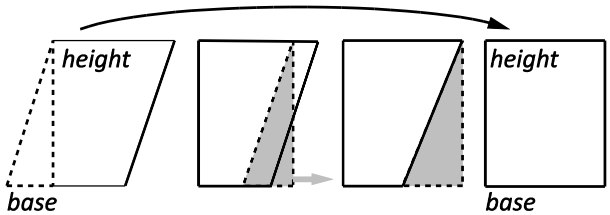
The height of an obtuse triangle may lie outside the triangle and therefore be measured by constructing a line segment outside of the triangle. The same is true for parallelograms—height may be measured either inside or outside of the shape. This is new to Mathematics 7.



Review these understandings about the area of parallelograms and triangles with students. Use various types of quadrilaterals and classifications of triangles to ensure students have vocabulary to communicate clearly about their learning. Students should not memorize mathematical formulas without this understanding.

PARALLELOGRAMS

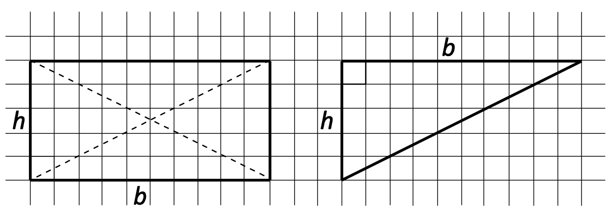
Introduce students to the area of a parallelogram by building on their prior knowledge of area. One way to do this is to transform a parallelogram into a rectangle by cutting a right triangle out of the parallelogram and sliding it to the other side of the parallelogram to create a rectangle. From this, students should recognize that the area of a parallelogram is the same as the area of a corresponding rectangle (one with the same base and height). Students should be able to determine the base or height, given the area and the other dimension, and recognize that a variety of parallelograms can have the same area.



An activity such as this explains why the area of a parallelogram is *A = (base)(height)*, and builds awareness of conservation of area. Give students the opportunity to explore a variety of parallelograms in various orientations to find the area of the parallelograms and generalize a formula. Students should generalize that any parallelogram can be rearranged to form a rectangle. Students should be able to determine the area when given base and height, when given the area and the other dimension, and recognize that a variety of parallelograms can have the same area.

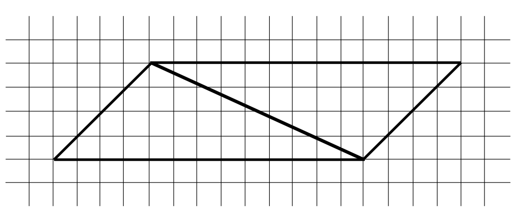
TRIANGLES

The area of a triangle is related to the area of a rectangle. When a rectangle is cut along a diagonal, two congruent right triangles are created. Each triangle has an area that is one-half of the area of the rectangle, or $\frac{1}{2}$ *b* x *h*. The base and height are obtained by measuring the sides of the rectangle. As students observe that two congruent right triangles form a rectangle, they will understand that $\frac{1}{2}$ *b* x *h* identifies the area of a triangle.



Following the same procedure as above, students can discover that the area of a triangle with the same base and perpendicular height as a corresponding parallelogram has one-half the area of that parallelogram. Therefore, the area of a triangle is $A = \frac{bh}{2}$ or $\frac{1}{2}bh$.

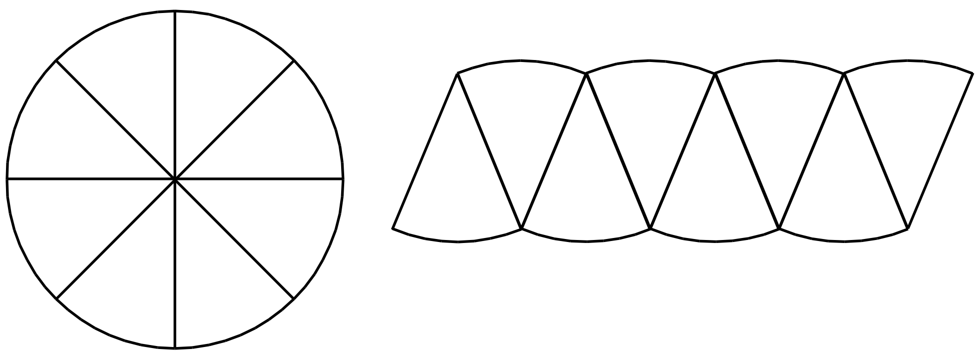
Students should also understand that, as long as the base and height are the same, the areas of different triangles are the same.



CIRCLES

Have students develop the formula for the area of a circle through investigations that connect a circle that has been cut into equal sectors to a quasi-parallelogram.

When a circle is sectioned through the centre, and the pieces are rearranged so that the centre alternates between pointing up and pointing down, the pieces will form a quasi- or near-parallelogram. This quasi-parallelogram can be used to estimate the area of a circle, and to explain the formula for finding the area of a circle.



To effectively develop the area of a circle with this activity, the measures of the circle (radius and half the circumference) should be transferred to the measures of the parallelogram. Since the base of the parallelogram is one-half the circumference of the circle ($C = 2\pi r$), the base can be represented by πr and the height of the parallelogram is r. Then the formula for the area of a parallelogram can be applied to create the formula for the area of a circle. Students have not been exposed to powers or exponents and will not be until Mathematics 9 (N01). When developing the formula for the area of a circle $A = \pi r^2$, it can be introduced as A = (π x r) x r. Students have only seen square notation when working with area units. To introduce the formula of A = πr^2, discuss the notation for a square metre as being m^2 instead of m x m, and relate it to r x r as being expressed as r^2 for efficiency in symbolic notation. A common error for students is to double the area instead of multiplying it by itself.

When students build their own generalizations from their own experiences, they will understand the connections in formulas. Building these connections provides the best opportunities for students to remember formulas and to apply them correctly. If students forget a formula, they are in a position to rebuild it, test it, and carry on using it.

Assessment, Teaching, and Learning

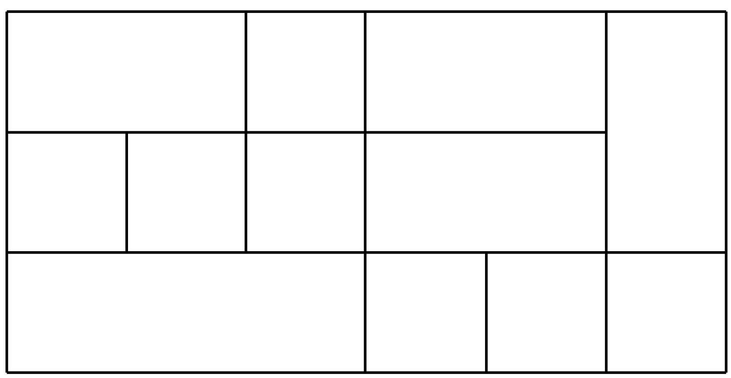
Assessment Strategies

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Tell students that the perimeter of a triangle is 18 cm. Have them describe and draw the possible side lengths. (A geoboard or digital geoboard could also be used. Specify each type of triangle—scalene, isosceles, and equilateral, etc.)

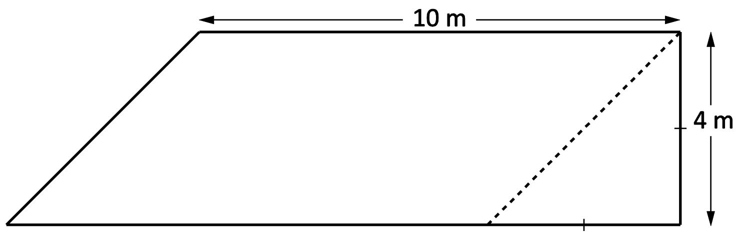
- Identify and sort the following quadrilaterals according to their attributes.
  - squares
  - rectangles
  - trapezoids
  - parallelograms
  - rhombuses
- Provide students with area tasks such as the following:
  - Kailey mowed two lawns. One lawn was 11 m × 12 m, and the other was 16 m × 10 m. Kailey charges $3 for each 10 m^2. How much did she charge to mow the two lawns?
  - Write a definition of area.
  - Draw a series of rectangles with the same area.
  - Explain the formula for area of a rectangle.
  - Find the area of the largest rectangle and record it using square units.



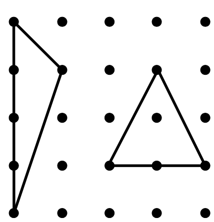
WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Create a parallelogram with an area of 24 cm^2. Then create three different parallelograms with the same base length and area and determine the height of each triangle. Draw a conclusion that connects the base and height when the area is constant. (This could be done on grid paper, a geoboard/geoboard app, or software.)
- Compare the areas of a triangle and a parallelogram with the same base and the same height. Ask them to include diagrams in their explanation.
- Estimate the area of a circular plate that has a radius of 10 cm and explain their thinking. Write the formula for the area of a circle. Calculate the area of the plate. Show all work.
- If a garden plot was created in the shape below, will the total area of the garden plot be greater than 40 m^2? Explain your thinking. Calculate the total area of the garden plot. Show all your work including the formulas you need.

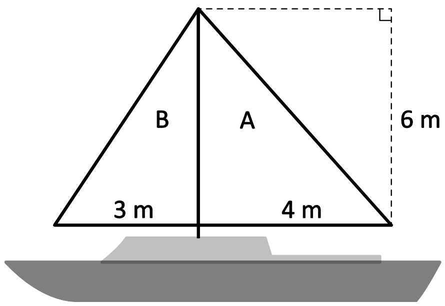


- On a geoboard, create as many different triangles as possible that have an area of 2 cm^2. You should discover that any triangle with base 4 and height 1, or base 2 and height 2 will have this area.

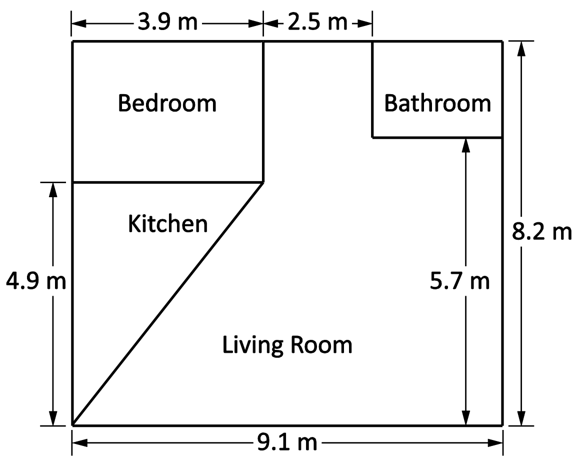


- On a geoboard, create as many triangles as possible with the same base and height. How many different triangles can you create? Compare the area of each triangle. (Triangles with same base and height will have the same area.)

- Daniel just bought a used sailboat with two sails that need replacing. How much sail fabric will Daniel need if he replaces sail A? Explain your thinking.

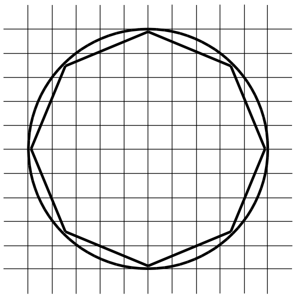


  - How much sail fabric will Daniel need if he replaces sail B?
- Daisy wants new flooring and carpeting for her rectangular apartment. A floor plan of her apartment is shown below.



  - If bathroom flooring costs $12.95 per square meter, how much will it cost Daisy to put new flooring on her bathroom?
  - If Daisy has $700 to spend on carpet in her living room and bedroom and carpet costs $9.98 a square meter, does she have enough money to carpet the two rooms?
- A triangle and a parallelogram have the same base and the same height. Explain how their areas compare. Include diagrams in your explanation.
- Explain how the formulas for the area of rectangles, parallelograms, and triangles are the same. Explain how they are different.

- Estimate the area of the circle using the octagon as a benchmark. (Notice that the octagon fills more of the circle than a square would.)



- Jackie's mom was decorating Jackie's bedroom and placed a round mat on the floor near the bed. Jackie had just learned about circles in mathematics class and wondered about the area of the mat. The tag on the mat said that it was 60 cm wide. She performed the following calculations:

d = 60 cm
r = 30 cm
A = π × r × r
A = 3.14 × 30 × 30
A = 3.14 × 9000
Area is around 28 260 cm^2

  - Are Jackie's calculations reasonable? Explain.
- Draw 3 different parallelograms each with an area of 48 cm^2.
- The radius of a circular pizza is 22.0 cm. If the pizza is to be equally divided among four people, what will be the area of each slice be?
- A circular skating rink has a radius of 20 m. How much area do skaters have to skate on?

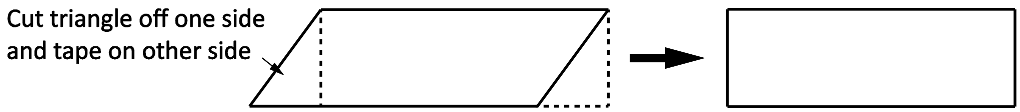
Planning for Instruction

Choosing Instructional Strategies

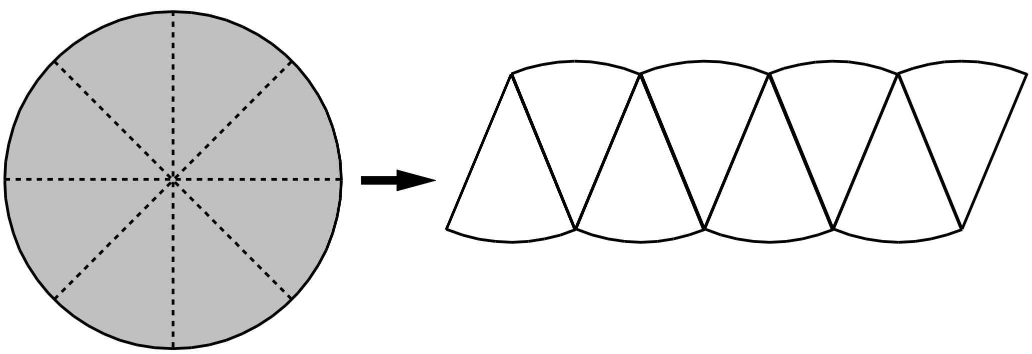
Consider the following strategies when planning daily lessons.

- Ensure students estimate before calculating the areas of parallelograms, triangles, and circles.
- Make a flexible rectangle using geo-strips or cardboard strips and brads. Begin to tilt the rectangle. Ask the students whether or not the area has changed. Keep tilting until students see that the area has decreased. Discuss how with each additional tilt, a new parallelogram was created with the same base, but less height; therefore, the area decreased.
- Have students construct area formulas by applying their knowledge of the area of rectangles and rearranging the shapes of triangles, parallelograms, and circles.

- Have students demonstrate conservation of area so that they know the area remains the same when shapes are rearranged. Have students construct parallelograms of different dimensions to test whether this might be true of all parallelograms.

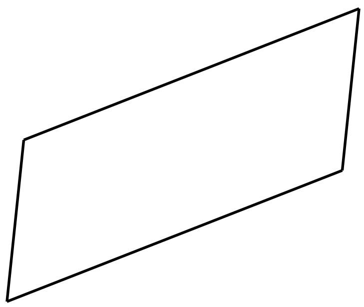


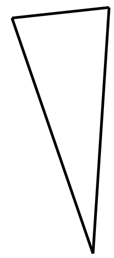
- Provide students with many types of triangles and parallelograms to construct meaning for the areas of these 2-D shapes.
- Allow students to recognize the connection between the area of a rectangle and the area of a parallelogram. Then have them construct meaning for the area of a triangle by connecting it to the area of a parallelogram. Finally, have students construct meaning for the area of a circle by rearranging it into a parallelogram or a rectangle. Emphasize the connections among the formulas.
- Have students cut out circles to develop the formula for the area of a circle. Have students draw a dark line around the circle so the circumference will be apparent. Cut out each sector and line the pieces up to form a rectangle. The smaller the sectors of the circles are, the closer it resembles a parallelogram. The radius of the circle represents the height of the rectangle and the base is represented by half of the circumference = πr. Have them write the formula they have discovered for the area of a circle.



Suggested Learning Tasks

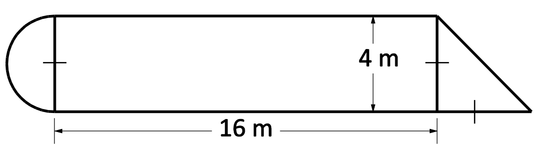
- Find the area of the largest rectangle and record it using square units.
- Draw two different parallelograms with the same area on grid paper (or using technology). Record the dimensions and area of each parallelogram.
- If a parallelogram has an area of 36 cm and the base of 9 cm, what is the height?
- Indicate the base and height of each of the following shapes.





- Draw two different parallelograms that have an area of 32 square units, or a base of 8 units and an area of 32 units.
- Use a tri-fold foldable to keep notes about the three shapes discussed in this unit: parallelograms, triangles, and circles. Topics such as estimation, definitions, and area formulas could be included.
- Respond to the following:
  - Mr. McGowan made an apple pie with diameter of 25 cm. He cut the pie into 6 equal slices. Find the approximate area of each slice.
  - The outer section on the Canadian Toonie has an outside radius of 14 mm, and an inside radius of 8 mm. What is the area of the outer section?

- A garden plot was made in the following shape:



  - Estimate the area of the garden plot. Explain your thinking.
  - Find the area of the plot.
  - If the width (4 m) of the plot is doubled, is the area of the plot doubled? Explain.

SUGGESTED MODELS AND MANIPULATIVES

- dot paper
- geo-boards*
- geo-strips and brads
- grid paper
- power polygons

*also available in *Interactive Math Tools* (Pearson n.d.)

MATHEMATICAL LANGUAGE

Teacher	Student
▪ area	▪ area

Teacher	Student
▪ base ▪ formula ▪ height ▪ horizontal ▪ length ▪ parallel ▪ parallelogram ▪ perpendicular ▪ polygon ▪ quadrilateral (parallelogram, rectangle, rhombus, square, trapezoid) ▪ rectangle ▪ square units ▪ triangle (acute, equilateral, isosceles, obtuse, right, scalene) ▪ vertex ▪ width	▪ base ▪ formula ▪ height ▪ horizontal ▪ length ▪ parallel ▪ parallelogram ▪ perpendicular ▪ polygon ▪ quadrilateral (parallelogram, rectangle, rhombus, square, trapezoid) ▪ rectangle ▪ square units ▪ triangle (acute, equilateral, isosceles, obtuse, right, scalene) ▪ vertex ▪ width

Resources

Print

Making Mathematics Meaningful to Canadian Students, K–8, Second Edition (Small 2013), 399–402

Teaching Student-Centered Mathematics, Grades 5–8, Volume Three (Van de Walle and Lovin 2006b), 255–257

Math Matters: Understanding the Math You Teach, Grades K–8, Second Edition (Chapin and Johnson 2006), 286–288, 290–291

Mathematics for Elementary Teachers, A Contemporary Approach, Seventh Edition (Musser, Burger, and Peterson 2006), 663–666

Math Makes Sense 7 (Garneau et al. 2007)

- Student Book Unit 4 Circles and Area (NSSBB #: 2001640)
  - Section 4.3 Area of a Parallelogram
  - Section 4.4 Area of a Triangle
  - Section 4.5 Area of a Circle
  - Game: Packing Circles
  - Unit Problem: Designing a Water Park
- *ProGuide* (CD; Word Files) (NSSBB #: 2001641)
  - Assessment Masters
  - Extra Practice Masters
  - Unit Tests
- *ProGuide* (DVD) (NSSBB #: 2001641)

- Projectable Student Book Pages
- Modifiable Line Masters

Digital

- "Unnamed [geo-board web app]," *Math Learning Center* (Math Learning enter 2013): www.mathlearningcenter.org/web-apps/geoboard/ (Also available for iPad and Windows tablets)
- "Discovering the Area Formula for Circles," *Illuminations: Resources for Teaching Math* (National Council of Teachers of Mathematics 2015): http://illuminations.nctm.org/Lesson.aspx?id=1852

Geometry (G)

GCO: Students will be expected to describe the characteristics of 3-D objects and 2-D shapes and analyze the relationships among them.

GCO: Students will be expected to describe and analyze position and motion of objects and shapes.

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

GUIDING QUESTIONS

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Follow-up on Assessment

The evidence of learning that is gathered from student participation and work should be used to inform instruction.

GUIDING QUESTIONS

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

GUIDING QUESTIONS

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SCO G01: Students will be expected to perform geometric constructions, including ▪ perpendicular line segments ▪ parallel line segments ▪ perpendicular bisectors ▪ angle bisectors [CN, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

G01.01 Describe examples of parallel line segments, perpendicular line segments, perpendicular bisectors, and angle bisectors in the environment.
G01.02 Identify on a given diagram line segments that are parallel or perpendicular.
G01.03 Draw and construct a line segment perpendicular to another line segment and explain why they are perpendicular.
G01.04 Draw and construct a line segment parallel to another line segment and explain why they are parallel.
G01.05 Draw and construct the bisector of a given angle, using more than one method, and verify that the resulting angles are equal.
G01.06 Draw and construct the perpendicular bisector of a line segment, using more than one method, and verify the construction.

Scope and Sequence

Mathematics 6	Mathematics 7	Mathematics 8
G01 Students will be expected to construct and compare triangles, including scalene, isosceles, equilateral, right, obtuse, or acute in different orientations.	**G01** Students will be expected to perform geometric constructions, including ▪ perpendicular line segments ▪ parallel line segments ▪ perpendicular bisectors ▪ angle bisectors	**G01** Students will be expected to draw and interpret top, front, and side views of 3-D objects composed of right rectangular prisms.

Background

While the terms draw and construct are sometimes used interchangeably, it is important to note that constructions are performed using a compass and straight edge (without markings) as tools. Drawings may use the same tools as constructions, as well as Mira, mirrors, patty paper, and rulers. Measurements can be used in drawings. A sketch is a drawing completed without drawing tools.

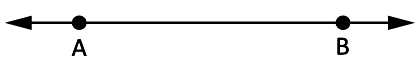
"What makes shapes alike and different can be determined by an array of geometric properties. For example, shapes have sides that are parallel, perpendicular, or neither…" (Van de Walle and Lovin 2006b, 179)
In Mathematics 5, students identified examples of perpendicular and parallel sides, edges, and faces in their study of 2-D shapes and 3-D objects (G01). They also identified examples of perpendicular and parallel line segments in the environment. Because Mathematics 5 students lack experience with measuring angles, the angle formed by perpendicular lines was identified as a right angle. Students measured and drew angles using protractors in Mathematics 6. In Mathematics 7, students will create parallel and perpendicular line segments and bisectors, as well as angle bisectors, using geometric constructions. Students should be exposed to a variety of methods when developing a concept such as perpendicular lines. Constructions, however, will be made with a straight edge and compass. Students should be able to describe how each construction was completed.

Since some of the constructions for parallel line segments involve perpendiculars, an option is to introduce perpendicular line segments and bisectors first. Quite often students don't distinguish between perpendicular line segments and perpendicular bisectors. Lines and angles can be bisected. In the word *bisectors*, *bi* means two and *sect* means to cut. When a line or an angle is bisected, it is cut into two pieces of equal size. We could say it is divided in half or divided down the middle. Students should come to understand the meaning of bisection through reference to familiar words with the same prefix like bicycle, biplane, and bilingual. Students should be able to explain similarities and differences between line bisectors and perpendicular line bisectors. Students also need to know the difference between line segments that intersect each other and those that bisect each other.

Lines, rays, and line segments are made up of sets of points and are straight and one-dimensional. Their only dimension is length.

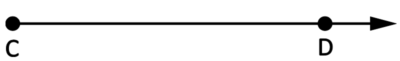
A *line* is a set of points that extends indefinitely in opposite directions.

The line AB:



A *ray* is a set of points that extends indefinitely in one direction.

The ray CD:



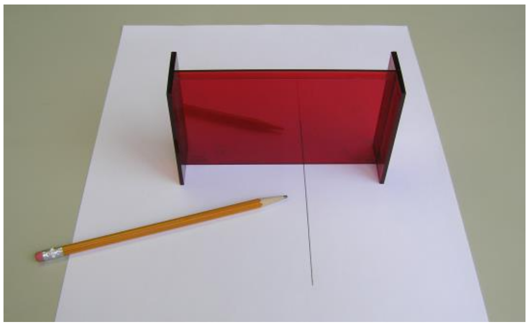
A *line segment* is a set of points along a line with two finite endpoints.

The line segment EF:

In Mathematics 5 and Mathematics 6, students will have explored methods of drawing **perpendicular lines** that include the following:

- *Method 1: Use a right angle.* Draw a line segment using a straightedge. Place the right angle of a right triangle or square on the line segment. Draw a line segment along the other side of the right angle of the triangle or square. Since these line segments intersect at 90° angles, or form a right angle, they are perpendicular to each other.
- *Method 2: Use a protractor.* Draw a line segment using a straightedge. Mark a point on the line segment. Align the point and line segment with the cross lines on the protractor. Mark the 90° measure. Use a straightedge to connect the original point and the 90° mark. Because the angle between the two line segments is 90°, they are perpendicular to each other.

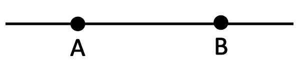
- *Method 3: Use a Mira.* Draw a line segment using a straightedge. Lay the Mira across the line segment and adjust its position until the reflection of the line segment in front of the Mira lines up on top of the line segment behind the Mira. Draw a line segment along the edge of the Mira. That line segment is perpendicular to the original line segment because the angles at the intersection of the line segments are right angles.



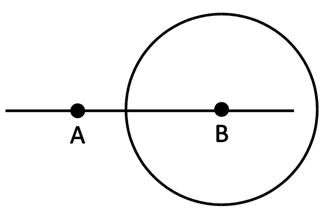
- *Method 4: Use tracing paper.* Use a straightedge to draw a line segment. Carefully fold the paper along the line segment so that the portion of the line segment on top falls directly on the part of the line segment underneath. When the line segments are aligned, crease the fold in the paper. Open it up and use a straightedge to draw the line segment along the crease. The line segments are perpendicular because the angle formed at the intersection measures 90°.

  In this outcome students are being asked to make constructions that require the use of a straightedge and a compass.

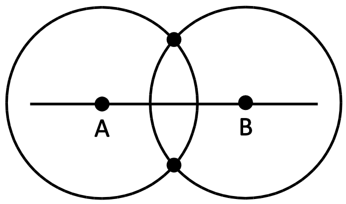
- *Method 5: Use a straightedge and a compass.* Draw a line segment using the straightedge. Mark any two points along the line segment.



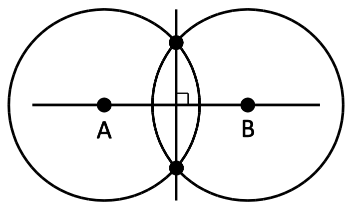
  – Use the compass to draw a circle with one of the points as the centre and a radius greater than one-half the distance between the two points.



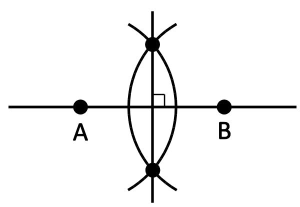
– Use the compass to draw a second circle with the same radius and the other point as centre.



– The circles will intersect at two points. Use the straightedge to connect those two points. The angle between the original line segment and the resulting line segment measures 90°; therefore, the line segments are perpendicular to each other.

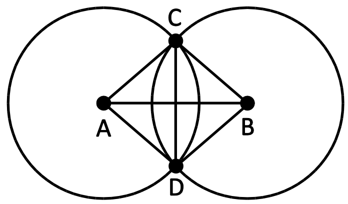


– To get a perpendicular line, it is not necessary to draw the entire circle; only an arc needs to be drawn.

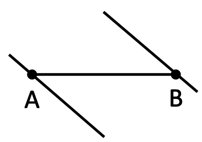


The use of circles emphasizes that the line segments from the centres to the points joined to form the perpendicular line segments are radii of congruent circles, and, therefore, are the same length.

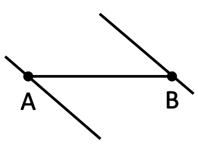
- This method used to construct perpendicular lines actually results in the construction of **perpendicular bisectors**. After drawing the circles, use the endpoints of the line segment, AB, as the centres for the circles. AC, CB, BD, and DA are congruent radii so ACBD is an equilateral quadrilateral. Therefore ACBD must be a rhombus. In a rhombus, the diagonals, in this case AB and CD, are perpendicular bisectors of each other.



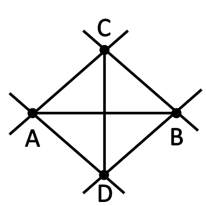
- A perpendicular bisector can also be constructed by creating a rhombus around a line segment. This will create the perpendicular bisector of the segment because the diagonals of a rhombus are perpendicular bisectors of each other.
- Lay a straightedge at an angle across a line segment with the endpoints on opposite sides of the straightedge. Adjust the straightedge until each endpoint touches the side of the straightedge. Trace lines through the endpoints along both sides of the straightedge.



- Rotate the straightedge until the end that was above the line is now below the line, and the end of the straightedge that was below is now above. Once again, adjust the straightedge so each endpoint touches the side of the straightedge, and trace both sides of the straightedge.



- Remove the straightedge. The intersecting lines create a rhombus. The diagonals of the rhombus are perpendicular bisectors.

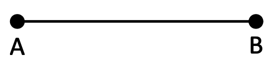


Methods for creating **parallel line segments** may include the following:

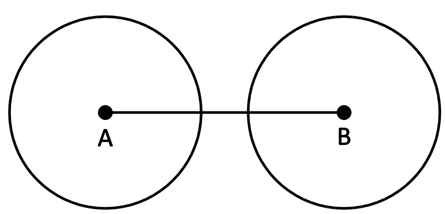
- Trace both edges of a straightedge, such as a ruler.
- Diagonals of a rectangle are the same length and they bisect each other. If the diagonals are connected in the centre to form an X, all four arms of the X are congruent to each other. Use two straws (or geo-strips, cardboard strips, stir sticks, etc.) to represent the diagonals, mark the

midpoint of each, and connect them with a push-pin. Lay the X on a sheet of paper. Connect the endpoints of two of the arms with a straightedge and trace a line segment. Mark the endpoints of the two remaining arms of the X and connect the points with a straightedge. The result is two parallel line segments. Stretch or collapse the X to adjust the distance between the parallel line segments.

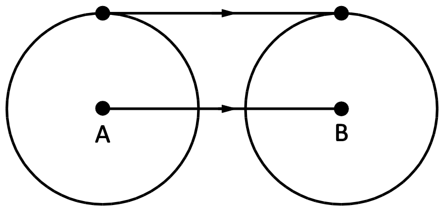
- Place a straightedge on a sheet of paper. Set the base of a right-angle triangle, or some other square corner, on the straightedge. Trace the side that creates a line segment perpendicular to the original line segment. Without moving the straightedge, slide the right angle along the straightedge to any desired position, and trace the same side of the right angle. All the line segments are parallel to one another.
- Draw a line segment. Connect points that are 90° and equidistant from the line segment. Use a right triangle or a protractor to draw two lines that are perpendicular to the original line segment. Measure and mark the same distance up each perpendicular line. Connect the marks to create a parallel line segment. The perpendicular lines are also parallel.
- Use a Mira to draw a line segment. Then use the Mira to draw perpendicular lines by making sure the original line segment is in line with its reflection in the Mira. The perpendicular lines are all parallel to each other.
- Fold a piece of paper carefully with the corners matching. Fold the paper again in the same direction. Crease the folds well. Open the paper and, using a straightedge, trace line segments along the creases. The resulting line segments are parallel.
- Use a compass and a straightedge.
  - Draw a line segment AB.



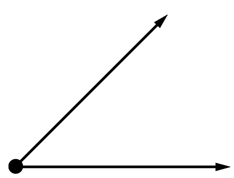
  - Use a compass to draw a circle around point A and a circle around point B with the same radius.



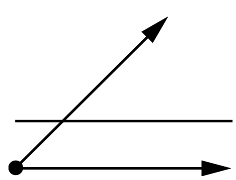
– Mark the highest (or lowest) points of each circle and connect them with a line. This line will be parallel to the original line segment AB.



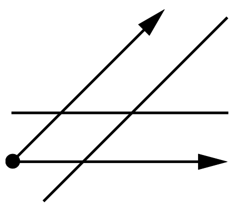
- Methods for generating **angle bisectors** may include the following:
  - Use a protractor.
    > Use a protractor to measure the original angle.
    > Divide the measurement in half.
    > Use the protractor to mark the new measure and draw the bisector.
  - Use tracing paper.
    > Copy the angle onto tracing paper.
    > Fold the paper so that the two original rays lie on top of each other.
    > Crease the paper along the fold.
    > Open the paper and use a straightedge to trace the crease.
  - Use a Mira.
    > Place the Mira so that part of its length lies over the vertex of the angle.
    > Adjust the angle of the Mira until each of the angle rays are reflected on top of each other.
    > Trace the edge of the Mira.
  - Use a straightedge.
    > Line up the edge of the straightedge along one of the rays in the angle so that the straightedge lies "inside" the angle.



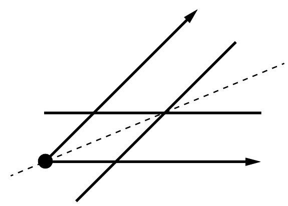
    > Trace the side of the straightedge not on the ray.



> Trace the side of the straightedge for the other ray.

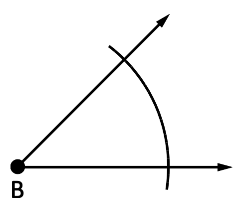


> Connect the vertex of the angle with the intersection of the lines just drawn.

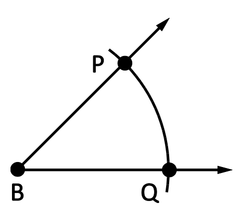


– Use a compass and a straightedge.

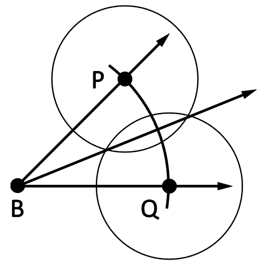
> Place the compass point on the vertex of the angle and draw an arc across the two rays.



> Place the compass point on one of the intersecting points, and draw a circle or an arc around the centre area of the angle. Keeping the same radius setting, draw a circle around the other point where the first arc intersects the other ray. It is not necessary to draw entire circles, just the intersection of the arcs of the circle approximately where the bisector will be.



> Use the straightedge to connect the intersection of the two circles with the vertex of the angle.



Encourage students to think critically about and comment on the ideas that are shared in class. They may acknowledge ideas they agree with, express appreciation for ideas that are explained, ask "how" or "why" questions, or offer further suggestions or support for an idea.

Assessment, Teaching, and Learning

Assessment Strategies

Assessing Prior Knowledge

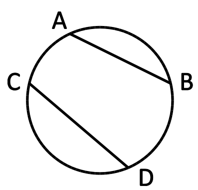
Tasks such as the following could be used to determine students' prior knowledge.

- Using a variety of 2-D shapes, have students identify parallel, intersecting, and perpendicular edges. Observe whether students use correct geometric terminology in their descriptions.
- Ask students to find a pattern block or Power Polygon that illustrates:
  - parallel sides and no right angles
  - parallel sides and right angles
- Have students represent parallel or perpendicular lines, or a variety of angles, including a right angle, using items like geo-strips, toothpicks, or straws. Some suggestions include:
  - parallel sides and no right angles
  - parallel to one another
  - intersecting
  - perpendicular at an end point of one straw
  - perpendicular at endpoints of each straw
  - one straw perpendicular to the other straw, but not at its end points

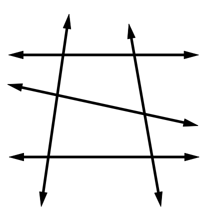
WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (which can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Draw a line segment approximately 10 cm in length. Construct the perpendicular bisector of this segment, and explain your method.
- Draw an acute angle. Construct the angle bisector. Explain your method.
- Construct the perpendicular bisectors of line AB and line CD. (These bisectors, if done correctly, should meet at the centre of the circle.)



- Explain the difference between a line bisector and a perpendicular line bisector.
- Describe a situation when an angle bisector and a line bisector are the same thing.
- Draw a line segment approximately 10 cm in length and to construct a parallel line segment. Explain their method.
- Identify the parallel lines in the following diagram.



- Define **perpendicular bisector** and to explain how a perpendicular bisector differs from a perpendicular line segment.
- Complete a project such as the following:
  - Replicate the floor plan for a sport facility (e.g., court, field, rink).
  - Design a map for a community, a fairground, a school campus, or a campground. Design major services to be equidistant from strategic points in the area.
  - Create a design for a fence, lattice, fabric pattern, or piece of artwork and include a certain number of lines that must be included (e.g., circles or half circles, parallel lines, perpendicular lines, perpendicular and angle bisectors)
- Use what you have learned about angles, parallel and perpendicular lines, and bisectors to create your project. Prepare a report on the types of lines included in the project and how you created the lines.

Planning for Instruction

Choosing Instructional Strategies

Consider the following strategies when planning daily lessons.

- Teach students to construct perpendicular bisectors of line segments and bisectors of angles using a variety of methods, such as paper folding, use of Miras, and compasses.
- Have students practise drawing perpendicular and parallel line segments. Then ask them to make a math journal entry commenting on which method they prefer, which method they believe to be most accurate, and which applications each method is best suited for.
- Discuss why and when people may use a perpendicular bisector. For example, a perpendicular bisector may be used to find the best place to put a single support under a beam, to find the division in a drawing, a design, or a building from which to build formal symmetry, to divide a piece of property into equal portions or to find a line that is the same distance from two points. The last application may be important in a variety of contexts, ranging from planning a city or a meeting spot to setting up a lemonade stand.
- Have students practise using various methods to create perpendicular bisectors. Individuals can create their own line segments or create line segments for their partners, or the teacher can assign line segments. After students have had sufficient practice in using the methods, ask them to make a math journal entry commenting on their preferred method, the method they think is most useful or most accurate, applications for the different methods, and so on.
- Have students practise drawing parallel line segments of different lengths and different distances apart. Partners, teachers, or the toss of a number cube can determine the length or distance, or students can choose their own measurements. After trying several methods to draw designated parallel line segments, students can choose the method they prefer and write about it in their math journals.
- Ask students to explain what an angle bisector is, and how they could test to see whether or not a line actually bisects an angle.
- Have students practise drawing angle bisectors. Supply students with angles or angle measures for which students can create bisectors, or have students generate their own measures. Ask students to identify and justify a preferred technique for bisecting angles and write about it in their math journals. Differentiation: Provide students with a handout outlining and illustrating methods for creating an angle bisector, along with a compass and templates for those students who may need them.
- Have students use geometry software to practice and investigate angle bisectors of different shapes.
- Assemble geometry kits for student use that contain protractors, rulers, compasses, right triangles, squares, Miras, tracing paper, etc.

Suggested Learning Tasks

- Give examples of parallel and perpendicular lines from their everyday life. Suggested examples may be:
  - Parallel lines:
    - opposite sides of picture frames

    - > railroad or roller coaster tracks
    - > lines on loose-leaf paper
    - > rows of siding on a house
    - > lines of latitude
    - > guitar strings
  - Perpendicular lines:
    - > railway tracks and railway ties
    - > fence posts and fence rails
    - > four way stops
    - > lines of latitude and longitude on a map
    - > a wall and a shelf
- Write the upper case letters of the alphabet that only use line segments. Find examples of bisectors of segments, perpendicular segments, and perpendicular bisectors.
- Make a list of as many pairs of parallel lines as you can find in the classroom in two minutes. After the time is up, pass your list to another student. Then, one at a time, read an entry from the list. Everybody who has that entry on their list will cross it off. At the end, the list with the most remaining entries will be the winner. (Students can be paired up for this activity.) Repeat this activity exploring perpendicular lines.
- Draw a line that is neither vertical nor horizontal. Then, using a method of your choice, draw a second line that is parallel to the first. Repeat this activity exploring perpendicular lines.
- Think of 2-D shapes (excluding quadrilaterals) that have parallel sides. Include diagrams to illustrate your thinking.
- Respond to the following:
  - Can two lines be both parallel and perpendicular?
  - Can a line have more than one line that is perpendicular to it?
  - Explain your reasoning. Provide examples.
- Create artwork composed of coloured lines and angles (perhaps similar to the work of Piet Mondrian) including labelled points. Create a key to go with your artwork that lists the parallel and perpendicular lines. The pictures and keys can be posted for display.
- Build an origami figure or shape, unfold it, trace the lines, and indicate parallel and perpendicular lines in the fold lines.
- Use items like geo-strips, toothpicks or straws to create the following:
  - one straw bisecting the other straw but not perpendicular;
  - each straw bisecting the other straw but not perpendicular;
  - one straw bisected by the other straw and perpendicular;
  - each straw bisecting the other straw and perpendicular.
- Provide students with a diagram of interconnected lines to play a game with a partner. Students each choose a different colour of highlighter or light-coloured pencil crayon. They pick either odd or even numbers on a number cube to represent parallel or perpendicular lines. Students take turns shaking the number cube and marking either a set of parallel lines or a set of perpendicular lines with their selected colour. If students cannot find a set of lines, they forfeit their turn. The player with the greatest number of sets of lines wins.
- Identify parallel and perpendicular lines in photographs (i.e., on paper or using an annotation/presentation app). Create a schematic with symbols to identity these lines.

- Investigate drawing perpendicular bisectors for each side of a triangle. Draw a circle using the point of intersection of the perpendicular bisectors as the centre of the circle and the radius as the distance from the centre to one of the vertices of the triangle. Continue this investigation for a variety of triangles, parallelograms, or other polygons.
- Make a graffiti wall. On a sticky note, write an example in the environment of either a pair of parallel lines, a pair of perpendicular lines, a perpendicular bisector, or an angle bisector. Post your sticky notes on the wall. Then choose a sticky note other than your own and determine which category it falls under. They then place it in the appropriate section under parallel lines, perpendicular lines, perpendicular bisector, or angle bisector.

SUGGESTED MODELS AND MANIPULATIVES

- compass
- computer software such as Geometer's Sketchpad
- dot/grid paper
- geo-boards*
- geo-strips
- Miras
- straight edge
- tracing paper

*also available in *Interactive Math Tools* (Pearson n.d.)

Mathematical Language

Teacher	Student
▪ angle ▪ angle bisector ▪ arc ▪ bisect ▪ bisector ▪ construct ▪ draw ▪ intersecting lines ▪ line ▪ perpendicular ▪ perpendicular bisector ▪ perpendicular lines ▪ parallel ▪ parallel lines ▪ ray ▪ segment ▪ sketch	▪ angle ▪ angle bisector ▪ arc ▪ bisect ▪ bisector ▪ construct ▪ draw ▪ intersecting lines ▪ line ▪ perpendicular ▪ perpendicular bisector ▪ perpendicular lines ▪ parallel ▪ parallel lines ▪ ray ▪ segment ▪ sketch

Resources

Print

Math Makes Sense 7 (Garneau et al. 2007)

- Unit 8: Geometry (NSSBB #: 2001640)
  - Section 8.1 Parallel Lines
  - Section 8.2 Perpendicular Lines
  - Section 8.3 Constructing Perpendicular Bisectors
  - Section 8.4 Constructing Angle Bisectors
  - Unit Problem: Design the Cover
- *ProGuide* (CD; Word Files) (NSSBB #: 2001641)
  - Assessment Masters
  - Extra Practice Masters
  - Unit Tests
- *ProGuide* (DVD) (NSSBB #: 2001641)
  - Projectable Student Book Pages
  - Modifiable Line Masters

SCO G02: Students will be expected to identify and plot points in the four quadrants of a Cartesian plane, using integral ordered pairs. [C, CN, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

G02.01 Label the axes of a four quadrant Cartesian plane and identify the origin.
G02.02 Identify the location of a given point in any quadrant of a Cartesian plane using an integral ordered pair.
G02.03 Plot the point corresponding to a given integral ordered pair on a Cartesian plane with units of 1, 2, 5, or 10 on its axes.
G02.04 Draw shapes and designs in a Cartesian plane using given integral ordered pairs.
G02.05 Create shapes and designs, and identify the points used to produce the shapes and designs, in any quadrant of a Cartesian plane.

Scope and Sequence

Mathematics 6	Mathematics 7	Mathematics 8
G05 Students will be expected to identify and plot points in the first quadrant of a Cartesian plane using whole number ordered pairs.	**G02** Students will be expected to identify and plot points in the four quadrants of a Cartesian plane, using integral ordered pairs.	**PR01** Students will be expected to graph and analyze two-variable linear relations. **G02** Students will be expected to demonstrate an understanding of the congruence of polygons under a transformation.

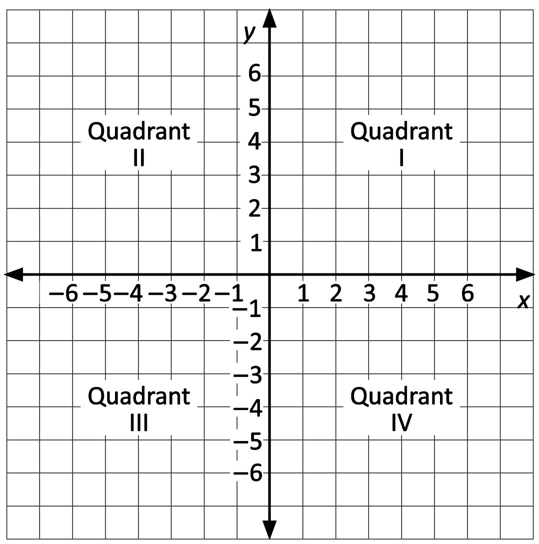
Background

In Mathematics 6 (G02), students were introduced to the Cartesian plane. They identified and plotted whole number ordered pairs, and drew shapes and designs limited to the first quadrant. Experiences with horizontal and vertical integral number lines in both Mathematics 6 and Mathematics 7 prepared students to extend plotting skills to work in all four quadrants of a Cartesian plane. Students should already be familiar with key terms such as coordinate plane, ordered pairs, origin, *x*-axis, *y*-axis, x-coordinates and y-coordinates. Continued use of appropriate terminology is important. Each of the achievement indicators associated with this outcome have previously been addressed in Mathematics 6, restricted to the first quadrant. Plotting ordered pairs accurately is an important skill for performing and describing transformations in learning outcome 7G03, and for graphing equations in Patterns and Relations.

Students should plot data points in all four quadrants. Each ordered pair of integers represents a position on a four quadrant Cartesian plane. The scale of the axes will need to be determined based on the magnitude of the coordinates. Students should be exposed to a variety of scales including those with intervals of 1, 2, 5, and 10.

The horizontal number line in the Cartesian number line is termed the *x*-axis, and the vertical number line the *y*-axis. The point at (0, 0) is called the "origin". The scale is chosen based on the numbers in the situation. The four main areas are named "quadrants". They are numbered 1 to 4 (often expressed as Roman numerals I to IV) in a counter-clockwise direction beginning with the positive coordinates.

A common error when identifying and plotting points is to reverse the order of the *x*-coordinate and the *y*-coordinate. To avoid making this mistake, students should label the *x*- and *y*-axes of a Cartesian plane.



Situations which might be modelled using 4-quadrant graphs include:

- daily high and low temperatures for different days plotted as coordinates;
- mathematical relationships (e.g., a number vs. its double) plotted as coordinates;
- locations, as blocks north, south, east, and west from the town centre plotted as coordinates.

Assessment, Teaching, and Learning

Assessment Strategies

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to plot points on grids with different scales (e.g., intervals of ones, twos, fives, tens).
- Have students create "join-the-dots" pictures on a coordinate grid to reinforce locating coordinates. After they draw their pictures on a grid, they list the coordinates in order of connection. The list of coordinates can be given to other students who then use them to recreate the picture.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Plot the following points on a grid: A (–3, 2), B (1, 2), C (–3, –2). Determine what the coordinates of a fourth point (D) would be in order to create the square ABCD when the 4 points are connected.
- Determine an appropriate grid scale for plotting the following points: (–35, 30), (15, 30), (–20, –20), and (30, –20). Create the grid and plot the points. Explain why you chose that scale.
- Plot points A: (–2, 4) and B: (3, 4). Join the points to create line segment AB. What is the distance between A and B?
- Write the coordinates to draw a simple picture. Trade coordinates to see if your partner can follow your instructions to produce the picture you have in mind.

Planning for Instruction

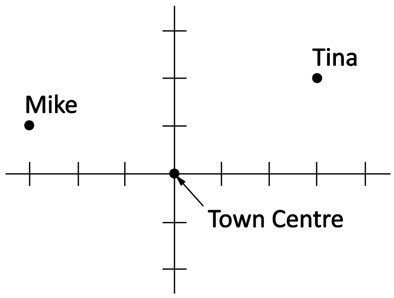
CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

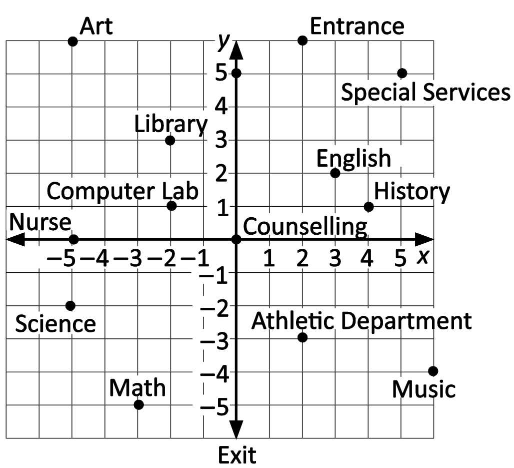
- Use four geo-boards linked together to represent the four quadrants.
- Provide students with a variety of points that may require them to change the scale on the coordinate grid. For example, (–35, 40) would require students to scale by 5 or 10 instead of a scale factor of 1.
- Create a grid on the floor where students can physically move to identified coordinates. You can integrate the Cartesian plane with the study of geography by using the coordinates on a map of the world. The equator could be represented as the *x*-axis, and 0° longitude could be represented as the *y*-axis. Using the map, ask students to determine the number of degrees (in relation to both the *x*-axis and the *y*-axis) between two cities. Coordinates can be determined with any type of map. You could, for example, use a highway map or a topological map of the area around your school or community.

SUGGESTED LEARNING TASKS

- Show a map (graph) like the one below. How many blocks north of town centre does Tina live? How many blocks east? Write Mike's location as an ordered pair.



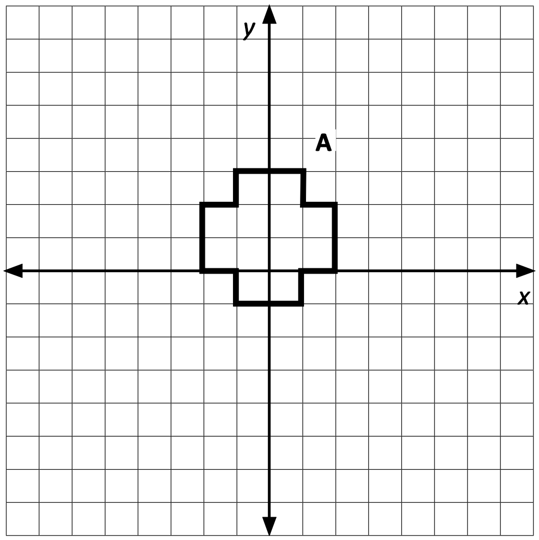
- Plot 10 points in quadrant 1 for which the difference between the first and second coordinate is 3. This will create a line. Find ordered pairs with one or more negative coordinates that are on that line.
- Plot 10 points for which the first coordinate is the opposite of the second (e.g., (5,–5)). Describe the pattern you see and explain why you might have expected that pattern.
- Plot 10 points on a Cartesian plane. In pairs, take turns trying to find each other's points similar to the game of Battleship.
- Create drawings using all four quadrants of the coordinate grid. Then provide other students with a list of the vertices, in order, for each drawing created. The other students would subsequently re-create the drawings.
- Research why coordinate planes are often called Cartesian planes. Write a brief paragraph explaining your findings.
- Use the coordinate plane below to answer the questions that follow. It shows a map of the rooms in a junior high school.



  – Jessica is in the room located at (5, 5). What room is she in? Describe in words how to get to the nurse from this point.
  – Jessica's next class is 8 units to the right and 2 units up on the map from the nurse. In what room is Jessica's next class? Find the ordered pair that represents the location of that room.

- Lucas is in the music room, but his next class is in the library.
- Give Lucas directions on how to get to the library.

- Answer questions similar to the following: Given sets of points as ordered pairs, such as A(1, 3), B(–1, 3), C(–1, 2), D(–2, 2), E(–2, –1), F(2, –1), G(2, 2) and H(1, 2), plot them on a coordinate plane and join those points to create a shape.
- Using shapes drawn on a coordinate plane, identify the locations of the vertices.



- Create a simple shape, such as a polygon, on quadrant I of a coordinate grid. Then uses the plot to generate a list of ordered pairs. Exchange lists with a partner, plot the points on the partner's list, and connect the points in the order given to create the polygon. Verify each other's work.
- Create your own designs on grid paper and list the ordered pairs. If needed help, use pattern blocks to build a design, and then tracing it onto grid paper. For a display of coordinate picture designs and a student gallery of completed pictures, refer to the *PlottingCoordinates.com* website for "CoordinArt News": (www.plottingcoordinates.com/coordinartnews.html).
- For each set of ordered pairs below, choose an appropriate scale and label the *x*-axis and the *y*-axis. Plot the ordered pairs, label the points, and draw a line to connect the points in order and connect the last point with the first. State the quadrant(s) in which the figure is located.
  - A(5, 5) B(3, 5) C(3, 3) D(5, 3) The figure is in quadrant(s) ________
  - A(–6, 8) B(–9, 3) C(1, 3) D(4, 8) The figure is in quadrant(s) ________
  - A(10, 12) B(–15, –8) C(22, –22) The figure is in quadrant(s) ________
  - A(–30, –25) B(25, 40) C(–30, 40) The figure is in quadrant(s) ________

Suggested Models and Manipulatives

- geo-boards
- graph paper
- maps

MATHEMATICAL LANGUAGE

Teacher	Student
▪ axes ▪ Cartesian plane ▪ coordinates ▪ ordered pair ▪ origin ▪ *x*-axis ▪ *y*-axis	▪ axes ▪ Cartesian plane ▪ coordinates ▪ ordered pair ▪ origin ▪ *x*-axis ▪ *y*-axis

Resources

Print

Math Makes Sense 7 (Garneau et al. 2007)

- Unit 8: Geometry (NSSBB #: 2001640)
  - Section 8.5 Graphing on a Coordinate Grid
  - Section 8.6 Graphing Translations and Reflections
  - Section 8.7 Graphing Rotations
  - Unit Problem: Design the Cover
- *ProGuide* (CD; Word Files) (NSSBB #: 2001641)
  - Assessment Masters
  - Extra Practice Masters
  - Unit Tests
- *ProGuide* (DVD) (NSSBB #: 2001641)
  - Projectable Student Book Pages
  - Modifiable Line Masters

Digital

- "Welcome to Graph Mole," *FunBased Learning* (Dun 2007): http://funbasedlearning.com/algebra/graphing/points2
- "CoordinArt News," *PlottingCoordinates.com* (PlottingCoordinates.com 2010): www.plottingcoordinates.com/coordinartnews.html

SCO G03: Students will be expected to perform and describe transformations (translations, rotations, or reflections) of a 2-D shape in all four quadrants of a Cartesian plane (limited to integral number vertices). [C, CN, PS, T, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

G03.01 Identify the coordinates of the vertices of a given 2-D shape on a Cartesian plane.
G03.02 Describe the horizontal and vertical movement required to move from a given point to another point on a Cartesian plane.
G03.03 Describe the positional change of the vertices of a given 2-D shape to the corresponding vertices of its image as a result of a transformation, or successive transformations, on a Cartesian plane.
G03.04 Determine the distance between points along horizontal and vertical lines in a Cartesian plane.
G03.05 Perform a transformation or consecutive transformations on a given 2-D shape, and identify coordinates of the vertices of the image.
G03.06 Describe the positional change of the vertices of a 2-D shape to the corresponding vertices of its image as a result of a transformation or a combination of successive transformations.
G03.07 Describe the image resulting from the transformation of a given 2-D shape on a Cartesian plane by identifying the coordinates of the vertices of the image.

Scope and Sequence

Mathematics 6	Mathematics 7	Mathematics 8
G03 Students will be expected to perform a combination of translation(s), rotation(s), and/or reflection(s) on a single 2-D shape, with and without technology, and draw and describe the image. **G04** Students will be expected to perform a combination of successive transformations of 2-D shapes to create a design and identify and describe the transformations.	**G03** Students will be expected to perform and describe transformations (translations, rotations, or reflections) of a 2-D shape in all four quadrants of a Cartesian plane (limited to integral number vertices).	**G02** Students will be expected to demonstrate an understanding of the congruence of polygons under a transformation.

G06 Students will be expected to perform and describe single transformations of a 2-D shape in the first quadrant of a Cartesian plane (limited to whole number vertices).		

Background

Students have studied transformational geometry in previous grades. They used the informal terms slides, flips, and turns in 2G04 but were later introduced to the formal mathematical language in 5G04. When students first began studying these transformations, they worked with concrete shapes on a flat surface. In Mathematics 5, students worked with a single transformation (G03, G04). In Mathematics 6, this was extended to a combination of transformations (G03, G04). They also worked with a single transformation in the first quadrant of a Cartesian plane (G06).

In translations, reflections, and rotations, the shapes and their images are congruent, but their orientation on the plane and/or their location on the plane may change, depending on the shape and the type of transformation. These transformations, which preserve size and shape, are known as isometries (from the Greek iso "shape" and metry "measure"). Transformations are studied using 2-D shapes, which are called "pre-images," and the resultant shape after a transformation, called the "image." The pre-images are named by their vertices (ABC) and the images are labelled (A′B′C′), read as A prime, B prime, C prime, and so on. Successive images are labelled with additional prime marks (A′′B′′C′′), read as A double-prime, B double-prime, C double-prime, and so on. Labelling the vertices should move clockwise after identifying the initial vertex.

In Mathematics 7, students extend their skills to work with successive transformations in all four quadrants of the Cartesian plane. Students regularly encounter 2-D transformations represented in design patterns and computer graphics. They are evident on logos, fabric patterns, frieze patterns, wallpaper, architectural design, landscape design, and so on. Transformations can be used to create interesting symmetrical patterns. In addition, 2-D transformations on Cartesian planes can be used to represent physical movements in a single plane (e.g., sports plays, rides at a fair, traffic routes). Movie animation is created using motion geometry. Examples can be viewed online. The learning experiences suggested for outcome G03 will help students develop their understanding and appreciation of the transformations existing around them, enhance their problem-solving skills and spatial sense, and prepare them for further studies in algebra and geometry.

Transformational changes can be described by identifying the type of transformation, the changes in orientation of the shape or position of the vertices of the shape, the horizontal and vertical movement, or the new *x*- and *y*-coordinates of the vertices of the image, or by stating the change in (*x*- and *y*-coordinates) between the pre-image and its image. When describing transformations, students should be able to recognize a given transformation as a reflection, a translation, a rotation, or some combination of these. In addition, when given a pre-image and its image, students should be able to describe

- a translation, using words and notation describing the translation (e.g., ΔA′B′C′ is the translation image of ΔABC) (Given a pre-image students should be able to say, for example that ΔABC has been translated 2 units to the right and 3 units up to produce its image ΔA′B′C′. Continue to remind students that they must describe the horizontal change first and the vertical change second.)

- a reflection, by determining the location of the line of reflection (Reflections should be limited to use of the *x*-axis or *y*-axis as reflection lines. When describing the reflection, the proper language to use is "a reflection in the *x*-axis" and not "a reflection across the *x*-axis.")
- a rotation, using degree or fraction-of-turn measures, both clockwise and counterclockwise, and identify the location of the centre of a rotation (A centre of rotation may be located on the pre-image (such as at a vertex) or off the shape.)

Successive transformations are defined as the same transformation being applied one after the other. This means that the transformation is first applied to the pre-image resulting in image_1 and then the same transformation is now applied to image_1 resulting in image_2. The prime and double prime notation will be used to label each image.

Consecutive transformation follows the same idea as successive transformation but uses a combination of different transformations. If A(2, –2) is reflected in the *x*-axis, for example, the resulting image is (A´(2, 2)). If A´ is then translated 4 units left and 2 units up, this results in A′′(–2, 4).

The language that is used when describing consecutive or successive transformations is "followed by." For the example, A (2,-2), the question to ask students would be, "What is the resulting image when A (2,-2) is reflected in the *x*-axis followed by a translation 4 units to the left and 2 units up? The common mistake students make is to apply both transformations to the pre-image. Using the terminology "followed by" will help students use the correct image for the second transformation.

Students should be able to describe the single positional change that maps the pre-image directly to image_2. In this example the single transformation that will map A onto A′′ is translation 4 units left and 6 units up. Students should also be able to describe the single positional change by comparing the vertices of the pre-image with their corresponding vertices in the final image.

Assessment, Teaching, and Learning

Assessment Strategies

Assessing Prior Knowledge

Tasks such as the following could be used to determine students' prior knowledge.

All of the following tasks are to be done in the first quadrant only:

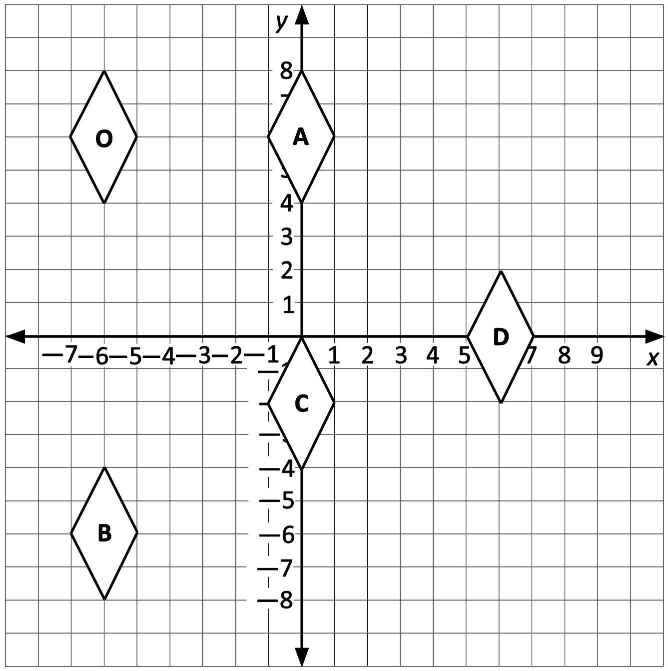
- Ask students to prove that a 2-D shape and its transformation image are congruent.
- Ask students to draw a shape (pre-image), translate it, and then describe the positional change that results in the image.
- Present students with three pictures (pre-image, image_1, and image_2) on grid paper after two transformations were performed on them. Ask students to predict what two transformations were performed. Could this have been done in more than one way? Could this have been done by a single transformation?
- Provide students with a 2-D shape and have them follow the given successive transformations to determine the image.

- Ask students to explain the transformations shown in a pattern, such as fabric, wallpaper, or other designs.
- Invite students to investigate such questions as,
  - If a shape undergoes a translation followed by another translation, does it matter in which order they take place?
  - Does a reflection followed by a translation produce the same result as the translation followed by the reflection?

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Sketch a quadrilateral (pre-image) on a Cartesian plane. Label and record the coordinates of its vertices.
  - Translate the quadrilateral 3 units to the right and 2 units up.
  - Label and record the coordinates of the corresponding vertices of the image.
  - Compare the coordinates of the pre-image with the coordinates of its image and record your observations.
  - Use your observations to predict the coordinates if the pre-image is translated 3 units left and 3 units down.
- Determine what happens to a plotted pre-image if all the first coordinates are switched with the corresponding second coordinates (e.g., A (3, –2) becomes A′(–2, 3)).
  - Describe where the image of each of these points would be located following a half-turn rotation about the origin: P (–3, –5), Q (3, 6), R (–2, 4).
- ΔABC has the following coordinates: A (1, 2), B (3, 5), and C (4, 0).
  - Reflect the triangle in the horizontal axis and label the coordinates for ΔA′B′C′.
  - Reflect ΔA′B′C′ in the vertical axis and label the coordinates for ΔA″B″C″.
  - Discuss ΔA″B″C″ in relation to ΔABC. Is Δ ABC congruent to Δ A″B″C″? Explain. Has the orientation of the transformation of ΔABC changed? Explain.
- If O is the original shape, and A, B, C, and D are images of O, ask students to do the following.
  - Identify the coordinate pairs of the vertices for object O and its images.
  - Describe the movement required to move from O to each of its images.



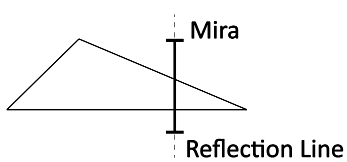
- Respond to the following: If a shape undergoes two transformations, one after the other, does it matter in what order they are applied? Will you get the same final image either way?

Planning for Instruction

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Have students use notation such as “the reflection image of A (3, 5) is A’ (–3, 5)” and say “the reflection image of A (3, 5) is A *prime* (–3, 5).”
- Use grid or dot paper to represent the four-quadrant coordinate plane. Paper folding and the Mira (transparent mirror) are encouraged in working with reflections. When using paper folding, students can fold on the reflection line and trace the image figure. The Mira can be placed on the reflection line, as shown, and students can trace the image from the reflection that appears in the Mira.



- Have students explore rotations where the turn centre of is located off the given pre-image. Many students may still require the use of tracing paper to assist them in placing the transformation of the pre-image.
- Use computer programs, such as Geometer’s Sketchpad or GeoGebra, when available, to assist work with transformations.

- Explore tessellations as a context for applying transformations. It may also be interesting to study art that involves repetitive patterning.

SUGGESTED LEARNING TASKS

- Use a geoboard to create a pre-image and a transformation of that image. Exchange geoboards with a partner, and have that partner explain the transformation, using specific transformation language, and to describe a transformation that would move the new image back to the pre-transformational position. This process can be repeated using different figures, transformations, and combinations of transformations.
- Reflect a triangle in a line and then reflect it in another line that is parallel to the first. Compare the final image with the pre-image. Describe *one* transformation that would move the transformation back to its original image position.
- Work in groups to complete this project: Select a piece of your favourite music, and choreograph a dance to accompany the music. The dance should include at least two examples of each transformation (translation, reflection, and rotation). Perform the dance in class while other students identify the various transformations.
- Using grid paper, answer the following: You are employed by a graphic design firm that creates graphic designs for companies who manufacture wallpaper, wrapping paper, tile, and fabric. Your supervisor has assigned you to develop a new design, using the following design elements:
  - it must use at least two types of transformations
  - it must use at least two colours
  - it must be extended to cover at least 75% of the grid

Create a design and write an explanation of how you created your design.

- Provide grid paper with some shapes already sketched on the paper and provide specifications for some transformations. Have students perform the transformations and label the images. In place of the sketches, provide the ordered pairs for the vertices of the pre-images and have students plot the pre-images and images.
- Have students sketch pre-images, specify transformations, prepare a key, and exchange papers with a partner, who will create the images. Then have students return the papers to each other, verify responses, and discuss any discrepancies.

SUGGESTED MODELS AND MANIPULATIVES

- coordinate graph paper
- geo-boards*
- geometry sets
- Geometer's Sketchpad
- grid paper
- Miras
- tracing paper/wax paper

*also available in *Interactive Math Tools* (Pearson n.d.)

MATHEMATICAL LANGUAGE

Teacher	Student
▪ 2-D	▪ 2-D

Teacher	Student
▪ Cartesian plane	▪ Cartesian plane
▪ centre of rotation	▪ centre of rotation
▪ clockwise	▪ clockwise
▪ combination	▪ combination
▪ congruent	▪ congruent
▪ coordinates	▪ coordinates
▪ counter-clockwise	▪ counter-clockwise
▪ image	▪ image
▪ isometries	
▪ line of reflection	▪ line of reflection
▪ quadrant	▪ quadrant
▪ reflection	▪ reflection
▪ rotation	▪ rotation
▪ shape	▪ shape
▪ successive	▪ successive
▪ transformation	▪ transformation
▪ translation	▪ translation
▪ vertex	▪ vertex
▪ vertices	▪ vertices

Resources

Print

Developing Thinking in Geometry (Johnston-Wilder and Mason 2005), 154–158

Teaching Student-Centered Mathematics, Grades 5–8, Volume 3 (Van de Walle and Lovin 2006b), 209–215

Math Makes Sense 7 (Garneau et al. 2007)

- Unit 8: Geometry (NSSBB #: 2001640)
  - Section 8.6 Graphing Translations and Reflections
  - Section 8.7 Graphing Rotations
  - Technology: Using a Computer to Transform Shapes
  - Unit Problem: Design the Cover
- *ProGuide* (CD; Word Files) (NSSBB #: 2001641)
  - Assessment Masters
  - Extra Practice Masters
  - Unit Tests
- *ProGuide* (DVD) (NSSBB #: 2001641)
  - Projectable Student Book Pages
  - Modifiable Line Masters

Digital

- "33.01 The Coordinate Plane." *Grade 7: The Learning Equation Math.* (Reed 2000): http://staff.argyll.epsb.ca/jreed/math7/strand3/3301.htm (Directions are displayed with the applet.)
- "Shape, Space and Measures," *KS3 Bitesize* (BBC 2015): www.bbc.co.uk/education/topics/zvhs34j. (In this computer game, students choose a mirror line or rotation points to reflect a pentagon house onto its shadow.)
- "Symbol Rotation Patterns," *Wolfram Demonstrations Project.* (Wolfram Demonstrations Project and Contributors 2015): http://demonstrations.wolfram.com/SymbolRotationPatterns/
- "Transmographer," *Interactive*. (Shodor 2015): www.shodor.org/interactivate/activities/Transmographer/

- GeoGebra (International GeoGebra Institute 2015): www.geogebra.org/cms/en
- The Geometer's Sketchpad (Key Curriculum Press 2013; NSSBB #: 50474, 50475, 51453)

Statistics and Probability (SP)

GCO: Students will be expected to collect, display, and analyze data to solve problems.

GCO: Students will be expected to use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

GUIDING QUESTIONS

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Follow-up on Assessment

The evidence of learning that is gathered from student participation and work should be used to inform instruction.

GUIDING QUESTIONS

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

GUIDING QUESTIONS

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SCO SP01: Students will be expected to demonstrate an understanding of central tendency and range by ▪ determining the measures of central tendency (mean, median, mode) and range ▪ determining the most appropriate measures of central tendency to report findings [C, PS, R, T]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

SP01.01 Determine mean, median, and mode for a given set of data, and explain why these values may be the same or different.
SP01.02 Determine the range for a given set of data.
SP01.03 Provide a context in which the mean, median, or mode is the most appropriate measure of central tendency to use when reporting findings.
SP01.04 Solve a given problem involving the measures of central tendency.

Scope and Sequence

Mathematics 6	Mathematics 7	Mathematics 8
SP02 Students will be expected to select, justify, and use appropriate methods of collecting data, including questionnaires, experiments, databases, and electronic media.	**SP01** Students will be expected to demonstrate an understanding of central tendency and range by ▪ determining the measures of central tendency (mean, median, mode) and range ▪ determining the most appropriate measures of central tendency to report findings	**SP01** Students will be expected to critique ways in which data is presented.

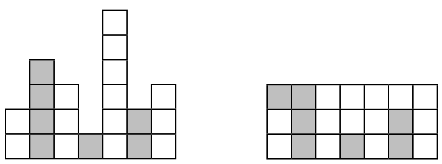
Background

Statistics help reduce large quantities of data to single values and provide an overall sense of data. The single value makes it much simpler to conceptualize and communicate about the information contained in the data. Statistics, however, are sometimes manipulated or presented in a manner that uses facts to mislead people and sway their opinions. By studying statistics, students develop their ability to understand and analyze information presented in advertising, politics, and news reports, and to communicate their experience with data.

Measures of central tendency allow us to describe a set of data with a single, meaningful value. The focus of this outcome is to determine mean, median, and mode and to understand that situational contexts will determine which measure is most meaningful. In Mathematics 6, students collected data first-hand and from digital or text sources and learned when to use each source. In Mathematics 7, students have their first introduction to three statistical measures of central tendency: mean, median, and mode. Each is a numeric value attempting to represent a set of data. Each measure is an average with its own purpose, strengths, and weaknesses. The different measures are best used in different situations, although sometimes all three measures provide meaningful information about the data. Students also learn about range—the difference between the greatest value and least value in a data set.

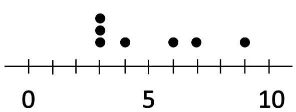
The mean (arithmetic mean) can be computed using an equal distribution (levelling off), finding a central point, or by using an algorithm. Students need experiences gathering and evaluating data, without using an algorithm in order to promote students' understanding of the concept of mean. Only after a conceptual understanding is established should the algorithm be introduced.

With equal distribution (levelling off), students can find the arithmetic mean of a data set using linking blocks. The numbers in the set of data are each represented with linking blocks. The set of blocks can then be rearranged so each column is the same height. When using this strategy, ensure the data sets used have a small number of elements with low numbers, and that they level off to a whole number. The data set 2, 4, 3, 1, 6, 2, 3, for example, has a mean of 3, as can be seen from the diagram below. Only after the blocks are equally distributed should students be told that they found the mean.

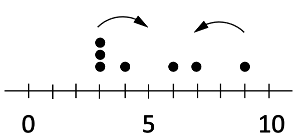


The mean can also be found by moving data points toward the central data point on a number line. This method is more suited to small data sets.

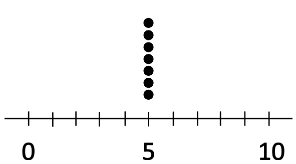
Given the set of data 3, 4, 6, 3, 3, 9, 7, plot the numbers on a number line:



Start moving the numbers toward the centre, allowing one move to the left for every one move to the right:



Continue this process until the numbers line up on one point:



The algorithm for calculating the mean is to sum all values in the set of data, and divide the combined value by the number of elements in the set. The mean of the data set 40, 51, 65, 75, 75, 90 is calculated below.

$$Mean = \frac{40 + 51 + 65 + 75 + 75 + 90}{6} = 66$$

The algorithm can be related to the equal distribution method by having students look at levelling off as the combining of all the values in the set of data and then evenly redistributing them.

The median is the middle value of a set of data that has been ordered. To find the median for an odd number of data values, place all the values in the set of data in order, including repeated numbers, and select the value in the middle. The ordered data set 35, 45, 60, 70, 75, 80, 80 has a median of 70. It is the fourth of the sorted seven values and has three values on either side. To find the median of an even number of data values, add the two middle values together and divide by two. Because the median is the middle value, half the values in a data set will be greater than the median and half the values will be less than the median. The median of 35, 45, 60, 70, 70, 80 is 65 (mean of the third and fourth).

The following set of data with an odd number of values has a median of 56:

12	16	31	42	48	56	63	64	78	83	91

The following set of data, containing an even number of values, has a median of 50 (46 + 54)/2:

12	16	27	31	42	46	50	54	54	63	64	78	82

A common error by students is to forget to arrange the numbers in order when calculating the median. Another common error occurs with data sets that have an even number of values. Students often use one of the two middle values, rather than the number halfway between them, as the median. Remind students that there must be the same number of data values above the median as below it. This provides a quick check that can identify errors in the calculation of the median.

The mode is the data value that occurs most frequently in a data set. A set of data may not have a mode (if each data value appears equally, there is no mode), or it may have one mode, be bimodal, or have multiple modes. The mode may or may not indicate the centre of the data it represents. Modes are very unstable and a small change in the data can drastically change the mode. Because the mode identifies the most typical item in a set, it is useful in predicting the case in a particular situation. For example, if the mode for shirts sold is size 10, the buyers for a store can use the mode to help them decide which sizes to stock in the store's inventory.

Although the main focus of this outcome is on the centre of data sets, students can get a better understanding of these sets by exploring how the data is dispersed. The simplest measure of this dispersion is **range**. The range describes a set of data by identifying the difference between the greatest value and the least value in a data set. Students sometimes incorrectly describe the range using the minimum and maximum data values. Remind students that, similar to the mean and median, the range is a single value.

When asked to calculate the average, students often choose the arithmetic mean, saying it is "more mathematical." In fact, all three measures of central tendency can represent the average or the centre of the data. Students need to learn that the appropriateness of each measure depends upon the context presented.

Understanding how and when to use the different statistical values gives students the ability to understand and communicate about data more clearly, and to use data wisely to make informed decisions. Students worked with discrete and continuous data in grade 6. Discrete data has finite values and, usually, is data that can be counted. The values between the points typically have no meaning given the context. Continuous data have an infinite number of values between data points, which all make sense in the context of a problem. When asked to identify a typical value, students may choose the mode, because they had previous work with bar graphs (5SP02). When discrete data is displayed as a bar graph, students often see the tallest bar standing out in the display. There are certain real-life situations where the mode is the appropriate measure. A shoe store, for example, might order new stock based on the shoe sizes it sells most often. In this case, a mean or median shoe size, such as 6.2, does not make sense in the context of ordering shoe sizes. The mean and median statistics can be chosen for continuous data including monetary values, temperatures, or test scores. The mean is affected by extreme values, whereas the median is not. The effects of these extreme measures will be elaborated on in the next outcome.

When planning for student learning experiences, choose learning activities that emphasize concepts and understanding. Have students gather data for the purpose of answering questions. Allowing students to ask their own questions and collect their own data provides contexts and purposes for analyzing data and for exploring the different statistics. Students may, for example, wish to compare their classmates' habits or physical skills, integrate science or social studies content, or answer questions about world conditions or trends.

Assessment, Teaching, and Learning

Assessment Strategies

Assessing Prior Knowledge

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to list the following numbers from least to greatest: 56, 23, 34, 99, 56, 56, 77, 89, 16
- Review the concepts of formulating questions, collecting first- or second-hand data, and preparing bar graphs.
- Have students work individually, or in pairs, to do the following: Formulate a survey question about peers that can be answered with numeric values. Some questions might be:
  - How many siblings are in your family?
  - How many pets (or cell phones, televisions) does your family have?
  - How many times a week do you eat a particular food, watch a movie, or participate in physical activity?
  - How many hours do you sleep per night?
  - How tall are you?
  - How many pairs of mittens (or shoes, pants) do you own?
  - How many countries have you visited?
  - Compare the lengths of names in the class.

Gather the information. Display the data in a bar graph. Formulate a question about the population of the survey that could be answered using the information from the graph. Include an answer key to the question. These data sets can be used for subsequent learning experiences.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (which can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Which of mean, median, or mode would be most helpful to know in each situation? Justify your choice.
  - You are ordering bowling shoes for a bowling alley.
  - You want to know if you read more or fewer books per month than most people in your class.
  - You want to know the “average” amount of money spent per week on food in your class.
- The mean of a set of five test scores is 80. One of the grades was erased, but the other four are 90, 95, 85, and 100. What is the missing score?
- Write another set of data that would have the same mean and median as 3, 4, 5, 6, 7.
- Between January and March one year, school was cancelled in Snowytown seven times due to blizzards. The following data gives the number of days each blizzard lasted. Find the mean, median, and mode for this data.

1 day	6 days
4 days	2 days
2 days	3 days
3 days	

- The mean of a set of data is much lower than the median. What do you know about the data?
- How can you determine the largest value in a data set if they are given the range and the smallest value? Use an example to explain your answers.
- Juanita, George, and Lee are captains of the school math teams. Their contest results are recorded in the table below.

	Juanita	George	Yin Lee
Contest 1	82	84	85
Contest 2	82	84	85
Contest 3	88	90	85
Contest 4	100	71	81
Contest 5	77	78	81
Contest 6	81	87	85
Contest 7	87	89	82
Contest 8	83	88	85
Contest 9	83	86	83

Ask students to respond to the following: Which measure would you choose to determine whose team is the best? Why? Why might someone disagree with you?

Planning for Instruction

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- To introduce the concept of mean, have students arrange themselves into two groups of 2, one group of 4, two groups of 3, one group of 1, and one group of 6, and then redistribute themselves into seven equal groups to determine how many students would be in each new group.
- Rolls of tickets may be used to introduce the concept of median. On a strip of tickets, students write a data value on each ticket in ascending order. If there is an odd number of tickets, the strip is folded at the median.

- Develop a better conceptual understanding of mean by building models of the data values with towers of linking cubes and having the students move the cubes so the towers are all the same height.
- Have a varying number of students line up in increasing order of height to help them understand and visualize the concept of median.
- Have a group of five students (and then six students) line up in increasing order of shoe size to help them understand and visualize the concept of mode.

SUGGESTED LEARNING TASKS

- Calculate the mean, median, and mode of a set of data from a bar graph.
- Create a tri-fold card to define and create examples of each of the measures of central tendency. On each of the outside panels, name and define mean, median, or mode. On the corresponding inside panel, create and solve an example of a problem using the measure of central tendency on the front.
- The following data was collected to represent the progress of two students in science class. Each of the students would have the same mark based on the calculation of the mean. Find the range of the data for each student and explain how the range can add valuable information to the representation of the progress of each student.
  - Student 1: 76%, 78%, 80%, 82%, 84%
  - Student 2: 60%, 70%, 80%, 90%, 100%
- Create a set of data for each of the following. Each set must have at least six pieces of data.
  - Situation 1: The mean, median, and mode are the same.
  - Situation 2: The mean, median, and mode are different.
- Create a set of five numbers where the median and mode are the same. Explain why you chose those numbers.

Suggested Models and Manipulatives

- grid paper
- rulers
- linking or stacking cubes
- ticket rolls

Mathematical Language

Teacher	Student
▪ average	▪ average
▪ bimodal	
▪ data	▪ data
▪ maximum value	▪ maximum value
▪ mean	▪ mean
▪ measure of central tendency	▪ measure of central tendency
▪ median	▪ median
▪ minimum value	▪ minimum value
▪ mode	▪ mode
▪ multimodal	
▪ range	▪ range
▪ statistics	▪ statistics

Resources

Print

Math Matters, Understanding the Math You Teach, Second Edition (Chapin and Johnson 2006), 94–300

Mathematics for Elementary Teachers, A Contemporary Approach (Musser, Burger, Peterson 2006), 455-457

Teaching Student-Centered Mathematics, Grades 5–8, Volume Three (Van de Walle and Lovin 2006b), 312–315

Math Makes Sense 7 (Garneau et al. 2007)

- Unit 7: Data Anaylsis (NSSBB #: 2001640)
  - Section 7.1 Mean and Mode
  - Section 7.2 Median and Range
  - Section 7.4 Applications of Averages
  - Technology: *Using Spreadsheets to Investigate Averages*
  - Unit Problem: *Board Games*
- *ProGuide* (CD; Word Files) (NSSBB #: 2001641)
  - Assessment Masters
  - Extra Practice Masters
  - Unit Tests
- *ProGuide* (DVD) (NSSBB #: 2001641)

- Projectable Student Book Pages
- Modifiable Line Masters

SCO SP02: Students will be expected to determine the effect on the mean, median, and mode when an outlier is included in a data set. [C, CN, PS, R]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

SP02.01 Analyze a given set of data to identify any outliers.
SP02.02 Explain the effect of outliers on the measures of central tendency for a given data set.
SP02.03 Identify outliers in a given set of data, and justify whether or not they are to be included in reporting the measures of central tendency.
SP02.04 Provide examples of situations in which outliers would and would not be used in reporting the measures of central tendency.

Scope and Sequence

Mathematics 6	Mathematics 7	Mathematics 8
–	**SP02** Students will be expected to determine the effect on the mean, median, and mode when an outlier is included in a data set.	**SP01** Students will be expected to critique ways in which data is presented.

Background

In statistics, extreme values in data sets, values that are significantly different from the other data values, are known as **outliers**. The presence of outliers may affect which measure of central tendency best represents the data. Students need to explore the effect various outliers have on central tendency.

Since the mean uses the values of all the numbers in the data set, it is the measure of central tendency most affected by outliers; therefore, the mean is best used when the range of values in the set of data is narrow. Outliers do not affect the median, as it is the middle value of an ordered set of data.

When students compare the measures of central tendency for a situation, they must consider the impact of outliers. This may affect which statistic to choose.

In some cases, the presence of outliers may not affect the measures of central tendency. As an example, if students explore the effect of 38 and 98 on the measures of central tendency for the data set: 38, 64, 68, 71, 72, 75, 98, they should conclude that the values on opposite extremes of the data set would have virtually no effect on the average score.

Students should also analyze cases where there is only one outlier or multiple outliers on the same extreme. When the data displays outliers, the median may be a better representation than the mean. If one is studying the average temperature of objects in a kitchen, for example, most would be at room temperature, between 20°C and 25°C. If one includes a warm oven at 300°C, the median would be close to room temperature, but the mean temperature would be much higher. For this situation, the median would be the better choice to represent the average temperature of objects in the kitchen.

Outliers may occur in data sets due to a human error (e.g., incorrect measurements or recordings). In these cases, outliers should be ignored when computing statistics. If no error has occurred, the extreme values should be included. At times, the occurrence of outliers may not be obvious. Their identification is then a matter of choice.

In pairs, have students discuss the following situation to determine if an outlier exists:

- Drag racing is usually done on a $\frac{1}{4}$ mile track and cars are timed over that distance. The data collected for a challenger SRT was: 9.11 s, 9.10 s, 9.54 s, 8.01 s, 9.76 s, 9.32 s.

Follow this up with a class discussion to determine if there is agreement about the identification of an outlier in this data, and if so, whether the outlier should be excluded before calculating measures of central tendency.

Outcome SP02 should be done in conjunction with Outcome SP01.

Assessment, Teaching, and Learning

Assessment Strategies

ASSESSING PRIOR KNOWLEDGE

See Outcome SP01.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (which can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Define the term “outlier.” Provide an example of a situation in which an outlier must be excluded from the data before calculating the measures of central tendency, and explain why it would be excluded.
- Answer questions such as the following: Tanya received the following scores on her first five math quizzes: 75%, 75%, 80%, 77%, 82%.
  - What is the mean, median, and mode?
  - On her next quiz, Tanya only achieved a mark of 25%. What effect, if any, did this mark have on the measures of central tendency calculated above?
- Simone writes a spelling quiz each week. Each quiz is scored out of 10.
  - So far, Simone has written seven spelling quizzes. Her scores for the seven quizzes are 8, 8, 7, 9, 6, 10, and 8.
  - What score would best represent Simone’s spelling performance? Explain why you chose that score.

- Simone writes three more quizzes, with scores of 3, 7, and 8. What score do you think best represents her performance now? Why?
- Simone's scores on the next three quizzes are 9, 10, and 0. How will you adjust her spelling score? Why?
- Identify the outliers in this set of data.
- What are some possible reasons for the outliers?
- How do the outliers affect the averages?
- Should the outliers be included in the averages? Why, or why not?

Planning for Instruction

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Use linking cubes to represent a set of data values. For example, with the data set 2, 3, 4, 5, 6, discuss the mode and the median. Have students build cube towers so it can be easily seen that if they take 2 cubes off the 6-story tower and put them on the 3-story tower, and 1 cube off of the 5 story tower and put it on the 3 story tower that all five towers are now all the same height and demonstrate the mean.

  Next add another tower with 25 cubes. Ask students to predict and then find all 3 measures of central tendency with the new value included in the data set. Discuss the significant increase in mean and minimal increase in median and no change in mode.
- Have students calculate the mean, median, and mode for a data set with and without an outlier to see the effect of outliers (change just the lowest or highest value to an outlier). They should see that the median is unaffected but the mean gets either much higher or lower. The mode will usually remain unchanged unless the number changed was the mode.
- Use activities where the outlier is an obvious error to illustrate situations where the outlier would not be used in calculating the averages. If the outlier is not an error, it should still be used in calculations, but recognize that the median, in this case, is a better measure of central tendency.

SUGGESTED LEARNING TASKS

- Answer the following: Players on the grade 7 basketball team were asked to record their height in cm on a chart. The data obtained were used to represent the height of the team.

  155 cm, 153 cm, 150 cm, 167 cm

  164 cm, 182 cm, 170 cm, 159 cm

  185 cm, 19 cm, 182 cm, 174 cm
  - What is the outlier in this data set?
  - Suggest a reason for this outlier. Should it be included in the calculations for the measures of central tendency? Why or why not?
  - Calculate the mean, median, and mode for these heights.
  - Which measure(s) of central tendency would you use to represent the height of the team? Why?
- When Mr. Brown gave a science test, he found the following:

- The mean for the test was 72%.
- The mode for the test was 65%.
- The median of the test was 65%.

When he gave back the test, it was determined that his answer key was wrong, and all of the students had a certain question correct which was valued at 5%. He was then compelled to increase all the marks by 5%.

- How did this affect the mean, median, and mode?
- What things might be concluded about the set of test scores that would account for the mean being so much higher than the mode or the median?
- Create two possible sets of test scores for ten students that would fit Mr. Brown's new mean, median, and mode.

SUGGESTED MODELS AND MANIPULATIVES

- calculators
- linking cubes

*also available in *Interactive Math Tools* (Pearson n.d.)

MATHEMATICAL LANGUAGE

Teacher	Student
▪ average ▪ data ▪ mean ▪ measure of central tendency ▪ median ▪ mode ▪ range ▪ statistics ▪ outlier	▪ average ▪ data ▪ mean ▪ measure of central tendency ▪ median ▪ mode ▪ range ▪ statistics ▪ outlier

Resources

Print

Math Makes Sense 7 (Garneau et al. 2007)

- Unit 7: Data Anaylsis (NSSBB #: 2001640)
  - Section 7.3 The Effects of Outliers on Average
  - Section 7.4 Applications of Averages
  - Technology: Using Spreadsheets to Investigate Averages
- *ProGuide* (CD; Word Files) (NSSBB #: 2001641)
  - Assessment Masters
  - Extra Practice Masters

  - Unit Tests
- *ProGuide* (DVD) (NSSBB #: 2001641)
  - Projectable Student Book Pages
  - Modifiable Line Masters

SCO SP03: Students will be expected to construct, label, and interpret circle graphs to solve problems. [C, CN, PS, R, T, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

SP03.01 Identify common attributes of circle graphs, such as
- title, label, or legend
- the sum of the central angles is 360°
- the data is reported as a percent of the total, and the sum of the percents is equal to 100%

SP03.02 Create and label a circle graph, with and without technology, to display a given set of data.
SP03.03 Find and compare circle graphs in a variety of print and electronic media, such as newspapers, magazines, and the Internet.
SP03.04 Translate percentages displayed in a circle graph into quantities to solve a given problem.
SP03.05 Interpret a given or constructed circle graph to answer questions.

Scope and Sequence

Mathematics 6	Mathematics 7	Mathematics 8
SP01 Students will be expected to create, label, and interpret line graphs to draw conclusions. **SP03** Students will be expected to graph collected data and analyze the graph to solve problems. **M01** Students will be expected to demonstrate an understanding of angles by ▪ identifying examples of angles in the environment ▪ classifying angles according to their measure ▪ estimating the measure of angles using 45°, 90°, and 180° as reference angles ▪ determining angle measures in degrees ▪ drawing and labelling angles when the measure is specified	**SP03** Students will be expected to construct, label, and interpret circle graphs to solve problems.	**SP01** Students will be expected to critique ways in which data is presented.

Background

The purpose of graphs is to display data. Students come to Mathematics 7 with experience in using line graphs to display continuous data, and bar graphs, double bar graphs, pictographs, and line plots to display discrete data. In Mathematics 7, students learn about circle graphs. Circle graphs are also informally referred to as pie charts. Various media use circle graphs to display comparative data.

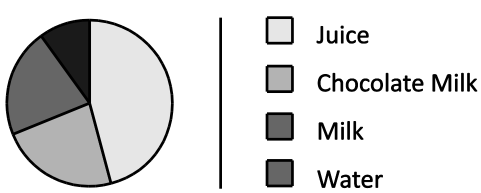
Comparisons can be made between circle graphs (part to whole) and bar graphs (actual data value measurements). Students constructed and interpreted bar graphs in Mathematics 4 (SP02), and double bar graphs in Mathematics 5 (SP02). The two are similar in that they provide information arranged in categories. On a circle graph, the categories are represented by sectors, while bars represent the categories on a bar graph. Since circle graphs display percents rather than quantities, the small set of data can be compared to the large set of data. That could not be done with bar graphs (Van de Walle and Lovin 2006b, 324).

A circle graph displays the distribution of data, not the actual data values. The set of data is grouped into categories, and each category is expressed as a percent of the whole set of data. Circle graphs are particularly useful for comparing the frequency of data in one category to the entire set of data, while still allowing for comparisons among categories.

Circle graphs may also be used to compare data sets of different size, as circle graphs compare ratios rather than definite quantities. The ratios regarding students' choices of beverage in the example below can be compared to choices made by students in other schools. The comparisons may be used to answer questions or to solve problems (e.g., which school to target for a nutrition education program).

Beverage Choice	Number of Students	Percent	Angle size
Juice	150	46%	166°
Chocolate milk	75	23%	83°
Milk	68	21%	75°
Water	32	10%	36°
Totals	325	100%	360°

Lunch Beverages Chosen by 325 Students at a Middle School Cafeteria



This circle graph shows that nearly half of the students eating in the school cafeteria choose juice as a lunch beverage, and that nearly equal numbers of students choose milk or chocolate milk.

When students interpret graphs constructed by others, they learn to appreciate the features that can help them make sense of a visual display of data. The title, legend, and labels are crucial to interpreting circle graphs. Each sector of the graph represents a part-to-whole ratio. Circle graphs emphasize the relation between a category and the whole set of data, as well as the relation between different categories within the data set. Data is partitioned into parts and the circle graph illustrates the ratio of each part to the whole. Data will typically be given as percentages or as raw data to be converted to percents. The percents represented by the sectors must total 100%, and the sum of the central angles must equal 360°. When rounding percents, the numbers may need to be adjusted slightly to ensure a total of exactly 100%. Each circle graph must have a descriptive title and must be labelled with the category names and corresponding percentages, or be accompanied by a legend. The mathematical convention for constructing circle graphs is to start with a vertical line from the centre of the circle to 12:00 (or 0°), and order the sectors in decreasing order in a clockwise direction. It is useful to colour-code the sectors for ease of comparison. Comparisons within circle graphs are most clear when the number of categories is small and when there is a definite variation in the size of the categories.

The ability to find the percent of a number and the ability to use a protractor are necessary skills when constructing circle graphs by hand from raw data. Once students have been engaged in generating circle graphs by hand, the focus should be on when a circle graph is the most appropriate form of data display and how to use technology to construct them. There are many ways to create circle graphs. Technology options include, but are not limited to, Microsoft Excel and Google Docs. A 3-D circle graph is not the best representation of data for interpretation purposes.

Whenever students construct data displays, these displays should be used for interpretation. The ability to organize and display data provides quick visual representations of the data, and the ability to predict future related events based on the data.

Assessment, Teaching, and Learning

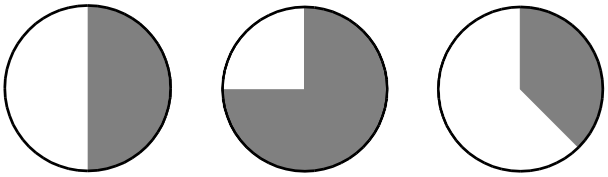
Assessment Strategies

Assessing Prior Knowledge

Tasks such as the following could be used to determine students' prior knowledge.

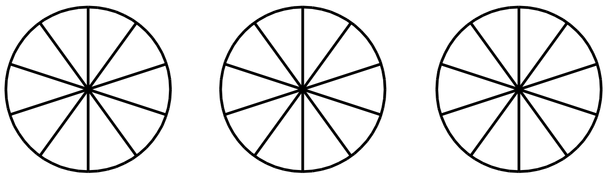
- A grade 7 class conducted a food drive to help the local food bank. The class collected 25 packages of pasta, 32 cans of soup, 14 cans of fruit, 7 cans of vegetables, 16 packages of cookies, and 6 packages of rice. What percent of the items collected consisted of fruit?
- Ask students to create and label a bar graph (with categories, title, and legend) using the data from the above food drive question.
- Use a class discussion to review the characteristics of graphs, including the visual display of data, descriptive titles, labelling of axes, scale, and plots.

- Ask students what percent of each of the following circles is shaded?



a) b) c)

- Ask students to shade the designated percent of each of the following circles.



a) 80% b) 10% c) 30%

- Remind students that the sum of the angles in a circle measure 360°. Ask them to:
  - Calculate the measure of the angle that would represent each percent of a circle.
    - > 50% of 360° = __________
    - > 25% of 360° = __________
    - > 13% of 360° = __________
  - Use a protractor to draw each of the angles.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (which can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask whether or not the various sections of a particular circle graph can be 35%, 25%, 30%, and 15%? Explain.
- Jay works part time in a shoe store. She was involved in completing the spring order. The following were ordered in relation to shoe size:

  5% size 5

  15% size 6

  45% size 7

  25% size 8

  5% size 9

  5% size 10
  - Construct a circle graph to display this data.
  - If Jay placed an order for 120 pairs of shoes, how many of each size should she be expected to order?
  - Create three questions that can be answered from the graph.
- Use the data in the table below; match the correct percentages with the correct sectors in a circle graph. Create an appropriate title and legend for the graph.

Mathematics	30%
Language Arts	25%
Science	20%
Social Studies	15%
French	10%

- Provide students with two circle graphs displaying similar data (such as population age distributions from different areas) and have students write comparison statements based on the data shown.
- How can a circle graph provide information about how parts of a whole are related?

Planning for Instruction

Choosing Instructional Strategies

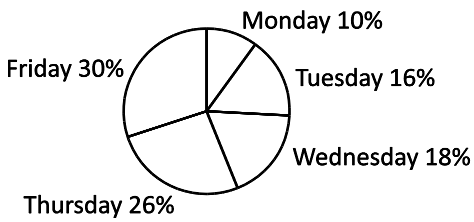
Consider the following strategies when planning daily lessons.

- Prior to constructing circle graphs, you may wish to use an informal activity such as the "Human Pie Graph" described in *Teaching Student-Centered Mathematics* (Van de Walle and Lovin 2006b, 324). Choose a topic, such as having students select their favourite hockey team in the Stanley Cup semi-final round, and line them up so that students favouring the same team are together. Other suggestions include having strips of paper, or wearing t-shirts, corresponding to eye colour. Form the entire group into a circle. You can tape the ends of four long strings in the centre of the circle, and extend them to the circle at each point where the team changes. This results in a circle graph with no measuring and no percentages. If students experience an activity such as the human circle graph, using their own calculations to make circle graphs should have more meaning.
- Use data that is real and interesting to students. Newspapers, magazines, and Internet sites, such as *Statistics Canada*, are good resources for data.
- Ensure that the construction of the graph and interpretation of the data are not addressed independently. When students take the time to construct circle graphs, they should be used for interpretation.
- Begin by having students draw circle graphs using a hundredths circle, before students learn the process of converting percentages to degrees.
- Integrate the use of technology to construct graphs after students have experience creating circle graphs with paper-and-pencil methods.

Suggested Learning Tasks

- Make a bar graph of a set of data. Once completed, cut out the bars themselves, and tape them together end to end. Next, tape the two ends together to form a circle. Estimate where the center of the circle is, draw lines to the points where different bars meet, and trace around the full loop to estimate percentages. (Van de Walle and Lovin 2006b, 324).
- With a graph from a text, magazine, or newspaper, convert the graph to some other form of display. Discuss which is the better way to display the data and why.

- Using the nutritional information found on food packages, create a circle graph showing the nutritional composition of one serving.
- Search newspapers, magazines, and the Internet for information that has been represented as a circle graph. Print or cut out a graph and glue or tape it into their journal. Analyze the graph according to criteria such as the following:
  - Is a title given? Does the title say what the graph is about?
  - Are sectors labelled or is a legend or key provided?
  - Do the percents add up to 100%?
  - Does the graph effectively get the reader's attention?
- Jan wants to show that the sales of chocolate milk are higher at the end of the week, so that more chocolate milk can be ordered for that time. She creates the circle graph below.



  Analyze the graph and answer these questions.
  - What percentage of the milk is sold on Wednesday?
  - Identify a group of days that accounts for about one-half of the total sales. (There is more than one possible answer.)
  - If Friday is a holiday, discuss how that would affect ordering chocolate milk for that week.
  - In a regular week, 500 cartons of chocolate milk are sold. How many cartons should be ordered if Friday was a holiday?
  - If weekly sales of chocolate milk are $200, how much money is made on Monday?
  - Why do you think chocolate milk sales increased steadily as the week progressed?
- Respond to the following: When you are studying a circle graph, what kind of questions should you ask yourself about the information it shows?
- Conduct surveys with your class and ask students to use the results to create circle graphs. Possible survey ideas include:
  - How many children are in your family?
  - What kind of pet do you have?
  - In what month were you born?
  - What colour are your eyes?
  - What is your favourite hockey team?
- Research the most recent population figures for Newfoundland and Labrador, Nova Scotia, New Brunswick, and Prince Edward Island. Record the recent population figures and figures from approximately 20 years ago in two circle graphs. Answer the following questions:
  - How can you tell from the circle graphs which provinces showed the greatest change in total population?
  - What are two questions that could be answered using the circle graphs you drew?

SUGGESTED MODELS AND MANIPULATIVES

- access to data sources
- calculators*
- compass, including bullseye compass
- grid paper
- hundredths circles
- magazines
- newspapers
- protractors
- spreadsheets or graphing computer programs

*also available in *Interactive Math Tools* (Pearson n.d.)

MATHEMATICAL LANGUAGE

Teacher	Student
▪ angle ▪ central angles ▪ circle graph, bar(s) ▪ collect, organize, display, interpret data ▪ key ▪ label ▪ legend ▪ sector ▪ sum	▪ angle ▪ central angles ▪ circle graph, bar(s) ▪ collect, organize, display, interpret data ▪ key ▪ label ▪ legend ▪ sector ▪ sum

Resources

Print

Teaching Student-Centered Mathematics, Grades 5–8, Volume Three (Van de Walle and Lovin 2006b), p. 324

Math Makes Sense 7 (Garneau et al. 2007)
- Unit 4: Circles and Area (NSSBB #: 2001640)
  - Section 4.6 Interpreting Circle Graphs
  - Section 4.7 Drawing Circle Graphs
  - Technology: *Using a Spreadsheet to Create Circle Graphs*
- *ProGuide* (CD; Word Files) (NSSBB #: 2001641)
  - Assessment Masters
  - Extra Practice Masters
  - Unit Tests
- *ProGuide* (DVD) (NSSBB #: 2001641)
  - Projectable Student Book Pages
  - Modifiable Line Masters

Digital

- "Circle Graphs." *Math Playground* (MathPlayground.com 2014): www.mathplayground.com/piechart.html
- "Kids' Zone, Learning with NCES: Create a Graph," *Institute of Education Sciences, National Center for Education Statistics* (US Department of Education 2015)*:* http://nces.ed.gov/nceskids/createagraph/
- "Data Analysis and Probability," *National Library of Virtual Manipulatives* (Utah State University 2015): http://nlvm.usu.edu/en/nav/topic_t_5.html
- "Circle Graph," *Interactive* (Shodor 2015): www.shodor.org/interactivate/activities/CircleGraph
- "Circle Grapher," *Illuminations: Resources for Teaching Math* (National Council of Teachers of Mathematics 2015): http://illuminations.nctm.org/Activity.aspx?id=4092
- "Pie Charts," *Google* (Google 2015): https://support.google.com/docs/answer/190726?hl=en
- "Present Your Data in a Pie Chart," *Microsoft Excel* (Microsoft 2014): http://office.microsoft.com/en-ca/excel-help/present-your-data-in-a-pie-chart-HA010211848.aspx (Microsoft Excel circle graphs.)

SCO SP04: Students will be expected to express probabilities as ratios, fractions, and percents. [C, CN, R, T, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

SP04.01 Determine the probability of a given outcome occurring for a given probability experiment, and express it as a ratio, fraction, and percent.

SP04.02 Provide an example of an event with a probability of 0 or 0% (impossible) and an example of an event with a probability of 1 or 100% (certain).

Scope and Sequence

Mathematics 6	Mathematics 7	Mathematics 8
SP04 Students will be expected to demonstrate an understanding of probability by ▪ identifying all possible outcomes of a probability experiment ▪ differentiating between experimental and theoretical probability ▪ determining the theoretical probability of outcomes in a probability experiment ▪ determining the experimental probability of outcomes in a probability experiment ▪ comparing experimental results with the theoretical probability for an experiment	**SP04** Students will be expected to express probabilities as ratios, fractions, and percents.	**SP02** Students will be expected to solve problems involving the probability of independent events.

Background

Probability is a measure of how likely an event is to occur. In Mathematics 6, students calculated the probability of a single event. Probability is about the prediction of an event over the long term rather than predictions of individual, isolated events. **Theoretical probability** can sometimes be obtained by carefully considering the possible outcomes and using the rules of probability. For example, in flipping a coin, there are only two possible outcomes, so the probability of flipping a head is, in theory, $\frac{1}{2}$. Often in real-life situations involving probability, it is not possible to determine theoretical probability. We must rely on observation of several **trials** (experiments) and a good estimate, which can often be made through a data collection process. This is called **experimental probability**.

$$\text{Experimental probability} = \frac{\text{Number of times an outcome occurs}}{\text{Number of times the experiment is conducted}}$$

As students gather data, they should learn that as the sample size increases, the experimental probability approaches the value of the theoretical probability.

Theoretical probability of an event is the ratio of the number of favourable outcomes in an event to the total number of possible outcomes, when all possible outcomes are equally likely. Simply stated, theoretical probability describes what "should" happen and helps predict the experimental probability.

$$\text{Theoretical probability} = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

In Mathematics 7, students are to express probabilities in a variety of ways, including ratios, fractions, and percents. Earlier in Mathematics 7, students expressed percents as decimals and fractions (N03). Probability is a number between 0 and 1 that measures the likelihood of an event. The probability of a single event occurring is the ratio of the number of favourable outcomes to the number of possible outcomes. For example, the probability of rolling a prime number on a regular deca die (10 sides) numbered 1 to 10 can be expressed in multiple ways.

- As a ratio: P(prime) = 4:10 = 2:5
- As a fraction: P(prime) = $\frac{4}{10} = \frac{2}{5}$
- As a percent: P(prime) = $\frac{4}{10} = \frac{40}{100}$ = 40%

In Mathematics 5, students determined the likelihood of an event as impossible, possible, or certain. Students will now express the probabilities of impossible events as 0 or 0% and of certain events as 1 or 100%.

Assessment, Teaching, and Learning

Assessment Strategies

Assessing Prior Knowledge

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to create a spinner for which there are three equally likely outcomes and another spinner for which the three outcomes are not equally likely. Have them predict the probability for each spinner's outcomes.
- Provide students with a bag with 10 green cubes and five blue cubes. Ask students to determine the theoretical probability of picking a blue cube.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (which can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Answer the following questions.

2	4	1	1	2
1	1	4	1	3
3	5	2	2	3
2	4	1	2	1
3	2	5	3	1

  - The table below shows the results of spinning a spinner. Find the following probabilities and express your answers as
    - > a ratio, as a fraction and as a percent each time
    - > P(spin of 2)
    - > P(spin of 5)
    - > P(spin of an even number)
    - > P(spin of 7)
    - > P(spin of 1, 2, 3, 4 or 5)
- Letter tiles for the word CANADIAN are placed in a bag.
  - What is the probability of drawing a letter A from the bag?
  - What is the probability of drawing a consonant from the bag?
- Provide an example of a situation with a probability of 0.
- Provide an example of a situation with a probability of 1.
- Find the theoretical probability for each of the following situations that involve a six-faced die:
  - the probability of tossing a 4 with the die
  - the probability of tossing an even number
  - the probability of tossing a number greater than 2

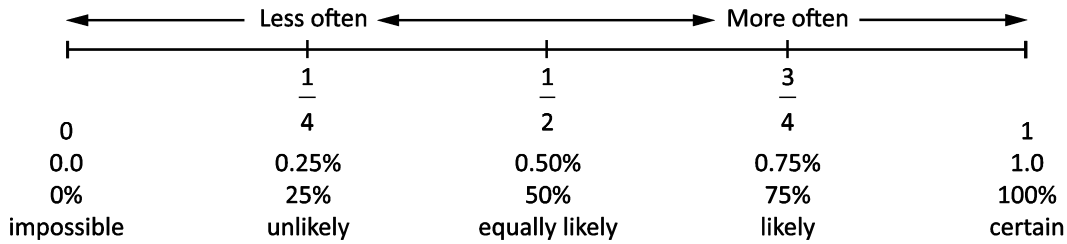
Planning for Instruction

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Ensure students acquire an understanding that probability can be represented in multiple forms. One means of accomplishing this understanding is by specifying a particular form for the answer.
- Occasionally give questions without specifying a particular form. Different groups can also be provided with the same problems but be asked to present the answers in different forms. When the class discusses the results in a large group, students should observe the variation in the answers and discuss or account for the differences. Through such experiences, students should come to the realization that the various forms are alternative representations of the same value.

- Review benchmarks and how they relate to probability with students.



SUGGESTED LEARNING TASKS

- Make a similar “area model diagram” for Jill, who is a 60% free-throw shooter in basketball, and use it to decide if she is more likely to get 0 points, 1 point, or 2 points. Make the diagram on grid paper and use it to determine the probability of getting 0 points, 1 point, or 2 points.
- Use the information in the table below that shows the results of spinning a spinner to find each of the following probabilities. Express your answer as a ratio, as a fraction, and as a percent each time.
  - P(spin of 2)
  - P(spin of 5)
  - P(spin of even number)

2	1	2	4	1
3	2	3	5	2
3	5	1	2	3
1	1	1	4	3
4	2	2	1	1

- What does it mean that an event has a probability of 79%? of $\frac{2}{3}$? of 1:5?
- A probability of 0 means an event is impossible. A probability of 1 means it is certain. Describe a situation that has a probability of 0.5 occurring. Explain your reasoning.
- A bag contains 30 marbles: 7 red, 6 black, 4 yellow, 5 orange and 8 green. What is the probability of drawing a red marble from the bag? Express your answer as a fraction, decimal, and percent.
- Kari says that the probability that a person’s birthday is in the winter is about $\frac{1}{4}$. Andy says it is about 250:1000, and Carson says it is about 25%. Who is right? Explain.
- Describe an event for each of the following probabilities using a single octahedron (8-sided die).

  0   0.25   50% $\frac{3}{4}$ 5:8 (hint: less than 6)

- What is the probability of Sarah rolling a factor of 6 if she rolled a six-sided die? Write your answer as a fraction, a ratio, and a percent.
- What is the theoretical probability:
  - of randomly pointing to a prime number on a hundreds chart?
  - that a 2-digit number that ends in 3 is also divisible by 3?

- Using a scale with benchmarks 0 (0%), $\frac{1}{4}$ (25%), $\frac{1}{2}$ (50%), $\frac{3}{4}$ (75%), and 1 (100%), assess the reasonable probability of the following. Explain your answers.
  - The next baby born in your town will be a boy.
  - It will snow at least once in the month of June.
  - A person will live 6 months without water.
  - The sun will set tomorrow.

Suggested Models and Manipulatives

- cubes
- 10 × 10 grid
- marbles
- number cubes
- spinners

Mathematical Language

Teacher	Student
▪ certain event	▪ certain event
▪ dependent event	▪ dependent event
▪ event	▪ event
▪ experimental probability	▪ experimental probability
▪ frequency table or chart	▪ frequency table or chart
▪ impossible event	▪ impossible event
▪ independent event	▪ independent event
▪ likely	▪ likely
▪ outcome	▪ outcome
▪ probability	▪ probability
▪ random	▪ random
▪ theoretical probability	▪ theoretical probability

Resources

Print

Teaching Student-Centered Mathematics, Grades 3–5, Volume Two (Van de Walle and Lovin 2006a), 334–336

Math Makes Sense 7 (Garneau et al. 2007)
- Unit 7: Data Analysis (NSSBB #: 2001640)
  - Section 7.5 Different Ways to Express Probability
  - Game: All the Sticks
  - Unit Problem: Board Games
- *ProGuide* (CD; Word Files) (NSSBB #: 2001641)
  - Assessment Masters

  - Extra Practice Masters
  - Unit Tests
- *ProGuide* (DVD) (NSSBB #: 2001641)
  - Projectable Student Book Pages
  - Modifiable Line Masters

Digital

- "Probability [applet]," *Macmillan Educatonl* (W. H. Freeman 2015): http://bcs.whfreeman.com/ips4e/cat_010/applets/Probability.html
- "Experimental Probability." *Interactive* (Shodor 2015): http://www.shodor.org/interactivate/activities/ExpProbability/
- "Data Analysis and Probability," *National Library of Virtual Manipulatives* (Utah State University 2015): http://nlvm.usu.edu/en/nav/topic_t_5.html. (In the Grades 6 to 8 section, select Coin Tossing or Spinners.)

SCO SP05: Students will be expected to identify the sample space (where the combined sample space has 36 or fewer elements) for a probability experiment involving two independent events. [C, ME, PS]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

SP05.01 Provide an example of two independent events, such as the following, and explain why they are independent.
- spinning a four-section spinner and an eight-sided die
- tossing a coin and rolling a twelve-sided die
- tossing two coins
- rolling two dice

SP05.02 Identify the sample space (all possible outcomes) for each of two independent events using a tree diagram, table, or other graphic organizer.

Scope and Sequence

Mathematics 6	Mathematics 7	Mathematics 8
SP04 Students will be expected to demonstrate an understanding of probability by ▪ identifying all possible outcomes of a probability experiment ▪ differentiating between experimental and theoretical probability ▪ determining the theoretical probability of outcomes in a probability experiment ▪ determining the experimental probability of outcomes in a probability experiment ▪ comparing experimental results with the theoretical probability for an experiment	**SP05** Students will be expected to identify the sample space (where the combined sample space has 36 or fewer elements) for a probability experiment involving two independent events.	**SP02** Students will be expected to solve problems involving the probability of independent events.

Background

Students were introduced to the concept of probability in Mathematics 5, where they conducted probability experiments, and defined probability as the chance of an event happening out of all possible outcomes.

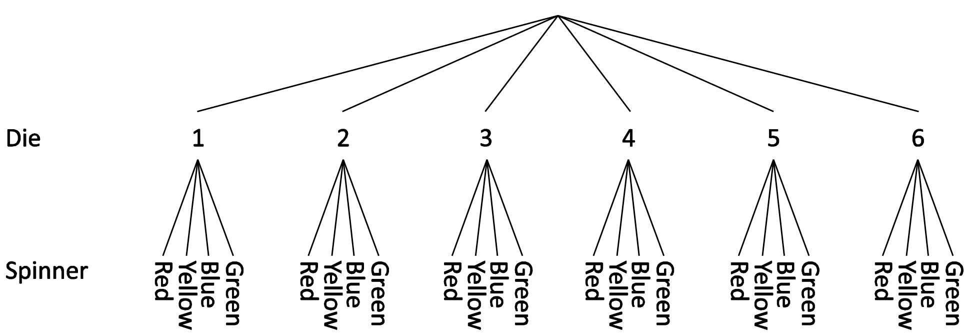
In Mathematics 6, students identified all the possible outcomes of a probability experiment of a single event. In Mathematics 7, the study of **sample space** (the list of all possible outcomes) is limited to two independent events. Two events are considered to be independent if the result of one does not depend on or influence the result of another. Students should understand that spinning a four section spinner, for example, does not in any way affect the number an eight-sided die will land on when tossed.

A common error for tossing two coins, or rolling two dice, is a failure to distinguish between the two events, especially when the outcomes are combined. When determining the probability of tossing two coins and getting heads and tails, some students might treat HT and TH as the same outcome. Identifying the sample space before calculating probability should help students avoid this error.

The possible outcomes can be represented in a tree diagram or table. Students will explore several ways to organize the sample space for two independent events.

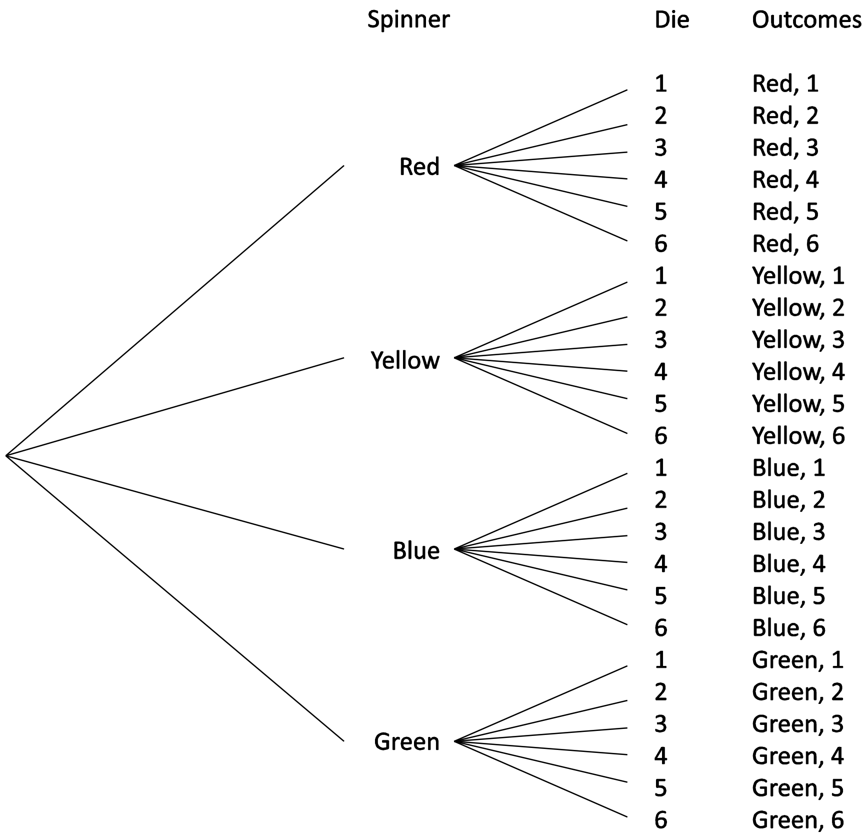
Example: Students can display sample space for a regular 6-sided die, numbered 1 to 6, and a 4-colour spinner, coloured red, yellow, blue, and green, in either a vertical or horizontal tree diagram, or a table.

Vertical Tree Diagram



Outcome 1R, 1Y, 1B, 1G, 2R, 2Y, 2B, 2G, 3R, 3Y, 3B, 3G,
4R, 4Y, 4B, 4G, 5R, 5Y, 5B, 5G, 6R, 6Y, 6B, 6G

Horizontal Tree Diagram



	Die						
		1	2	3	4	5	6
Spinner	Red	R,1	R,2	R,3	R,4	R,5	R,6
	Yellow	Y,1	Y,2	Y,3	Y,4	Y,5	Y,6
	Blue	B,1	B,2	B,3	B,4	B,5	B,6
	Green	G,1	G,2	G,3	G,4	G,5	G,6

Assessment, Teaching, and Learning

Assessment Strategies

Assessing Prior Knowledge

Tasks such as the following could be used to determine students' prior knowledge.

- Provide students with a 10-sided die and have them determine the theoretical probability of rolling a prime number (2, 3, 5, 7). Have students roll the die 5 times, 10 times, and 50 times and compare the experimental probability result of each with the theoretical probability. Ask them to explain why it is important to have more than a few trials in a probability experiment.
- Invite students to explain how a scientific experiment is like a probability experiment. They should focus on the differences between theory/hypothesis and experimental results.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (which can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Determine the sample space (all possible outcomes) for the following, using a tree diagram or table.
  - Jesse has three sweaters and two pairs of shorts. How many different outfits can she create?
  - A menu offers a lunch special of a hot dog or a hamburger with a choice of an apple, banana, or orange for dessert. How many different combinations of sandwich and dessert could be ordered?
  - Ling purchased a new cell phone. There is a choice of a hard plastic or leather case and a choice of colours: black, green, blue, or red. How many different combinations of case and colour are possible?
- Answer questions such as the following: Bob loves wearing interesting colour combinations. In his closet, he has a variety of shirts and pants to choose from. He has shirts that are blue, green, yellow, red, orange, and pink. As pants, he chooses between shorts, jeans, dress pants, and casual pants.
  - What are the two independent events?
  - Explain why these events are independent.
  - Using an appropriate method, identify the sample space that describes all possible combinations of shirts and pants Bob can create.
  - Bob's mom buys him a new purple shirt. How many different outfits can he now create?

Planning for Instruction

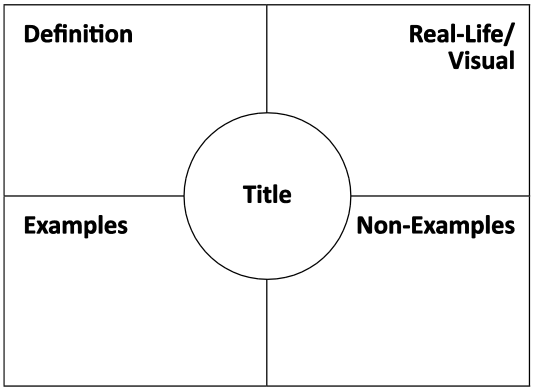
CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Clarify terminology used in probability; e.g., sample space, outcomes, events, equally likely outcomes, unequally likely outcomes, and trials.
- Provide examples and non-examples of independent events to deepen students' understanding of independent events.
- Provide a variety of manipulatives for illustrating independent events; e.g., coins, dice, spinners, drawing cards from a deck, objects from a bag.

Suggested Learning Tasks

- With a Frayer Model template, fill in the sections, individually or as a group, to consolidate your understanding of independent events.



- Predict the probability of getting all questions correct on a test that had five multiple-choice questions with four options for each question if they answered the questions randomly. Determine the sample space by creating a tree diagram or a table.
- You are helping their little sister pick out an outfit to wear. In her closet, she has a variety of tops and bottoms from which to choose. As tops, she has t-shirts that are blue, green, yellow, red, orange and pink. As bottoms, she has a skirt, a pair of shorts, a pair of capri pants, and a pair of jeans.
  - What are the two independent events in this example? Explain why these events are independent.
  - Using an appropriate method, identify the sample space which describes all possible combinations of tops and bottoms you can create for your little sister.
  - Your mom buys your sister a new purple shirt. How many different outfits can you now create?
- Decide whether each pair of events are independent or not, and to explain your reasoning.
  - Roll a die and then roll a different die.
  - Roll a die and then roll the same die again.
  - Choose a name from a hat and then choose a second name from the hat without replacing the first name.
  - Choose a student from grade 7 and choose a student from grade 8.

SUGGESTED MODELS AND MANIPULATIVES

- cards
- coins
- colour tiles or linking cubes
- dice with a variety of number of sides
- number cubes
- SmartBoard and Mimio spinners and dice
- various spinners, including digital spinners

MATHEMATICAL LANGUAGE

Teacher	Student
▪ certain event ▪ dependent event ▪ event ▪ experimental probability ▪ equally likely outcomes ▪ frequency table	▪ certain event ▪ dependent event ▪ event ▪ experimental probability ▪ equally likely outcomes ▪ frequency table

▪ impossible event ▪ independent event ▪ likely ▪ outcome ▪ possible outcome ▪ probability ▪ random ▪ sample size ▪ sample space ▪ theoretical probability ▪ tree diagram ▪ trials ▪ unequally likely events	▪ impossible event ▪ independent event ▪ likely ▪ outcome ▪ possible outcome ▪ probability ▪ random ▪ sample size ▪ sample space ▪ theoretical probability ▪ tree diagram ▪ trials ▪ unequally likely events

Resources

Print

Teaching Student-Centered Mathematics, Grades 3–5, Volume Two (Van de Walle and Lovin 2006a), 341–342

Math Makes Sense 7 (Garneau et al. 2007)

- Unit 7: Data Analysis (NSSBB #: 2001640)
  - Section 7.6 Tree Diagrams
  - Game: All the Sticks
  - Unit Problem: Board Games
- *ProGuide* (CD; Word Files) (NSSBB #: 2001641)
  - Assessment Masters
  - Extra Practice Masters
  - Unit Tests
- *ProGuide* (DVD) (NSSBB #: 2001641)
  - Projectable Student Book Pages
  - Modifiable Line Masters

Digital

- "Adjustable Spinner," *Illuminations: Resources for Teaching Math* (National Council of Teachers of Mathematics 2015): http://illuminations.nctm.org/adjustablespinner
- "Unnamed [virtual spinner]," *Math Playground* (MathPlayground.com 2015): www.mathplayground.com/probability.html
- "Unnamed [virtual spinner]," *National Library of Virtual Manipulatives* (Utah State University 2015): http://nlvm.usu.edu/en/nav/frames_asid_186_g_1_t_1.html?open=activities
- "Virtual Dice," *Birmingham Grid for Learning* (Birmingham City Council 2015): www.bgfl.org/bgfl/custom/resources_ftp/client_ftp/ks1/maths/dice/index.htm

SCO SP06: Students will be expected to conduct a probability experiment to compare the theoretical probability (determined using a tree diagram, table, or other graphic organizer) and experimental probability of two independent events. [C, PS, R, T]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

SP06.01 Determine the theoretical probability of a given outcome involving two independent events.
SP06.02 Conduct a probability experiment for an outcome involving two independent events, with and without technology, to compare the experimental probability with the theoretical probability.
SP06.03 Solve a given probability problem involving two independent events.

Scope and Sequence

Mathematics 6	Mathematics 7	Mathematics 8
SP04 Students will be expected to demonstrate an understanding of probability by ▪ identifying all possible outcomes of a probability experiment ▪ differentiating between experimental and theoretical probability ▪ determining the theoretical probability of outcomes in a probability experiment ▪ determining the experimental probability of outcomes in a probability experiment ▪ comparing experimental results with the theoretical probability for an experiment	**SP06** Students will be expected to conduct a probability experiment to compare the theoretical probability (determined using a tree diagram, table, or other graphic organizer) and experimental probability of two independent events.	**SP02** Students will be expected to solve problems involving the probability of independent events.

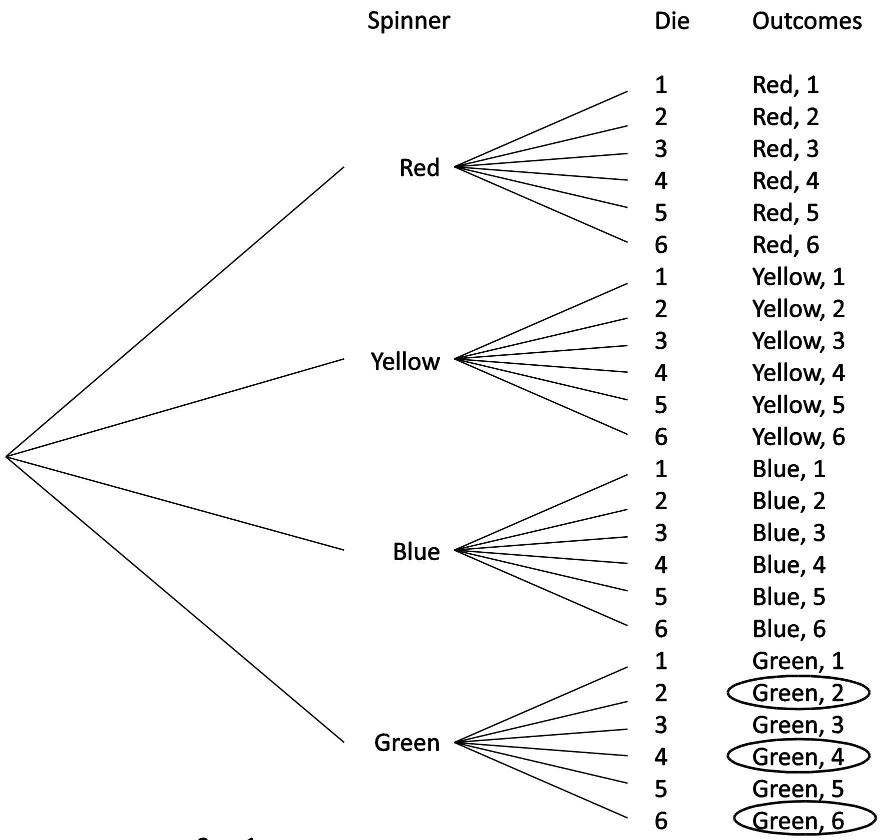
Background

In Mathematics 6, students determined both the theoretical and experimental probability of a single event. In Mathematics 7, students will conduct experiments to compare the theoretical probability of two independent events to the experimental probability and will record their work using graphic organizers.

The theoretical probability of an event is the ratio of the number of favourable outcomes in an event to the total number of possible outcomes, when all possible outcomes are equally likely. Students use the sample space to determine the number of favourable outcomes and the number of possible outcomes. They then display this proportion as a ratio, percent, or fraction. In Mathematics 8, students will determine the probability of independent events as the product of the probabilities of each event occurring separately, after they study multiplication of fractions.

Using the die-spinner example from the previous outcome, if students were asked the theoretical probability of rolling an even number and spinning the colour green, they would indicate or circle all of the possible outcomes of the sample space using any graphic organizer.

Using Tree Diagram



P(even, green) = $\frac{3}{24} = \frac{1}{8}$ = 1.8 = 0.125 = 12.5%

For example, students should realize that the probability in many situations cannot be characterized as equally likely. The theoretical probability of a thumbtack landing with the point up or down, , is more difficult to determine. In such cases, experiments or simulations are conducted to determine the "experimental probability."

Before conducting experiments, students should predict the probability whenever possible, and use the experiment to verify or refute the prediction. Materials such as spinners, dice, coins, or coloured marbles may be used to conduct experiments. Simulations with graphing calculators, virtual simulations, or computer software could also be used.

Assessment, Teaching, and Learning

Assessment Strategies

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

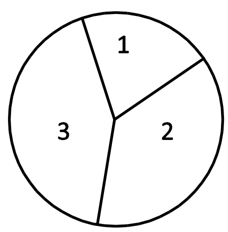
- Provide pairs of students with 24 linking cubes of different colours and a paper bag. Have them determine the theoretical probability of selecting each colour from the bag. Next, have them conduct the experiment by drawing and replacing one cube at a time for 50 trials. Compare the theoretical and experimental probabilities and discuss.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (which can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- A probability experiment was performed that consisted of tossing a "fair coin" and spinning a spinner like the one shown below. The outcomes of the probability experiment are shown in the tally chart below.

Outcomes	Tallies
H1	𝍸 𝍸 𝍸 //
H2	𝍸 𝍸 ////
H3	𝍸 𝍸 𝍸 ////
T1	𝍸 𝍸 𝍸
T2	𝍸 𝍸 𝍸 ///
T3	𝍸 𝍸 𝍸 //



  How many trials were in the experiment? Explain.
  - What is the experimental probability of tossing a head and spinning an odd number? Explain.
  - What is the theoretical probability of tossing a head and spinning an odd number? Explain.
  - Compare the answers above and explain any discrepancy. What would be the theoretical probability of tossing a head and spinning an odd number if the spinner showed unequally likely outcomes as illustrated above? Show all your work.

  This experiment could be modified to include a spinner with more than three sections.
- A probability experiment consists of tossing two six-sided fair dice.
  - Does this experiment describe two independent events? Explain.
  - Draw a tree diagram or create a table to show all the possible outcomes for this experiment.
  - Find the theoretical probability of obtaining a sum of 5 on the two dice in this experiment. Show all of your work.
  - Describe how you could conduct this experiment by using two spinners instead of two six-sided dice.

- Describe how you could determine the experimental probability of getting 7 out of 10 questions correct on a true-or-false test by guessing alone.
- Three students play a coin toss game in which points are awarded depending on the following rules:
  - Player A gets a point if two tosses result in two heads.
  - Player B gets a point if two tosses result in two tails.
  - Player C gets a point if two tosses result in one head and one tail.

  The students play the game twenty times. The player who has the most points wins.

  Discussion around this activity should focus on questions such as:
  - Is there a favoured player? How do you know? Why is this player favoured? Is this player likely to win the next game? Is this player guaranteed to win the next game?
  - How many ways can two heads occur? Two tails? One head and one tail?
  - Is this game fair? It will be useful to consider both the theoretical and experimental probabilities.

- Samantha's little brother removed all the labels from the cans of soup in the pantry. He also removed the labels from the canned fruit. There are four cans of tomato soup, two cans of chicken noodle soup, and one can of cream of mushroom soup. There are two cans of peaches and one can of pears. The soup and fruit come in cans of different sizes. If one can of soup is opened, and one can of fruit is opened, what is the probability that the combination will be chicken noodle soup and peaches?
- Design your own probability experiment involving two independent events. Include the solution to the problem.

Planning for Instruction

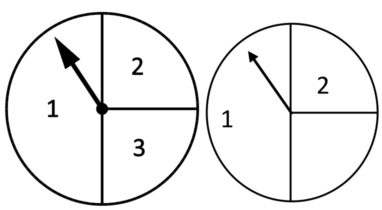
Choosing Instructional Strategies

Consider the following strategies when planning daily lessons.

- Conduct systematic experiments where
  - the problem and any underlying assumptions are clearly defined
  - a model is selected to generate the necessary outcomes
  - a large number of trials are conducted and recorded
  - the information is summarized to draw a conclusion
- Build on students' understanding of experimental and theoretical probability from the previous grade focusing on a single action and extend it to include two independent events (two separate actions).
- Ensure students use proper terminology used in probability.
- Integrate the use of technology after students have done hands-on work in carrying out experiments with independent events.
- Provide a variety of manipulatives in illustrating independent events; e.g., coins, dice, spinners and drawing cards from a deck or objects from a bag with replacement.
- Have students predict the results of any experiment with independent events by using theoretical probability.

SUGGESTED LEARNING TASKS

- Provide students with various two-player dice games in which two dice are rolled and rules are given that relate the two numbers rolled. Have students predict whether the games are fair. Encourage students to justify their predictions and then play the games with at least 30 trials to explore**.**
- Tell students that an experiment of tossing two fair coins was conducted. Ask them to estimate how many times in an experiment with 64 trials might you expect to get two heads? Have student explain their thinking. Have students work in pairs to carry out the experiment with each group doing 10 or 20 trials. Collate the results to obtain 64 trials and then add more trials as needed to show that experimental probability approaches theoretical probability as the number of trials increases. Have them calculate the experimental probability of getting two heads when two coins are tossed. Have students compare the experimental probability to the theoretical probability. This activity can be extended by having students toss three fair coins, and explore how many times they would get two coins with heads.
- Conduct an experiment where they spin a spinner like the one shown below, twice, and find the sum of the numbers from the two spins. Ask them to predict which sum will appear most often. Have students work in pairs to carry out the experiment with each pair doing 10 or 20 trials. Collate the results to obtain at least 100 trials. Have students compare the experimental results to their prediction and explain why there may be differences.



- Give students a paper cup and ask them to find the probability of it landing on its bottom if dropped. They should see that this is an example of a situation in which they are unable to find the theoretical probability, and so will have to conduct an experiment to find the probability.
- Have students respond to the following:
  - Matthew's brand new iPod has only five songs. They are all different. He hits the shuffle button to randomly select a song. Matthew's favourite song begins to play. At the end of that song, he hits the shuffle button again to get another random selection.
    - Organize the sample space (possible outcomes) for choosing two songs at random.
    - What is the probability that Matthew will hear his favourite song two times in a row? Show clearly how you obtained your answer.
  - At a carnival, you will receive a prize if you toss two dice and the sum is a prime number. Which of the following options provides the greatest likelihood of winning a prize? Explain your thinking.
    - tossing two six-sided dice
    - tossing one six-sided die and one four-sided die
  - A probability experiment consists of tossing two six-sided fair dice.
    - Does this experiment describe two independent events? Explain.
    - Draw a tree diagram or create a table to show all possible outcomes for this experiment.

> Find the theoretical probability of obtaining a sum of five on the two dice in this experiment. Show all your work.
> Describe how you could conduct this experiment by using two spinners instead of two six-sided dice.

SUGGESTED MODELS AND MANIPULATIVES

- colour tiles*
- dice with a variety of number of sides
- electronic versions of spinners and dice
- linking cubes
- number cubes
- SmartBoard and Mimio spinners and dice
- various spinners

*also available in *Interactive Math Tools* (Pearson n.d.)

MATHEMATICAL LANGUAGE

Teacher	Student
▪ certain event	▪ certain event
▪ dependent event	▪ dependent event
▪ event	▪ event
▪ experimental probability	▪ experimental probability
▪ favourable outcome	▪ favourable outcome
▪ frequency table or chart	▪ frequency table or chart
▪ impossible event	▪ impossible event
▪ independent event	▪ independent event
▪ likely	▪ likely
▪ outcome	▪ outcome
▪ possible outcome	▪ possible outcome
▪ probability	▪ probability
▪ random	▪ random
▪ relative frequency	▪ relative frequency
▪ sample size	▪ sample size
▪ sample space	▪ sample space
▪ theoretical probability	▪ theoretical probability
▪ tree diagram	▪ tree diagram

Resources

Print

Teaching Student-Centered Mathematics, Grades 3–5, Volume Two (Van de Walle and Lovin 2006a), 341–342

Math Makes Sense 7 (Garneau et al. 2007)

- Unit 7: Data Analysis (NSSBB #: 2001640)

  - > Section 7. 6 Tree Diagrams
  - > Unit Problem: *Board Games*
- *ProGuide* (CD; Word Files) (NSSBB #: 2001641)
  - > Assessment Masters
  - > Extra Practice Masters
  - > Unit Tests
- *ProGuide* (DVD) (NSSBB #: 2001641)
  - > Projectable Student Book Pages
  - > Modifiable Line Masters

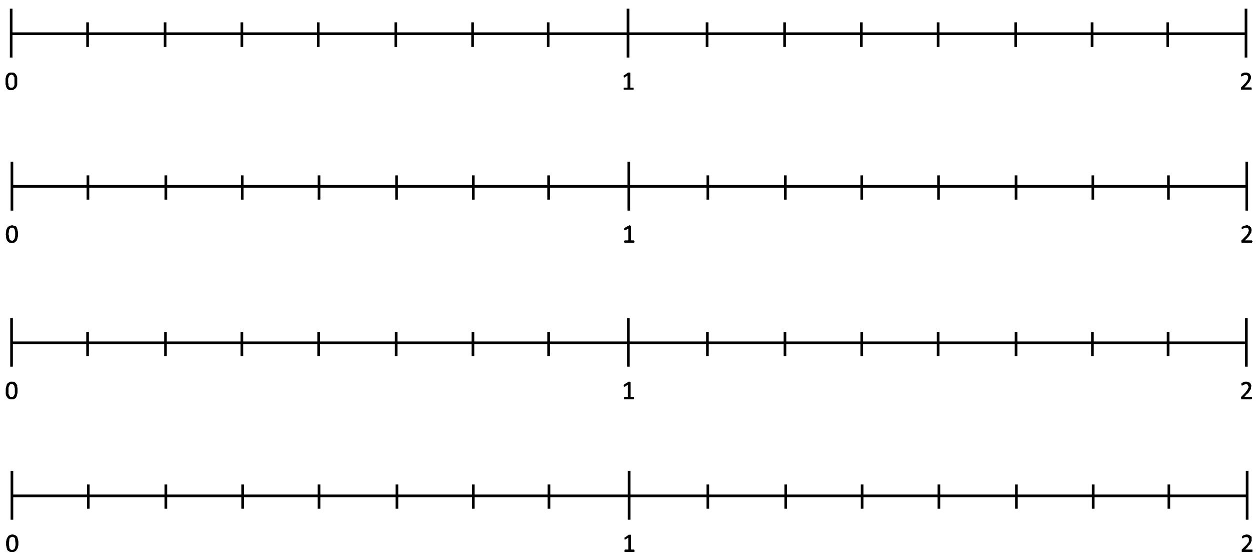
Digital

- "Adjustable Spinner," *Illuminations: Resources for Teaching Math* (National Council of Teachers of Mathematics 2015): http://illuminations.nctm.org/adjustablespinner
- "Unnamed [virtual spinner]," *Math Playground* (MathPlayground.com 2015): www.mathplayground.com/probability.html
- "Unnamed [virtual spinner]," *National Library of Virtual Manipulatives* (Utah State University 2015): http://nlvm.usu.edu/en/nav/frames_asid_186_g_1_t_1.html?open=activities
- "Virtual Dice," *Birmingham Grid for Learning* (Birmingham City Council 2015): www.bgfl.org/bgfl/custom/resources_ftp/client_ftp/ks1/maths/dice/index.htm

Appendices

Appendix A: Fraction Number Lines

Fraction Number Lines



Appendix B: Connect Three—Addition of Fractions

Material

- 2 paper clips
- approximately 20 counters per player, each set a different colour
- game board

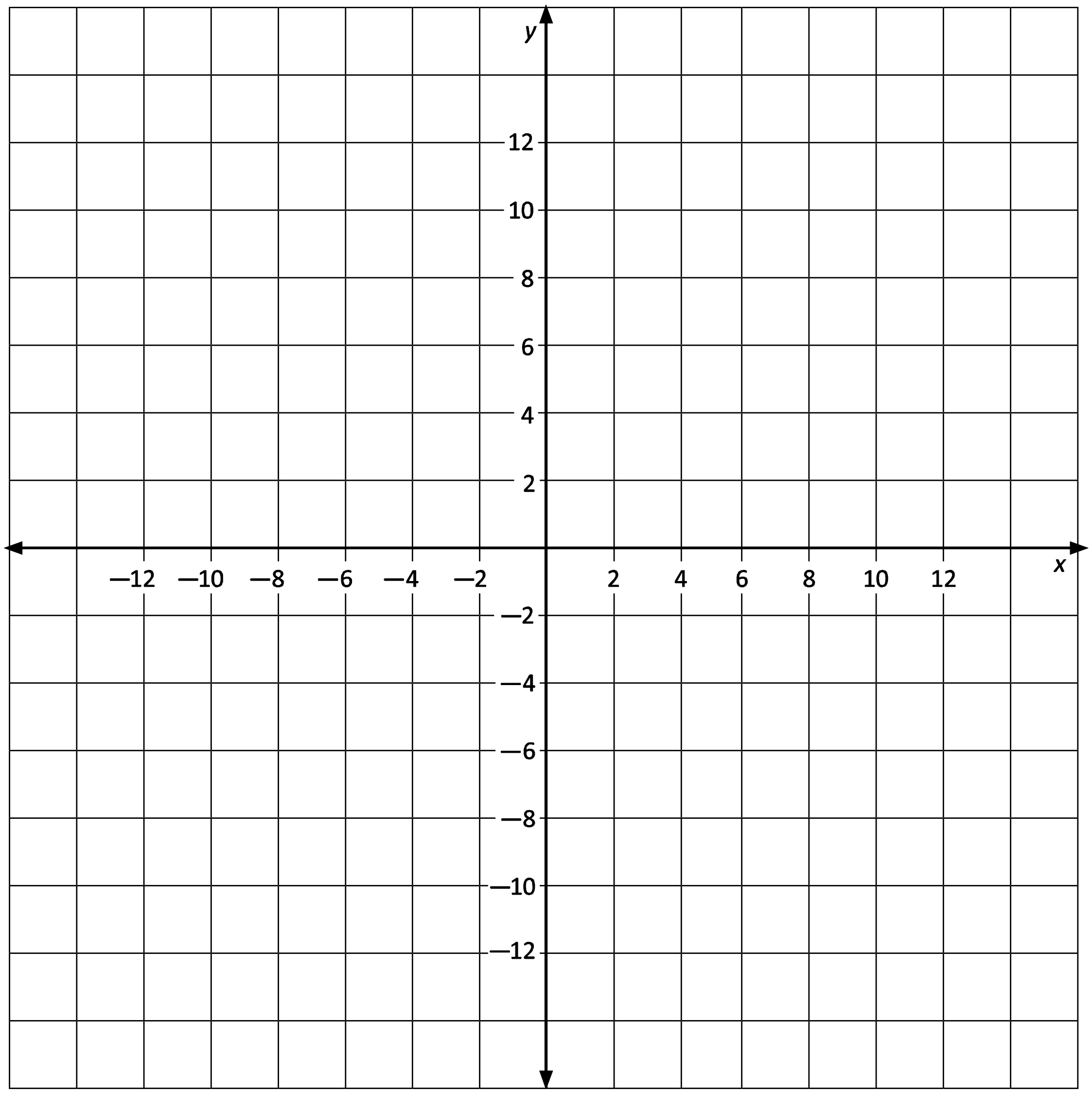
How to Play

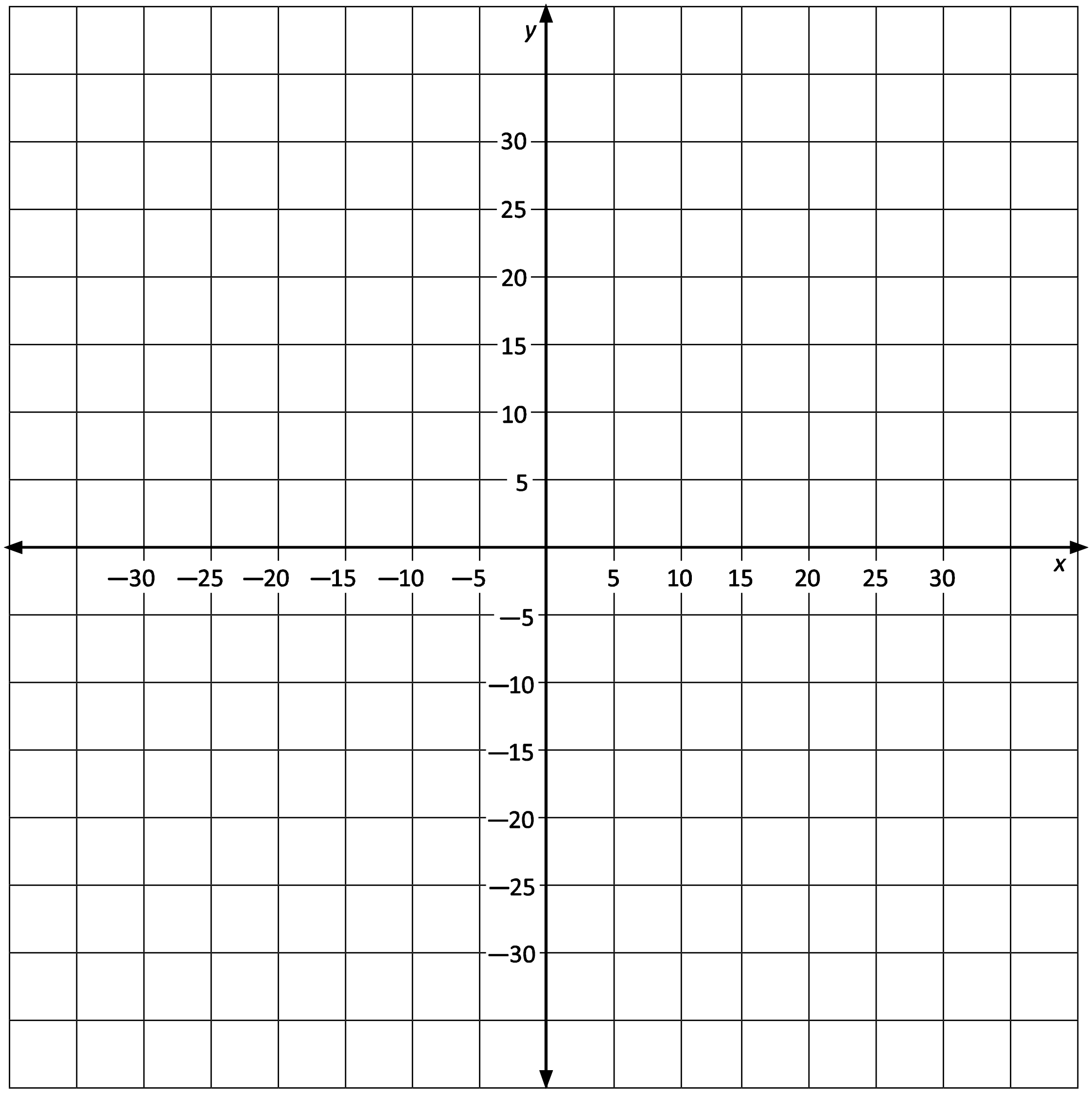
The starting player places the paper clips on two fractions on the strip of fractions below the game board. That player then uses one of the coloured counters to cover the sum of those two fractions on one square of the game board.

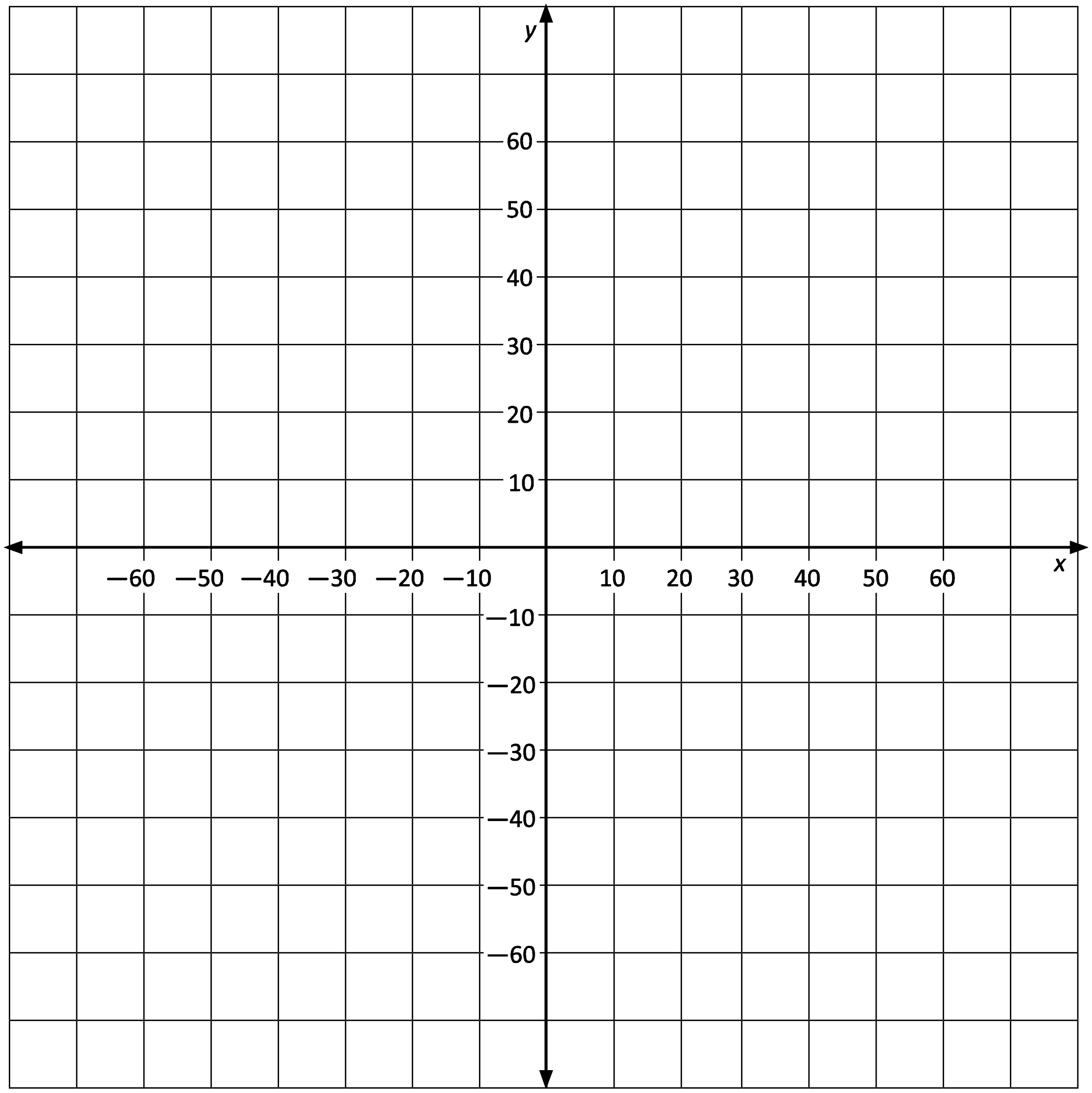
The second player moves exactly one of the paper clips to make a second sum (two paper clips can be on the same fraction). The second player then places a counter on the sum of the two fractions on the game board.

The play alternates until one player connects three of their own colour either horizontally, vertically, or diagonally. Of course, players will want to block each other so there can be a lot of strategy involved in playing the game.

Appendix C: Cartesian Planes: Increments of 2, 5, and 10




y
30
25
20
15
10
5
−30 −25 −20 −15 −10 −5
5 10 15 20 25 30
x
−5
−10
−15
−20
−25
−30


y
60
50
40
30
20
10
x
—60
—50
—40
—30
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—10
10
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50
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—10
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—30
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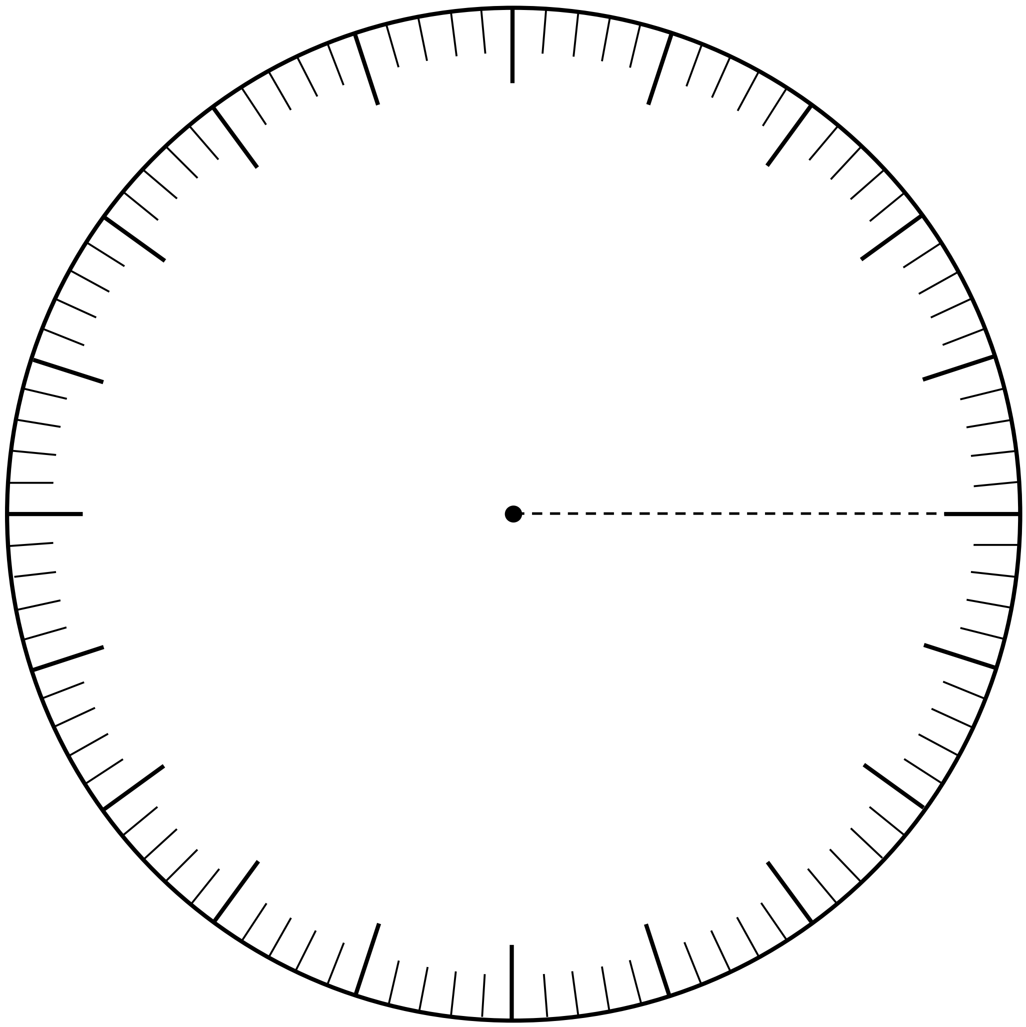
—50
—60

Appendix D: Hundredths Circle



References

Alberta Education. 2007. *The Alberta K–9 Mathematics Program of Studies with Achievement Indicators*. Edmonton, AB: Province of Alberta.

American Association for the Advancement of Science. 1993. *Benchmark for Science Literacy*. New York, NY: Oxford University Press.

Armstrong, T. 1999. *Seven Kinds of Smart: Identifying and Developing Your Many Intelligences*. New York, NY: Plume.

Birmingham City Council. 2015. "Virtual Dice," *Birmingham Grid for Learning.* www.bgfl.org/bgfl/custom/resources_ftp/client_ftp/ks1/maths/dice/index.htm.

Black, Paul, and Dylan Wiliam. 1998. "Inside the Black Box: Raising Standards through Classroom Assessment." *Phi Delta Kappan* 80, No. 2 (October 1998), 139–144, 146–148.

British Columbia Ministry of Education. 2000. *The Primary Program: A Framework for Teaching*. Victoria, BC: Province of British Columbia.

Caine, Renate Numella, and Geoffrey Caine. 1991. *Making Connections: Teaching and the Human Brain*. Reston, VA: Association for Supervision and Curriculum Development.

Chapin, Suzanne H., and Art Johnson. 2006. *Math Matters: Understanding the Math You Teach*, 2nd ed. Sausalito, CA: Math Solutions.

Davies, Anne. 2000. *Making Classroom Assessment Work*. Courtenay, BC: Classroom Connections International, Inc.

Dun, Sulan. 2007. "Welcome to Graph Mole." *FunBased Learning*. http://funbasedlearning.com/algebra/graphing/points2.

Frankenstein, Marilyn. 1995. "Equity in Mathematics Education: Class in the World outside the Class." *New Directions for Equity in Mathematics Education*. Cambridge, MA: Cambridge University Press.

Gardner, Howard E. 2007. *Frames of Mind: The Theory of Multiple Intelligences*. New York, NY: Basic Books.

Garneau, M., S. Ludwig, and John Pusic. 2007. *Math Makes Sense 7.* Toronto, ON: Pearson Education Canada. (NSSBB #: 2001640)

Garneau, M., S. Ludwig, and John Pusic. 2007. *Math Makes Sense 7 ProGuide* [with CD and DVD]. Toronto, ON: Pearson Education Canada. (NSSBB #: 2001641)

GeoGebra. 2013. GeoGebra. International GeoGebra Institute. Computer Software. www.geogebra.org/cms/en.

Ginger Booth. 2013. "Pattern Blocks." *Ginger Booth*. http://gingerbooth.com/flash/patblocks/patblocks.php#.VAM4BrxdW7w.

Google. 2015. "Pie Charts." *Google*. https://support.google.com/docs/answer/190726?hl=en.

Gutstein, Eric. 2003. "Teaching and Learning Mathematics for Social Justice in an Urban, Latino School." *Journal for Research in Mathematics Education* 34, No. 1. Reston, VA: National Council of Teachers of Mathematics.

Herzig, Abbe. 2005. "Connecting Research to Teaching: Goals for Achieving Diversity in Mathematics Classrooms." *Mathematics Teacher*, Volume 99, No. 4. Reston, VA: National Council of Teachers of Mathematics.

Hope, Jack A., Larry Leutzinger, Barbara Reys, and Robert Reys. 1988. *Mental Math in the Primary Grades*. Palo Alto, CA: Dale Seymour Publications.

Houghton Mifflin Harcourt School Publishers. 2015. "iTools: Fractions." *Houghton Mifflin Harcourt School Publishers*. www-k6.thinkcentral.com/content/hsp/math/hspmath/na/common/itools_int_9780547584997_/fractions.html.

Hume, Karen. 2011. *Tuned Out: Engaging the 21st Century Learner*. Don Mills, ON: Pearson Education Canada.

Institute of Education Sciences, National Center for Education Statistics. 2015. "Kids' Zone, Learning with NCES: Create a Graph." *US Department of Education.* http://nces.ed.gov/nceskids/createagraph/.

Johnston-Wilder, Sue, and John Mason. 2005. *Developing Thinking in Geometry*. London, UK: Sage Publications Ltd.

Key Curriculum Press. 2013. The Geometer's Sketchpad. Columbus, OH: McGraw-Hill Education. Computer Software. (NSSBB #: 50474, 50475, 51453)

Kroon, Cindy. 2006. "Teacher to Teacher: Playing around with Mono-pi-ly," *Mathematics Teaching in the Middle School*, Volume 11, Issue 6, February 2006, 294–297. Reston, VA: National Council of Teachers of Mathematics. www.nctm.org/Publications/mathematics-teaching-in-middle-school/2006/Vol11/Issue6/Teacher-to-Teacher_-Playing-around-with-Mono-pi-ly/.

Ladson-Billings, Gloria. 1997. "It Doesn't Add Up: African American Students' Mathematics Achievement." *Journal for Research in Mathematics Education* 28, No. 6. Reston, VA: National Council of Teachers of Mathematics.

Manitoba Education, Citizenship and Youth. 2009. *Kindergarten Mathematics: Support Document for Teachers*. Winnipeg, MB: Government of Manitoba.

——. 2009. *Kindergarten to Grade 8 Mathematics Glossary: Support Document for Teachers*. Winnipeg, MB: Government of Manitoba.

Math Learning Center. 2013. "Unnamed [geo-board web app]." *Math Learning Center*. www.mathlearningcenter.org/web-apps/geoboard.

MathPlayground.com. 2014. "Circle Graphs," *Math Playground.* www.mathplayground.com/piechart.html.

——. 2015. "Unnamed [virtual spinner]," *Math Playground*. www.mathplayground.com/probability.html.

Microsoft. 2014. "Present Your Data in a Pie Chart," *Microsoft Excel*. http://office.microsoft.com/en-ca/excel-help/present-your-data-in-a-pie-chart-HA010211848.aspx.

Musser, Gary L., Blake E. Peterson, and William F. Burger. 2006. *Mathematics for Elementary Teachers: A Contemporary Approach*, 7th ed. Milton: Queensland, Australia.

——. 2014. *Mathematics for Elementary Teachers: A Contemporary Approach*, 10th ed. [Kindle Edition]. Milton: Queensland, Australia.

National Council of Teachers of Mathematics, Inc. 2000. *Principles and Standards for School Mathematics*. Reston, VA: The National Council of Teachers of Mathematics, Inc.

——. 2000. *Principles and Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics.

——. 2001. *Mathematics Assessment: A Practical Handbook*. Reston, VA: National Council of Teachers of Mathematics.

——. 2005. "Computation, Calculators, and Common Sense: A Position of the National Council of Teachers of Mathematics" (position paper, May 2005). Reston, VA: National Council of Teachers of Mathematics.

——. 2015. "Adjustable Spinner," *Illuminations: Resources for Teaching Math*. http://illuminations.nctm.org/adjustablespinner.

——. 2015. "Circle Grapher,"*Illuminations: Resources for Teaching Math*. http://illuminations.nctm.org/Activity.aspx?id=4092.

——. 2015. "Discovering the Area Formula for Circles." *Illuminations: Resources for Teachers* (National Council of Teachers of Mathematics 2015). http://illuminations.nctm.org/lesson.aspx?id=1852.

——. 2015. "Equivalent Fractions." *Illuminations: Resources for Teachers* (National Council of Teachers of Mathematics 2015). http://illuminations.nctm.org/Activity.aspx?id=3510.

National Library of Virtual Manipulatives. 2015. "Algebra (Grades 6–8)." *Utah State University*. http://nlvm.usu.edu/en/nav/category_g_3_t_2.html.

Neuschwander, Cindy. 1999. *Sir Cumference and the Dragon of Pi*. St. Louis, MO: Turtleback Books.

New Brunswick Department of Education. 2008. *Mathematics Grade 7 Curriculum*. Fredericton, NB: New Brunswick Department of Education.

Newfoundland and Labrador Department of Education. 2013. *Mathematics: Grade 7.* St. John's, NL: Government of Newfoundland and Labrador.

Nova Scotia Department of Education and Early Childhood Development. 2015. "Mathematics Blackline Masters Grades P to 9, Table of Contents. *Province of Nova Scotia*. http://lrt.ednet.ns.ca/PD/BLM/table_of_contents.htm.

——. 2010. *Gifted Education and Talent Development*. Halifax, NS: Province of Nova Scotia.

NRICH Enriching Mathematics. 2015. "Cuisenaire Environment." *University of Cambridge*. http://nrich.maths.org/4348.

——. 2015. "Matching Fractions, Decimals, Percentages," *University of Cambridge*. http://nrich.maths.org/1249

——. 2015. “Unnamed [interactive circular geoboard].” *University of Cambridge*. http://nrich.maths.org/content/id/2883/circleAngles.swf.

OECD Centre for Educational Research and Innovation. 2006. *Formative Assessment: Improving Learning in Secondary Classrooms*. Paris, France: Organization for Economic Co-operation and Development (OECD) Publishing.

PBS LearningMedia. 2015. “Modelling Fractions with Cuisenaire Rods.” *PBS and WGBH*. www.pbslearningmedia.org/resource/rttt12.math.cuisenaire/modelling-fractions-with-cuisenaire-rods.

Pearson. n.d. *Interactive Math Tools*. Pearson Canada Inc. http://nsvs.ednet.ns.ca/nsps/nsps26/course/view.php?id=3875.

PlottingCoordinates.com. 2010. “CoordinArt News.” *PlottingCoordinates.com*. www.plottingcoordinates.com/coordinartnews.html.

Reed, Jim. 2000. “33.01 The Coordinate Plane.” *Grade 7: The Learning Equation Math.* http://staff.argyll.epsb.ca/jreed/math7/strand3/3301.htm.

Rubenstein, Rheta N. 2001. “Mental Mathematics beyond the Middle School: Why? What? How?” *Mathematics Teacher*, September 2001, Vol. 94, No. 6. Reston, VA: National Council of Teachers of Mathematics.

Shaw, J. M., and Cliatt, M. F. P. 1989. “Developing Measurement Sense.” In P.R. Trafton, ed., *New Directions for Elementary School Mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Shodor. 2015. “Circle Graph,” *Interactive*. www.shodor.org/interactivate/activities/CircleGraph.

——. 2015. “Experimental Probability,” *Interactive*. www.shodor.org/interactivate/activities/ExpProbability.

——. 2015. “Transmographer,” *Interactive*. www.shodor.org/interactivate/activities/Transmographer/.

Small, Marian. 2009. *Making Math Meaningful to Canadian Students, K–8*. Toronto, ON: Nelson Education Ltd.

——. 2013. *Making Math Meaningful to Canadian Students, K–8*, 2nd ed. Toronto, ON: Nelson Education Ltd.

Steen, L. A. ed. 1990. *On the Shoulders of Giants: New Approaches to Numeracy*. Washington, DC: National Research Council.

Tate, William F. 1995. “Returning to the Root: A Culturally Relevant Approach to Mathematics Pedagogy.” *Theory into Practice* 34, Issue 3. Florence, KY: Taylor & Francis.

Utah State University. 2015. “Data Analysis and Probability,” *National Library of Virtual Manipulatives.* http://nlvm.usu.edu/en/nav/topic_t_5.html.

——. 2015. “Fraction Pieces.” *Utah State University*. http://nlvm.usu.edu/en/nav/frames_asid_274_g_2_t_1.html?open=activities.

——. 2015. “National Library of Virtual Manipulatives: Number & Operations (Grades 3–5).” *Utah State University*. http://nlvm.usu.edu/en/nav/category_g_2_t_1.html.

——. 2015. “National Library of Virtual Manipulatives.” *Utah State University*. http://nlvm.usu.edu/en/nav/vlibrary.html.

——. 2015. “Unnamed [virtual spinner],” *National Library of Virtual Manipulatives.* http://nlvm.usu.edu/en/nav/frames_asid_186_g_1_t_1.html?open=activities.

Van de Walle, John A., and LouAnn H. Lovin. 2006a. *Teaching Student-Centered Mathematics, Grades 3–5,* Volume Two. Boston, MA: Pearson Education, Inc.

——. 2006b. *Teaching Student-Centered Mathematics, Grades 5–8,* Volume Three. Boston, MA: Pearson Education, Inc.

Western and Northern Canadian Protocol (WNCP) for Collaboration in Education. 2006. *The Common Curriculum Framework for K–9 Mathematics*. Edmonton, AB: Western and Northern Canadian Protocol (WNCP) for Collaboration in Education.

W.H. Freeman. 2015. “Probability [applet],” *Macmillan Education.* http://bcs.whfreeman.com/ips4e/cat_010/applets/Probability.html.

Wolfram Demonstrations Project. 2015. “Symbol Rotation Patterns.” *Wolfram Demonstrations Project and Contributors.* http://demonstrations.wolfram.com/SymbolRotationPatterns/.